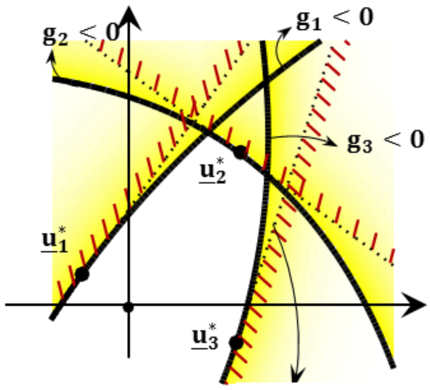
# 457.646 Topics in Structural Reliability

## In-Class Material: Class 17

### ◎ FORM approximation (Hohenbichler & Rackwitz 1983)

**① Series system**




$n \rightarrow$ # rv's

$m \rightarrow$ # comp's

$$P(E_{sys}) = P(\bigcup_{i=1}^{m} E_i)$$

$$= P(\bigcup_{i=1}^{m} g_i(\mathbf{x}) \le 0)$$

$$\overset{FORM}{\cong} \; P(\bigcup_{i=1}^{m} \qquad \le 0)$$

$$G_i(\mathbf{u}) \square\, G_i(\mathbf{u}_i^*) + \nabla G_i(\mathbf{u}_i^*)(\mathbf{u} - \mathbf{u}_i^*)$$

$$= \nabla G_i(\mathbf{u}_i^*)(\mathbf{u} - \mathbf{u}_i^*) \le 0$$

$$\Leftrightarrow \qquad\qquad \le 0$$

Let $Z_i = \hat{\boldsymbol{\alpha}}_i \mathbf{u}$ , $i = 1, \cdots, m$

$E[Z_i] =$

$Var[Z_i] = \| \quad \|^2 =$

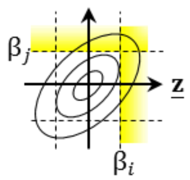
Therefore, $Z_i \sim \quad ( \quad , \quad )$

$$\rho_{Z_i Z_j} = \frac{[ \quad , \quad ]}{} = E[ \qquad ] - E[ \quad ] \cdot E[ \quad ]$$

$$= E[ \quad \cdot \quad ^{\mathrm{T}}] = E[ \qquad ] = \quad E[\mathbf{u}\mathbf{u}^T] \quad =$$

$\therefore \mathbf{Z} \sim \quad ( \quad , \quad )$, $\rho_{Z_i Z_j} =$

$$\therefore P(E_{sys}) \overset{FORM}{\cong} P(\bigcup_{i=1}^{m} \qquad \le 0)$$




$$= 1 - P(\bigcap_{i=1}^{m} \qquad \le \qquad )$$

$$= 1 - \Phi_m( \quad , \cdots , \quad ; \mathbf{R})$$

Joint normal CDF of $\mathbf{Z} \sim N(\mathbf{0}; \mathbf{R})$

Where $\Phi_m(\boldsymbol{\beta}; \mathbf{R}) = \int_{-\infty}^{\beta_1} \cdots \int_{-\infty}^{\beta_m} \varphi_m(\mathbf{Z}; \mathbf{R}) d\mathbf{z}$

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## ② Parallel system

$$P(E_{sys}) = P(\bigcap_{i=1}^{m} E_i)$$

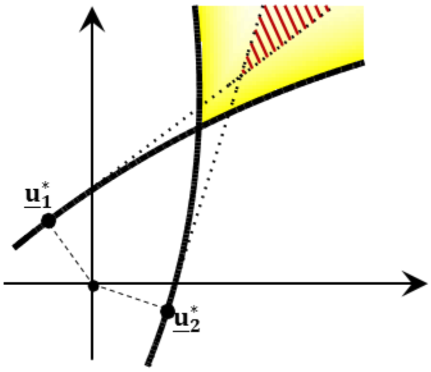
$$= P(\bigcap_{i=1}^{m} g_i(\mathbf{x}) \le 0)$$

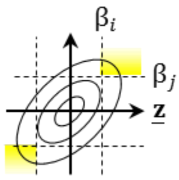
$$\overset{FORM}{\cong} P(\bigcap_{i=1}^{m} \qquad\qquad \le 0)$$

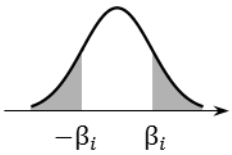
$$= P(\bigcap_{i=1}^{m} \qquad \ge \qquad )$$

$$\overset{sym}{=} P(\bigcap_{i=1}^{m} \qquad \le \qquad )$$

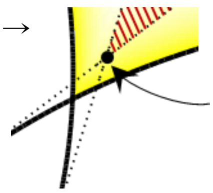
$$= \Phi_m( \qquad , \cdots , \qquad ; \mathbf{R})$$




→ may have huge errors due to curvatures

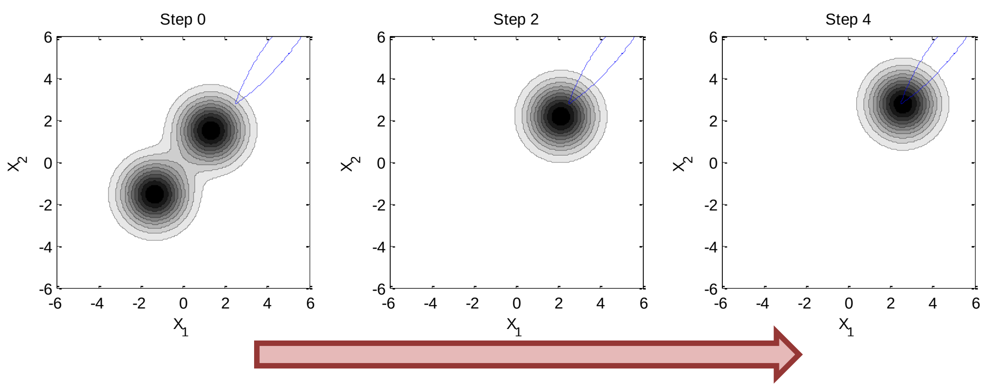
→  better linearization point?

"joint design point"

Hard to find or may not exist

**Note:** One could find such important domain using an adaptive sampling technique

Kurtz, N., and J. Song (2013). Cross-entropy-based adaptive importance sampling using Gaussian mixture. *Structural Safety*. Vol. 42, 35-44.



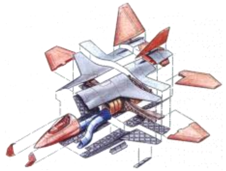
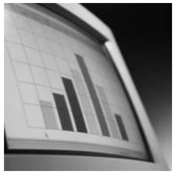

## ③ General system?

⇒ No direct FORM approximation

# Risk-quantification of Complex Systems by Matrix-based System Reliability Method



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**Seoul National University**

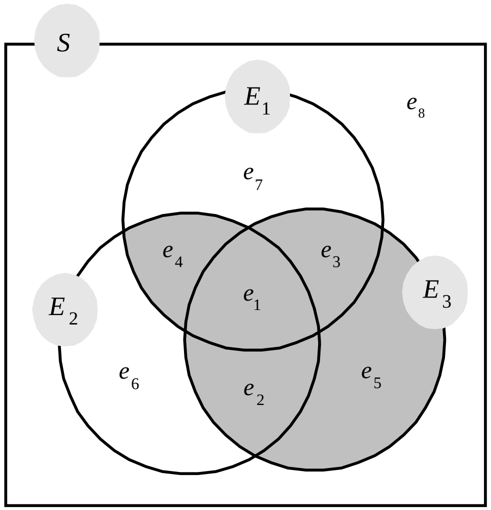
# Matrix-based Formulation

- Matrix-based formulation of system failure:

$$P(E_{sys}) = \mathbf{c}^{\mathrm{T}}\mathbf{p}$$

* Example:

$$P(E_1E_2 \cup E_3) = p_1 + p_2 + p_3 + p_4 + p_5$$
$$= [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0] \cdot [p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5 \quad p_6 \quad p_7 \quad p_8]^{\mathrm{T}}$$




- **c**: "**event**" vector
  ~ describes the system event of interest
- **p**: "**probability**" vector
  ~ likelihood of component joint failures

# Identification of event vector, c

- Matrix-based event operations:

$$\mathbf{c}^{\bar{E}} = \mathbf{1} - \mathbf{c}^{E}$$

$$\mathbf{c}^{E_1 \cdots E_n} = \mathbf{c}^{E_1} .* \mathbf{c}^{E_2} .* \cdots .* \mathbf{c}^{E_n}$$

$$\mathbf{c}^{E_1 \cup \cdots \cup E_n} = \mathbf{1} - (\mathbf{1} - \mathbf{c}^{E_1}) .* (\mathbf{1} - \mathbf{c}^{E_2}) .* \cdots .* (\mathbf{1} - \mathbf{c}^{E_n})$$

  - Efficient and easy to implement by matrix-based computing languages, e.g. Matlab®, Octave
  - Can construct directly from event vectors of components and other system events
  - Can develop/use problem-specific algorithms to identify event vectors

# Identification of event vector, c

- Event vectors for component events:

$$\mathbf{C}_{[1]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \mathbf{C}_{[i]} = \begin{bmatrix} \mathbf{C}_{[i-1]} & \mathbf{1} \\ \mathbf{C}_{[i-1]} & \mathbf{0} \end{bmatrix} \quad \text{for } i = 2,\ldots,n$$

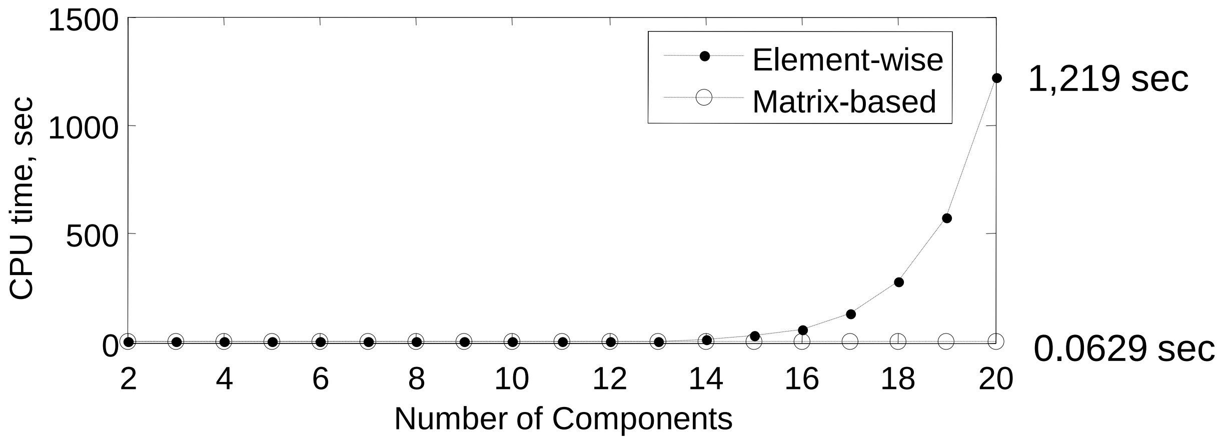
  - **0** and **1** denote the column vectors of $2^{(i-1)}$ zeros and ones
  - After $\mathbf{C}_{[n]}$ is constructed, the $i$-th column of the matrix is the event vector of the $i$-th component event.

# Computation of probability vector, p

- Iterative matrix-based procedure for statistically independent (s.i.) components

$$\mathbf{p}_{[1]} = \begin{bmatrix} P_1 & 1 - P_1 \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{p}_{[i]} = \begin{bmatrix} \mathbf{p}_{[i-1]} \cdot P_i \\ \mathbf{p}_{[i-1]} \cdot (1 - P_i) \end{bmatrix} \quad \text{for } i = 2, \ldots, n$$



# Statistical dependence b/w components

- By total probability theorem,

$$P(E_{sys}) = \int_{\mathbf{s}} P(E_{sys} \mid \mathbf{s}) f_{\mathbf{S}}(\mathbf{s})\, d\mathbf{s}$$
$$= \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}} \mathbf{p}(\mathbf{s}) f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s}$$
$$= \mathbf{c}^{\mathrm{T}} \tilde{\mathbf{p}}$$

  - Utilize **conditional s.i.** of components given an outcome of random variables **S** causing component dependence e.g. Earthquake magnitude for a bridge system
  - Event vector **c** is independent of this consideration ~ no need to construct the probability vector for new system events

# "What if not explicitly identified?"

- Example: approximation by Dunnett-Sobel (DS) correlation matrix (1955)

$$Z_i \sim N(\mathbf{0}, \mathbf{R}), \ \rho_{ij} = r_i \cdot r_j$$

$$Z_i = \sqrt{1 - r_i^2} \cdot U_i + r_i S,$$

  - $Z_i$, $i=1,\ldots,n$ are conditional s.i. given $S=s$
  - Fit the given correlation matrix with a DS correlation matrix with the least square error
  - Generalized DS model (Song and Kang, Structural Safety)

$$Z_i \sim N(\mathbf{0}, \mathbf{R}), \quad \rho_{ij} = \Sigma_{k=1}^{m}(r_{ik} r_{jk})$$

$$Z_i = \sqrt{1 - \Sigma_{k=1}^{m} r_{ik}^2} \cdot U_i + \Sigma_{k=1}^{m}(r_{ik} S_k)$$

# Conditional prob./importance measure

- Conditional probability Importance Measure (CIM)

$$CIM_i = P(E_i \mid E_{sys}) = \frac{P(E_i E_{sys})}{P(E_{sys})}$$

- Fussell-Vesely IM

$$FV_i = \frac{P(\bigcup_{k:C_k \supseteq E_i} C_k)}{P(E_{sys})}$$

  - $P(E_{sys}')/P(E_{sys}) = (\mathbf{c}'^T\mathbf{p}) / (\mathbf{c}^T\mathbf{p})$
  - Once the system reliability is done, only additional task is to find the event vector for a new system event

# Parameter sensitivity of system reliability

- Statistically independent components

$$P(E_{sys}) = \mathbf{c}^{\mathrm{T}}\mathbf{p}$$

$$\Rightarrow \quad \frac{\partial P_{sys}}{\partial \theta} = \mathbf{c}^{\mathrm{T}} \frac{\partial \mathbf{p}}{\partial \theta}$$

- Statistically dependent components

$$P(E_{sys}) = \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}}\mathbf{p}(\mathbf{s}) f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s}$$

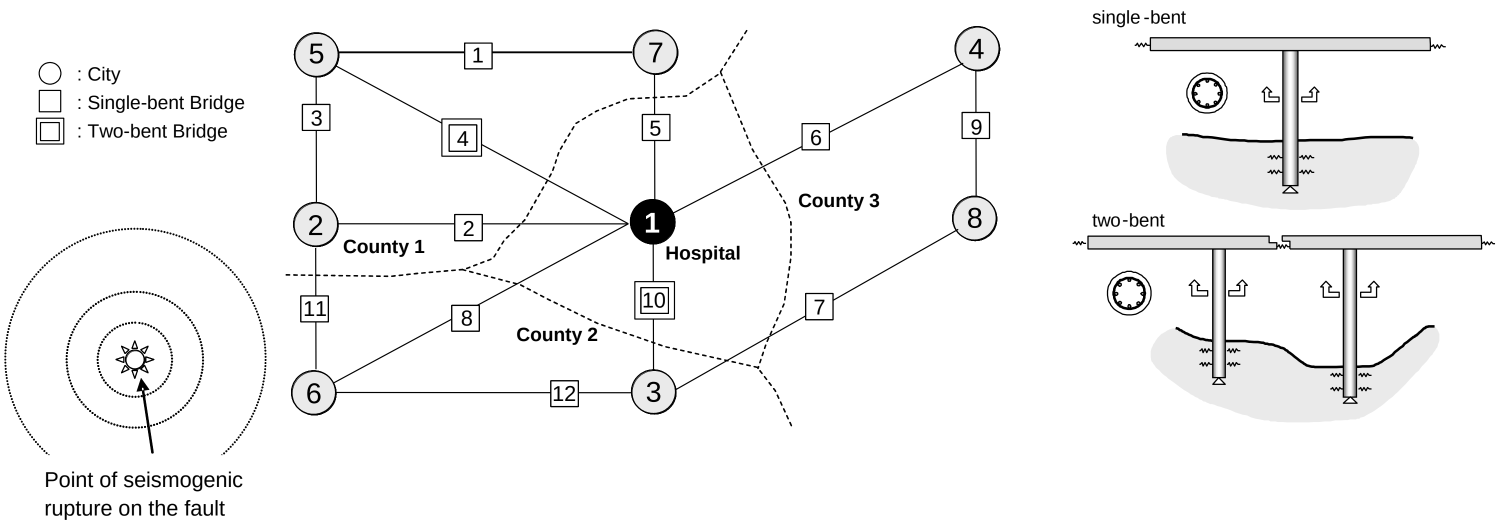
Zero unless $\theta$ is a parameter related to common source.

$$\Rightarrow \quad \frac{\partial P_{sys}}{\partial \theta} = \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}} \left[ \frac{\partial \mathbf{p}(\mathbf{s})}{\partial \theta} f_{\mathbf{S}}(\mathbf{s}) + \mathbf{p}(\mathbf{s}) \frac{\partial f_{\mathbf{S}}(\mathbf{s})}{\partial \theta} \right] d\mathbf{s}$$

* Song, J. and W.-H. Kang "System Reliability and Sensitivity under Statistical Dependence by Matrix-based System Reliability Method," *Structural Safety*, Vol. 31(**2**), 148-156.

