재료의 기계적 거동
(Mechanical Behavior of Materials)

Plastic Deformation of Crystalline Solids

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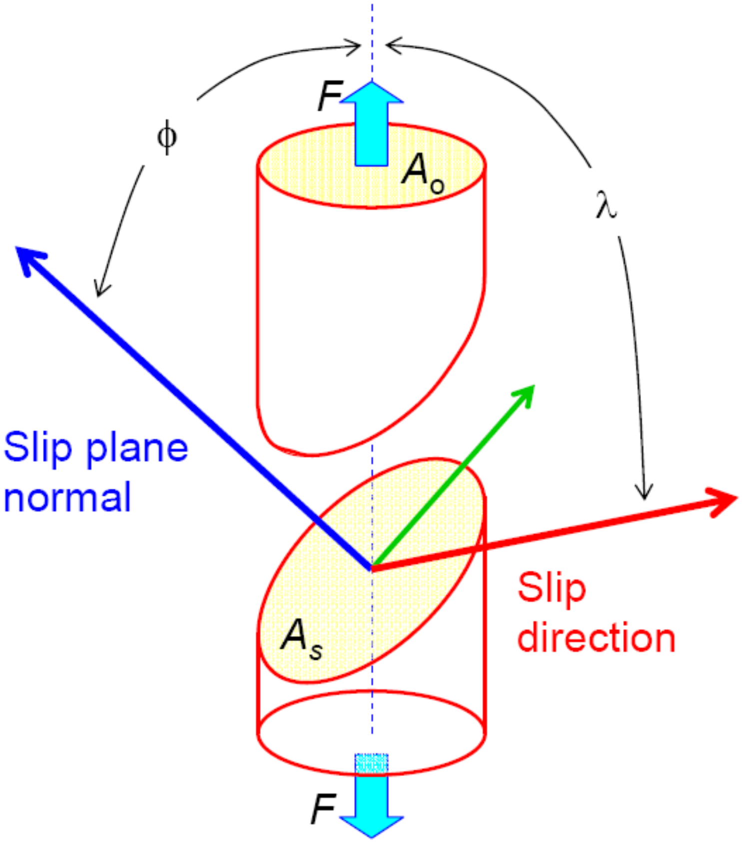
Critical Resolved Shear Stress

- Plastic deformation is initiated at a critical stress the critical resolved shear stress (CRSS).
- The CRSS is the stress at which dislocations begin to move.



Resolved Shear Stress

Talyor factor

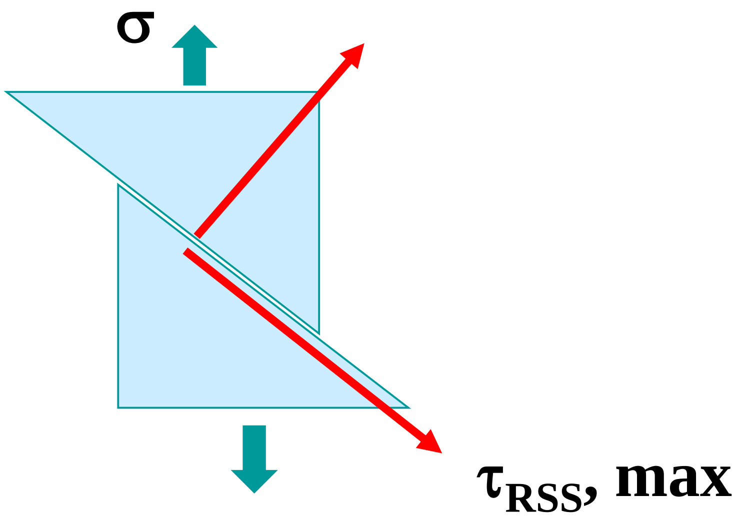
$$\tau_{RSS} = \frac{F}{A_o} \underbrace{\cos\phi \cos\lambda}_{\text{Schmid Factor}} = \sigma / m$$

Plastic flow is initiated when τ_{RSS} reaches a critical value, characteristic of the material, called *critical RSS*, when $m\ \tau_{CRSS} = \sigma_{ys}$ *(Schmid law)*.

Critical Resolved Shear Stress

MAXIMUM Resolved Shear Stress occurs when **$\phi = \lambda = 45^o$** called $\tau_{RSS,max}$. Slip is on the planes 45° from the applied stress.

Then, $\boldsymbol{\tau_{RSS,\ max} = \sigma \cos^2\phi = \sigma /2}$ at $\boldsymbol{\phi = \lambda = 45^o}$.



Slip occurs when 1/m is maximum.

Critical Resolved Shear Stress

Example problem I

Calculate the tensile yield stress that is applied along the $[1\bar{2}0]$ axis of a gold crystal to cause slip on the $(1\bar{1}\bar{1})[0\bar{1}1]$ slip system. The critical resolved shear stress is 10 MPa.

Table 4-4 Room-temperature slip systems and critical resolved shear stress for metal single crystals

Metal	Crystal structure	Purity, %	Slip plane	Slip direction	Critical shear stress, MPa	Ref.
Zn	hcp	99.999	(0001)	$[11\bar{2}0]$	0.18	[a]
Mg	hcp	99.996	(0001)	[1120]	0.77	[b]
Cd	hcp	99.996	(0001)	$[11\bar{2}0]$	0.58	[c]
Ti	hcp	99.99	(1010)	$[11\bar{2}0]$	13.7	[d]
		99.9	(1010)	$[11\bar{2}0]$	90.1	[d]
Ag	fcc	99.99	(111)	[110]	0.48	[e]
		99.97	(111)	[110]	0.73	[e]
		99.93	(111)	[110]	1.3	[e]
Cu	fcc	99.999	(111)	[110]	0.65	[e]
		99.98	(111)	[110]	0.94	[e]
Ni	fcc	99.8	(111)	[110]	5.7	[e]
Fe	bcc	99.96	(110) (112) (123)	[111]	27.5	[f]
Mo	bcc	. . .	(110)	[111]	49.0	[g]

[a] D. C. Jillson, *Trans. AIME*, vol. 188, p. 1129, 1950.

[b] E. C. Burke and W. R. Hibbard, Jr., *Trans. AIME*, vol. 194, p. 295, 1952.

[c] E. Schmid, "International Conference on Physics," vol. 2, Physical Society, London. 1935.

