unsteady, non-uniform flow, low Re

$$\underbrace{m}_{\text{particle mass}} \frac{d\overbrace{v_i}^{\text{part. vel.}}}{dt} = \underbrace{m g_i}_{\text{body force}} + \overbrace{V_d}^{\text{vol. particle}}\left(-\frac{\partial p}{\partial x_i} + \frac{\partial \overbrace{\tau_{ij}}^{\text{by undisturbed flow}}}{\partial x_j}\right)$$

$$+ 3\pi\mu_c D\left[(\overbrace{u_i}^{\text{flow vel.}} - v_i) + \frac{D^2}{24}\nabla^2 u_i\right]$$

$$+ \frac{1}{2}\rho_c V_d \frac{d}{dt}\left[(u_i - v_i) + \frac{D^2}{40}\nabla^2 u_i\right]$$

$$+ \frac{3}{2}\pi\mu_c D^2 \int_0^t \left[\frac{\frac{d}{d\tau}\left(u_i - v_i + \frac{D^2}{24} \times \nabla^2 u_i\right)}{\pi\nu_c (t-\tau)^{0.5}}\right] d\tau$$

Faxen force

drag force

added mass

Basset (history) force.

④ Basset (history) force

- FVM accounts for the form drag due to acceleration.

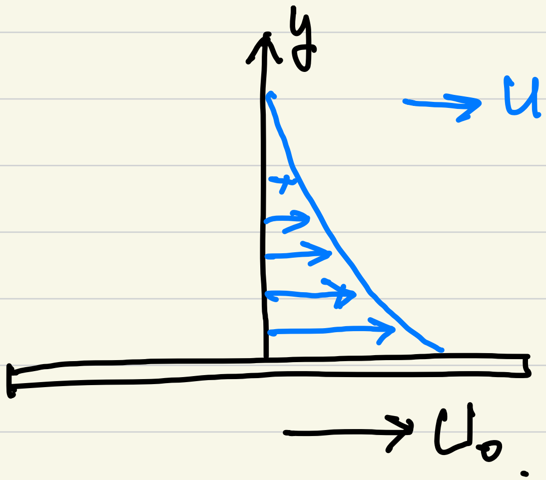
→ the Basset force accounts for

the viscous effect

→ temporal delay in the boundary-layer development. → history term

called as

e.g.) Impulsively accelerated flat plate.



$u(0, y) = 0$.

$u(t, 0) = U_0$

$u(t, \infty) = 0$

$$\frac{\partial u}{\partial t} = \nu_c \frac{\partial^2 u}{\partial y^2}$$ → similarity solution.

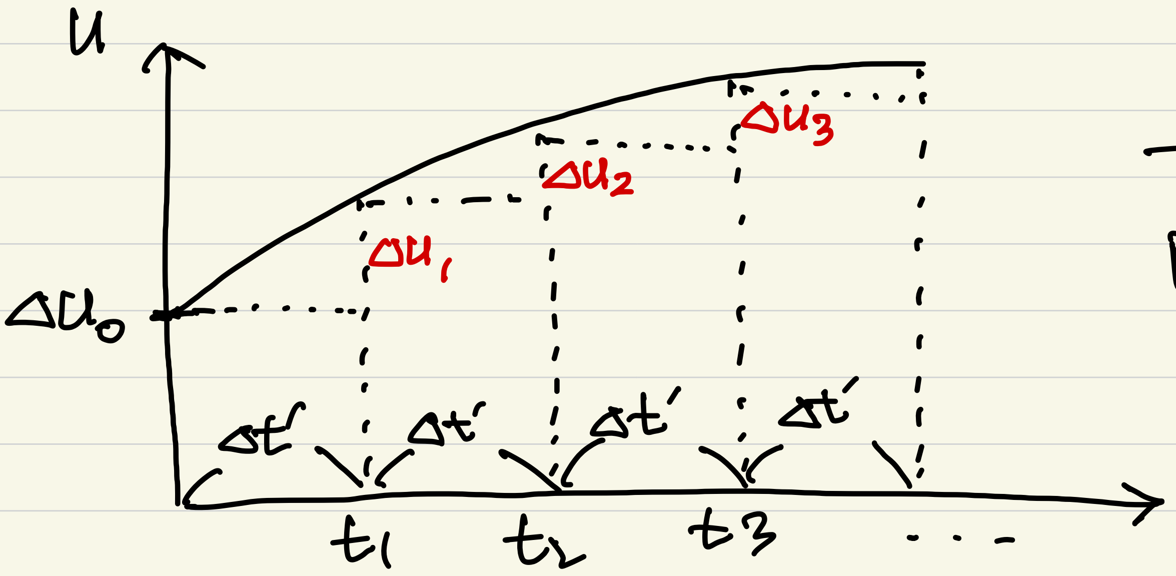
from the Pi-theorem, similarity variable