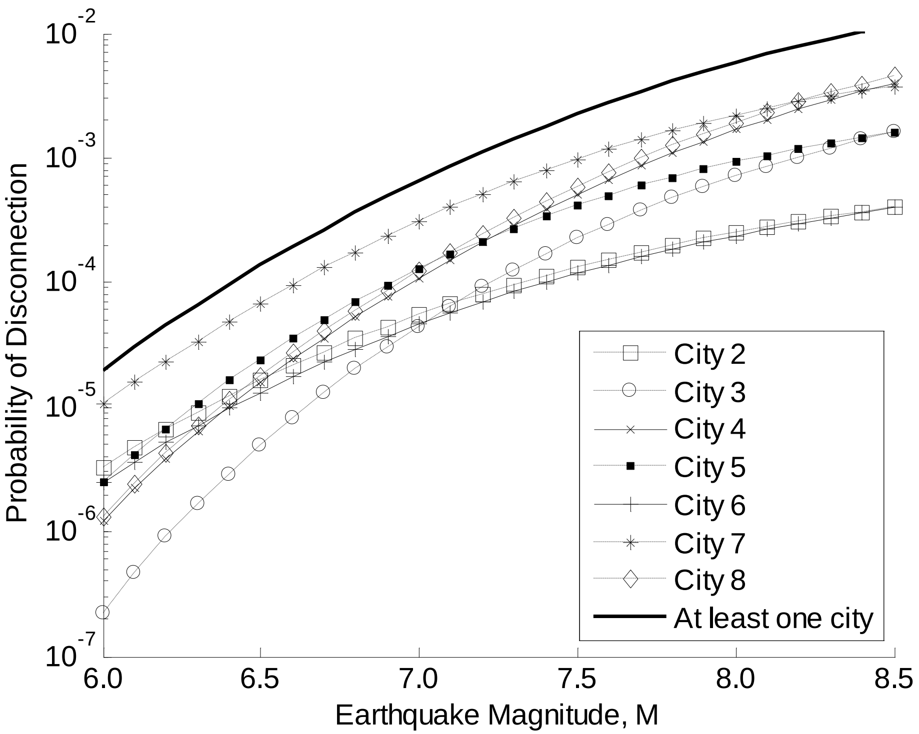
# Appl. I: Connectivity of a transportation network

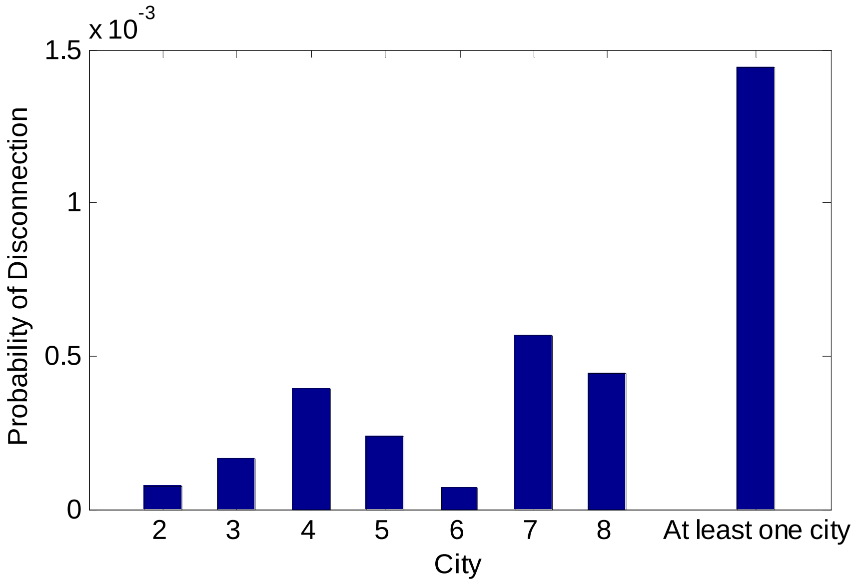
* Kang, W.-H., J. Song, and P. Gardoni (2008) "Matrix-based system reliability method and applications to bridge networks," *Reliability Engineering & System Safety*, Vol. 93, 1584-1593.




- Post-earthquake disconnection from the critical facility
- Fragilities for bridges (Gardoni et al. 2003)
- Deterministic attenuation relationship used
- For given magnitude, the bridge component failures are conditional s.i.

# Connectivity of a transportation network



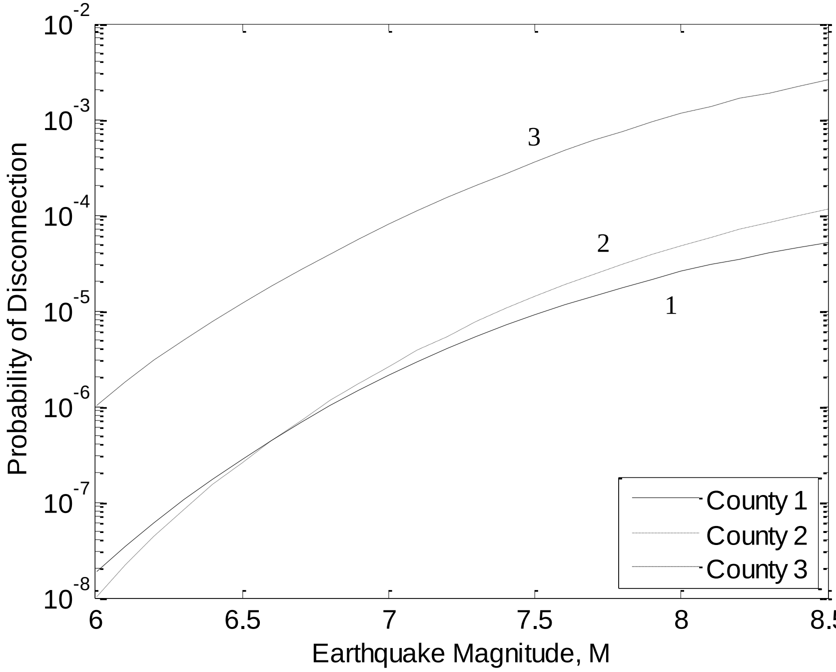

$$P(E_{sys} \mid M = m) = \mathbf{c}^{\mathrm{T}} \mathbf{p}(m)$$

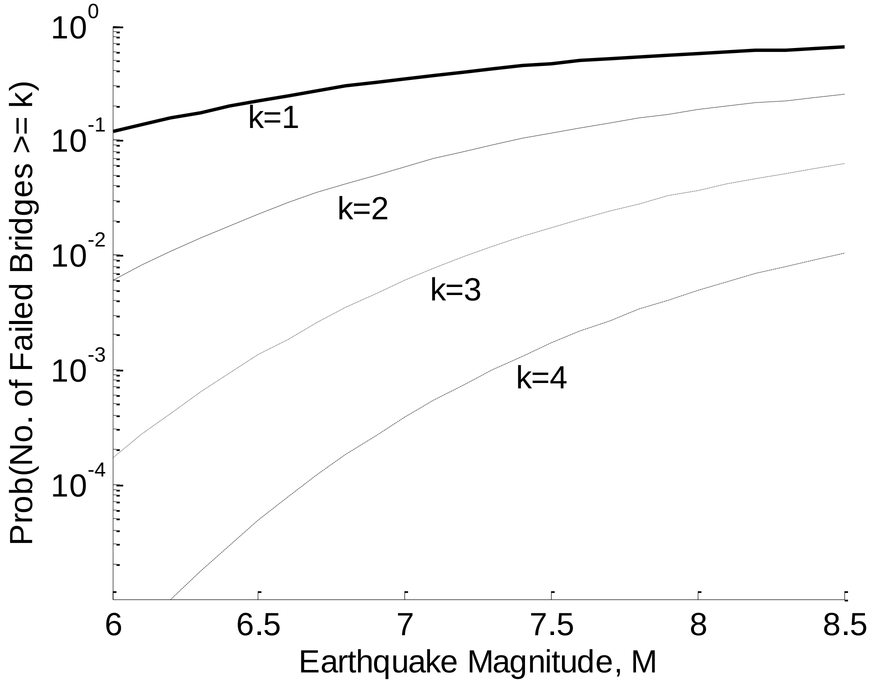
$$P(E_{sys}) = \int_{m_0}^{m_c} \mathbf{c}^{\mathrm{T}} \mathbf{p}(m)\, f_M(m) dm = \mathbf{c}^{\mathrm{T}} \tilde{\mathbf{p}}$$

Conditional probability of disconnection of cities

Probability of disconnection of cities

# Connectivity of a transportation network



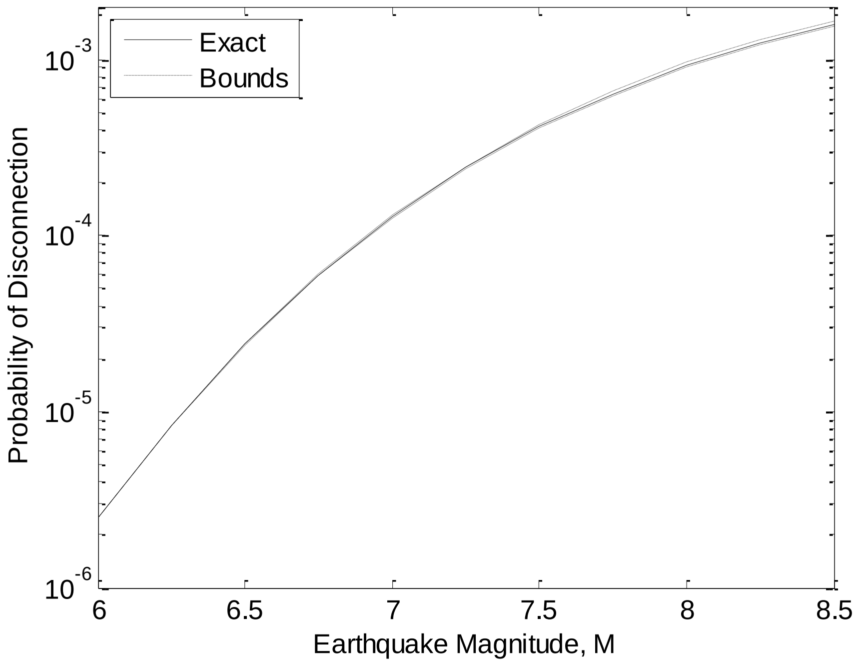

$$P(E'_{sys}) = \mathbf{c}'^{\mathrm{T}}\mathbf{p}(m)$$

Conditional probability of disconnection of counties

$$P(E'_{sys}) = \mathbf{c}''^{\mathrm{T}}\mathbf{p}(m)$$

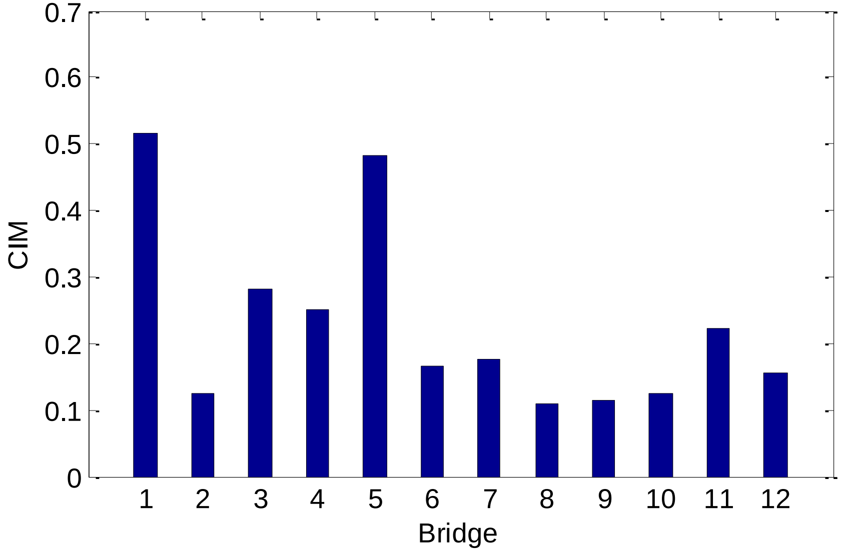
Prob (No. of failed bridges ≥ *k*)

# Connectivity of a transportation network




$$\min(\max) \quad \mathbf{c}^{\mathrm{T}}\mathbf{p}(m)$$

Bounds on P(City 5 disconnected)
(No information on Bridge 12)

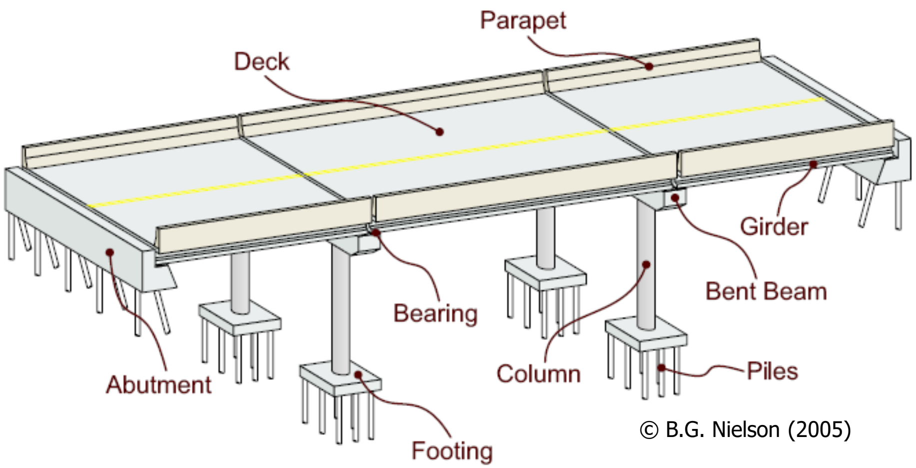


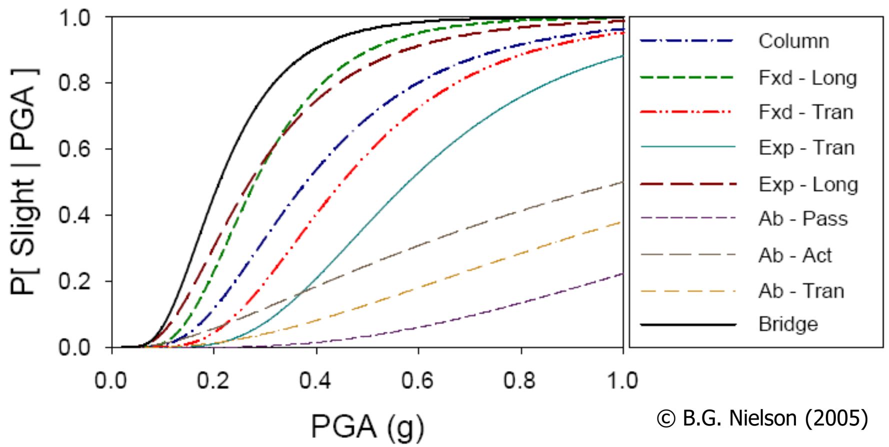

$$P(E_i \mid E_{sys}) = \frac{P(E_i E_{sys})}{P(E_{sys})} = \frac{\mathbf{c}'^{\mathrm{T}}\tilde{\mathbf{p}}}{\mathbf{c}^{\mathrm{T}}\tilde{\mathbf{p}}}$$

Importance measure of components
w.r.t. the likelihood of at least a disconnection

# Appl. II: Damage of a bridge structural system

* Song, J. and W.-H. Kang "System Reliability and Sensitivity under Statistical Dependence by Matrix-based System Reliability Method," *Structural Safety*, Vol. 31(**2**), 148-156.