[d] A. T. Churchman, *Proc. R. Soc. London Ser. A*, vol. 226A, p. 216, 1954.

[e] F. D. Rosi, *TRans., AIME*, vol. 200, p. 1009, 1954.

[f] J. J. Cox, R. F. Mehl, and G. T. Horne, *Trans. Am. Soc. Met.*, vol. 49, p. 118, 1957.

[g] R. Maddin and N. K. Chen, *Trans. AIME*, vol. 191, p. 937, 1951.

Example II: FCC Cu with Loading axis [112]

- What is most likely initial slip system?
- If CRSS is 50 MPa, what is the tensile yield stress at which Cu will start to deform plastically?

Slip Plane **n**	**Slip direction** **s**	**n*l** **cosϕ**	**s*l** **cosλ**	**Schmid factor** **cosϕcosλ**	**σ (MPa)**
(111)	$[\bar{1}10]$	$2\sqrt{2}/3$	0	0	Not def.
	$[\bar{1}01]$		$\sqrt{3}/6$	$\sqrt{6}/9$	184
	$[0\bar{1}1]$		$\sqrt{3}/6$	$\sqrt{6}/9$	184
$(\bar{1}11)$	$[110]$	$\sqrt{2}/3$	$\sqrt{3}/3$	$\sqrt{6}/9$	184
	$[101]$		$-\sqrt{3}/2$	$-\sqrt{6}/6$	- 122
	$[0\bar{1}1]$		$\sqrt{3}/6$	$\sqrt{6}/18$	367
$(1\bar{1}1)$	$[110]$	$\sqrt{2}/3$	$\sqrt{3}/3$	$\sqrt{6}/9$	184
	$[\bar{1}01]$		$-\sqrt{3}/6$	$-\sqrt{6}/18$	- 367
	$[011]$		$\sqrt{3}/2$	$\sqrt{6}/6$	122
$(11\bar{1})$ $=(\bar{1}\bar{1}1)$	$[\bar{1}10]$	0	0	0	Not def.
	$[101]$		$\sqrt{3}/2$	0	Not def.
	$[011]$		$\sqrt{3}/2$	0	Not def.

Smallest stress to cause slip (yielding)

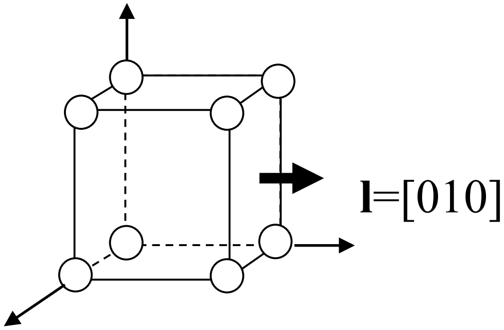
Initial Slip Systems (plane, direction) are then $(\bar{1}11)[101], (1\bar{1}1)[011]$

Example III:

Crystal with simple cubic structure :
slip planes {100} and slip directions <010>

Load is applied along [010].
Determine Schmid factor and what slip occurs.

slip plane **n**	φ, cosφ **n*l**	slip dir. **s**	λ, cosλ **s*l**	1/m cosφcosλ
(100)	90°, 0	[010] [001]	0°, 1 90°, 0	0
(010)	0°, 1	[100] [001]	90°, 0 90°, 0	0
(001)	90°, 0	[100] [010]	90°, 0 0°, 1	0

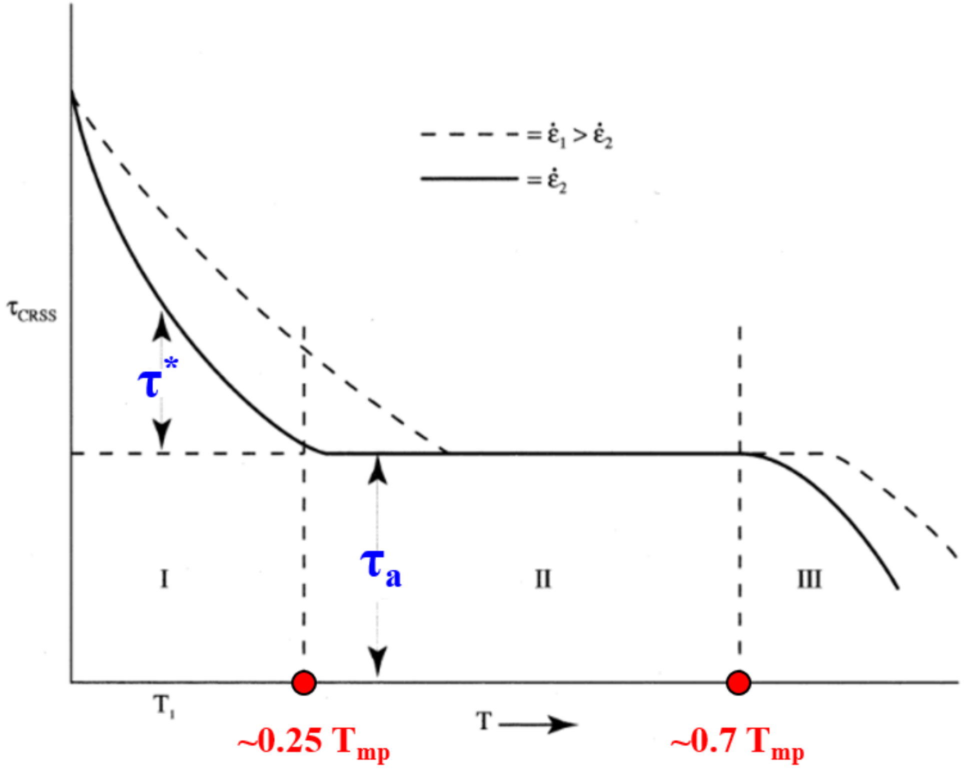


Is there any slip? Why?
If no slip, what must happen finally to material as load is increased?

Factors influencing τ_{CRSS}

- Temperature
  - Increase $T \rightarrow$ decrease τ_{CRSS}.
- Impurity level
  - Decrease impurity content $\rightarrow$ decrease τ_{CRSS}.
- Strain rate
  - Decrease $\dot{\varepsilon} \rightarrow$ decrease τ_{CRSS}.
- Dislocation density
  - Decrease $\rho_{\perp} \rightarrow$ decrease τ_{CRSS}.

