interaction of particles w/ turbulent small scales due to inertia.

IV. TWO-FLUID MODEL (conservation law)

- Review of single-phase conservation eqs.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k}(\rho u_k) = 0 \Rightarrow \frac{\partial u_k}{\partial x_k} = 0 \quad \text{(incompressible)}.$$

mass conservation

$$\parallel \quad \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}.$$

$$\underline{\rho \frac{\partial u_j}{\partial t}} + \underline{\rho u_k \frac{\partial u_j}{\partial x_k}} = \underline{\frac{\partial \sigma_{ij}}{\partial x_i}} + \underline{\rho f_j} \quad \text{mtm conserv.}$$

$$= \underline{-\frac{\partial P}{\partial x_j}} + \underline{\frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]} + \rho f_j$$

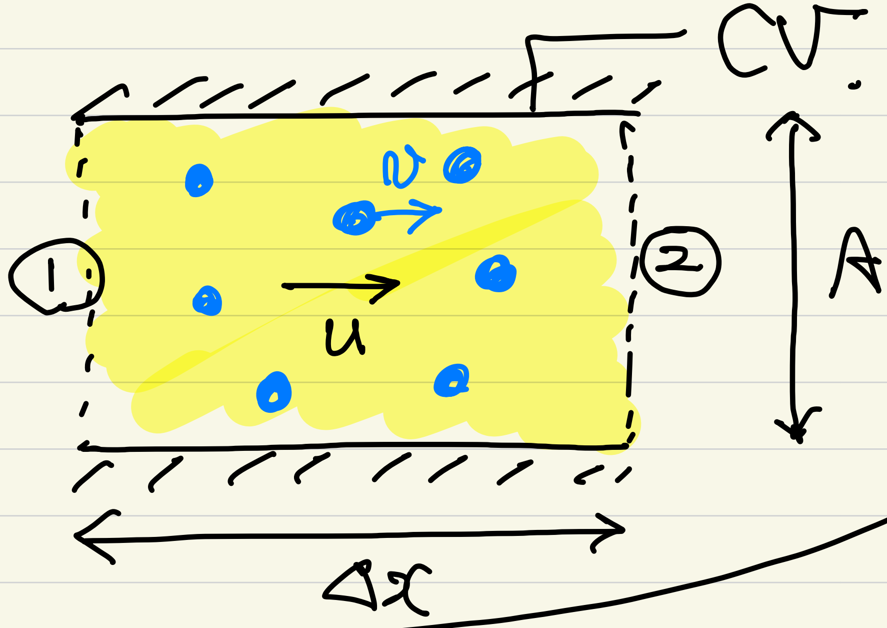
(N-S eq.)

$$\rho \frac{\partial e}{\partial t} + \rho u_k \frac{\partial e}{\partial x_k} = \sigma_{ij} \frac{\partial u_j}{\partial x_i} - \frac{\partial q_j}{\partial x_j}$$ heat added by conduction.

mech e. → thermal e by σ_{ij} (role of μ)

$$= \underline{\mu} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_j}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\underline{k \frac{\partial T}{\partial x_j}} \right).$$

- Mass conservation



·for continuous phase

: change rate of mass

= net mass flux

+ mass coupling

mass released by particles.

(Here, $\Delta_t = \frac{\Delta(\,)}{\Delta t}$, $\Delta = \frac{\partial(\,)}{\partial x}\cdot\Delta x$. $V = A\cdot\Delta x$)

$$V\cdot\Delta_t(\alpha_c\rho_c) = -\Delta(\alpha_c\rho_c uA) + \underline{S_{mass}}$$

$$= -n\dot{m}V$$

number density

$$\frac{\partial}{\partial t}(\alpha_c\rho_c V) + \frac{\partial}{\partial x}(\alpha_c\rho_c uA)\Delta x = -n\dot{m}V$$

$$\frac{\partial}{\partial t}(\alpha_c \rho_c) + \cancel{\frac{1}{A}} \frac{\partial}{\partial x}(\alpha_c \rho_c u \cancel{A}) = -n\dot{m}$$

$$\frac{\partial}{\partial t}(\alpha_c \rho_c) + \frac{\partial}{\partial x}(\alpha_c \rho_c u) = -n\dot{m} \quad (1D).$$

or $$\frac{\partial}{\partial t}(\alpha_c \rho_c) + \frac{\partial}{\partial x_i}(\alpha_c \rho_c u_i) = -n\dot{m} \quad (\text{multi-dimension})$$