- Nielson (2005) developed analytical fragilities of bridge components such as bearings, abutments and columns
- Identified the statistical dependence between demands
- Probability that at least one component fails (series system)
- Performed MCS to account for component dependence

# Damage of a bridge structural system

* Safety Factor $F_i = \ln C_i - \ln D_i$

* Fragility

$$P(LS_i \mid IM) = P(F_i \le 0 \mid IM)$$

$$= P\left( Z_i \le -\frac{\mu_{F_i}}{\sigma_{F_i}} \mid IM \right)$$

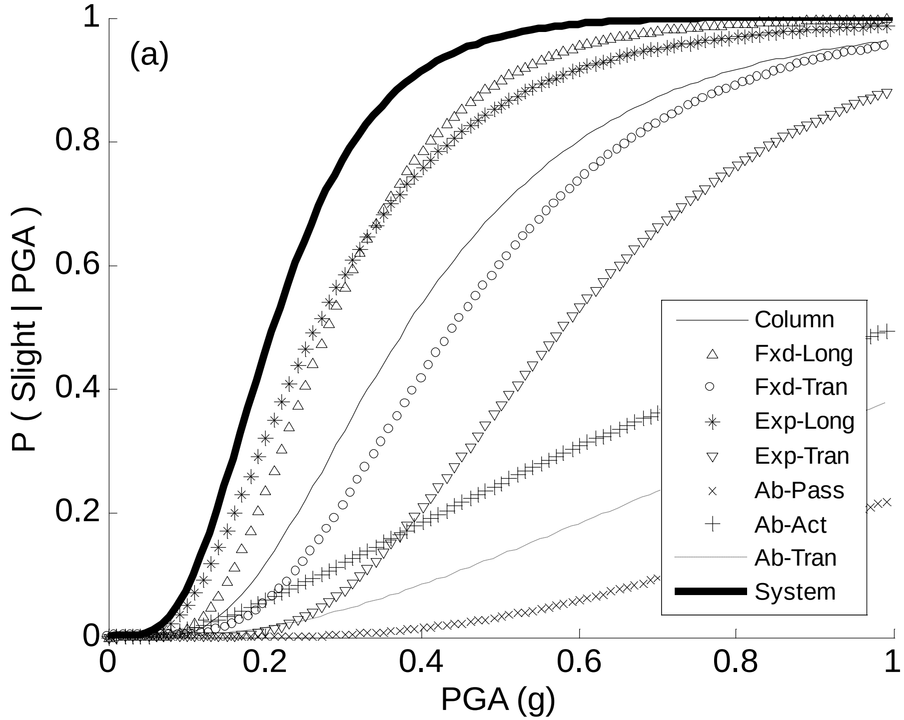
$$= \Phi\left[ -\frac{\mu_{F_i}(IM)}{\sigma_{F_i}(IM)} \right]$$

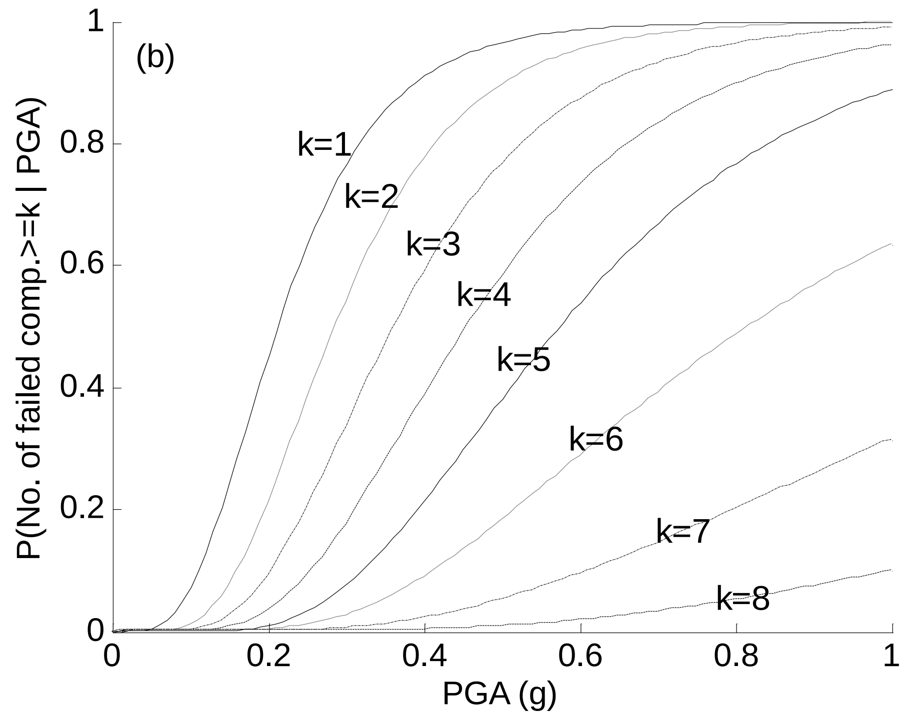
* Correlation

$$\rho_{Z_i Z_j} = \rho_{F_i, F_j} = \frac{(\zeta_{D_i} \cdot \zeta_{D_j})}{(\zeta_{C_i}^2 + \zeta_{D_i}^2)^{1/2} (\zeta_{C_j}^2 + \zeta_{D_j}^2)^{1/2}} \cdot \underline{\rho_{\ln D_i, \ln D_j}}$$

* Fitting by DS-class corr. matrix: average of percentage error ~ 3%

# Damage of a bridge structural system




$$P(E_{sys} \mid PGA = pga) = \mathbf{c}^{\mathrm{T}}\mathbf{p}(pga)$$
$$= \int \mathbf{c}^{\mathrm{T}}\mathbf{p}(pga, x)\varphi(x)dx$$

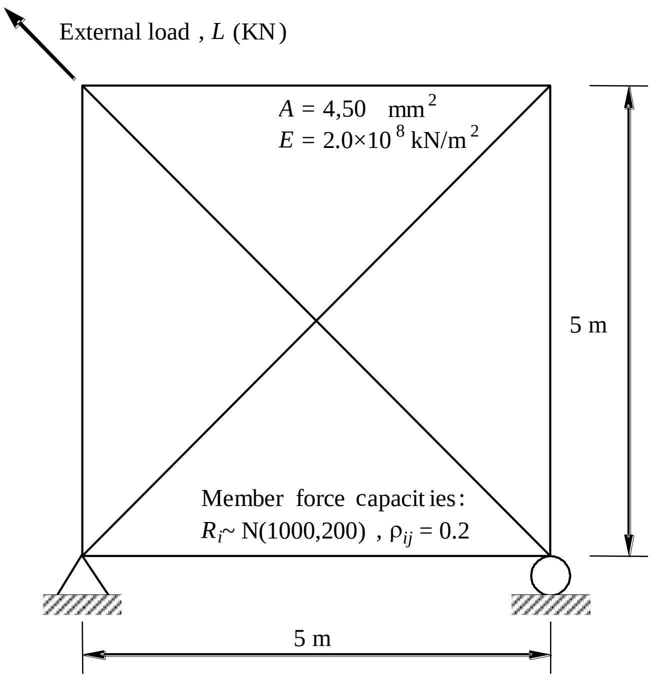
System fragility (at least one)

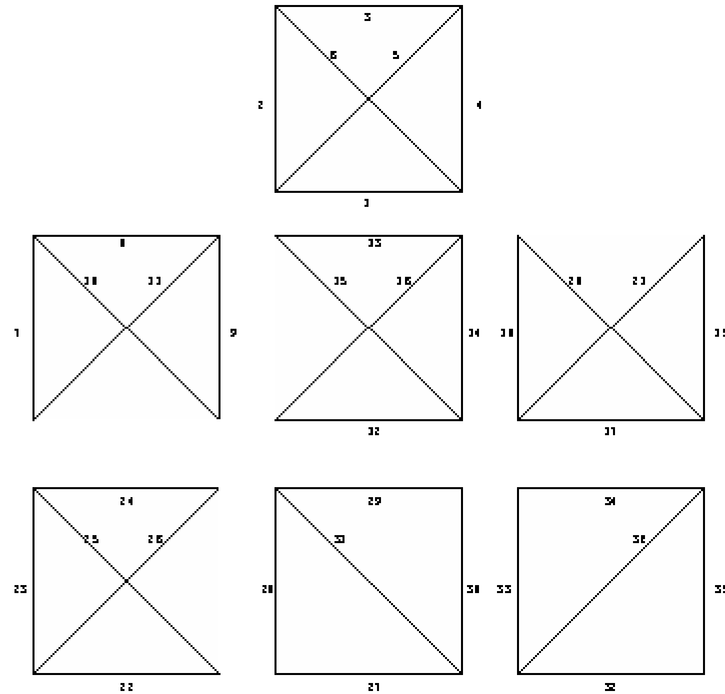
$$P(E_{sys} \mid PGA = pga) = \mathbf{c}'^{\mathrm{T}}\mathbf{p}(pga)$$

P(No. of failed components ≥ k)

# Appl. III: Progressive failure of a truss structure

* Song, J. and W.-H. Kang "System Reliability and Sensitivity under Statistical Dependence by Matrix-based System Reliability Method," *Structural Safety*, Vol. 31(**2**), 148-156.



$$P(\overline{E}_{sys}) = P[\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6 \cup (E_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_7\overline{E}_8\overline{E}_9\overline{E}_{10}\overline{E}_{11})$$

$$\cup (\overline{E}_1E_2\overline{E}_3\overline{E}_4\overline{E}_5\overline{E}_6)(\overline{E}_{12}\overline{E}_{13}\overline{E}_{14}\overline{E}_{15}\overline{E}_{16}) \cup \cdots$$

$$\cup (\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4\overline{E}_5E_6)(\overline{E}_{32}\overline{E}_{33}\overline{E}_{34}\overline{E}_{35}\overline{E}_{36})]$$

# Progressive failure of a truss structure

$$P(\bar{E}_{sys}) = P[\bar{E}_1\bar{E}_2\bar{E}_3\bar{E}_4\bar{E}_5\bar{E}_6 \cup (E_1\bar{E}_2\bar{E}_3\bar{E}_4\bar{E}_5\bar{E}_6)(\bar{E}_7\bar{E}_8\bar{E}_9\bar{E}_{10}\bar{E}_{11})$$

$$\cup (\bar{E}_1 E_2\bar{E}_3\bar{E}_4\bar{E}_5\bar{E}_6)(\bar{E}_{12}\bar{E}_{13}\bar{E}_{14}\bar{E}_{15}\bar{E}_{16}) \cup \cdots$$

$$\cup (\bar{E}_1\bar{E}_2\bar{E}_3\bar{E}_4\bar{E}_5 E_6)(\bar{E}_{32}\bar{E}_{33}\bar{E}_{34}\bar{E}_{35}\bar{E}_{36})]$$

Disjoint link sets (36→11)

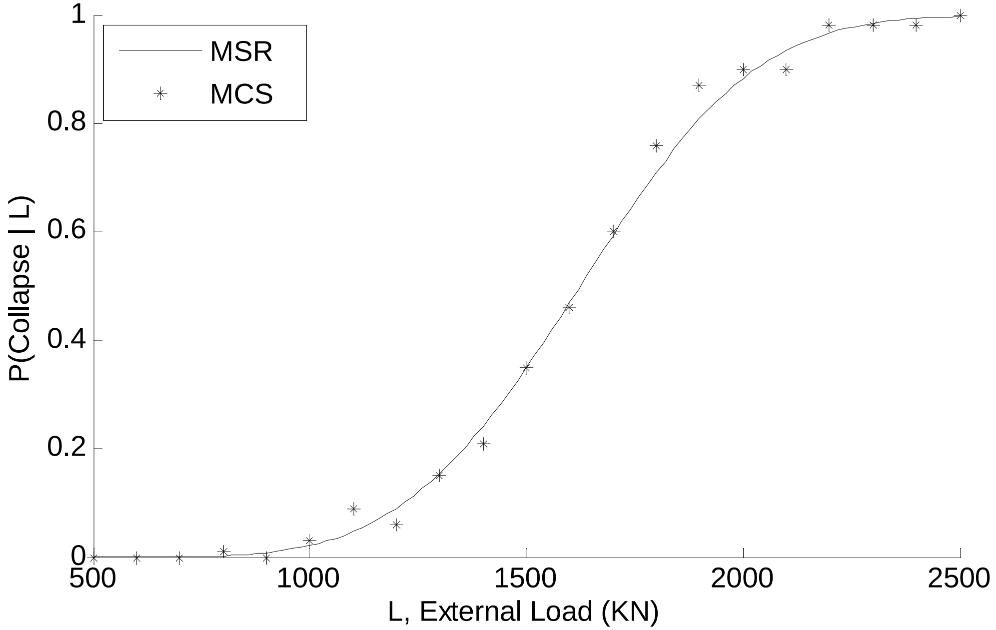
$$P(\bar{E}_{sys}) = P(\bar{E}_1\bar{E}_2\bar{E}_3\bar{E}_4\bar{E}_5\bar{E}_6) + P(E_1\boxed{\bar{E}_2}\bar{E}_3\bar{E}_4\bar{E}_5\bar{E}_6\boxed{\bar{E}_7}\bar{E}_8\bar{E}_9\bar{E}_{10}\bar{E}_{11})$$

$$\cdots + P(\bar{E}_1\bar{E}_2\bar{E}_3\bar{E}_4\bar{E}_5 E_6\bar{E}_{32}\bar{E}_{33}\bar{E}_{34}\bar{E}_{35}\bar{E}_{36})$$

Perfect correlation

7 systems with 6 components

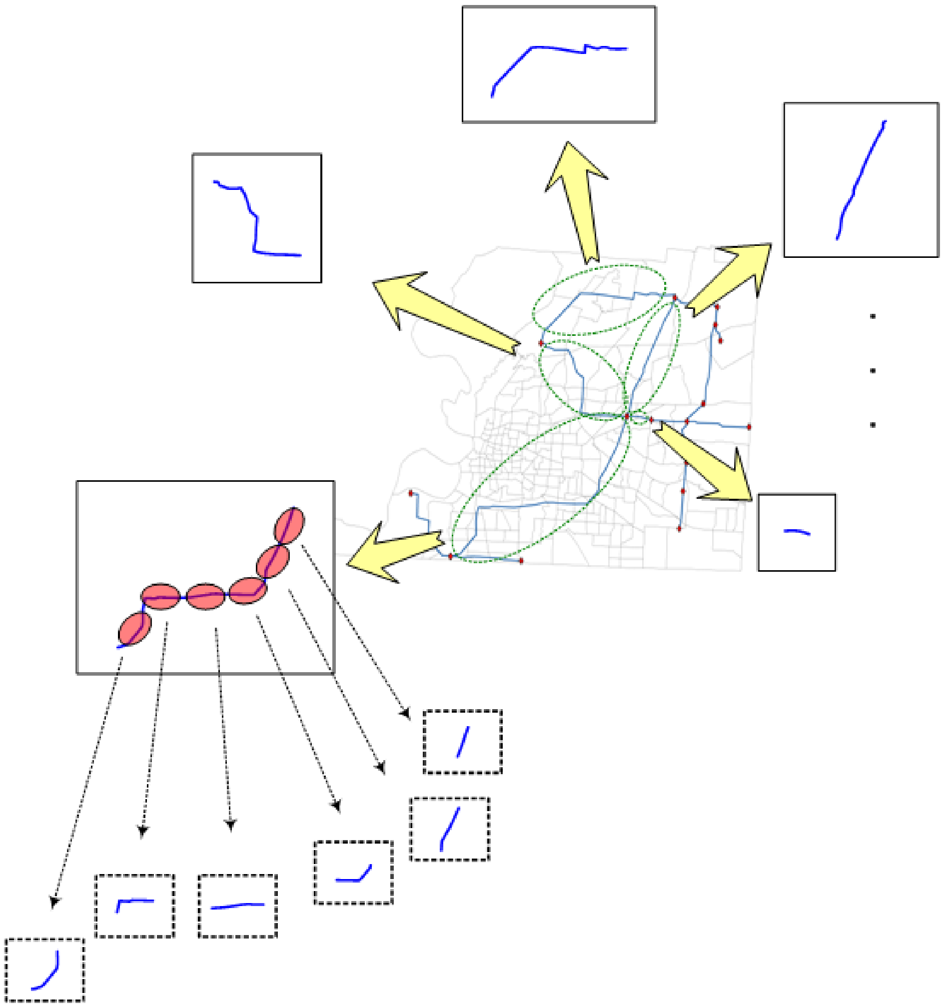
# Progressive failure of a truss structure




- System collapse fragility curve given abnormal load
- Verified through MCS
- Importance of members (components)
- Sensitivity of fragility w.r.t. design parameters

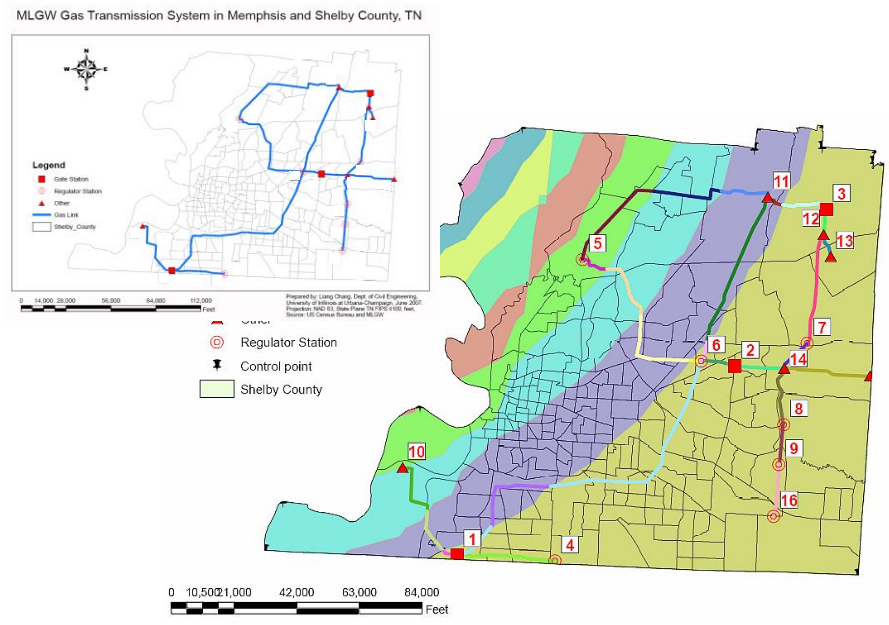
# Appl. IV: Multi-scale SRA of lifeline networks

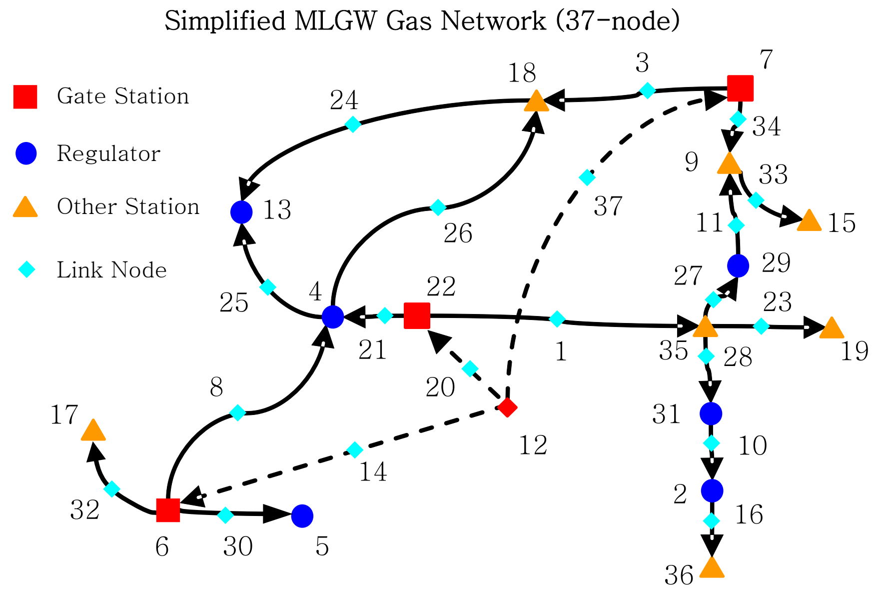
* Song, J., and S.-Y. Ok (2010). Multi-scale system reliability analysis of lifeline networks under earthquake hazards. *Earthquake Engineering and Structural Dynamics*, Vol. 39(**3**), 259-279.