τ_{CRSS} as a function of temperature and strain rate



$$\boxed{\tau_{CRSS} = \tau_a + \tau^*}$$

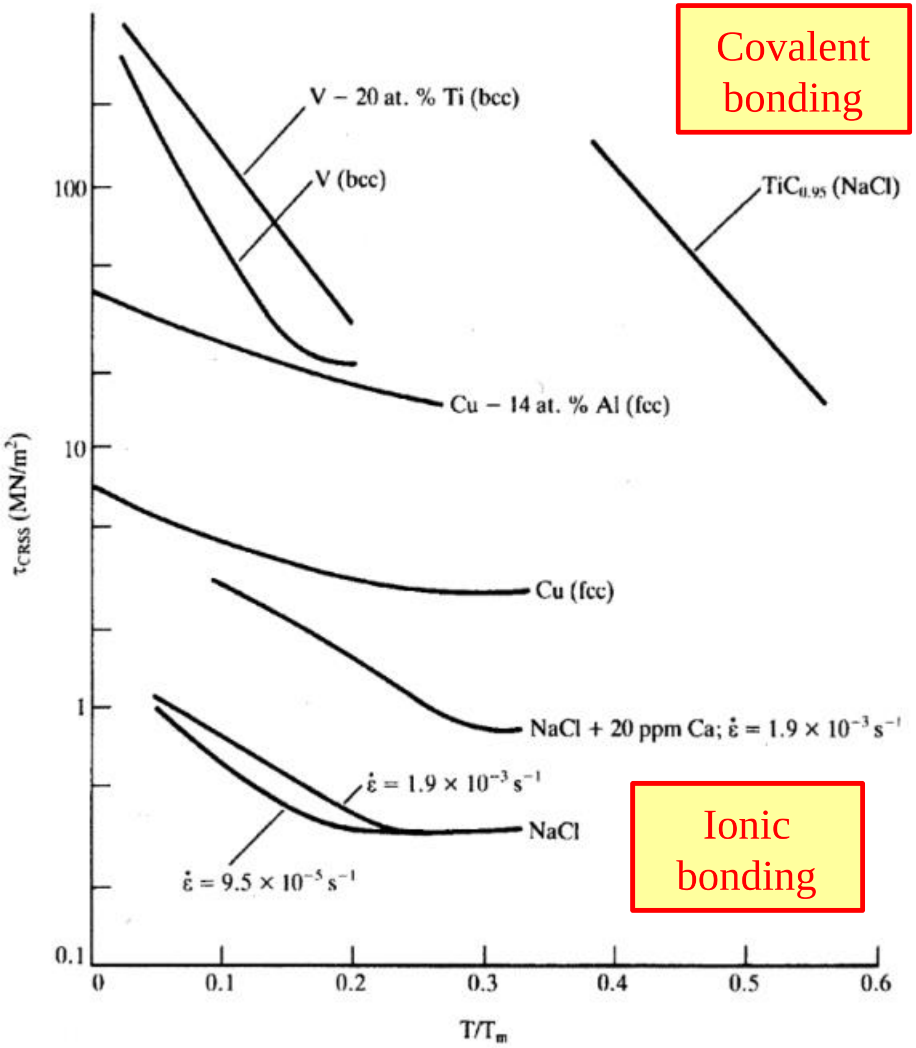
τ_a is the athermal component of flow stress ($\tau_a \neq F(T)$)

τ^* is the thermally dependent component of flow stress ($\tau^* \downarrow$ as $T \uparrow$)

τ_{CRSS} as a function of temperature and strain rate

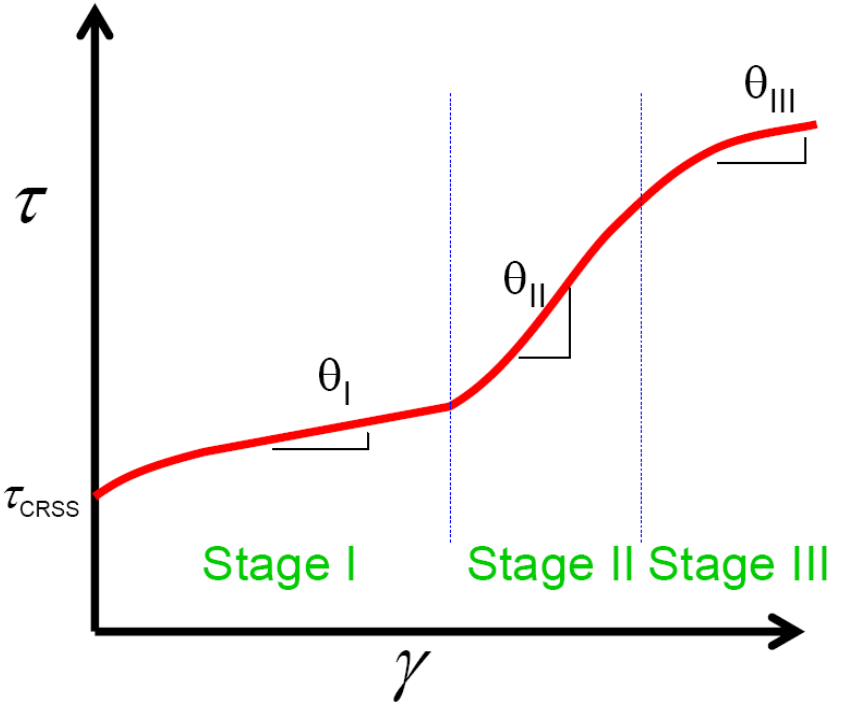
- Region I ($T \leq 0.25T_{mp}$)
  - $\tau_{CRSS} \uparrow$ with $\downarrow T$.
  - $\tau_{CRSS} \uparrow$ with $\uparrow \dot{\varepsilon}$.
  - Athermal component of flow stress is large. Difficult for dislocations to surmount short-range barriers.
- Region II ($0.25T_{mp} < T < 0.7T_{mp}$)
  - In this temperature range, $\tau_a \neq F(T)$ & $\tau^* \cong 0$; $\therefore$ τ_{CRSS} = constant.
  - Temperature is too low for diffusion to influence permanent deformation, thus $\tau \cong \tau_a \neq F(T)$.
- Region III ($T \geq 0.7T_{mp}$)
  - In this temperature range, diffusive processes become important. Diffusion aids dislocation motion (i.e., makes it easier for dislocations to surmount barriers to their motion). Both τ_a & $\tau^* \downarrow$ as $T \uparrow$.
  - $\tau_{CRSS} \downarrow$ with $\uparrow T$.
  - $\tau_{CRSS} \uparrow$ with $\uparrow \dot{\varepsilon}$.

