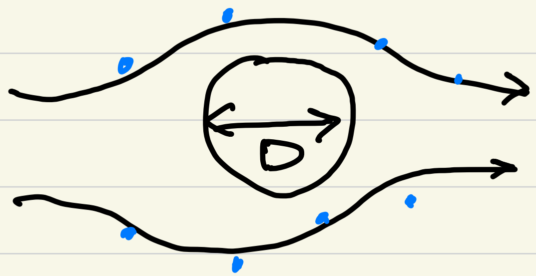
- Other forces on the particles in the non-uniform flow

① Effect of rarefied flow.



Knudsen number,

$$Kn = \frac{\lambda}{D} = \frac{\text{mean free path}}{\text{particle size}}$$

$$= \mu / c\rho_c$$

↳ speed of sound.

$$Kn = \frac{\mu}{c\rho_c D} = \frac{1}{cD/\nu_c} = \frac{1}{UD/\nu_c} \cdot \frac{U}{c} = \frac{Ma}{Re}.$$

kinematic viscosity

charac. vel. scale

$Kn < 10^{-3}$: continuum.

$10^{-3} < Kn < 0.25$: slip flow.

$0.25 < Kn < 10$: transitional flow

1. $Kn > 10$: free molecular flow

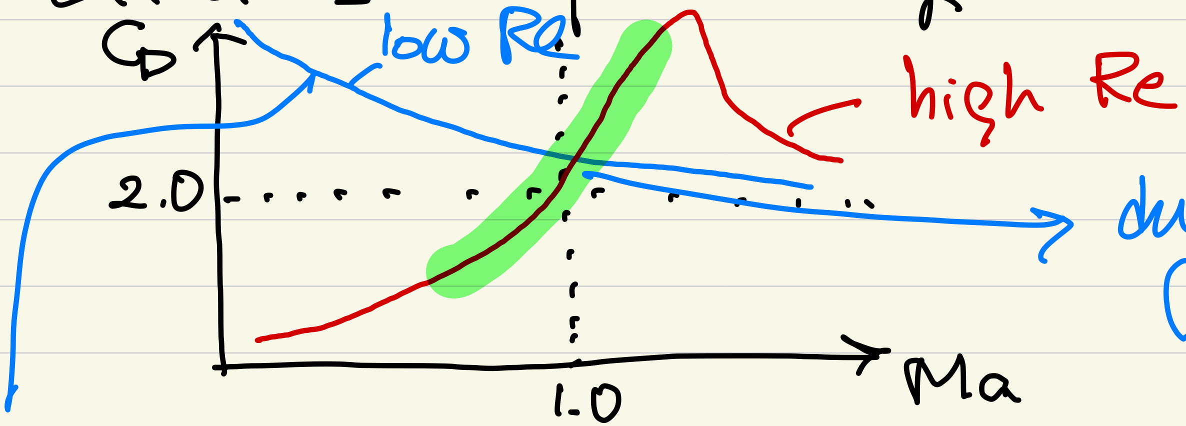
$$F_D = F_{D0} \cdot \frac{1}{1 + Kn\{2.49 + 0.84\exp(-1.74/Kn)\}}$$: empirical Millikan (1923).

$$F_D = F_{D0} \cdot \left\{ 1 - 1.01619\,k + \left(0.5 + 0.2349\frac{K}{K+1}\right)k^2 \right\}$$: theoretical Sone & Aoki (1977).

$k \rightarrow \frac{\sqrt{\pi}}{2} Kn$.

$K \rightarrow k_g / k_s$ (thermal conductivity of gas and solid).

※ Effect of compressibility



due to the shock wave. (form drag)

No peak due to the rarefied flow effect.

$$C_D = 2 + (C_{D0} - 2)\exp\left(-\frac{3.07\sqrt{k}\,g(Re)\cdot Ma}{Re}\right) + \frac{h(Ma)}{\sqrt{k}\,Ma}\exp\left(-\frac{Re}{2Ma}\right).$$

Crowe et al (1972)

$$\begin{cases} g(Re) = \dfrac{1+Re\,(12.278+0.548Re)}{1+11.278Re} & \text{Hermsen (1979)} \\ h(Ma) = \dfrac{5.6}{1+Ma} + 1.7\sqrt{\dfrac{T_d}{T_c}} \end{cases}$$

temperature of particle and gas.

Used for solid propellant rocket nozzle.

② Blowing effect. (phase change)

- surface of a burning (combustion) or evaporating droplet tends to reduce the C_D.

$$C_D = \frac{C_{D0}}{1+B} \quad \text{(Eisenklam et al. 1967)}$$

B ← transfer number.

- for an evaporating droplet

$$B = C_p \Delta T / h_L$$

h_L ← latent heat for evaporation.