→ $\eta = y / 2\sqrt{\nu_c t}$

error fcn

$$\frac{u}{U_0} = \mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\lambda^2} d\lambda .$$

Wall shear stress, $\tau = \mu_c \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{\sqrt{\rho_c \mu_c}\, U_0}{\sqrt{\pi t}}$.



temporal change is broken into a series of time step.

$$\rightarrow \tau = \sqrt{\frac{\rho_c \mu_c}{\pi}} \left[\frac{\Delta U_0}{\sqrt{t}} + \frac{\Delta u_1}{\sqrt{t-t_1}} + \frac{\Delta u_2}{\sqrt{t-t_2}} + \cdots \right]$$

for $\Delta t'$, the change in velocity is $\frac{du}{dt'} \cdot \Delta t'$

$$\tau = \sqrt{\frac{\rho_c \mu_c}{\pi}} \sum_{n=0}^{N} \frac{du/dt'}{\sqrt{t - n\Delta t'}} \Delta t' .$$

as $\Delta t' \to 0$, $n\Delta t' \to t'$.

$$\therefore \tau = \sqrt{\frac{\rho_c \mu_c}{\pi}} \int_0^t \frac{du/dt'}{\sqrt{t-t'}} dt' . \to \text{force due to viscosity.}$$

↓ Apply the same approach to the impulsively moving sphere.

$$F_{Basset,i} = \frac{3}{2} D^2 \sqrt{\pi \rho_c \mu_c} \int_0^t \frac{\frac{d}{dt'}(u_i - v_i)}{\sqrt{t-t'}} dt' .$$

acceleration history, up to the present time (t).

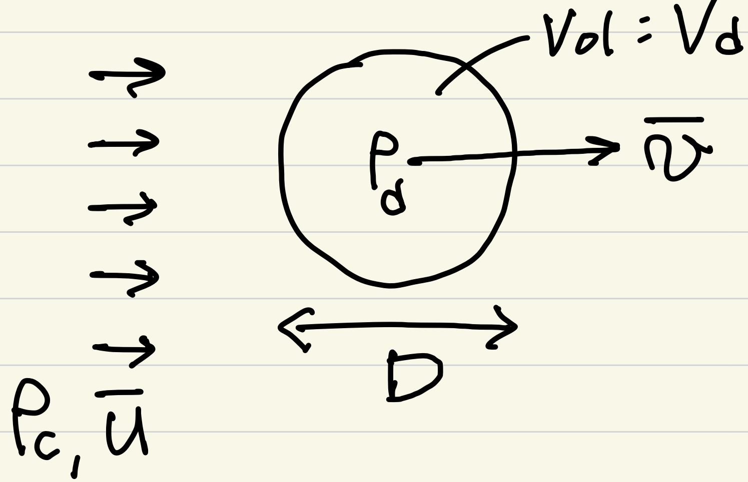
relative

ϕ

for small ρ_c/ρ_d, it is insignificant.

But it is important in many unsteady situations!.

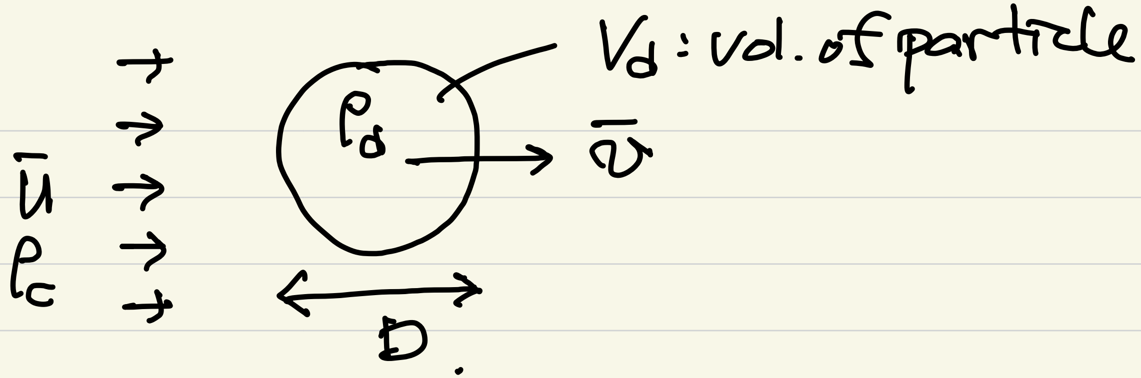
- Basset-Boussinesq-Oseen (BBO) Eq.
  - Eq. of single spherical particle at low Re, w/o flow curvature effect.
  - particle rotation (X)