Temperature variation of τ_{CRSS} for various materials



- FCC metals have low resistance to plastic deformation (i.e., they are weaker)
- BCC metals have much higher resistances to plastic deformation (i.e., they are generally stronger)
- T dependence of τ_{CRSS} :
  BCC–high ; FCC –lower
- Impurities ↑τ_{CRSS}, sometimes greatly (principle behind solid solution hardening)
- Ionically bonded materials have low τ_{CRSS} (e.g., NaCl, CsCl, etc.)
- Covalently bonded materials have high τ_{CRSS} (e.g., TiC, diamond, etc.)

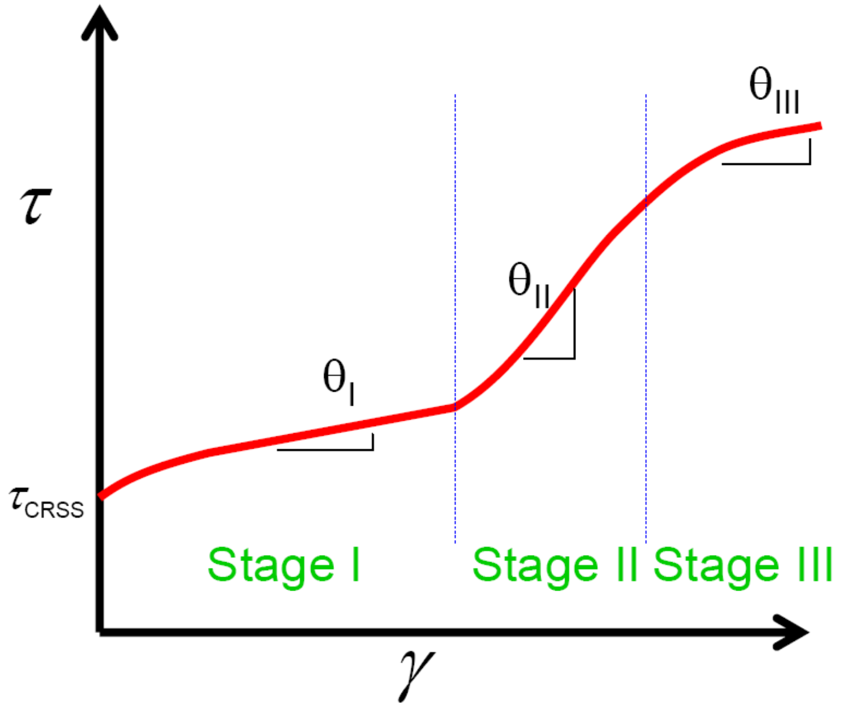
Deformation of single crystals



Stage I:

- After yielding, the shear stress for plastic deformation is essentially constant. There is little or no work hardening.
- This is typical when there is a single slip system operative. Dislocations do not interact much with each other. "Easy glide"
- Active slip system is one with maximum Schmid factor.

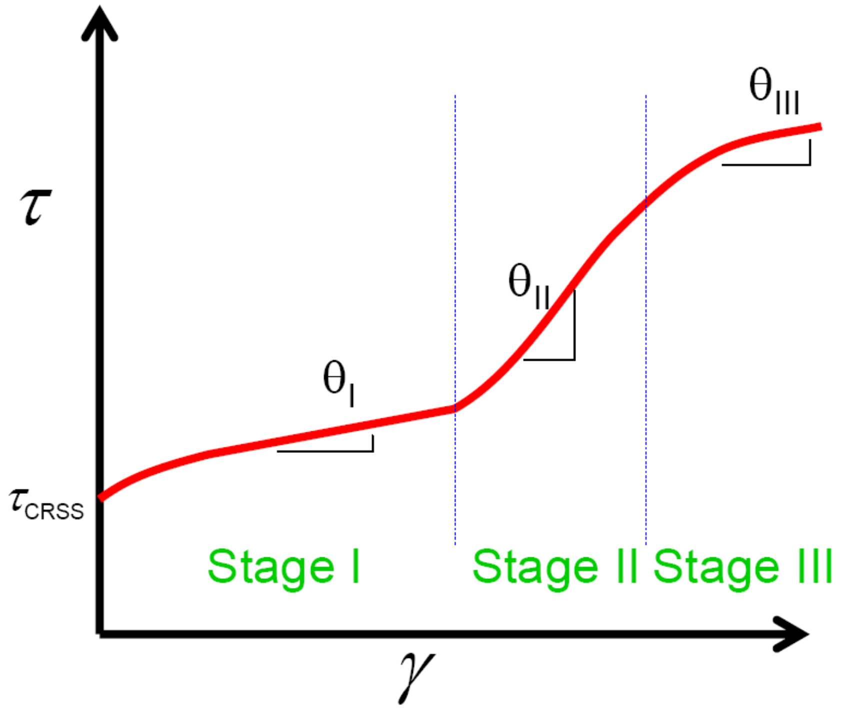
Deformation of single crystals



Stage II:

- The shear stress needed to continue plastic deformation begins to increase in an almost linear fashion. There is extensive work hardening ($\theta \cong G/300$).
- This stage begins when slip is initiated on multiple slip systems.
- Work hardening is due to interactions between dislocations moving on intersecting slip planes.