- **"Divide and Conquer" approach**
  - Lower-scale system reliability analyses are performed for "supercomponents" and followed by higher-scale system reliability analyses
  - Proposed to facilitate the use of **LP bounds method** (Song and Der Kiureghian, 2003) for large-size systems
  - **MSR method** is a good tool for SRA at multiple scales
- **Advantages**
  - **Multi-scale modeling** of a system – seeing big picture without disregarding the details
  - Helps identify **important** components and parameters at **multiple scales**
  - **Collaborative** risk management
  - Facilitates parallel computing

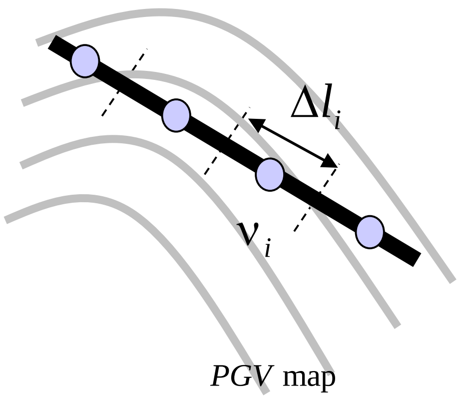
# Example: MLGW gas network




- Gas pipeline network of Memphis Light, Gas, and Water (MLGW), Shelby County, TN
- A simplified network in Chang et al. (1996) was modified based on comments from R. Bowker (MLGW)
- 37-node and 40-arc network: nodes representing pipelines and stations
- Earthquake hazard scenarios: Epicenter at N35.54$^{o}$-W90.43$^{o}$ at Blytheville, AR
- Fragilities of pipelines and stations – *HAZUS-MH*
- PGV and PGA maps from *MAEviz*

# Failure prob. of pipeline segments




- Failure probability of the *i*-th segment of a pipeline

$$P_i = 1 - \exp(-\nu_i \cdot \Delta l_i)$$

- Failure occurrence rate of a pipeline (HAZUS-MH: FEMA 2003)

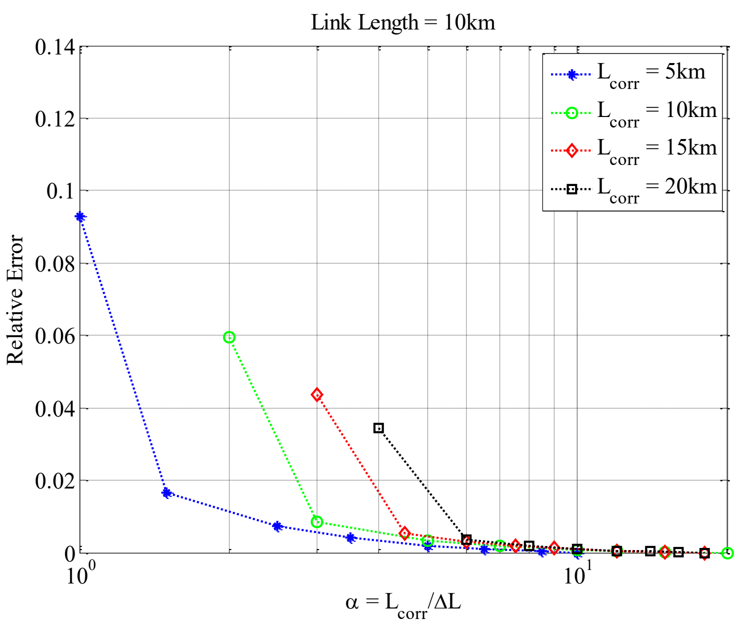
$$\nu_i = k \cdot (PGV_i)^{\gamma}$$

- Uncertainty in PGV (Adachi & Ellingwood, 2007)

$$PGV_i = \overline{PGV_i} \times \varepsilon_i$$

Lognormal r.v. (median = 1, c.o.v. = 0.6)

Attenuated PGV (Fernandez and Rix 2006)




- Spatial Correlation (Wang & Takada, 2005)

$$\rho_{\ln PGV_i, \ln PGV_j} = \rho_{\ln \varepsilon_i, \ln \varepsilon_j} = \exp(-\| \mathbf{x}_i - \mathbf{x}_j \| / L_{corr})$$

- Generalized Dunnett-Sobel (Song and Kang, 2008)

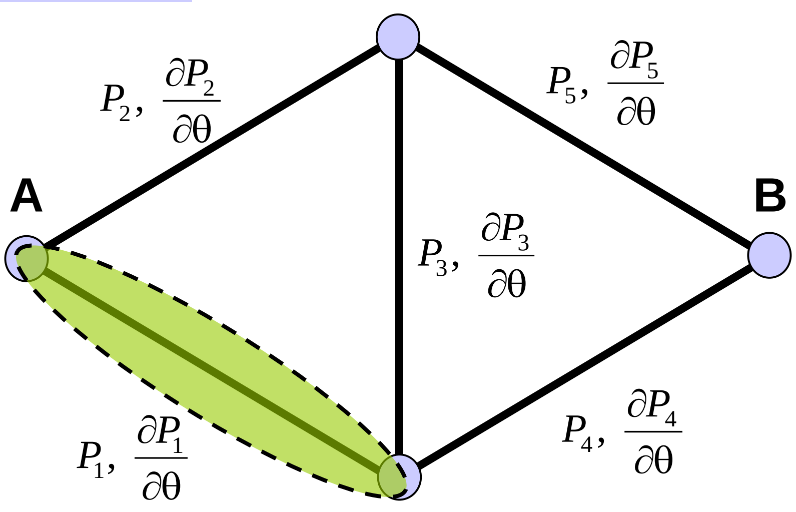
$$Z_i = \ln \varepsilon_i / \zeta_i \sim N(\mathbf{0}, \mathbf{R})$$ → Find gDS that fits best

- (←) Discretization error

choose number of segments considering corr. length

# Multi-scale SRA using MSR Method

## Higher-scale

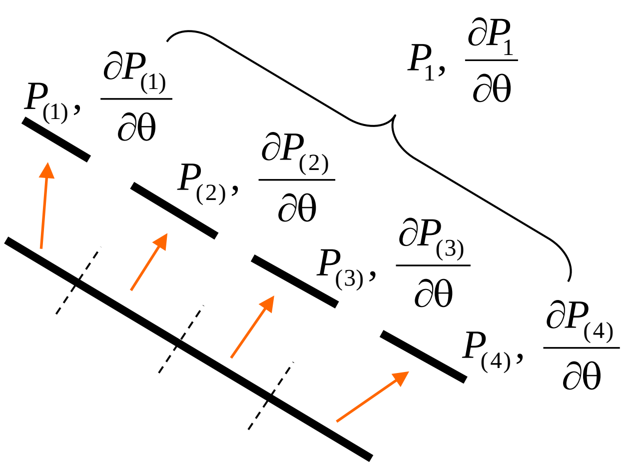



$$P(E_{sys}) = \mathbf{c}^{\mathrm{T}}\mathbf{p}$$

$$\frac{\partial P(E_{sys})}{\partial \theta} = \mathbf{c}^{\mathrm{T}} \frac{\partial \mathbf{p}}{\partial \theta} = \mathbf{c}^{\mathrm{T}} \hat{\mathbf{P}} \frac{\partial \mathbf{P}}{\partial \theta}$$

→ MSR analysis using failure probability and sensitivity of links $P_i, \frac{\partial P_i}{\partial \theta} \quad i = 1, \ldots, n_{link}$

## Lower-scale

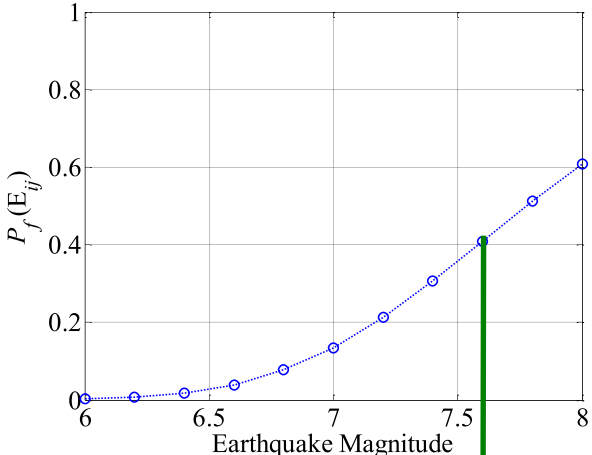



$$P_1 = \mathbf{c}_1^{\mathrm{T}}\mathbf{p}_1$$

$$\frac{\partial P_1}{\partial \theta} = \mathbf{c}_1^{\mathrm{T}} \frac{\partial \mathbf{p}_1}{\partial \theta} = \mathbf{c}_1^{\mathrm{T}} \hat{\mathbf{P}}_1 \frac{\partial \mathbf{P}_1}{\partial \theta}$$

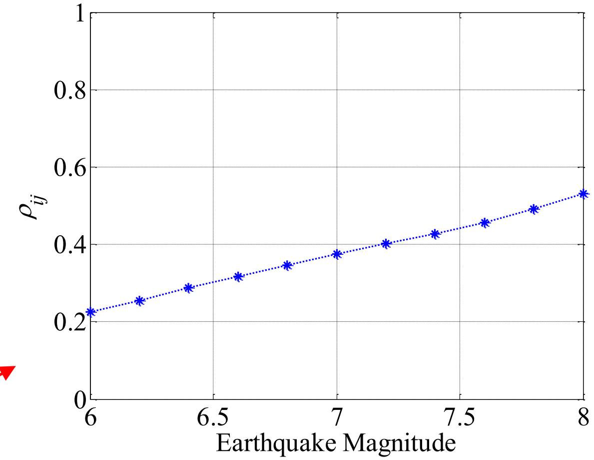
→ MSR analysis using failure probability and sensitivity of segments $P_{(i)}, \frac{\partial P_{(i)}}{\partial \theta} \quad i = 1, \ldots, n_{seg}$

# Correlation between pipelines



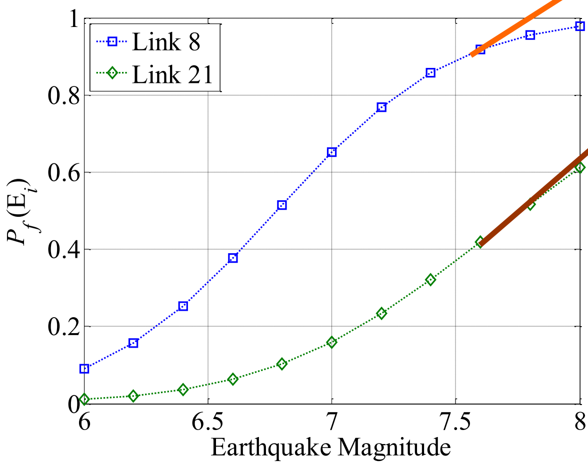

**MSR for joint failure of pipelines**

$$(E_{i,1}\cup\ldots\cup E_{i,N_i})\cap(E_{j,1}\cup\ldots\cup E_{j,N_j})$$



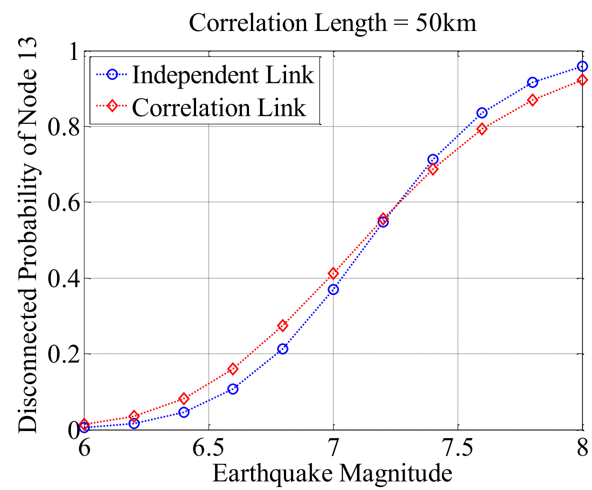

$$\Phi_2(-\beta_1,-\beta_2,\rho)=\Phi(-\beta_1)\Phi(-\beta_2)+\int_0^{\rho}\varphi(-\beta_1,-\beta_2,\rho')\,d\rho'$$