- for the dispersed phase.

$$V \cdot \Delta_t (\alpha_d \rho_d) = -\Delta(\alpha_d \rho_d \tilde{v} A) - S_{mass}$$

$$\tilde{v} = \sum_k m_k v_k \Big/ \sum_k m_k \,.$$

: mass averaged velocity.

$$\frac{\partial}{\partial t}(\alpha_d \rho_d) + \frac{\partial}{\partial x}(\alpha_d \rho_d \tilde{v}) = n\dot{m}$$

(1D)

(in case of different particle mass)

$$\partial/\partial t\,(\alpha_d \rho_d) + \underline{\frac{\partial}{\partial x_i}(\alpha_d \rho_d \tilde{v}_i)} = n\dot{m}$$

or, $\frac{\partial}{\partial t}(\alpha_d \rho_d) + \frac{\partial}{\partial x_i}(\alpha_d \rho_d v_i)$ fluctuating component.

$$= -\frac{\partial}{\partial x_i}\left(\sum_k \delta\bar{\rho}_{d,k} \cdot \delta v_{i,k}\right) + n\dot{m} \quad \text{(multi-dim.)}$$

dispersion term arising from turb. fluctuation in the flow.

$$\sum_k \delta\bar{\rho}_{d,k} \cdot \delta v_{i,k} \simeq -D \cdot \frac{\partial \bar{\rho}_d}{\partial x_i}$$

dispersion coeff. (mostly, determined empirically)

$$\left(-\overline{u'v'} \simeq \mu_t \frac{\partial u}{\partial y}\right).$$

$$\boxed{\tau = \mu \frac{\partial u}{\partial y}}$$

in total,

$$\Rightarrow \frac{\partial}{\partial t}(\alpha_c \rho_c + \alpha_d \rho_d) + \frac{\partial}{\partial x_i}(\alpha_c \rho_c u_i + \alpha_d \rho_d v_i) = 0$$

if both phases are in the equilibrium,

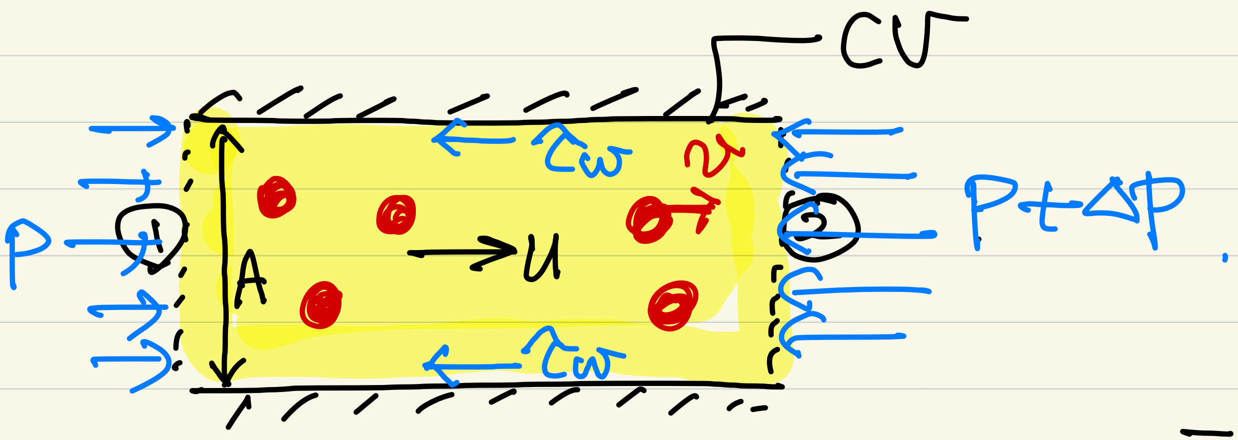
$$\frac{\partial}{\partial t}(\rho_m) + \frac{\partial}{\partial x_i}(\rho_m U_{m,i}) = 0$$

mixture.

if there is no mass coupling ($S_{mass} = 0$) and the same material density as constant,

$$\Rightarrow \frac{\partial}{\partial x_i}(\alpha_c U_i + \alpha_d V_i) = 0 \quad (\alpha_c + \alpha_d = 1.0)$$

- Momentum Conservation.



