far away from the droplet

$$\frac{dm}{dt} = Sh \cdot \rho_c \cdot D_v \cdot \pi D \left( \omega_{A,\infty} - \omega_{A,s} \right)$$

" convective mass tr./diffusion rate.

- effect of relative velocity between the droplet and the conveying flow to increase the evaportion or condensation rate. → Re effect.

$$Sh = 2 + 0.6\, Re^{0.5} \cdot Sc^{0.33} \quad \text{(Ranz-Marshall)}.$$

$$\begin{cases} Re = D|v_r|/\nu, & v_r = u - v \\ Sc = \nu/D_v & \text{(Schmidt number)}. \end{cases}$$

diffusion coeff.

- Energy eq. for a droplet w/ mass transfer.

(heat transfer w/ phase change)

$$m C_d \frac{dT_d}{dt} = Nu \cdot \pi k_c D (T_c - T_d) + \dot{m} \cdot \underline{h_L}$$

$\dot{m}$ ← $dm/dt$.

$h_L$ ↑ latent heat.

$$\Rightarrow m C_d \frac{dT_d}{dt} = \underline{Nu} \cdot \pi k_c D (T_c - T_d)$$

$$+ \underline{Sh} \cdot \rho_c \cdot D_v \pi (\omega_{A,\infty} - \omega_{A,s}) h_L .$$

- $D^2$-Law : evaporation of a droplet is represented such that the square of ~~the~~ <u>droplet diameter</u> varies linearly w/ time

$$m = \rho_d \cdot \frac{\pi}{6} D^3 \rightarrow \frac{dm}{dt} = \frac{\pi}{2} \rho_d \cdot D^2 \frac{dD}{dt}$$

$$= Sh \cdot \rho_c \cdot D_v \cdot \pi D (\omega_{A,\infty} - \omega_{A,s})$$

$$D\frac{dD}{dt} = -2\underbrace{\frac{Sh\cdot\rho_c\cdot D_v}{\rho_d}\underbrace{(\omega_s-\omega_\infty)}}_{\text{assumed to be constant.}}$$

↓ integrate.

$$\boxed{D^2 = D_0^2 - \lambda t.} \qquad \lambda = \frac{4\cdot Sh\cdot\rho_c D_v}{\rho_d}(\omega_s-\omega_\infty).$$

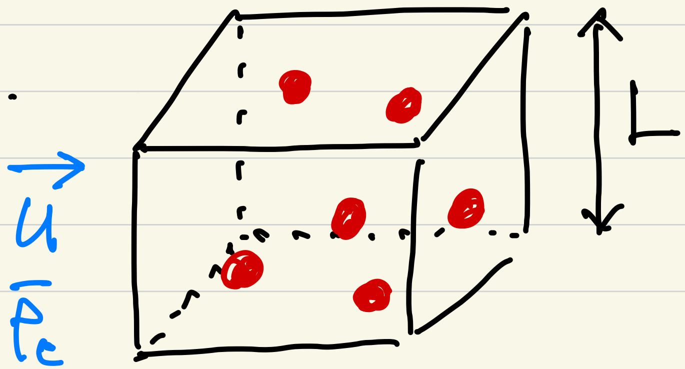
($D_0$ : initial droplet size)

↓

let $D=0$. $\rightarrow D_0^2 - \lambda t = 0 \rightarrow \tau_{life} = D_0^2/\lambda$.

(lifetime of a droplet, or evaporation time)

– Mass coupling.




- droplets evaporating in a volume

- mass generated by dispersed phase (per unit time)

$$\dot{M}_d = n \cdot L^3 \cdot \dot{m}$$

$\dot{m}$ : evap. rate of single droplet.