**MSR w/ correlation**

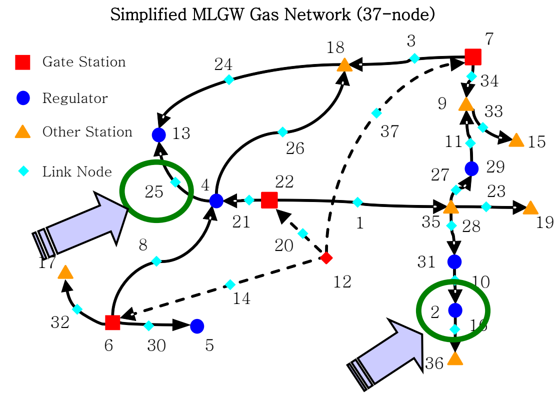



**Lower-scale MSR**

$$(E_{i,1}\cup\ldots\cup E_{i,N_i})$$
$$(E_{j,1}\cup\ldots\cup E_{j,N_j})$$

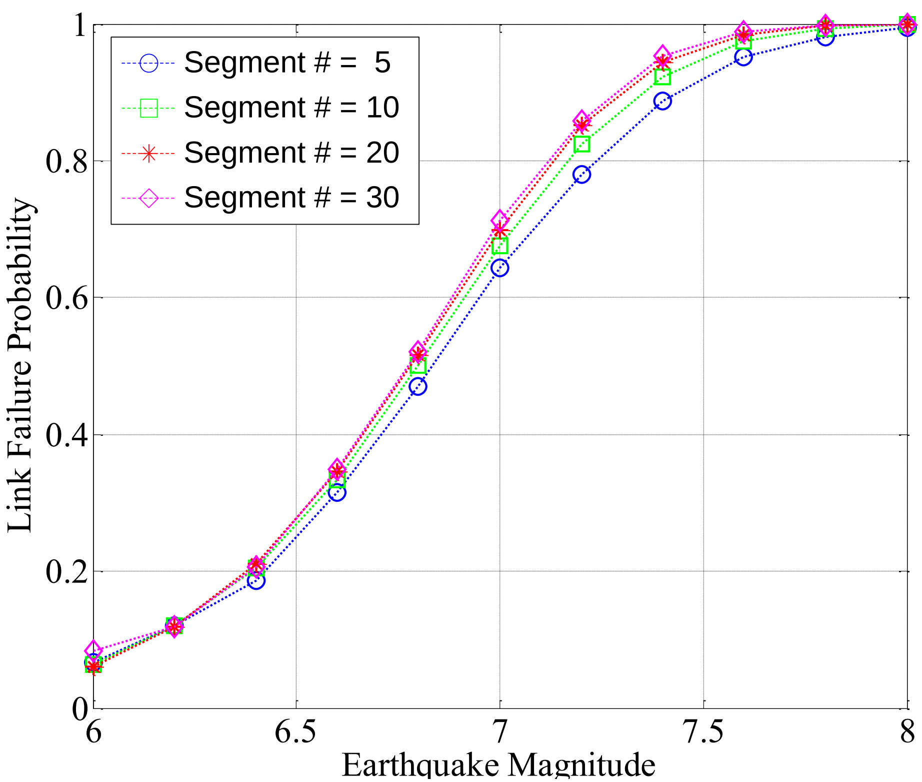


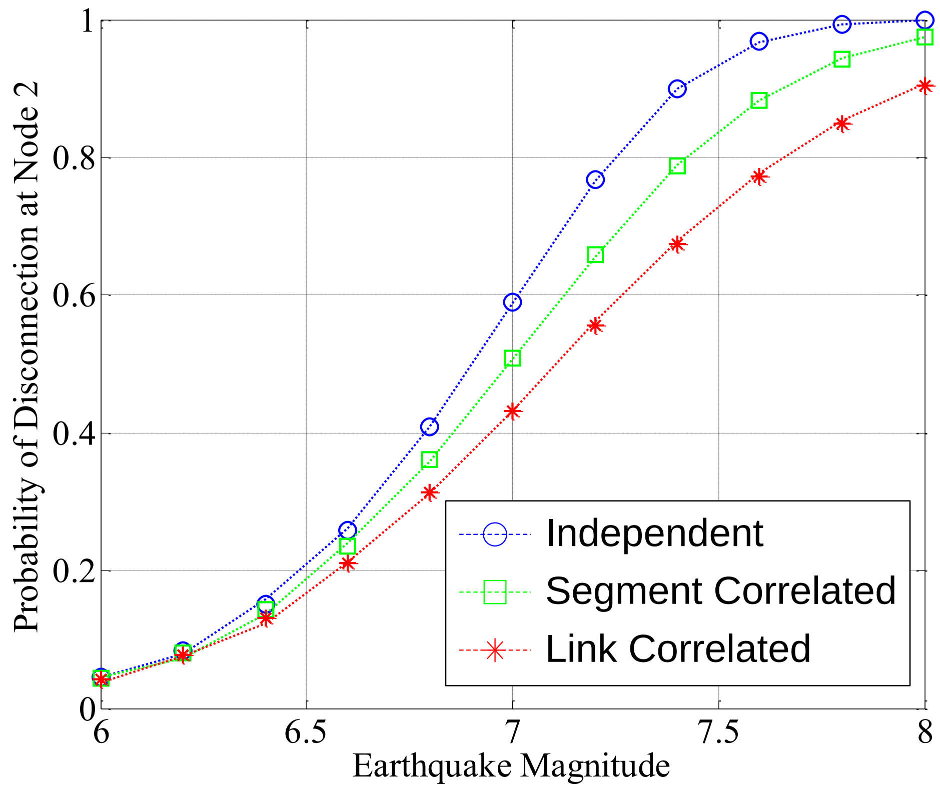
# Risk at multiple scales




**Lower**-scale: pipelines

Failure probability of **Link** 25



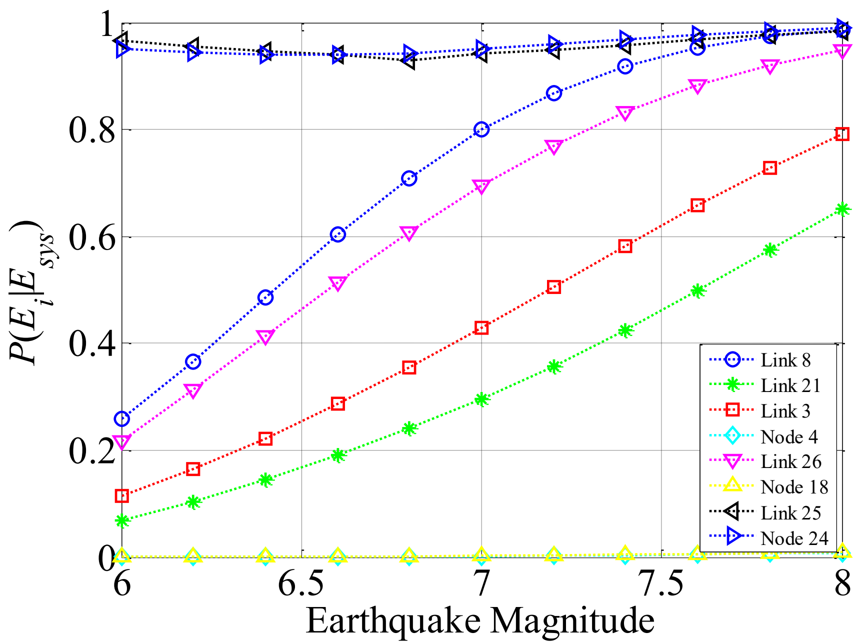

**Higher**-scale: service nodes

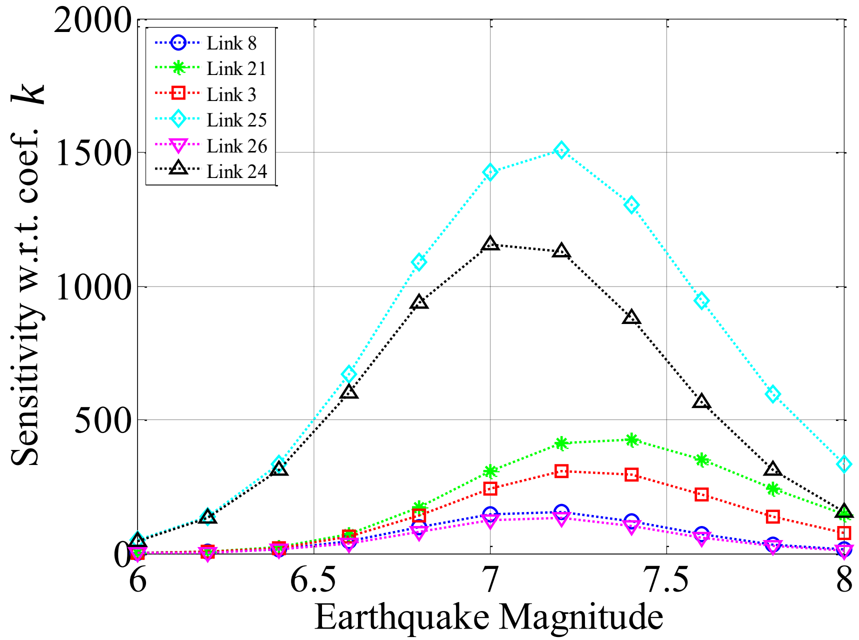
Prob. of Disconnection at **Node** 2

# Probabilistic inference and sensitivity

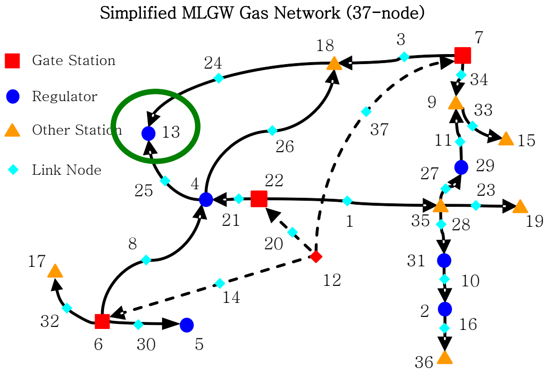
## Conditional Probabilities

## Parameter Sensitivity

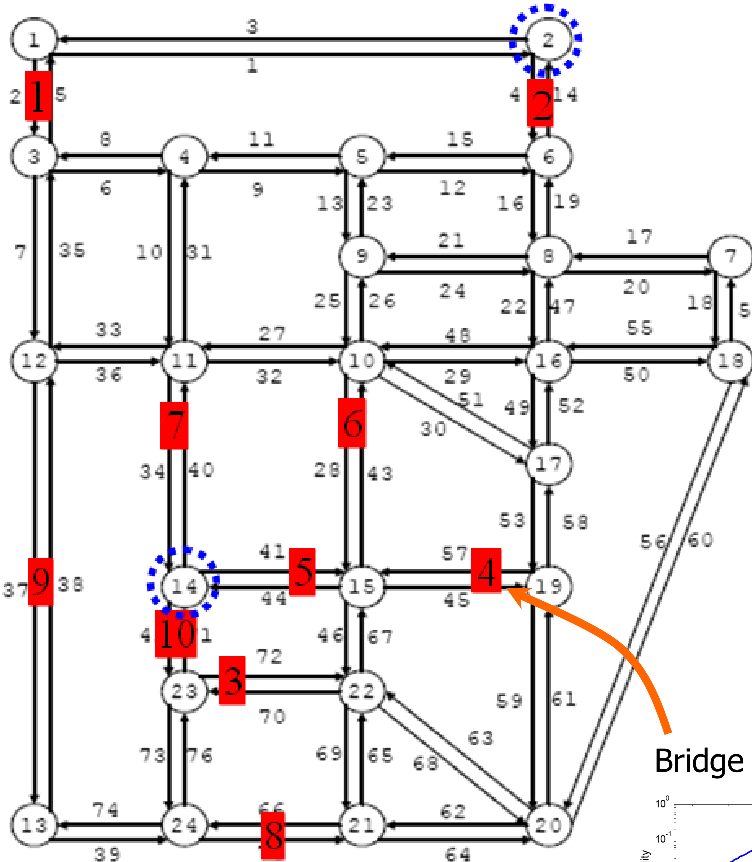


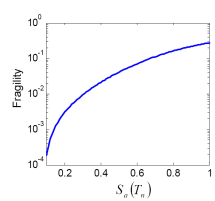

- Conditional probability of link failure probability given observed system event (e.g. disconnection)
- Sensitivity of system failure probability with respect to parameters in PGV-based model for failure occurrence rate: $\nu_i = k \cdot (PGV_i)^{\gamma}$



# Appl. V: Post-hazard flow capacity of a network




Bridge fragility




Example: Modified Sioux-Falls network

**Red**: bridges; **Circles**: Starting & Ending points

- Traffic flow **capacity** between two points in a network → determined by combinations of bridge damage

**q** : a vector of network flow capacity for bridge failure combinations (obtained by maximum flow capacity analysis)

$$\mu_Q = \mathbf{q}^{\mathrm{T}}\mathbf{p}$$

: average post-hazard flow capacity

$$\sigma_Q^2 = (\mathbf{q}.*\mathbf{q})^{\mathrm{T}}\mathbf{p} - (\mathbf{q}^{\mathrm{T}}\mathbf{p})^2$$

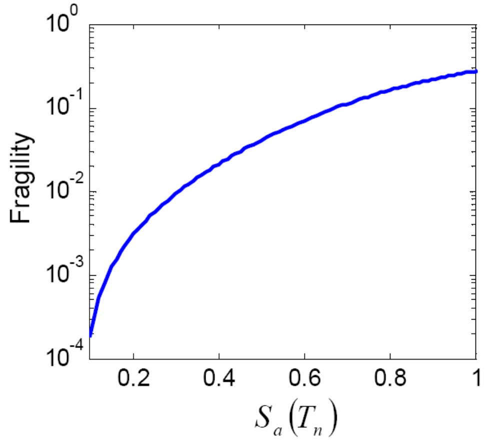
: variance of post-hazard flow capacity

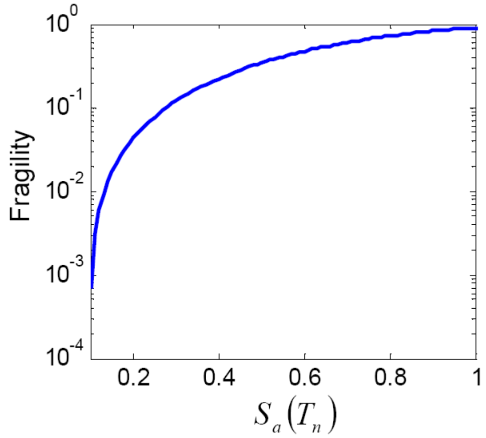
$$P(Q < a) = \sum_{\forall i: q_i < a} p_i$$

: probability that flow capacity is lower than $a$

# Multi-state Fragility

- Fragility curves (Gardoni *et al*. 2002, 2003)




⇒ **Only two states, "connected" or "disconnected"**

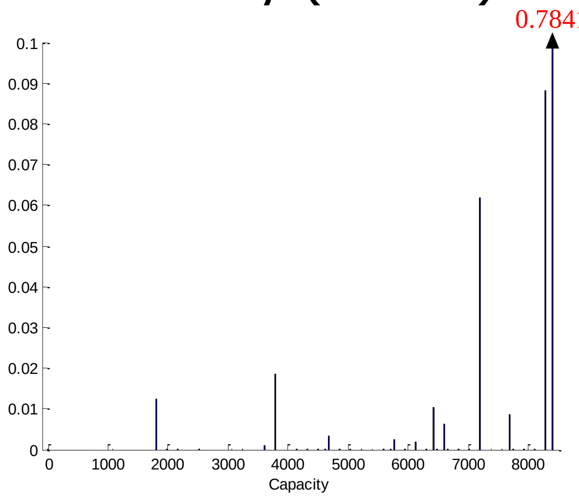
P(*Complete failure*) = 0.3 × $P_f$
P(*Heavy damage*) = 0.45 × $P_f$
P(*Moderate damage*) = 0.25 × $P_f$
P(*No damage*) = 1-$P_f$

F(*Complete failure*) = 0
F(*Heavy damage*) = 0.3 × Full capacity
F(*Moderate damage*) = 0.7 × Full capacity
F(*No damage*) = 1.0 × Full capacity

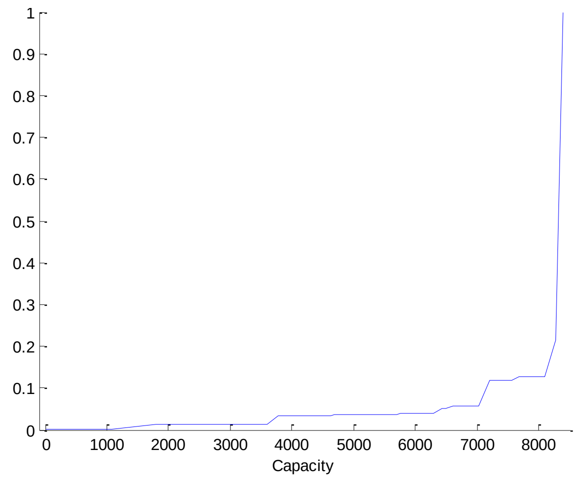
# Uncertainty quantification of flow capacity

- Capacity distribution for a given seismic intensity (M=7.0)

PMF




CDF

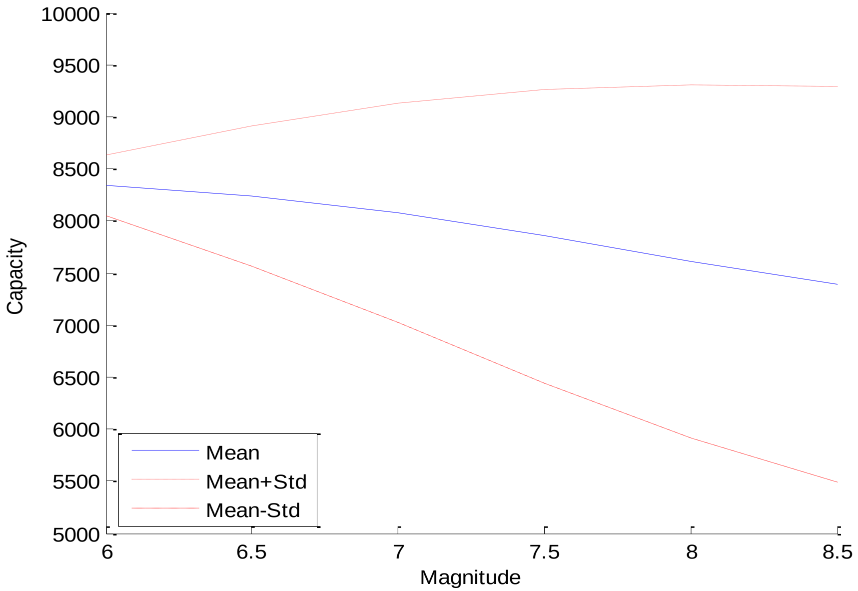



- Statistical parameters of flow capacity (M=6.0~8.5)

$$\mu_Q = \mathbf{p}^{\mathrm{T}}\mathbf{f}$$

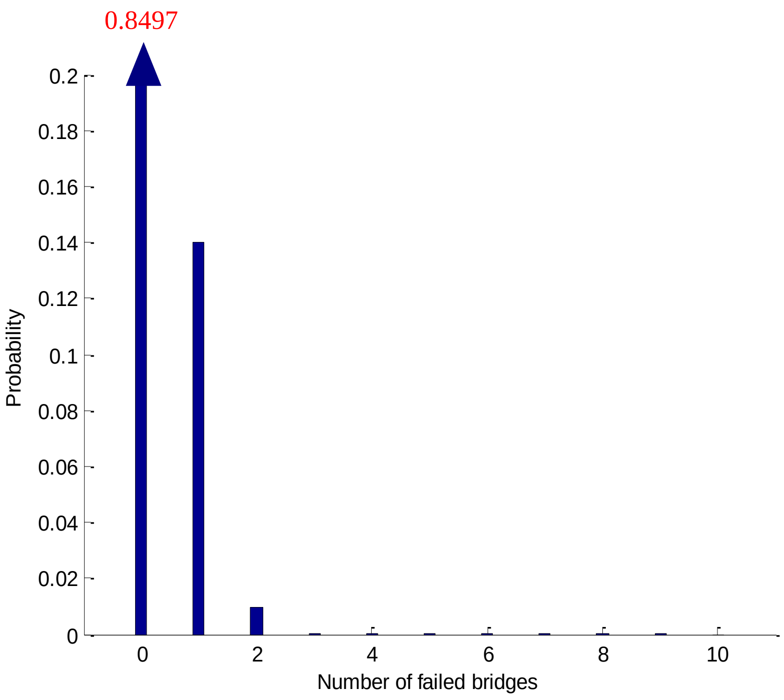
$$\sigma_Q = (\mathbf{p}^{\mathrm{T}}(\mathbf{f}.*\mathbf{f}) - \mu_Q{}^2)^{1/2}$$