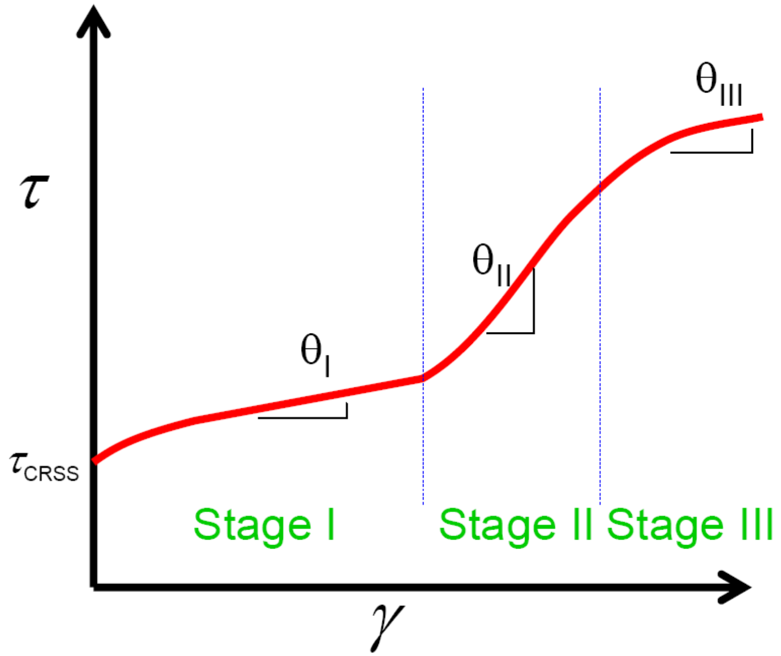
Deformation of single crystals



Stage III:

- There is a decreasing rate of work hardening.
- This decrease is due to an increase in the degree of cross slip resulting in a parabolic shape to the curve.

Deformation of single crystals



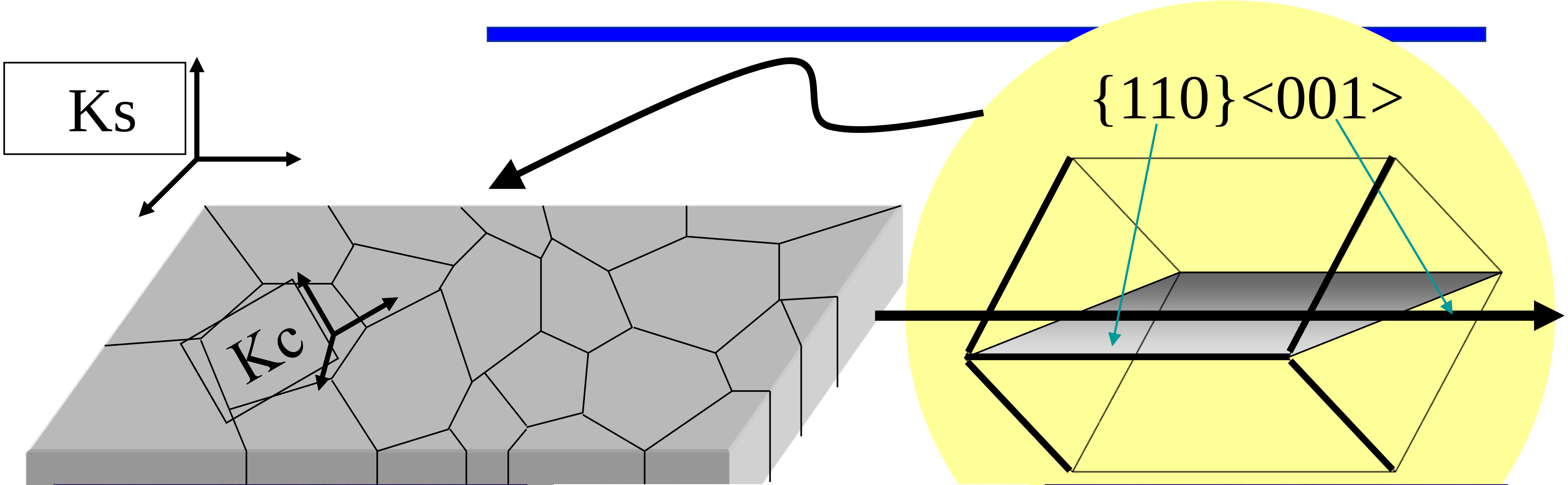
Effect of Temperature:

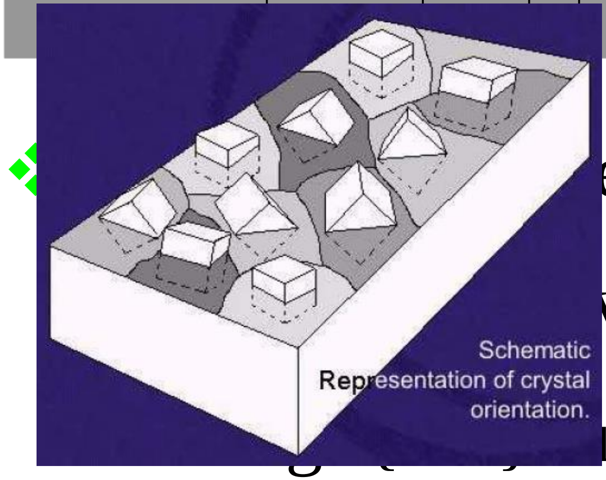
- Increasing T results in a decrease in the extent of Stage I and Stage II.
- Stage I:–Initiation of secondary slip systems is easier.
- Stage II:–Cross slip is easier.

Stacking Fault Energy (SFE) in FCC metals:

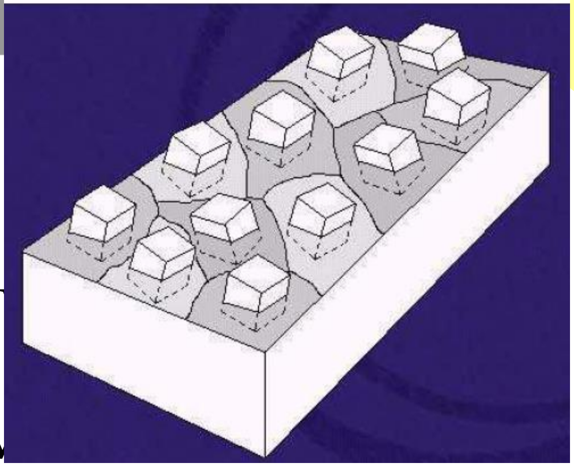
- Decrease SFE, decrease cross slip
- This increases the stress level needed to have a transition from Stage II to Stage III.