$n$ : number density

- mass flux of the continuous phase through the vol.

$$\dot{M}_c = \bar{\rho}_c u L^2$$

⇒ mass coupling parameter,

$$\Pi_{mass} = \dot{M}_d / \dot{M}_c = \frac{n \cancel{L^3} \dot{m}}{\bar{\rho}_c u \cancel{L^2}} = \frac{\overbrace{n m}^{\bar{\rho}_d}}{\bar{\rho}_c} \cdot \frac{\dot{m} L}{m u}$$

$$= \frac{\bar{\rho}_d}{\bar{\rho}_c} \cdot \frac{\dot{m}}{m} \frac{L}{u}$$

$C$ = $\frac{\bar{\rho}_d}{\bar{\rho}_c}$ : mass concentration.

$\tau_F = L/u$ (flow time scale)

$\tau_m = \frac{m}{\dot{m}}$

( characteristic time scale for

evaporation, condensation, or combustion, ... )

$$\Pi_{mass} = C \cdot \frac{\hat{\tau}_F}{\hat{\tau}_m} = C \cdot St_{mass}^{-1} \quad \left( St_{mass} = \frac{\hat{\tau}_m}{\hat{\tau}_F} \right)$$

(Stokes number w/ mass transfer).

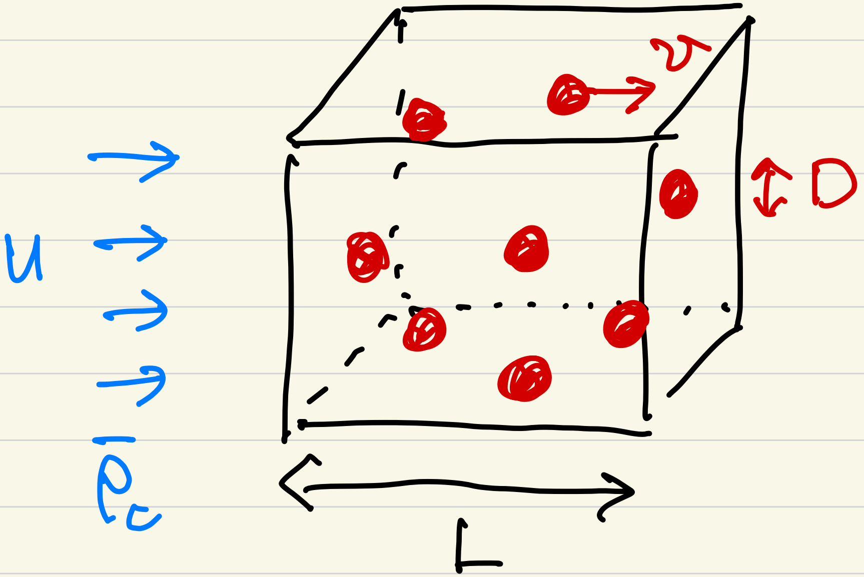
loading, $Z$ ( mass flow ratio)

if the velocities of each phase is comparable, the loading is the measure of the concentration. $\rightarrow Z \simeq C$.

$$\therefore \Pi_{mass} \simeq Z \cdot St_{mass}^{-1}.$$

($\Pi_{mass} \ll 1$ : one-way coupling).

- Momentum Coupling.




· Considering the drag force only.

$$F_D = \underbrace{n \cdot L^3}_{\text{total \# of particles}} \cdot 3\pi\mu D(v-u)$$

· mtm flux of the fluid through the CV.

$$\text{mtm} = \bar{\rho}_c u^2 L^2.$$

$$\Rightarrow \Pi_{mtm} = \frac{nL^3 \cdot 3\pi\mu D(v-u)}{\bar{\rho}_c u^2 L^2}$$

$$m = \rho_d \cdot \frac{\pi}{6} D^3, \quad \tau_v = \frac{\rho_d D^2}{18\mu}, \quad \Rightarrow \frac{m}{\tau_v} = 3\pi\mu D.$$

$$\tau_F = L/u \rightarrow St_v = \tau_v/\tau_F.$$

$$\Pi_{mtm} = \frac{\overbrace{nm}^{\bar{\rho}_d} L}{\bar{\rho}_c \tau_v u}\left(1 - \frac{v}{u}\right) = \frac{\bar{\rho}_d}{\bar{\rho}_c} \cdot \frac{\tau_F}{\tau_v}\left(1 - \frac{v}{u}\right).$$

$$= C \cdot \frac{1}{St_v}\left(1 - \frac{v}{u}\right) \longrightarrow \frac{0}{0}\ ?$$

$St_v \to 0$ : '$v$' approaches '$u$'

· eq. of particle motion.

$$\frac{dv}{dt} = \frac{u - v}{\tau_v},$$

assume that the particle velocity is proportional to the fluid velocity. (i.e., constant lag).