$$\delta_Q = \sigma_Q / \mu_Q$$

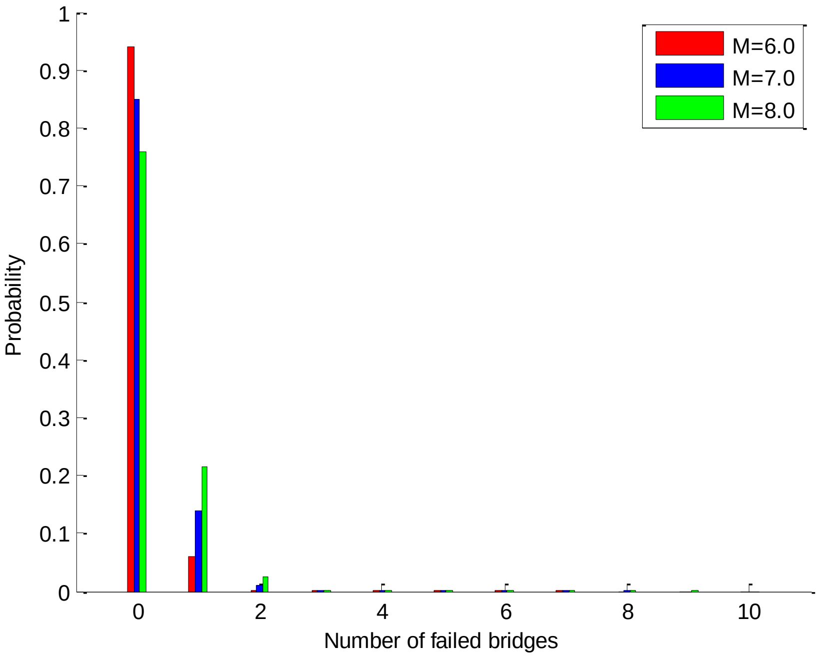


# Analysis Results

- Probability with number of failed bridges




**M=7.0**

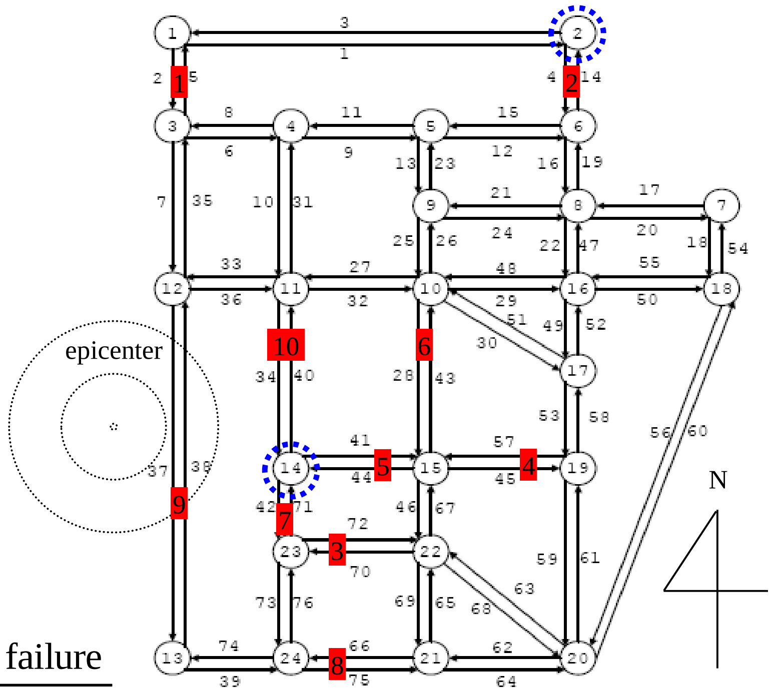



**M=6.0~8.0**

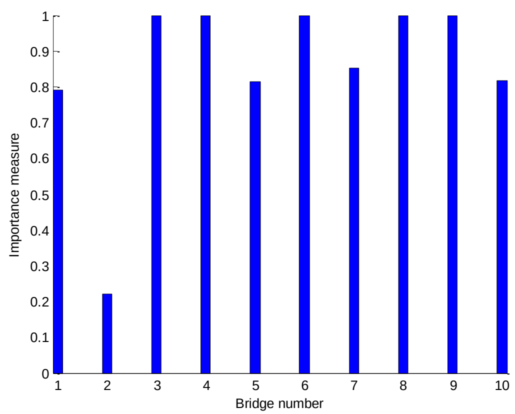
# Analysis Results

- Conditional flow capacity (For 10th bridge, M=7.0)

| Parameter | | Value |
|---|---|---|
| Mean | $\mu_{Q\|10th}$ | 6591.9 **(8076.3)** |
| Standard deviation | $\sigma_{Q\|10th}$ | 1268.9 **(1056.6)** |
| C.O.V. | $\delta_{Q\|10th}$ | 0.1925 **(0.1308)** |




- Importance measure for all bridges (M=7.0)

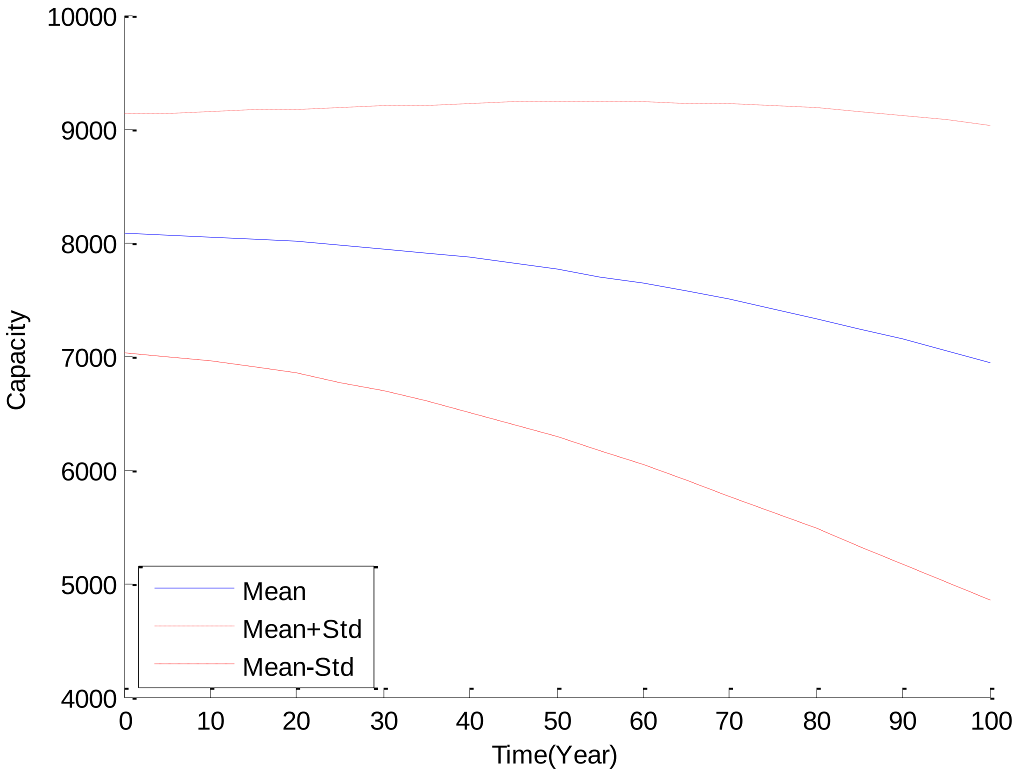



$$RF = 1 - \frac{\mu_{Q|\text{bridge failure}}}{\mu_Q}$$

**1st, 2nd, 5th, 7th, and 10th bridges are most important**

# Analysis Results

- Flow capacity with deterioration




- Assumptions

P(T, *Complete failure*)
= P(*Complete failure*) × (1.0+0.0005 × $T^2$)
P(T, *Heavy damage*)
= P(*Heavy damage*) × (1.0+0.015 × T)
P(T, *Moderate damage*)
= P(*Moderate damage*) × (1.0-0.015 × T)
P(T, *No damage*) = 1 - P(T, *Complete failure*)
- P(T, *Heavy damage*)
- P(T, *Moderate damage*)

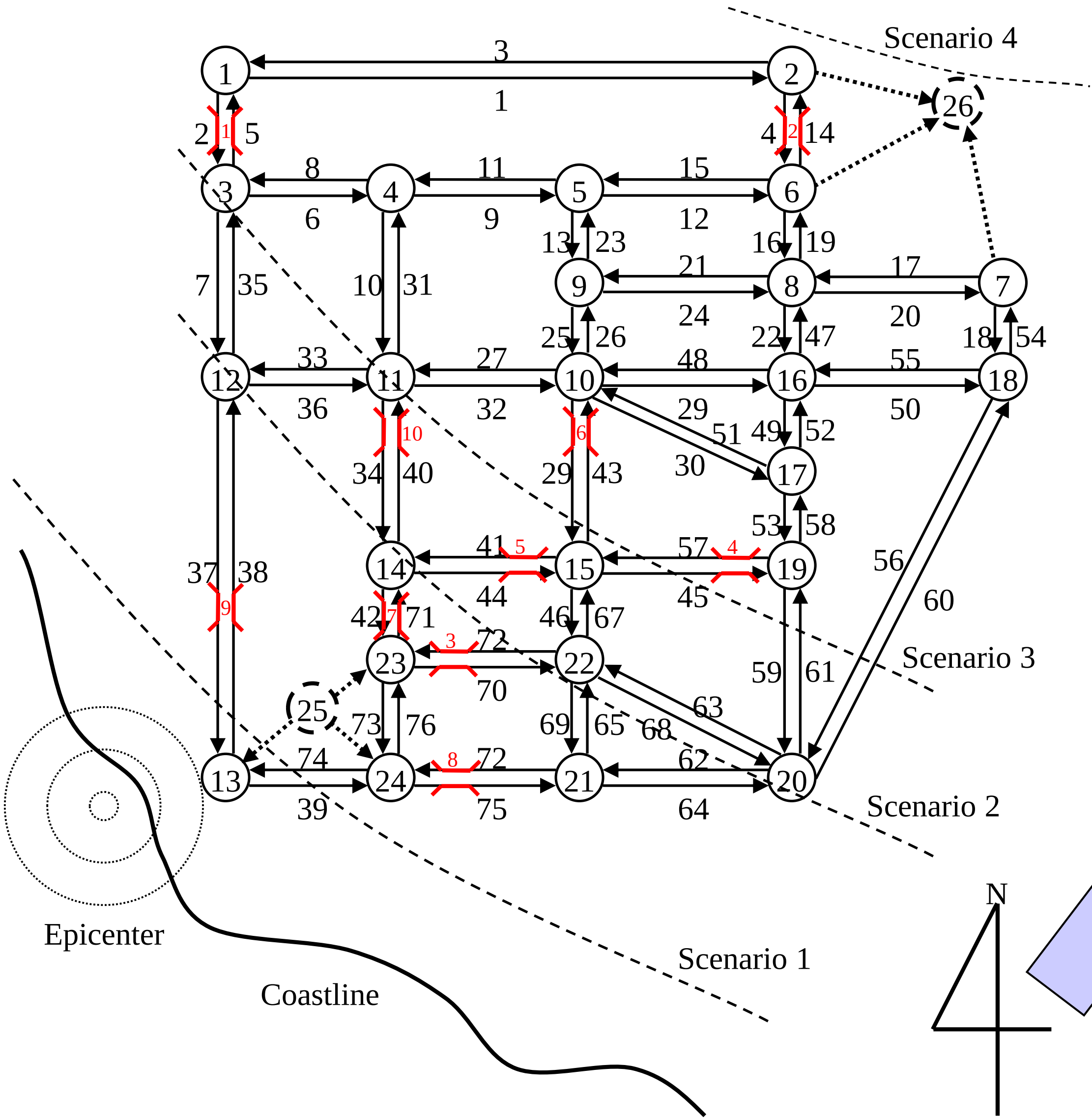
**, where T:[Years]**

$$\mu_Q(t) = \mathbf{q}^{\mathrm{T}}\mathbf{p}(t)$$

$$\sigma_Q(t) = \sqrt{(\mathbf{q}.*\mathbf{q})^{\mathrm{T}}\mathbf{p}(t) - \mu_Q^2(t)}$$

# Extension to multi-hazard environment

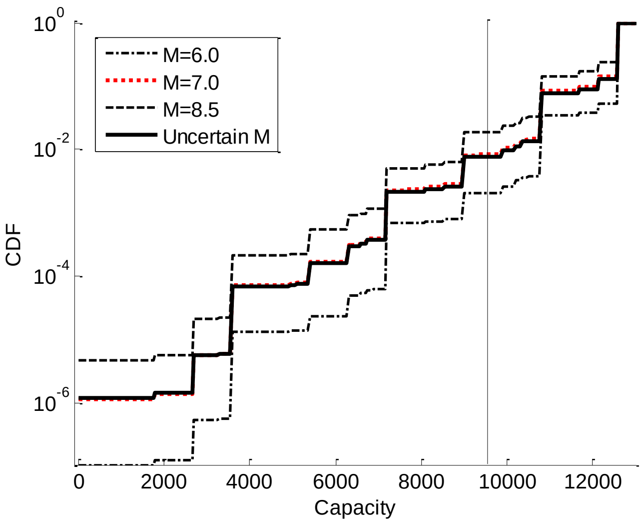
* Lee, Y.-J., J. Song, P. Gardoni, and H.-W. Lim. (2010). Post-hazard flow capacity of bridge transportation network considering structural deterioration of bridges, *Structure and Infrastructure Engineering*, Accepted for Publication.

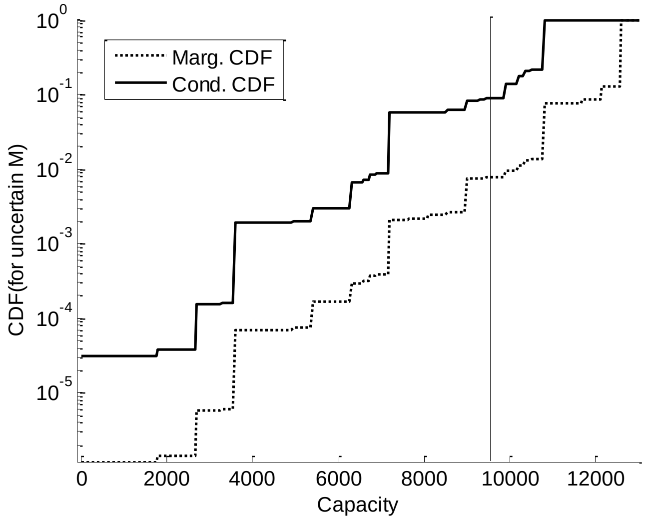


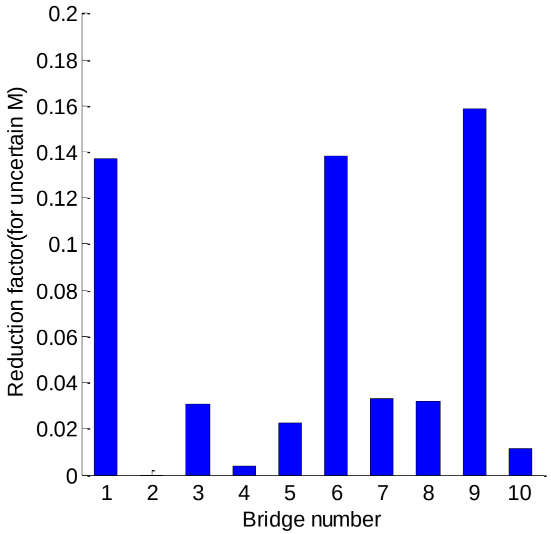

- More realistic assumptions
  - Multi-state fragility estimates w.r.t. drift capacity levels
  - Attenuation relationship (PSA & PGV)
  - Deterioration fragility estimates (Choe *et al*. 2007)
  - Multi-state flow capacity level proportional to number of open lanes
  - Deterioration scenarios
- Area-to-area flow capacity
- Further analysis for uncertain earthquake magnitude

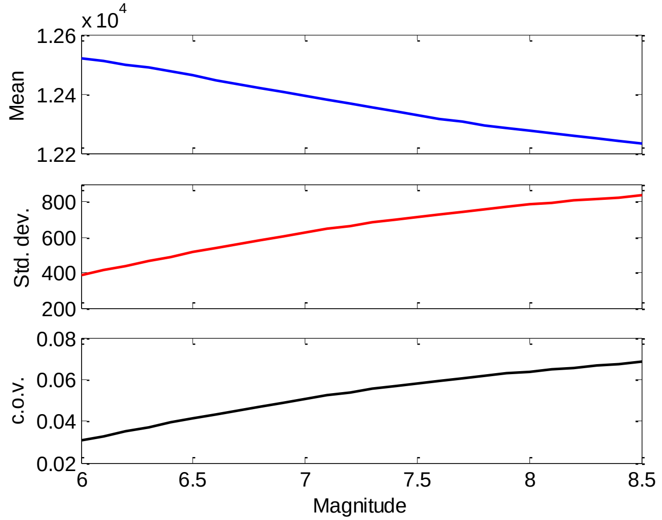
**Progress of Structural Deterioration (Corrosion) by Sea Air**