· for continuous phase.

: Change rate of mtm in CV

= net mtm flux + force

on the fluid + mtm coupling.

$$\rightarrow V \cdot \Delta_t(\alpha_c \rho_c u) = -\Delta(\alpha_c \rho_c u^2 A) + F + S_{mass} v$$

· force on fluid (F)

$$F_p = -A \cdot \Delta P$$

perimeter

$$F_f = -\tau_w \cdot P \cdot \Delta x$$

$$F_g = \alpha_c \rho_c g A \cdot \Delta x.$$

$$F_d = -N \left\{ \underbrace{3\pi \mu_c D \boxed{f (u-v)}}_{\text{viscous drag (Stokes)}} - V_d \frac{\partial p}{\partial x} \right\}$$

# of particles.

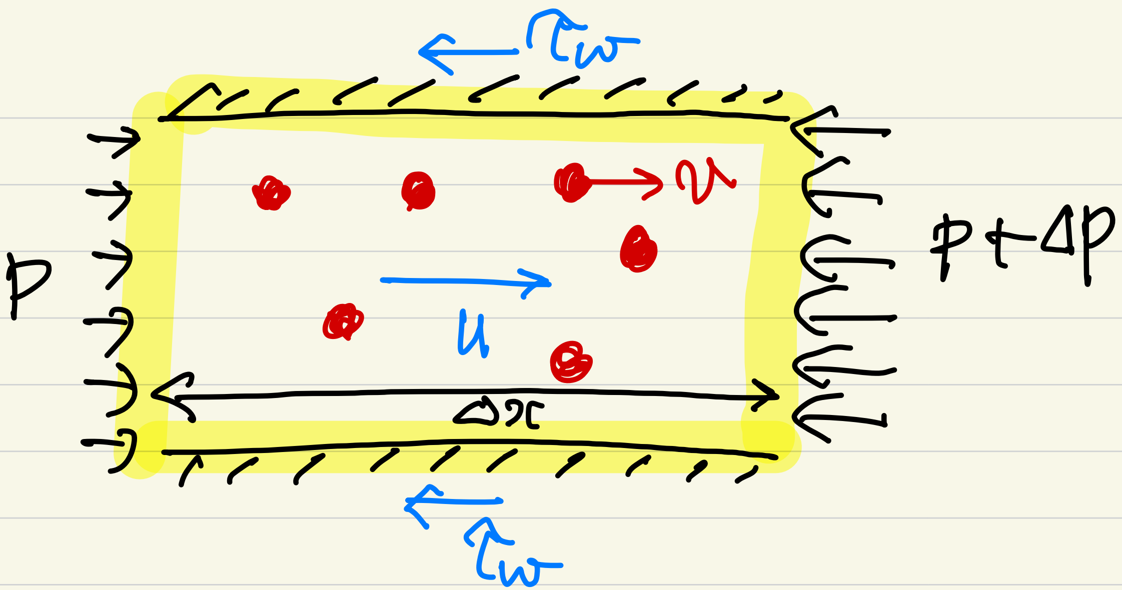
? form drag (pressure)

drag correction factor, $f \equiv \dfrac{C_D \cdot Re_r}{24}.$

$$\underline{N \cdot V_d} \frac{\partial p}{\partial x} - \underbrace{\alpha_d A \Delta x}_{=V} \frac{\partial p}{\partial x} = \underline{\alpha_d A \Delta P}.$$

$$\therefore V \Delta_t (\alpha_c \rho_c u) + \Delta (\alpha_c \rho_c u^2 A) = S_{mass} v - (1-\alpha_d) A \Delta p$$

$$- 3\pi \mu_c N D f (u - v) - \tau_w P \Delta x + \alpha_c \rho_c g V$$



· for continuous phase:

$$V\Delta_t(\alpha_c\rho_c u) + \Delta(\alpha_c\rho_c u^2 A) = S_{mass} v - (1-\alpha_d)A\Delta p$$

$$\boxed{-3\pi\mu_c N D f(u-v)} - \tau_w P\Delta x + \alpha_c\rho_c g V$$

perimeter.