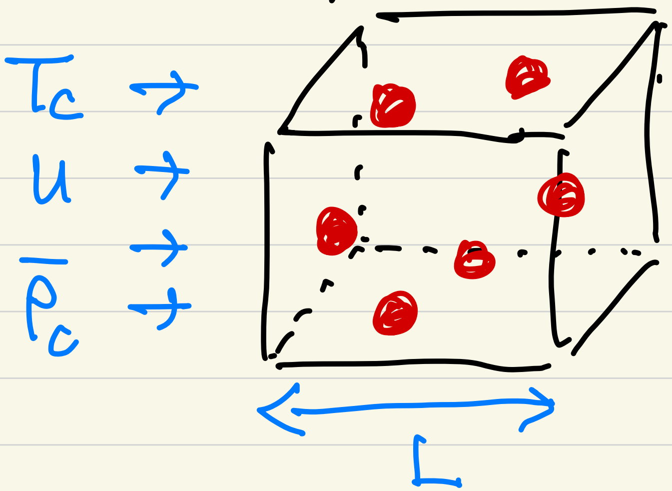
$$\frac{v}{u} = \phi. \Rightarrow \phi \boxed{\frac{du}{dt}} = \frac{u}{\tau_v}(1 - \phi).$$

$$\frac{du}{dt} \sim \frac{u}{\tau_F}$$

$$\frac{\tau_v}{\tau_F} = \frac{1-\phi}{\phi} = St_v.$$

$$\therefore \Pi_{mtm} = C \cdot \frac{1}{St_v}(1-\phi) = \frac{C}{St_v+1} \simeq \boxed{\frac{Z}{St_v+1}}.$$

– Energy Coupling.




∴ heat tr. from particle to fluid

$$\dot{Q} = nL^3 \times Nu \cdot \pi k_c D (T_d - T_c)$$

· energy flux of fluid through the CV.

$$\dot{E} = \bar{\rho}_c L^2 u C_p T_c.$$

$$\to \Pi_e = \frac{\dot{Q}}{\dot{E}} = \frac{nL^3 \cdot Nu \cdot \pi k_c D (T_d - T_c)}{\bar{\rho}_c L^2 u C_p T_c}$$

$$= \frac{\rho_d}{\bar{\rho}_c} \cdot \frac{\tau_F}{\tau_v} \left( \frac{T_d - T_c}{T_c} \right) \simeq \boxed{\frac{Z}{St_T+1}}.$$

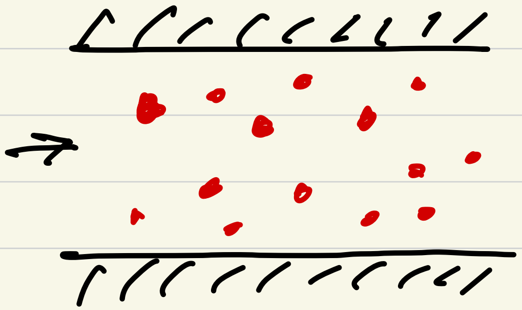
$\therefore$ As $St_T \to 0$, $\Pi_{mtm} \simeq \Pi_e \; (\simeq Z)$

(for most of gaseous flows).

$$\left.\begin{array}{l} \Pi_{mass} \ll 1 \\ \Pi_{mtm} \ll 1 \\ \Pi_e \ll 1 \end{array}\right\}$$

Numerical model or experimental measurement only accounts for one-way coupling effect.

→ effect of dispersed phase on the carrier phase is neglected.



hot particles are injected into a cooler gas flow in a pipe.