- for a burning droplet (combustion)

$$B = (C_p \Delta T + x_{O_2} H / s) / h_L$$

H: heat of combustion

s: stochastic rate for oxygen.

x_{O_2}: oxygen concentration

Yuen & Chen (1976) suggested that the blowing effects can be accounted for adjusting the Re based on the viscosity change and

→ film temperature (1/3 rule).

$$T_f = T_d + \frac{1}{3}(T_c - T_d) \Rightarrow \text{determine } \mu.$$

$$\rightarrow Re = \rho D U_r / \mu . \rightarrow C_D = f(Re).$$

③ Thermophoresis (due to temperature gradient) in particular, for aerosol.

temperature is a measure of fluctuating KE of fluid molecule. → more momentum exchange

→ force along the decreasing T.

- for a small particle ($D \ll \lambda$) — λ: mean free path

$$F_{th} = -\frac{p_c \lambda D^2 \Delta T_c}{T_d} \rightarrow V_{th} = -\frac{0.55 \mu_c}{\rho_c T_d} \Delta T_c .$$

- for a large particle ($D > \lambda$)

temperature gradient is established within the particle depending on the conductivity of the particle.

$$F_{th} = -\frac{9\pi D^2 \mu_c^2 H \Delta T_c}{2\rho_c T_d}$$

$$H = \left(\frac{1}{1+6\lambda/D}\right)\left(\frac{k_c/k_d + 4.4\lambda/D}{1+2k_c/k_d + 8.8\lambda/D}\right) \sim \frac{k_c}{k_d}.$$

(thermal conductivity ratio)

$$\simeq -\frac{9\pi D^2 \mu_c^2 \Delta T_c}{2\rho_c T_d} \cdot \frac{k_c}{k_d}.$$

$$V_{th} = -\frac{3\underbrace{C_c}_{\text{gas slip correction factor.}}\mu H \Delta T_c}{2\rho_c T_d}$$

④ Photophoresis (due to the gradient in the incident radiation)

- similar to the thermophoresis
- particle motion under the influence of asymmetric light (photons) absorption (momentum exchange) in a radiation beam.

⑤ Bjerknes force (due to the pressure gradient
∟ sound wave in a liquid)

- for a gas bubble in an acoustic pressure field, it can undergo volume pulsation.

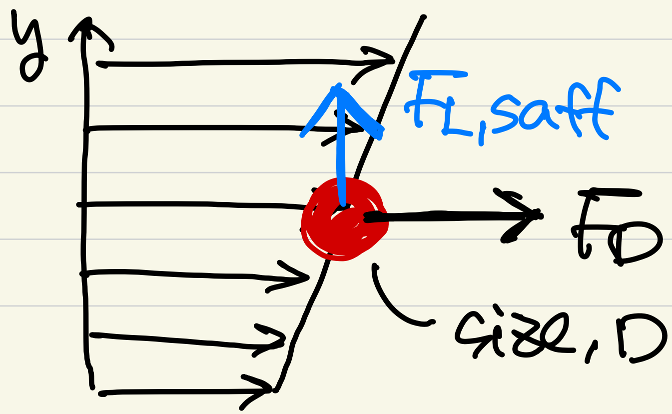
· for non-zero ΔP_r, it can couple with the bubble oscillations to produce a translational force.

→ Primary Bjerknes force (1909)

$$F_{Bj} = -V_d \cdot \nabla P(r,t)$$

⑥ Lift force (perpendicular to the major movement direction)

- shear-induced lift force (Saffman, 1965)
(uniform shear flow of an unbounded viscous fluid)



$u = u(y)$

$$F_{L,saff} = 1.61\,\mu_c D\,|\bar{u}_r|\sqrt{Re_G}.$$

$\bar{u}_r = \bar{u} - \bar{v}$: relative velocity

$$Re_G \equiv \frac{D^2}{\nu_c}\cdot\boxed{\frac{du_r}{dy}}.$$

$$F_{L,saff} = 1.61\,D^2\sqrt{\mu_c\rho_c}\,|u_r|\cdot\left|\frac{du_r}{dy}\right|^{0.5}$$ (shear Reynolds number)