Crystal orientation: Miller Indices

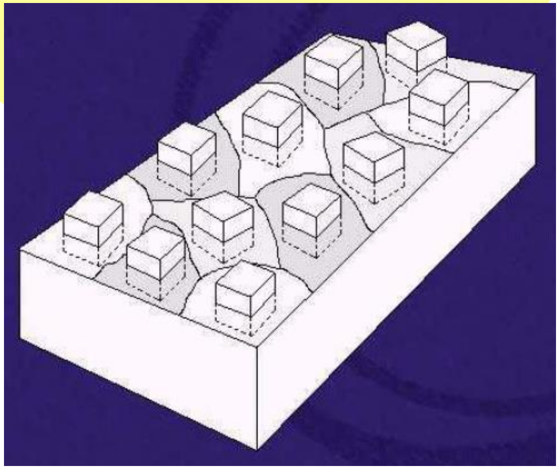




Random orientation

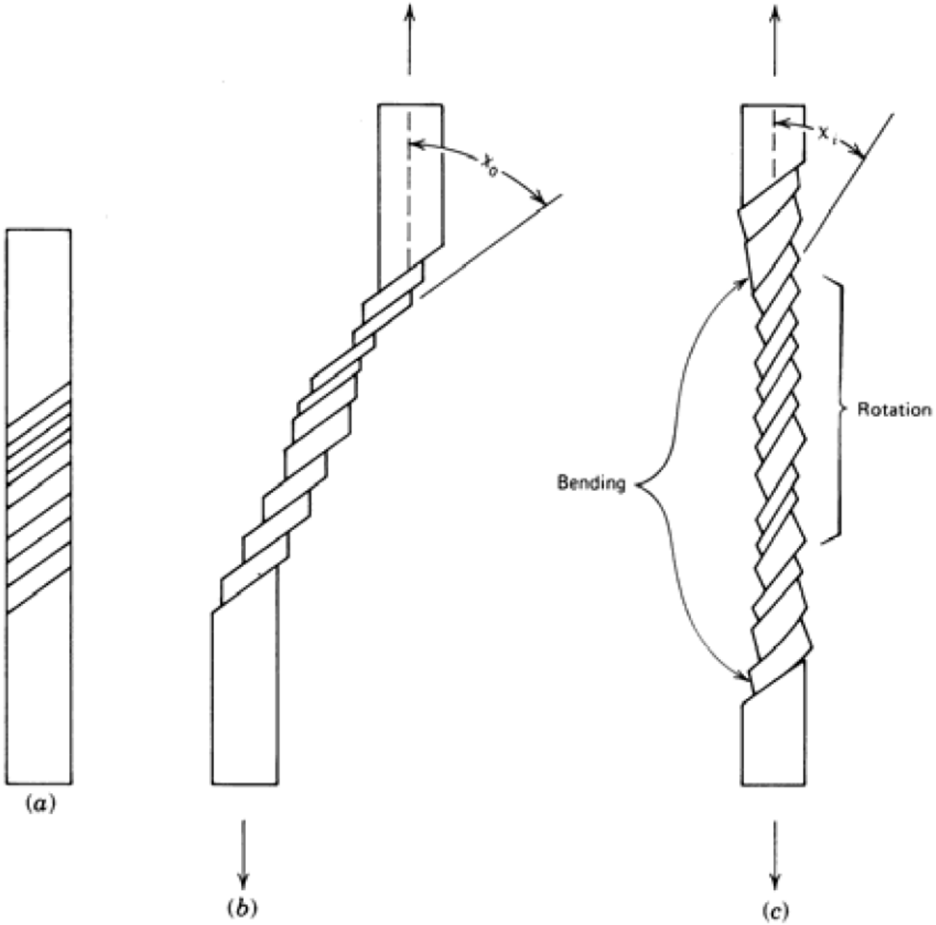


Preferred orientation



Highly oriented - close to a single crystal

What happens to a single crystal when it starts to yield?

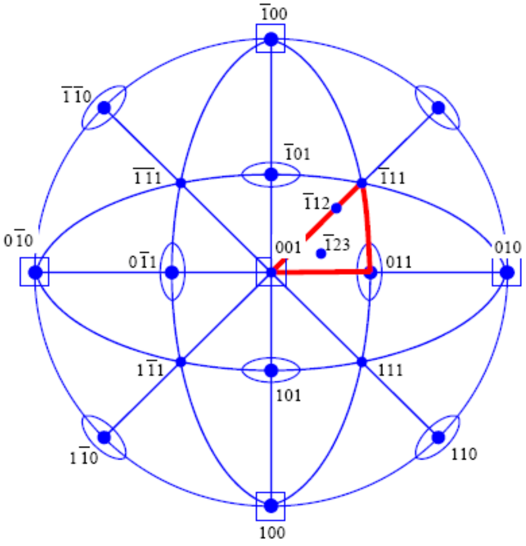


- Consider a single crystal oriented for slip on planes oriented χ degrees from the tensile axis.

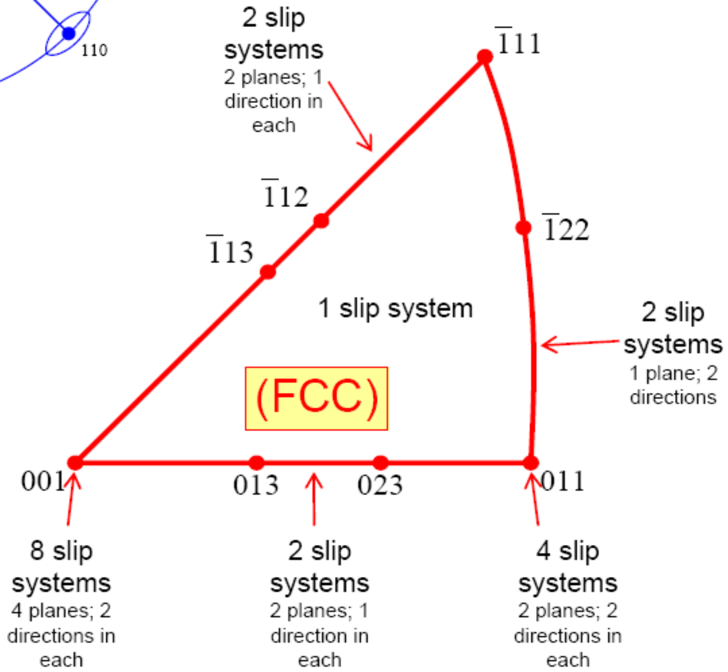
- Ideally, crystal planes will “glide” over one another without changing their relative orientation to the load axis.

- However, during tensile testing, the ends of the tensile bar are constrained. Thus, the crystal planes cannot glide freely. They are forced to rotate towards the tensile axis $(\chi_i<\chi_o)$.