$$m_d = \rho_d \cdot \frac{\pi}{6}D^3, \quad \tau_v = \rho_d D^2/18\mu_c.$$

$$\frac{m_d}{\tau_v} N f(u-v) = \rho_d V_d N \frac{f}{\tau_v}(u-v).$$

$$= \bar{\rho}_d V \frac{f}{\tau_v}(u-v) = \boxed{\beta_v V (u-v)}$$

$$\beta_v \equiv \bar{\rho}_d f / \tau_v$$

(parameter for mtm coupling)

$$V \Delta_t [\alpha_c \rho_c u) + \Delta(\alpha_c \rho_c u^2 A) = S_{mass} v - \underline{\alpha_c A \Delta P}$$
$$\underline{- \beta_v V (u-v)} - \tau_w P \Delta x + \alpha_c \rho_c g V.$$

$$\rightarrow \frac{\partial}{\partial t}(\alpha_c \rho_c u) + \frac{\partial}{\partial x}(\alpha_c \rho_c u^2) = S_{mass} v - \underline{\alpha_c \frac{\partial p}{\partial x}} \qquad (1D)$$
$$- \beta_v (u-v) - \frac{\tau_w}{R_H} + \alpha_c \rho_c g.$$

$\hookrightarrow$ hydraulic diameter ($\equiv 4A/P$).

or in the case of multi-dimensional case.
(w/ fluctuating velocity)

$$\frac{\partial}{\partial t}(\alpha_c \rho_c u_i) + \frac{\partial}{\partial x_j}(\alpha_c \rho_c u_j u_i) = S_{mass} v - \alpha_c \frac{\partial p}{\partial x_j} - \beta_v (u_i - v_i)$$
$$+ \underline{\alpha_c \frac{\partial}{\partial x_j}(\tau_{ij}^e)} + \alpha_c \rho_c g_i$$

effective

$$\tau_{ij}^e = \tau_{ij} - \langle \rho_c u_i' u_j' \rangle$$: effective vol-averaged stress

$\langle \rho_c u_i' u_j' \rangle \to$ $\underbrace{\text{volume average}}$

$\Downarrow$

equivalent to the Reynolds stress.

if $Re \gg 1$, the drag on the fluid due to the particle drag

$$\rightarrow F_d = n V \left[V_d \frac{\partial p}{\partial x} + \underbrace{C_D \frac{1}{2} \rho_c |u-v| (u-v) \frac{1}{4}\pi D^2}_{(\neq \text{stokes drag})} \right]$$

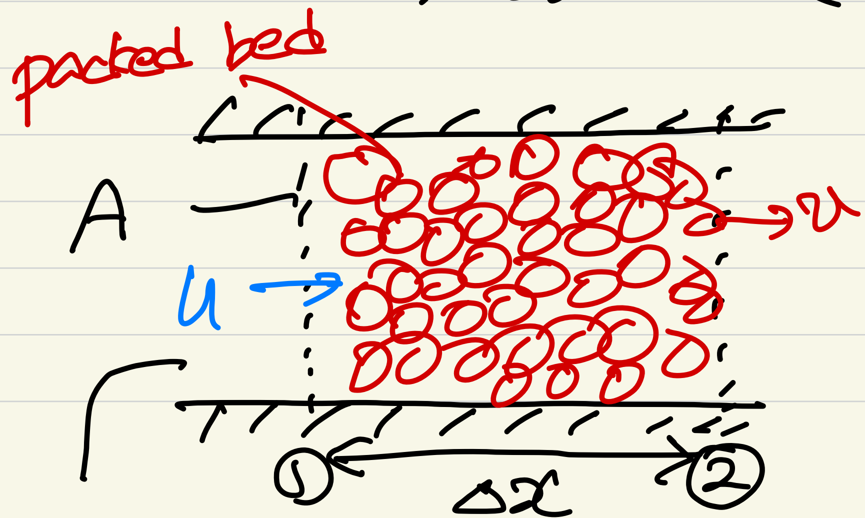
$$= V \left[\alpha_d \frac{\partial p}{\partial x} + n \cdot C_D \frac{1}{2} \rho_c \underbrace{|u-v| (u-v) \frac{1}{4}\pi D^2}_{} \right]$$

$$= \beta_V = \frac{3}{4} C_D \frac{\rho_c (1-\alpha_c)|u-v|}{D}$$

$$= V \left[\alpha_d \frac{\partial p}{\partial x} + \beta_V (u-v) \right]$$

※ for very large volume fraction (particle clouds)

→ contact (collision) dominant.



- superficial velocity
: calculated as if the given phase (of fluid) were the only one