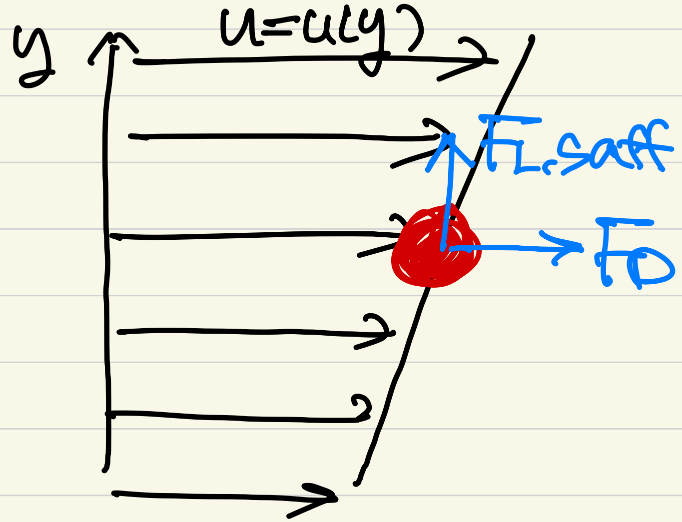
$$= 1.61\,D^2\sqrt{\mu_c\rho_c}\cdot\frac{\bar{u}_r\times\bar{\omega}_c}{\sqrt{|\bar{\omega}_c|}},\quad \bar{\omega}_c = \nabla\times\bar{u}.$$

$u_r > 0$: $F_{L,saff}$ acts to the higher velocity region

$u_r < 0$: " lower "

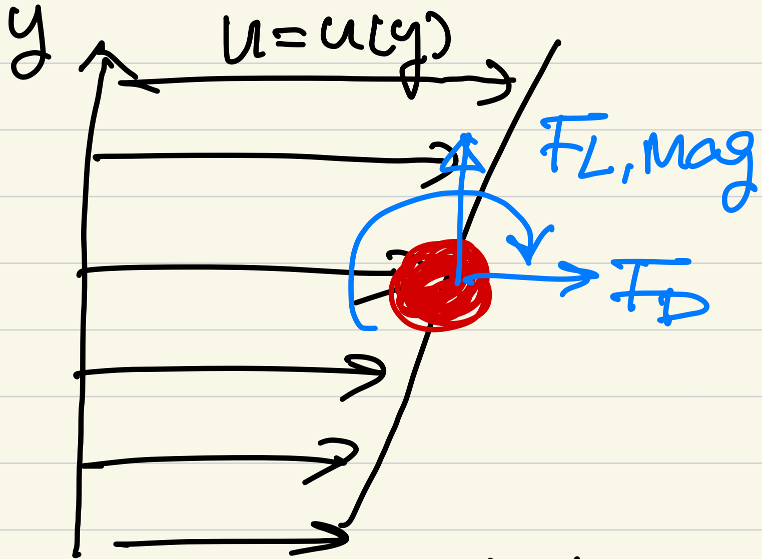
- Saffman lift is valid when,
  $Re_r = u_r D/\nu \ll 1$, $Re_G \ll 1$, and $Re_r \ll Re_G$.

- Aerodynamic lift : the magnus effect appears when a particle is moving in a fluid w/ a rotation, caused by external sources other than the fluid.



particle rotation
= fluid rotation.

$$\bar{\omega}_d = \bar{\omega}_c = \frac{1}{2}(\nabla \times \bar{u})$$



particle rotation
≠ fluid rotation

$$\bar{\omega}_d \neq \bar{\omega}_c \left(= \frac{1}{2}\nabla \times \bar{u}\right)$$

relative vorticity

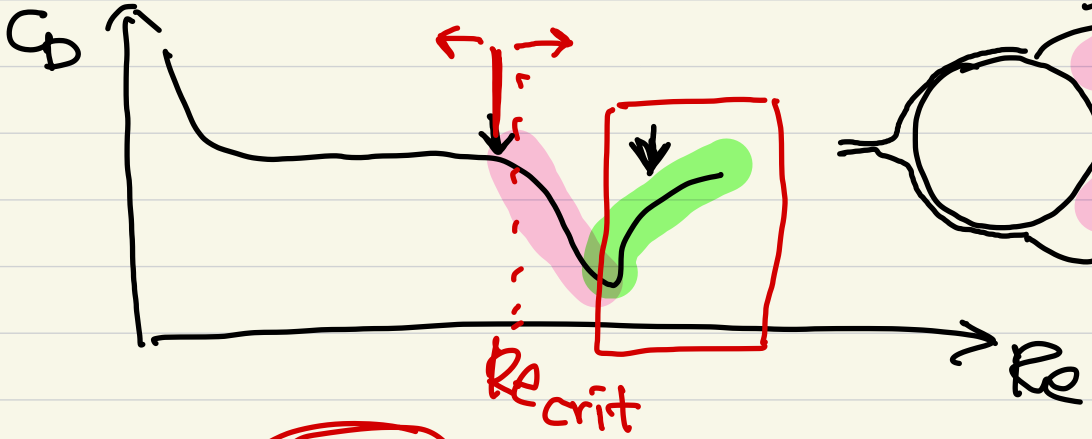
$$\bar{\omega}_r = \bar{\omega}_d - \bar{\omega}_c.$$