[001] stereographic projection of cubic crystal



Standard Triangle



2 slip systems
2 planes; 1 direction in each

1 slip system

2 slip systems
1 plane; 2 directions

8 slip systems
4 planes; 2 directions in each

2 slip systems
2 planes; 1 direction in each

4 slip systems
2 planes; 2 directions in each

What happens to a single crystal when it starts to yield?

- When (as is usual) testing constrains the upper and lower ends keeping them aligned, the crystal will rotate such that the angle between the stress axis and the slip direction decreases.

- Thus, the Schmid factor changes! This can lead to the initiation of slip on a different system.

Overshoot caused by latent hardening

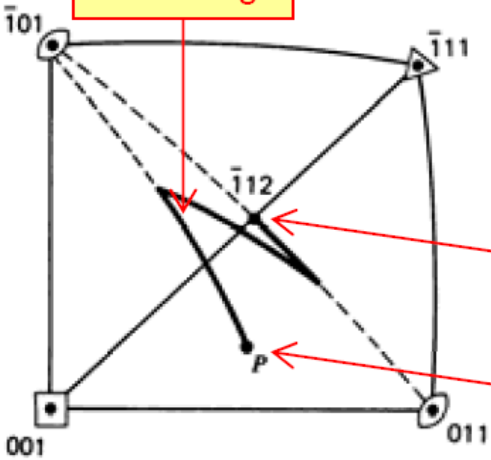


FIGURE 3.9 Lattice rotation of FCC crystal involving "overshoot" of primary and conjugate slip systems.

Final stress axis orientation

$$[\bar{1}12] = \sum [\bar{1}01] + [011]$$

Initial stress axis orientation

- The crystal will continue to rotate with deformation occurring on alternating slip systems.

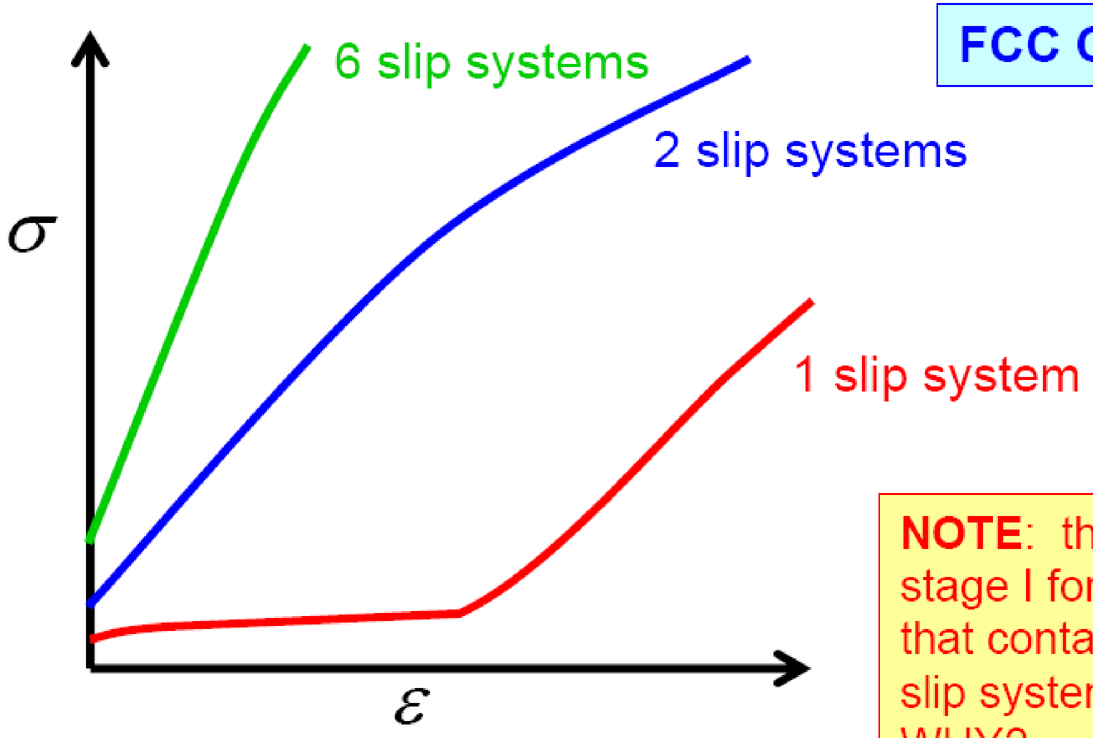
- This will continue until the load axis reaches [-112] where the crystal will neck down until failure without changing orientation.

Influence of stress axis orientation

The stress axis orientation plays a **major role** in the stress-strain behavior of a single crystal

More slip systems means a "harder"material.

FCC Crystal

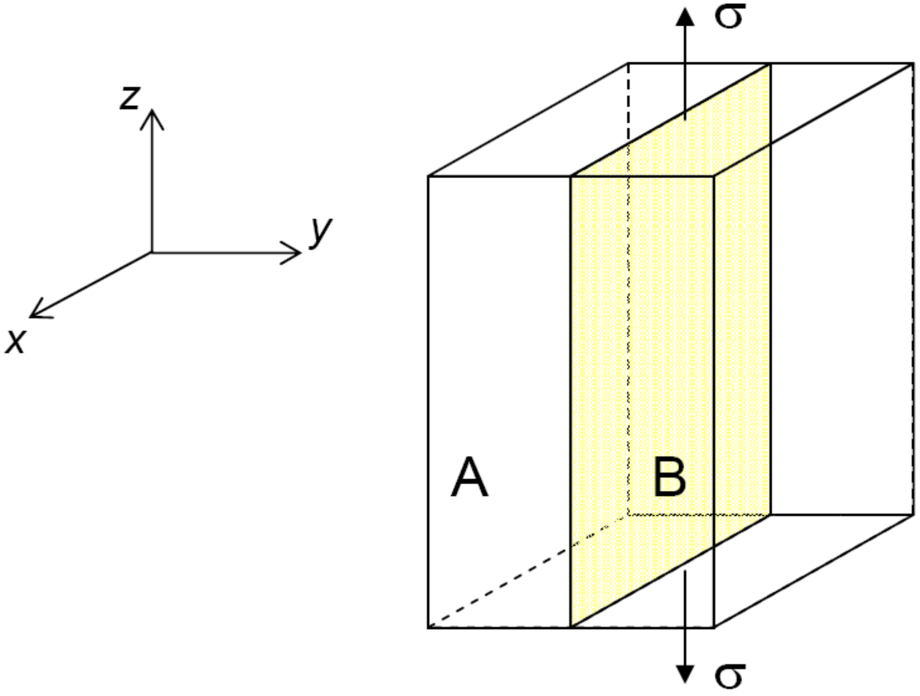


NOTE: there is no stage I for crystals that contain multiple slip systems? WHY?

