⇓

superfical relative velocity : $\underline{U_{rs} = \alpha_c(u - v)}$

flowing or present in the given cross-sectional area.

↓

particle force term needs to be considered additionally (empirically).

$P_1 - P_2 = \underline{\Delta P'}$ ← additional pressure difference due to particle collision.

∴ Net pressure force, $F_p = -A\Delta P - A\Delta P'$

Fluid force by drag, $F_d = nV\left[V_d \frac{\partial p}{\partial x} + V_d \frac{\partial p'}{\partial x}\right]$

$\rightarrow F_p + F_d = -V\alpha_c \frac{\partial p}{\partial x} - V\alpha_c \frac{\partial p'}{\partial x}$.

↑ fluid

↑ particles (empirical relation)

By Ergun (1952), experiments of packed bed.

$$\frac{\Delta P'}{\Delta x} = 150 \frac{\alpha_d^2 \mu_c (u-v)}{\alpha_c^2 D^2} + 1.75 \frac{\alpha_d \rho_c (u-v)|u-v|}{\alpha_c D}$$

↓ pressure loss due to viscous drag.

↓ " due to inertial drag.

. mtm conservation for the dispersed phase

: change rate of mtm in CV

= net mtm flux + (mtm coupling)

$$\frac{\partial}{\partial t}(\alpha_d \rho_d v) + \frac{\partial}{\partial x}(\alpha_d \rho_d v^2) = -\alpha_d \frac{\partial P}{\partial x} + \beta_V (u-v) - \frac{\tau_w}{R_H} - \frac{\partial}{\partial x}\left[\sum_k \bar{\rho}_{dk} (\delta v_k)^2\right] \quad (1D)$$

$$\sum_k \bar{\rho}_{dk} v_k^2 = \sum_k \bar{\rho}_{dk} (\bar{v} + \delta v_k)^2 = \bar{\rho}_d \bar{v}^2 + \sum_k \bar{\rho}_{dk} (\delta v_k)^2$$

mass averaged

analogous to the Reynolds normal stress.

As the particles impact and rebound from the wall, there is a mtm loss that is represented by $\boxed{\tau_w}$ and must be estimated by modeling.

or in multi-dimensional case,

$$\frac{\partial}{\partial t}(\alpha_d \rho_d v_i) + \frac{\partial}{\partial x_j}(\alpha_d \rho_d v_j v_i) = -\alpha_d \frac{\partial p}{\partial x_i} + \beta_V (u_i - v_i)$$

$$+ \alpha_d \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial}{\partial x_j}\left[\underline{\sum_k \bar{\rho}_{dk} \delta v_{ik} \delta v_{jk}} \right]$$

Reynolds stress from the

fluctuations of the velocity
against the mass-averaged
velocity.

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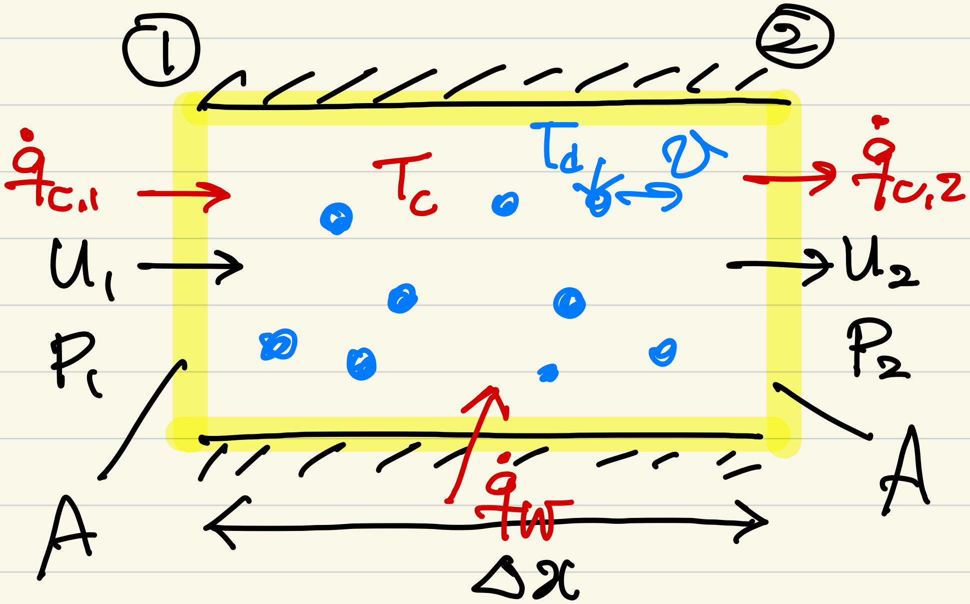
(30)

- Boundary condition at the phase interface.
  - the gov. eq can be applied to each phase up to the interface.
  - Across the interface ? a particular form of the balance eq. is needed.
    - mass : phase changes are pure exchange of mass between phases. → no capacity of mass at the interface.
    - mtm : balance between the mtm flux from the carrier fluid and interfacial tension. (deformation and shape of fluid particle, in particular)

· energy : change rate of surface energy

~ energy transfer from the bulk at each side and the work done by the surface tension.