$$m\frac{d\bar{v}}{dt} = 3\pi\mu_c D(\bar{u}-\bar{v}) + V_d(-\nabla P + \nabla\tau)$$
$$+ \frac{1}{2}\rho_c V_d\left(\frac{d\bar{u}}{dt} - \frac{d\bar{v}}{dt}\right)$$
$$+ \frac{3}{2}D^2\sqrt{\pi\rho_c\mu}\int_0^t \frac{d/dt(\bar{u}-\bar{v})}{\sqrt{t-t'}}\,dt' + m\bar{g}$$

BBO Eq.

- BBO EQ.



$$m\frac{d\bar{v}}{dt} = 3\pi\mu_c D(\bar{u}-\bar{v}) + V_d(-\nabla P + \nabla\tau) + \frac{1}{2}\rho_c V_d\left(\frac{d\bar{u}}{dt} - \frac{d\bar{v}}{dt}\right)$$

$$+ \frac{3}{2}D^2\sqrt{\pi\rho_c\mu}\int_0^t \frac{d(\bar{u}-\bar{v})/dt}{\sqrt{t-t'}}\,dt' + m\bar{g}$$

$$\left(1+\frac{1}{2}\frac{\rho_c}{\rho_d}\right)\frac{d\bar{v}}{dt} = \frac{1}{\tau_v}(\bar{u}-\bar{v}) + \frac{1}{\rho_d}(-\nabla P + \nabla\tau) + \frac{1}{2}\frac{\rho_c}{\rho_d}\frac{d\bar{u}}{dt}$$

$$+ \sqrt{\frac{9}{2\pi}}\left(\frac{\rho_c}{\rho_d}\right)^{0.5}\frac{1}{\sqrt{\tau_v}}\int_0^t \frac{d(\bar{u}-\bar{v})/dt}{\sqrt{t-t'}}\,dt' + \bar{g}$$

↳ $\rho_d D^2/18\mu_c$.

from N-S eq.

$$-\nabla p + \nabla \tau = \rho_c \frac{D\bar{u}}{Dt} - \rho_c \bar{g}$$

$$\left(1 + \frac{1}{2}\cancel{\frac{\rho_c}{\rho_d}}\right)\frac{d\bar{v}}{dt} = \frac{1}{\tau_v}(\bar{u}-\bar{v}) + \cancel{\frac{3}{2}\frac{\rho_c}{\rho_d}\left(\frac{D\bar{u}}{Dt}\right)} + \cancel{\sqrt{\frac{9}{2\pi}\left(\frac{\rho_c}{\rho_d}\right)}^{0.5}}\cdot\frac{1}{\sqrt{\tau_v}}\int_0^t dt'$$

$$+ \bar{g}\left(1 - \cancel{\frac{\rho_c}{\rho_d}}\right)$$

if $\rho_c/\rho_d \ll 1$: heavy particles (solid or liquid) in gas.

$$\Rightarrow \frac{d\bar{v}}{dt} = \frac{1}{\tau_v}(\bar{u}-\bar{v}) + \bar{g}$$

previously, $\tau_v = \rho_d D^2 / 18\mu_c$.

$$V_{st} = \frac{1}{18}\cdot\frac{D^2 g}{\nu_c}\left(\frac{\rho_d}{\rho_c}-1\right)$$

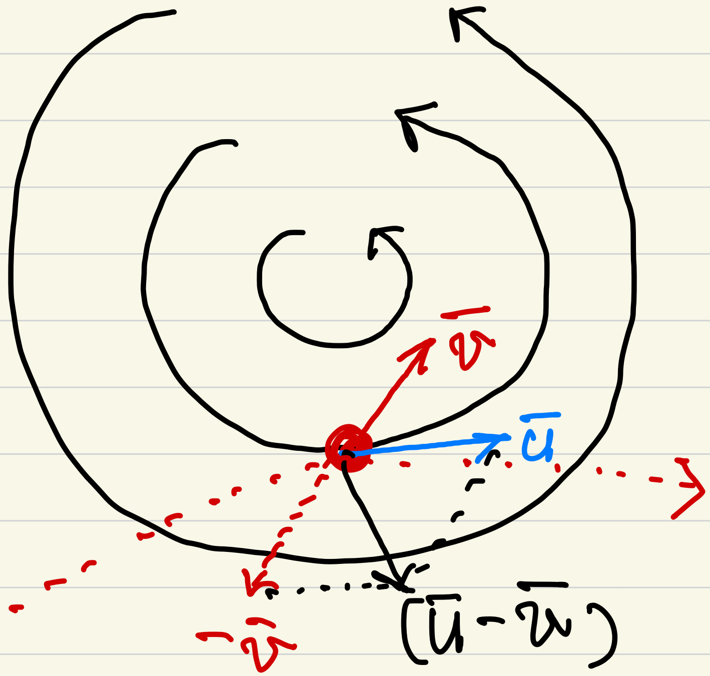
for $\rho_c/\rho_d \ll 1$.