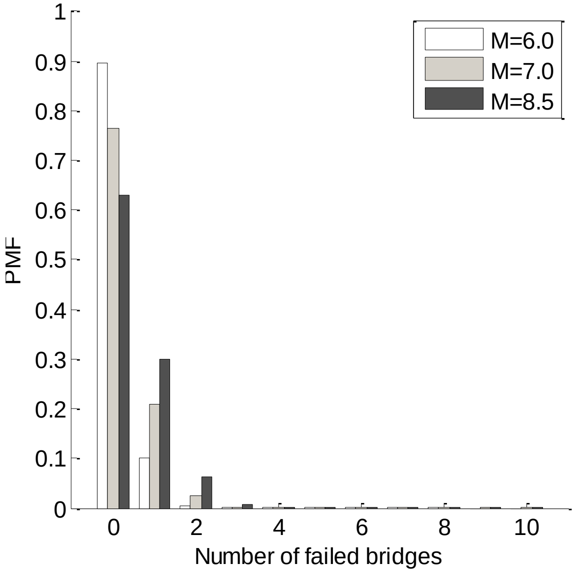
# Analysis Results

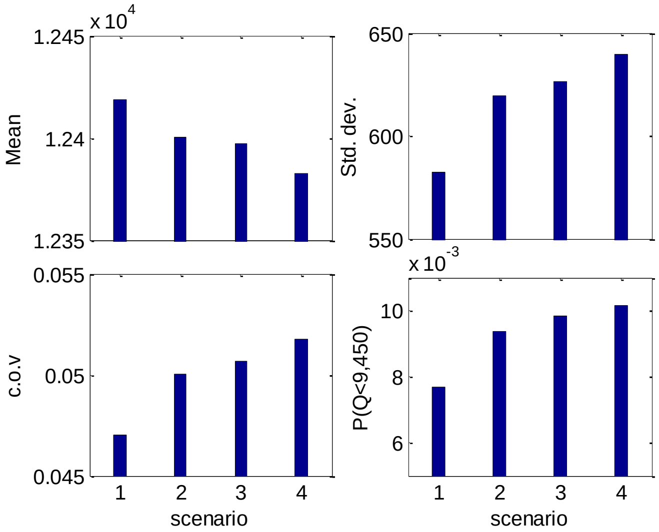


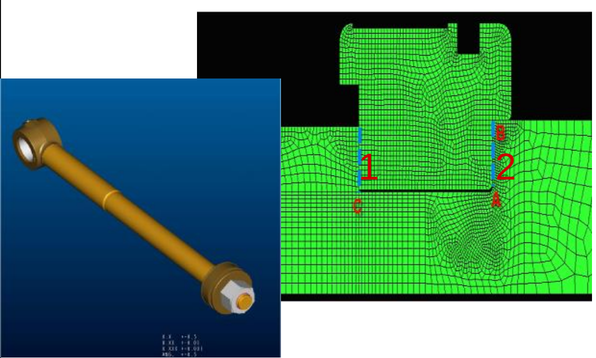



# Application VI: FE system reliability analysis

* Lee, Y.-J., J. Song, and E.J. Tuegel (2008). Finite element system reliability analysis of a wing torque box. *Proc. 10th AIAA NDA*, April 7-10, Schaumburg, IL.

- FE reliability analysis: component vs. system
  - System-level risk is a logical function of multiple component events characterized by **failure modes**, **locations** and **load cases**
  - Using MSR methods, the **system-level risk and parameter sensitivities** are estimated based on the results of FE "component" reliability analysis.

1. Mechanical structures (single-nut piston)

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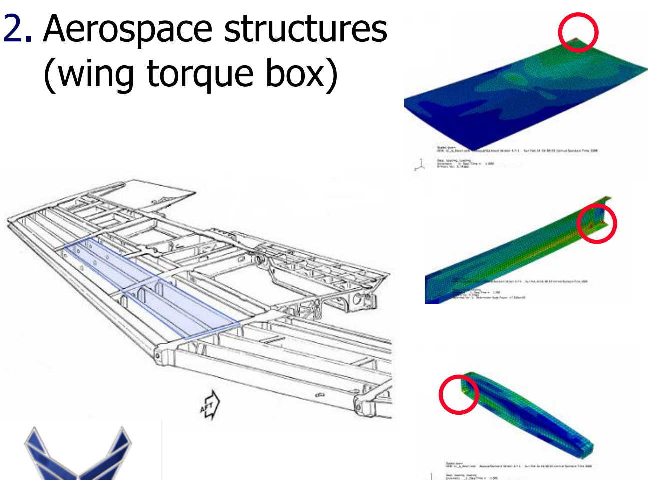
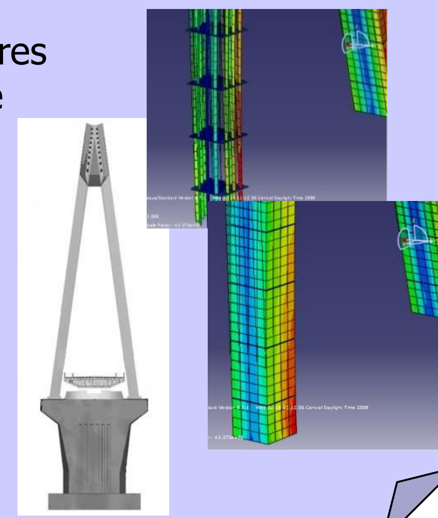
2. Aerospace structures (wing torque box)

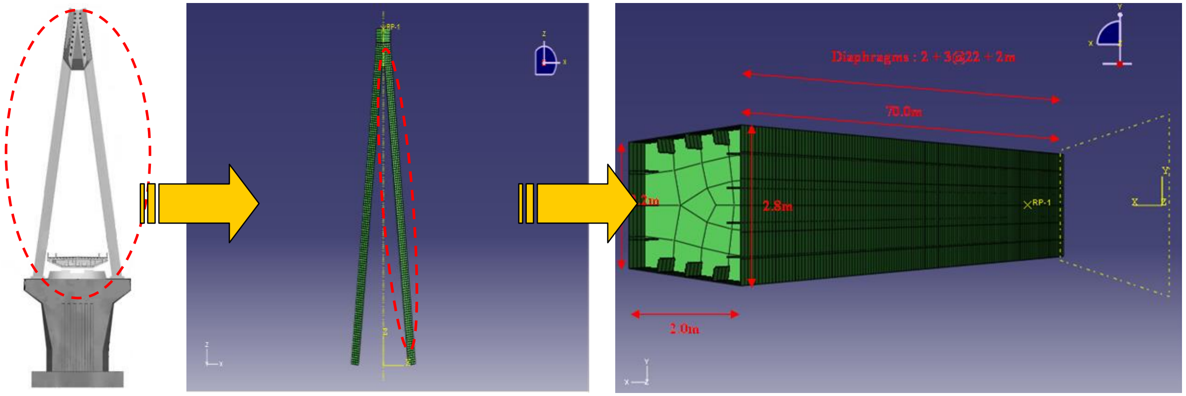


U.S. AIR FORCE

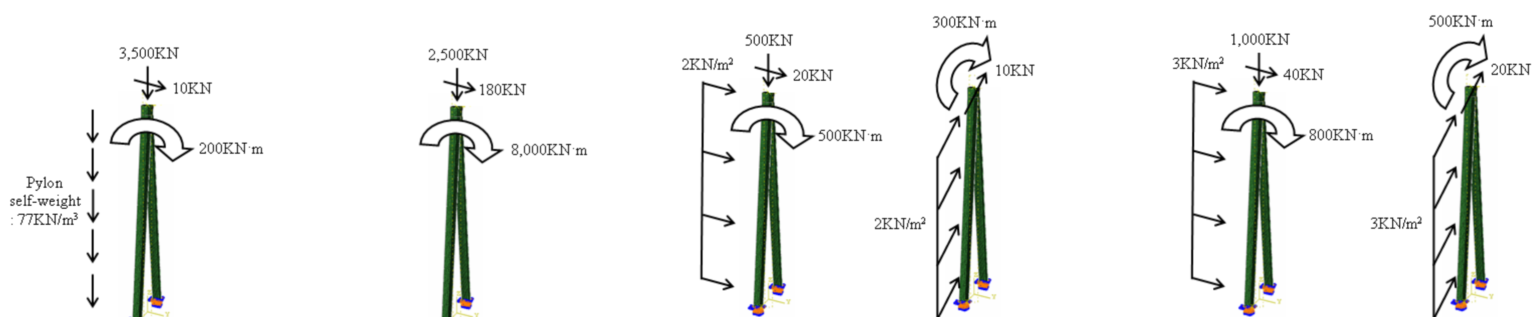
3. Civil structures (Bridge pylon)



# Example: FE-SRA of bridge pylon system




- Bridge pylon system
  - Consists of 2 arms – each has 13 stiffeners and 23 diaphragms
  - Yielding failure considered in this example
  - Uncertainties in <u>Young's modulus</u>, <u>yield strength</u> and <u>scale factors of load cases</u> (dead, live, in-service wind and out-of-service wind loads) considered
  - Two load combinations considered: LC1 = D+L+Wi, LC2 = D+Wo



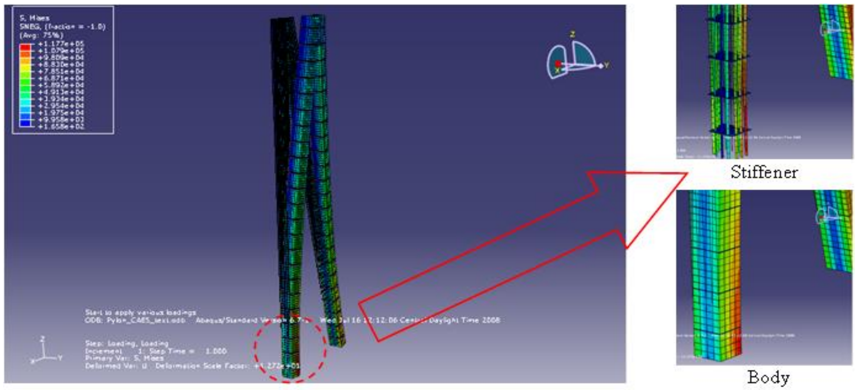

(i) Dead load(D)

(ii) Live load(L)

(iii) In-service wind load
(Wi, In-plane & Out-of-plane)

(iv) Out-of-service wind load
(Wo, In-plane & Out-of-plane)

# FE component reliability analysis




- Identification of significant components
  - Deterministic FE analysis using the mean values of random variables → identify "hot spots" for each load combination
  - FE reliability analysis for identified "hot spots" by FORM → neglect if (1) Pf is too low or (2) highly correlated with other (more likely) component events

| | Component event | Failure probability ($\times 10^{-4}$) |
|---|---|---|
| **Components identified** | $E_1$ (LC1; 1st spot on right body) | 1.295 |
| | $E_2$ (LC1; 1st spot on left body) | 1.295 |
| | $E_3$ (LC1; 1st spot on right stiffener) | 0.606 |
| | $E_4$ (LC1; 1st spot on left stiffener) | 0.606 |
| | $E_5$ (LC2; 1st spot on right body) | 6.996 |
| | $E_6$ (LC2; 1st spot on left body) | 6.996 |
| | $E_7$ (LC2; 1st spot on right stiffener) | 2.445 |
| | $E_8$ (LC2; 1st spot on left stiffener) | 2.445 |
| **Truncated due to high correlation** | $E_9$ (LC1; 2nd spot on right body) | 0.430 |
| | $E_{10}$ (LC1; 2nd spot on left body) | 0.430 |
| | $E_{11}$ (LC2; 2nd spot on right body) | 4.044 |
| | $E_{12}$ (LC2; 2nd spot on left body) | 4.044 |

- Correlation between components
  - Correlation b/w components are computed by

$$\rho_{ij} = \hat{\alpha}_i^{T} \hat{\alpha}_j$$

| Correlation | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | $E_7$ | $E_8$ |
|---|---|---|---|---|---|---|---|---|
| $E_1$ | 1 | 0.814 | 0.708 | 0.744 | 0.646 | 0.502 | 0.448 | 0.476 |
| $E_2$ | | 1 | 0.744 | 0.708 | 0.502 | 0.646 | 0.476 | 0.448 |
| $E_3$ | | | 1 | 0.683 | 0.423 | 0.451 | 0.680 | 0.429 |
| $E_4$ | | | | 1 | 0.451 | 0.423 | 0.429 | 0.680 |
| $E_5$ | | | | | 1 | 0.887 | 0.820 | 0.842 |
| $E_6$ | | | | | | 1 | 0.842 | 0.820 |
| $E_7$ | | | Symmetric | | | | 1 | 0.801 |
| $E_8$ | | | | | | | | 1 |

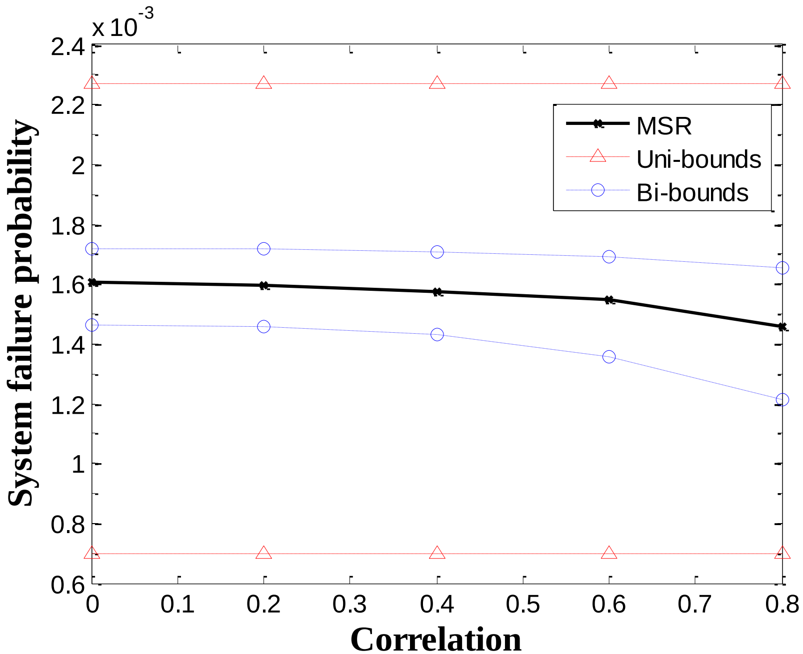
# FE system reliability analysis by MSR

- FE-SRA by MSR
  - Probability of most dominant component: $6.996\text{x}10^{-4}$ vs. system failure probability $1.550\text{x}10^{-3}$ → component reliability analysis may underestimate the risk significantly
  - Using component failure probability and sensitivity, the MSR method computes the system level parameter sensitivity
  - Can analyze other system events just by replacing event vector **c**

$$P(E_{sys}) = P\left(\bigcup_{i=1}^{8} E_i\right) \cong P\left[\bigcup_{i=1}^{8} \beta_i - Z_i \le 0\right]$$

$$= \int_{\Omega} \varphi_N(\mathbf{z};\mathbf{R})\, d\mathbf{z}$$

$$= \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}} \mathbf{p}(\mathbf{s})\, f_{\mathrm{S}}(\mathbf{s})\, d\mathbf{s}$$




| Random variables | | $\delta_i = \frac{\partial P_1}{\partial \mu_i}\sigma_i$ | $\eta_i = \frac{\partial P_1}{\partial \sigma_i}\sigma_i$ |
|---|---|---|---|
| Young's modulus | Diaphragm (Left) | −0.0004 | 0 |
| | Diaphragm (Right) | −0.0003 | 0 |
| | Body (Left) | −0.6480 | 1.8018 |
| | Body (Right) | −0.6624 | 1.8159 |
| | Stiffener (Left) | 0.3463 | 1.3114 |
| | Stiffener (Right) | 0.3558 | 1.3198 |
| Load scale factor | Dead load | 0.5130 | 0.0171 |
| | Live load | 2.1175 | 1.8348 |
| | In-service wind load (In-plane) | 2.9923 | 14.873 |
| | In-service wind load (Out-of-plane) | 0.4900 | 1.9121 |
| | Out-of-service wind load (In-plane) | 13.989 | 66.648 |
| | Out-of-service wind load (Out-of-plane) | 2.3301 | 8.599 |
| Yield strength | Body (Left) | −8.0319 | 8.8381 |
| | Stiffener (Left) | −2.5299 | 2.925 |
| | Body (Right) | −8.0583 | 8.8729 |
| | Stiffener (Right) | −2.5132 | 2.9001 |