- Energy Conservation (1st Law of Thermodynamics)



change rate of the energy contained in the CV

= net energy flux

+ heat tr. to the CV

− work done by the flow

− energy release from particles.

$$V\Delta_t\left(\alpha_c\rho_c\left(i_c+\frac{1}{2}u^2\right)\right) = -\Delta\left(\alpha_c\rho_c uA\left(i_c+\frac{1}{2}u^2\right)\right) + \dot{Q} - \dot{W} \leftarrow$$

$$- N\dot{m}\left(i_s+\frac{1}{2}v^2\right)$$

specific internal energy.

internal energy of the fluid on the particle surface.

wall.

dispersed phase

$$\dot{Q} = \dot{Q}_c + \dot{Q}_w + \dot{Q}_d$$

perimeter

$$= -\Delta(\dot{q}_c A) - \dot{q}_w \underline{P\Delta x}. + N\cdot Nu\cdot\pi k_c D(T_d - T_c)$$

surf. pressure

$$\dot{W} = \dot{W}_f + \dot{W}_d.$$

$$= \underbrace{\Delta(\alpha_c PAu) + \Delta(\alpha_d PAv)}_{\text{work with flow across ① and ②.}} + \underbrace{N\dot{m}\frac{p_s}{\rho_s}}_{\text{work with mass flux from particle}} - \underbrace{p_s N\dot{V}_d}_{\text{work by particle vol. change}}$$

$$- \underbrace{N\cdot F_D\cdot v}_{\text{work by particle}} - \underline{\alpha_c\rho_c g V u}$$

drag

$$F_D = 3\pi\mu_c D f (u-v) - V_d \frac{\Delta P}{\Delta x}$$

surface

work done against the gravity

$$\therefore V\Delta_t\left(\alpha_c\rho_c\left(i_c+\frac{1}{2}u^2\right)\right)+\Delta\left(\alpha_c\rho_c u A\left(i_c+\frac{1}{2}u^2\right)\right)+N\dot{m}\left(i_s+\frac{1}{2}v^2\right)=$$

$$-\Delta(\alpha_c P A u)-\Delta(\alpha_d P A v)-N\dot{m}\frac{P_s}{\rho_s}+P_s N\dot{V}_d$$

$$-N\left[3\pi\mu_c D f(u-v)v - vV_d\frac{\Delta P}{\Delta x}\right]$$

$$+\alpha_c\rho_c g V u-\Delta(\dot{q}_c A)-\dot{q}_{w}P\Delta x+N\cdot Nu\,\pi k_c D(T_d-T_c)$$

🟨 $= \Delta\left[\alpha_c\rho_c A u\left(h_c+\frac{1}{2}u^2\right)\right]$. $h_c = i_c + \frac{P}{\rho_c}$: enthalpy.

🟩 $= -N_{mass}\left(h_s+\frac{1}{2}v^2\right)$, $h_s = i_s + \frac{P_s}{\rho_s}$

🟥 $= -P\Delta(\alpha_d v A)$.

$$\downarrow \ \underline{V}\Delta_t\left(\alpha_c\rho_c\left(i_c+\frac{1}{2}u^2\right)\right)+\Delta\left(\alpha_c\rho_c A u\left(h_c+\frac{1}{2}u^2\right)\right)-S_{mass}\left(h_s+\frac{1}{2}v^2\right)$$

$$= -P\Delta(\alpha_d A u)+\rho_s N\dot{V}_d-\beta_V V(u-v)v+\alpha_c\rho_c g V u$$

$$-\Delta(\dot{q}_c A)-\dot{q}_w P\Delta x+\underline{\beta_T}V(T_d-T_c)$$

↳ parameter for energy coupling.

↓

$$\frac{\partial}{\partial t}\left(\alpha_c\rho_c\left(i_c+\frac{1}{2}u^2\right)\right)+\frac{\partial}{\partial x}\left(\alpha_c\rho_c u\left(h_c+\frac{1}{2}u^2\right)\right)-S_{mass}\left(h_s+\frac{1}{2}v^2\right)$$

$$= -p\frac{\partial}{\partial x}(\alpha_d v)+\rho_s n\dot{V}_d-\underbrace{\beta_V(u-v)v}_{\text{drag}}+\underline{\alpha_c\rho_c g u}\ \text{gravity}$$

$$-\frac{\partial\dot{q}_c}{\partial x}-\frac{\dot{q}_w}{R_H}+\beta_T(T_d-T_c)$$

(1D total energy eq.)

Thermal energy eq = $\underline{\text{total energy eq}}$ − ($\underline{\text{mtm eq.}}$)×u

$$\frac{\partial}{\partial t}(\alpha_c \rho_c i_c) + \frac{\partial}{\partial x}(\alpha_c \rho_c u h_c) = \alpha_c u \frac{\partial P}{\partial x} + S_{mass}\left(h_s + \frac{|u-v|^2}{2}\right)$$

$$- p \frac{\partial}{\partial x}(\alpha_d v) + \rho_s n \dot{V}_d + \beta_V |u-v|^2 + \frac{1}{R_H}(u \tau_w - \dot{q}_w)$$

$$- \frac{\partial \dot{q}_c}{\partial x} + \beta_T (T_d - T_c)$$

MIDTERM. (Ch.1 – Ch.4)

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