$\bar{g} \simeq V_{st}/\tau_v$.

$$\therefore \frac{d\bar{v}}{dt} = \frac{(\bar{u}-\bar{v}) + V_{st}}{\tau_v}$$

: drag and gravity is the main source of the particle motion.

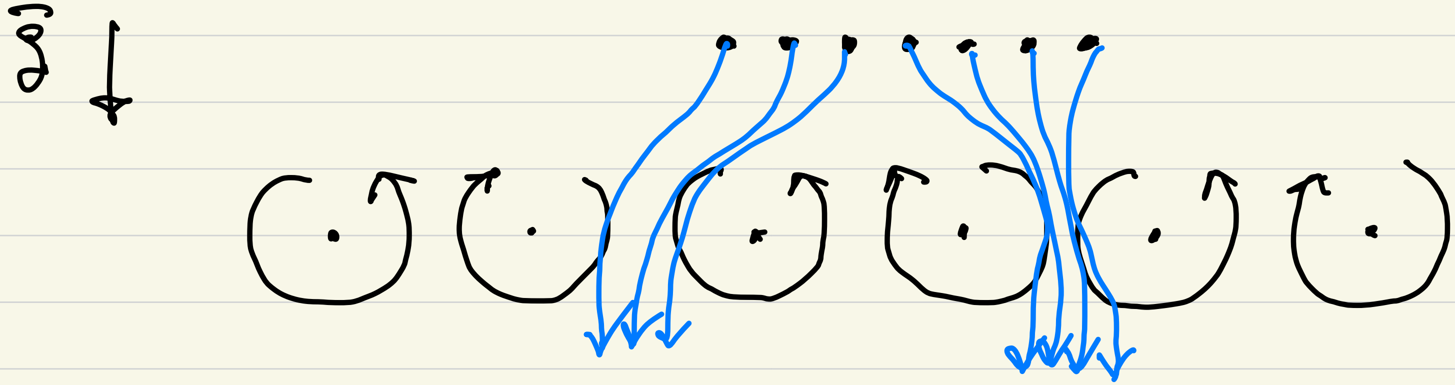
→ $(\bar{u}-\bar{v})$ cause a force that pushes the particle away from the vortex core, and into the region of lower vorticity (or, higher strain rate) ⟶ "preferential concentration".



$d\bar{v}/dt = (\bar{u}-\bar{v})/\tau_v$.

W/ the presence of gravity, this pref. concent. is coupled with V_{st}, $d\bar{v}/dt = (\bar{u}-\bar{v}+V_{st})/\tau_v$.

↳ preferential sweeping.
(crossing trajectories)



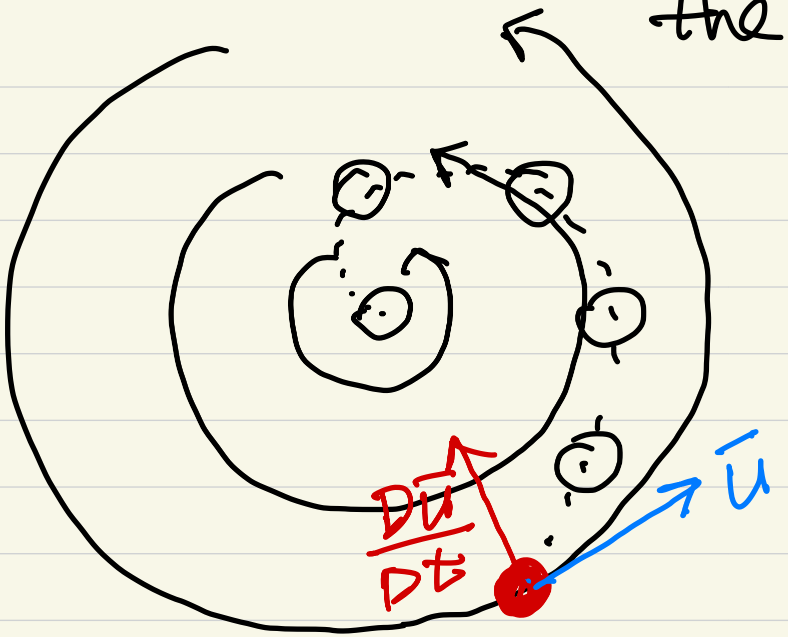
if $\rho_c/\rho_d \gg 1$. (light particles like gas bubbles in a liquid)