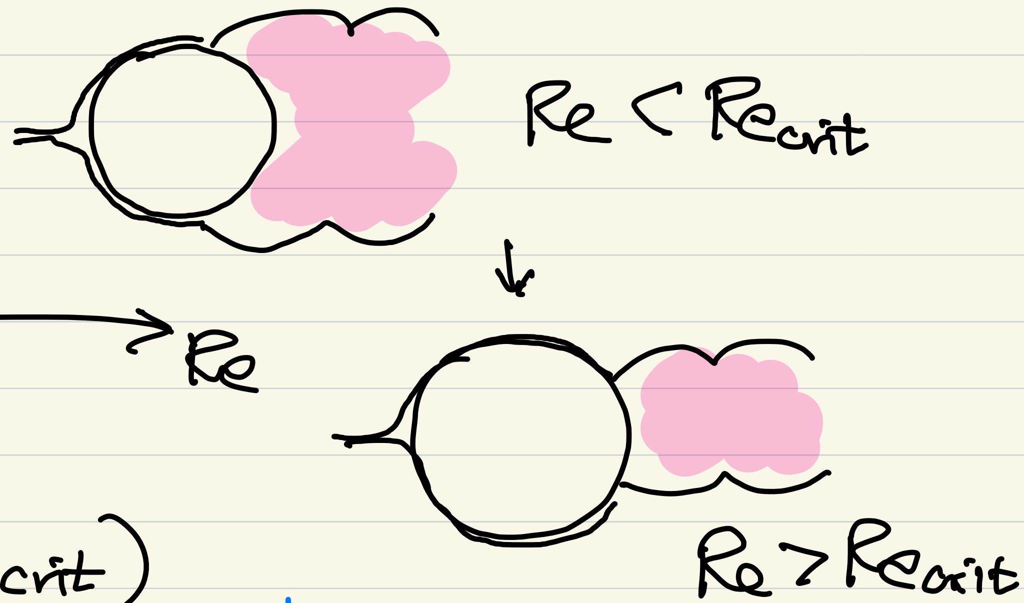
- For low Reynolds number.

$$F_{L,Mag} = \frac{\pi}{8} D^3 \rho_c (\bar{u}_r \times \bar{\omega}_r),$$

$$\bar{\omega}_r = \bar{\omega}_d - \frac{1}{2}(\nabla \times \bar{u})$$

- For higher Re's $(O(10^5 - 10^6))$, the asymmetric separation of the boundary layer is responsible for the magnus effect. → depends on the critical Reynolds number, Re_{crit} → (drag crisis)





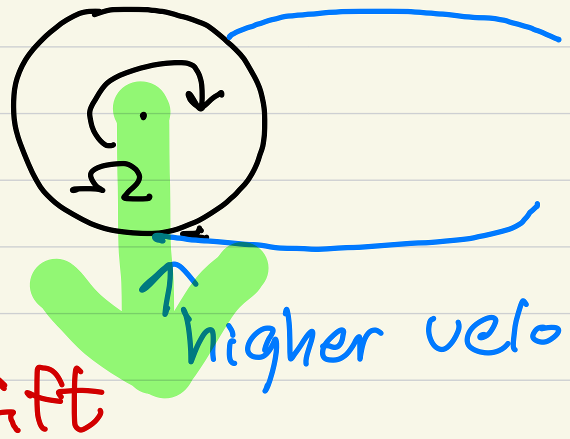
if $Re < Re_{crit}$ (but $Re \simeq Re_{crit}$)

turb. sp.

sep. points move upstream as $Re\uparrow$.

if $Re > Re_{crit}$ (supercritical Re)

: both upper and lower sides have a turb. separation. → lower side separates earlier w/ higher relative velocity



$u = \bar{u} + u'$.

⑦ Turbulence effect

- relative turbulence intensity. $I_r \equiv \dfrac{u'_{rms}}{|\bar{u} - \bar{v}|}$.
- turb. length scale – particle size ratio

: drag coefficient increases w/ increasing the ratio at a given I_r.

※ To apply the eq. of particle motion in this chapter to the particle in a turb. flow, different scales in turbulence should be considered.

- Large scale : L_I (integral length scale)
  u' (velocity fluctuation)
- Small scale : η_k and v_k (Kolmogorov vel.)
  (Kolmogorov length)

We will be back later on this!

$$v_k / u' \sim O(Re_\lambda^{-1/2}), \quad \eta_k / L_I \sim O(Re_\lambda^{-3/2})$$

Re_λ : Re based on the Taylor microscale)

particle size (inertia subrange)

- To apply the eq. we need $D/\eta_k \ll 1$, since η_k is the smallest scale at which the velocity is not uniform.
- Turbulence diffusion is dominated by large-scale "stirring", so the velocities of the order of u' over a long time lead to the statistically asymptotic behavior.
- For the velocity distribution, small-scale motion has to be considered to understand the