# App. VII: Reliability-Based Design Optimization

## >> Deterministic Optimization

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

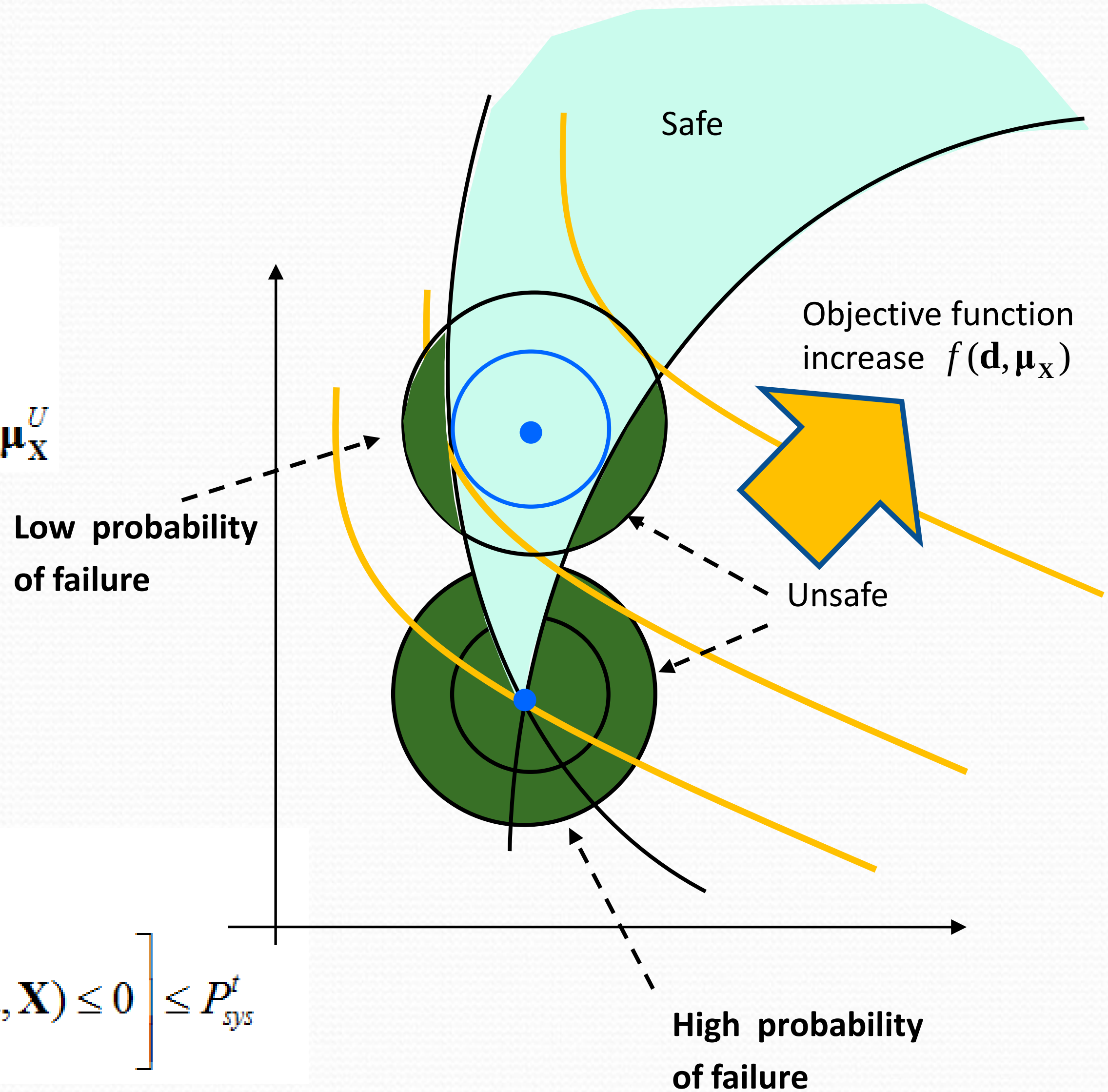
$$s.t. \quad g_i(\mathbf{d},\mathbf{X}) > 0$$

$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \qquad \boldsymbol{\mu}_\mathbf{X}^L \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}_\mathbf{X}^U$$

## >> Reliability-Based Design Optimization (RBDO)

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

$$s.t. \quad P_{sys} = P(E_{sys}) = P\left[\bigcup_k \bigcap_{i\in C_k} g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_{sys}^t$$

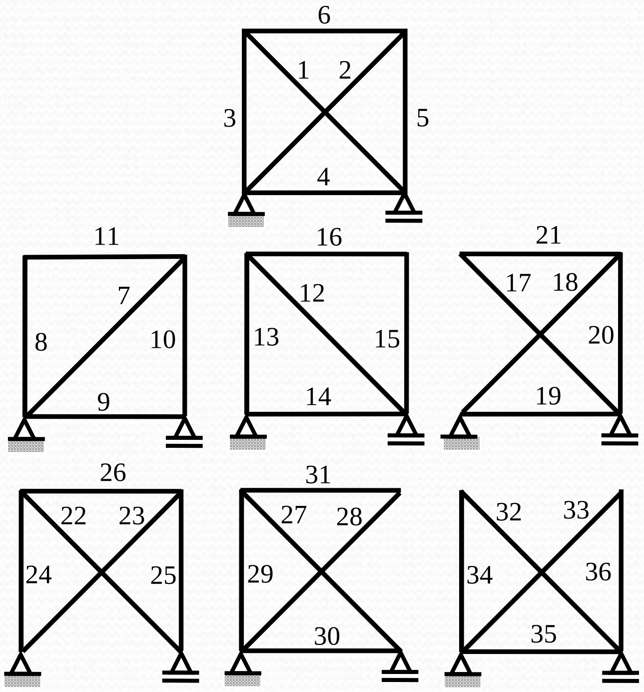


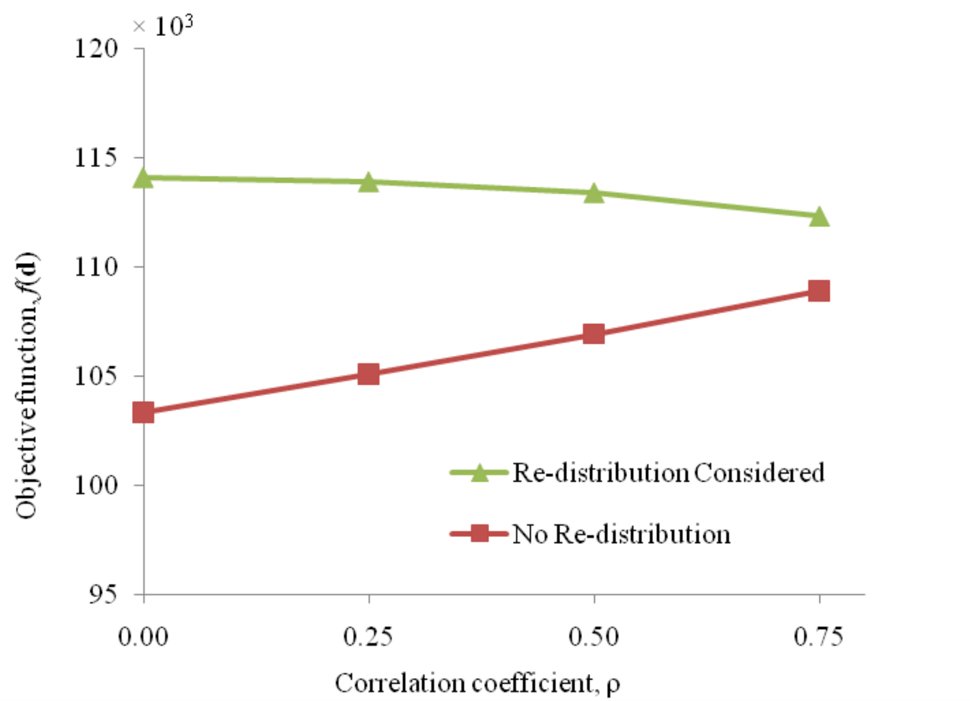
# System RBDO by MSR method

**RBDO of Truss system**: Minimize the cross section areas under target failure probability of system collapse

Using MSR method, we can consider

- Effects of **load re-distributions** (sequential failures)
- Effects of **correlation between components**

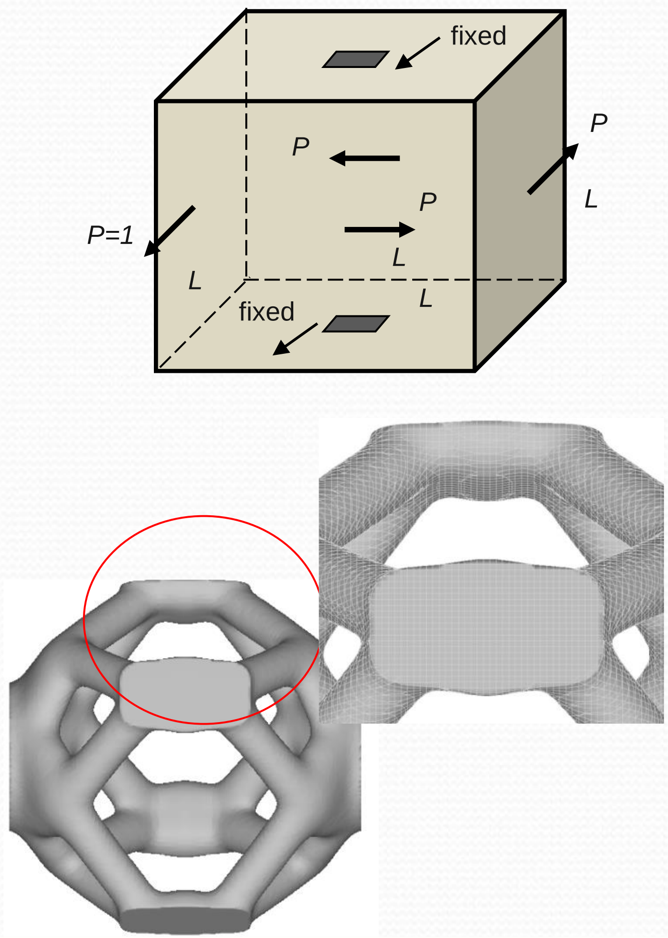


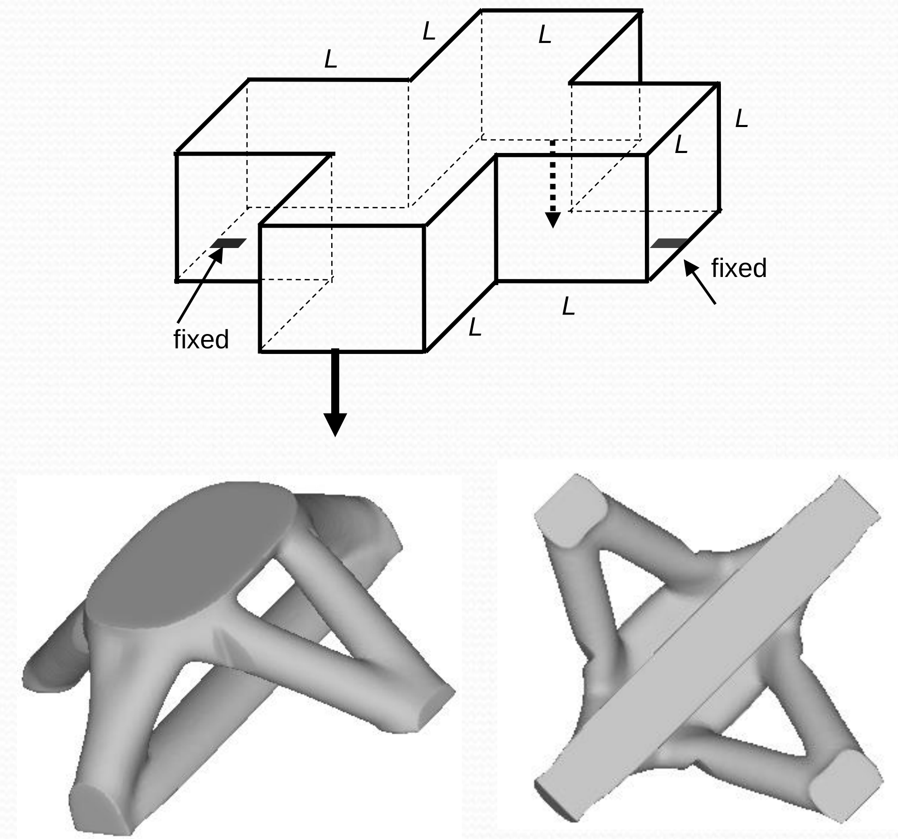




Nguyen, T.H., J. Song, and G.H. Paulino (2010). "Single-loop system reliability-based design optimization using matrix-based system reliability method: theory and applications," *J. of Mechanical Design*, ASME, Vol. 132, 011005-1~11.

# System RBTO by MSR method

**RBTO of 2D or 3D continuum**: Minimize the volume or compliance under target failure probability of *system* failure




Nguyen, T.H., Paulino, G.H., and Song, J., and Le, C.H., "A Computational Paradigm for Multiresolution Topology Optimization (MTOP)," *Structural and Multidisciplinary Optimization,* vol. 41(**4**), 525-539.

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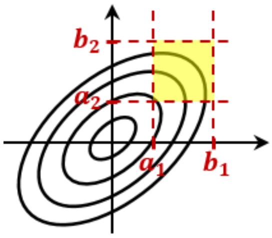
# 457.646 Topics in Structural Reliability
## In-Class Material: Class 18

### ◎ Multivariate normal integrals




$\mathbf{Z} \sim N(\mathbf{0};\mathbf{R})$

$$F(\mathbf{a},\mathbf{b};\mathbf{R}) = \int_{a_1}^{b_1} \cdots \int_{a_m}^{b_m} \qquad d\mathbf{z}$$

If $a_i = -\infty$, $i = 1,\cdots,m$, it becomes Joint of $\mathbf{Z} \sim N(0;\mathrm{R})$

$$\Phi_m(b_1,\cdots,b_m;\mathbf{R}) = \int_{-\infty}^{b_1} \cdots \int_{-\infty}^{b_m} \qquad d\mathbf{z}$$

I) **Ditlevsen & Madsen (1996)**

$$m = 2: \; \Phi_2(b_1,b_2;\rho_{12}) = \qquad\qquad + \int_0^{\rho_{12}} \varphi_2(b_1,b_2; \quad)d\rho$$

________ assumption     error by ________ assumption

Note: double-fold integral involving $(-\infty, b_i)$ $\Rightarrow$ single-fold integral in $(0,\rho_{12})$

Note: $\rho_{12} > 0$ : s.i assumption under/overestimate

$\rho_{12} < 0$ : s.i assumption under/overestimate

※ $m = 3$ Song & ADK (2005) double-fold integral

II) **Sequentially Conditioned Importance Sampling (SCIS)**

**(Ambartzumian et al. 1998)**

~sequentially sampling based on conditional PDF

given sampled value

~"scis.m" (developed by Prof. Young Joo Lee at UNIST
available at http://systemreliability.wordpress.com/software/

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### Ⅲ) Product of Conditional Marginals (Pandey & Sarkar 2002)

$$\Phi_m(\mathbf{b};\mathbf{R}) \cong \prod_{k=1}^{m} \Phi\left(\frac{b_k - \mu_{k|k-1}}{\sigma_{k|k-1}}\right)$$

→ reasonable accuracy & very efficient

→ parallel or series

→ error↑ as $m$↑

→ Improved PCM (Yuan & Pandey 2006)

### Ⅳ) Sequential Compounding Method (Kang & Song 2010)

$$\left\{(Z_1 < -\beta_1) \bigcup (Z_2 < -\beta_2)\right\} \bigcap (Z_3 < -\beta_3)$$

$$\underbrace{(Z_1 < -\beta_1) \cup (Z_2 < -\beta_2)}_{Z_A < -\beta_A,\ \rho_{A3}} \qquad \underbrace{(Z_3 < -\beta_3)}_{Z_B < -\beta_B}$$

→ applicable to general system

→ efficient and accurate

→ handle large $m$

→ when the same component event appears multiple times → difficult

→ parameter sensitivity of system reliability using SCM (Chun, Song, and Paulino, 2015, *Structural Safety*)

### Ⅴ) Matrix-based System Reliability (MSR) Method (Kang & Song 2008) (Kang et al. 2012)

### Ⅵ) Method by Genz (1992) http://www.math.wsu.edu/faculty/genz/homepage

Transformations to uniform hypercube

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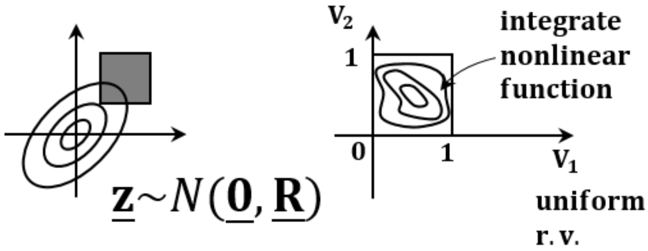


→ Parallel system

→ Very accurate & efficient even for large-size system

→ Integration by qusai-MCS

→ `mvncdf.m` in Matlab

Genz, A., and Bretz, F. (2009) *Computation of Multivariate Normal and t Probabilities, Lecture Notes in Statistics*, Springer-Verlag, NY.