$$\left(1+\frac{1}{2}\frac{\rho_c}{\rho_d}\right)\frac{d\bar{v}}{dt} = \frac{1}{\tau_v}(\bar{u}-\bar{v}) + \frac{3}{2}\frac{\rho_c}{\rho_d}\left(\frac{D\bar{u}}{Dt}\right) + \sqrt{\frac{9}{2\pi}}\left(\frac{\rho_c}{\rho_d}\right)^{0.5}\frac{1}{\sqrt{\tau_v}}\int_0^t dt' + \bar{g}\left(1-\frac{\rho_c}{\rho_d}\right)$$

$$\rightarrow \frac{d\bar{v}}{dt} \simeq \frac{(\bar{u}-\bar{v})+V_{st}}{\tau_v} + 3\frac{D\bar{u}}{Dt}.$$

; effect of added mass, pressure, viscous stress on the surface.

↑ pressure term acts to attract the particle to

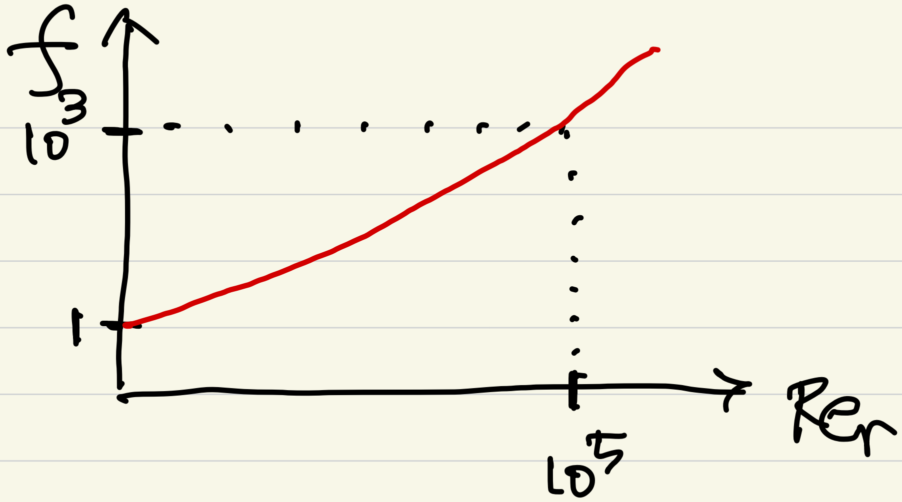


the lower pressure region (e.g. vortex core)

· w/ gravity (V_{st}), the particles will stay longer at the downward side of the vortex.
→ slower rise velocity.

- BBO eq. at higher Re.

· $\bar{F}_D = f \cdot 3\pi \mu_c D(\bar{u}-\bar{v})$
↳ drag factor (ratio of C_D to Stokes drag)
$\left(\equiv C_D / (24/Re_r)\right)$



$$\bar{F}_{vm} = \underline{C_{vm}} \frac{1}{2} \rho_c V_d \left(\frac{d\bar{u}}{dt} - \frac{d\bar{v}}{dt} \right)$$

$$\bar{F}_{Basset} = C_B \cdot \frac{3}{2} D^2 \sqrt{\pi \rho_c \mu_c} \int_0^t \frac{d(\bar{u} - \bar{v})/dt}{\sqrt{t - t'}} dt'$$

$$\Downarrow$$

$$\left(1 + \frac{C_{vm}}{2} \cdot \frac{\rho_c}{\rho_d}\right) \frac{d\bar{v}}{dt} = \frac{1}{\tau_v}(\bar{u} - \bar{v}) + \left(1 + \frac{C_{vm}}{2}\right) \frac{\rho_c}{\rho_d} \left(\frac{D\bar{u}}{Dt}\right)$$

$$+ C_B \sqrt{\frac{9}{2\pi}} \left(\frac{\rho_c}{\rho_d}\right)^{0.5} \frac{1}{\sqrt{\tau_v}} \int_0^t \frac{d(\bar{u} - \bar{v})/dt}{\sqrt{t - t'}} dt' + \bar{g}\left(1 - \frac{\rho_c}{\rho_d}\right).$$