Implications for polycrystalline materials

•Plastic deformation within an individual grain is constrained by the neighboring grains.

•Since plastic deformation of a single grain is restrained by its neighboring grain, a polycrystalline material will have an intrinsically greater resistance to plastic flow than would a single crystal.



$$\varepsilon_x^A = \varepsilon_x^B$$

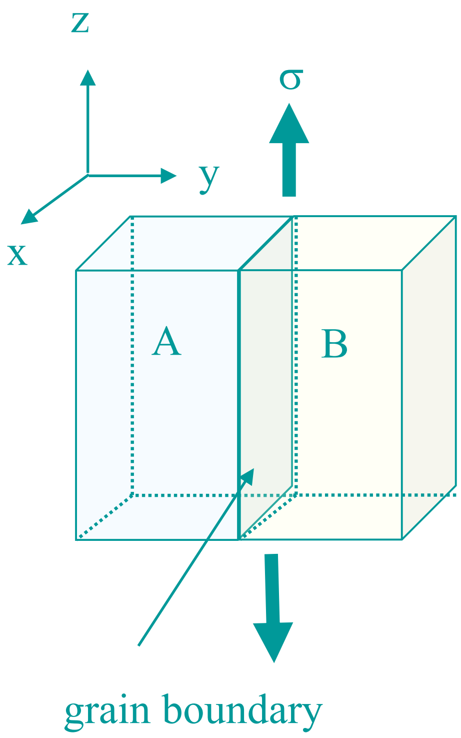
$$\varepsilon_z^A = \varepsilon_z^B$$

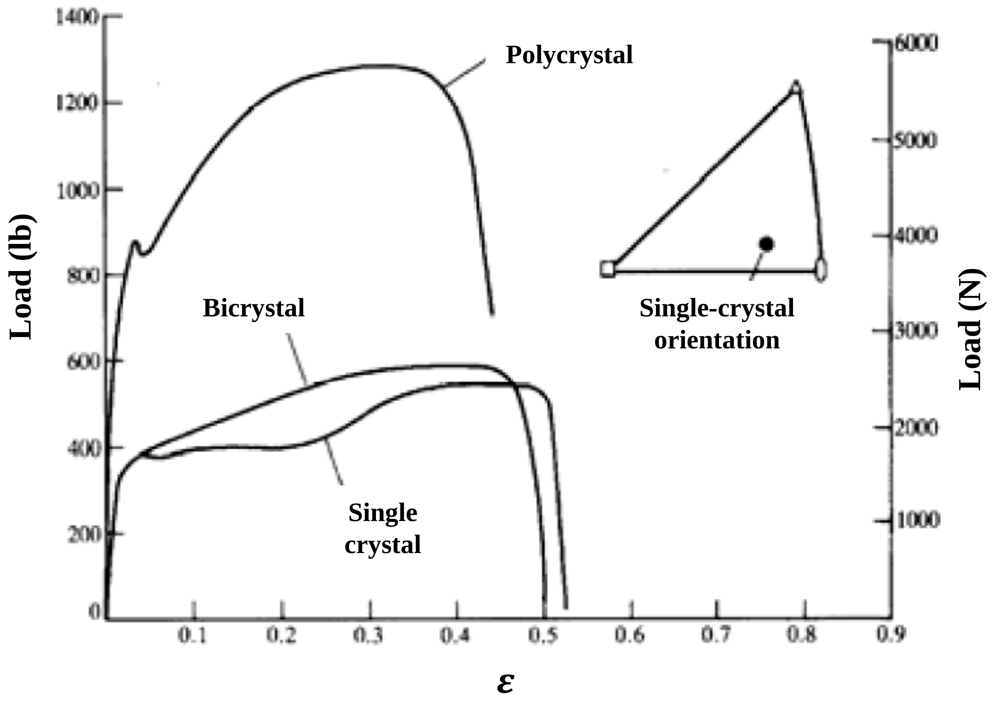
$$\gamma_{xz}^A = \gamma_{xz}^B$$

Required to maintain continuity of the grain boundary

Implications for polycrystalline materials

Because *one grain* has a **larger value of cos ϕ cos λ** [smaller Taylor factor (1/m)], the above constraints restrict the deformation of this more favorably oriented grain and result in a *higher Yield Strength* (greater work-hardening response of the bicrystal.

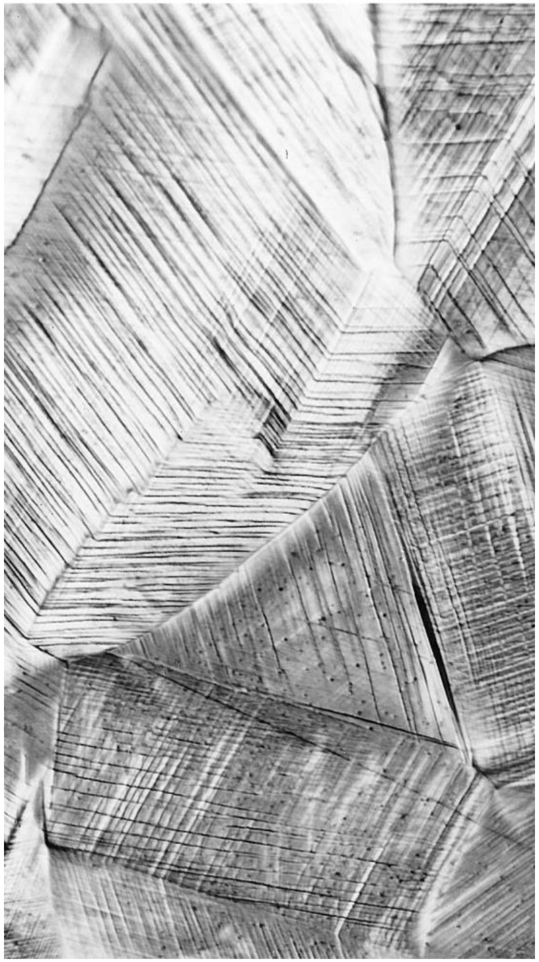




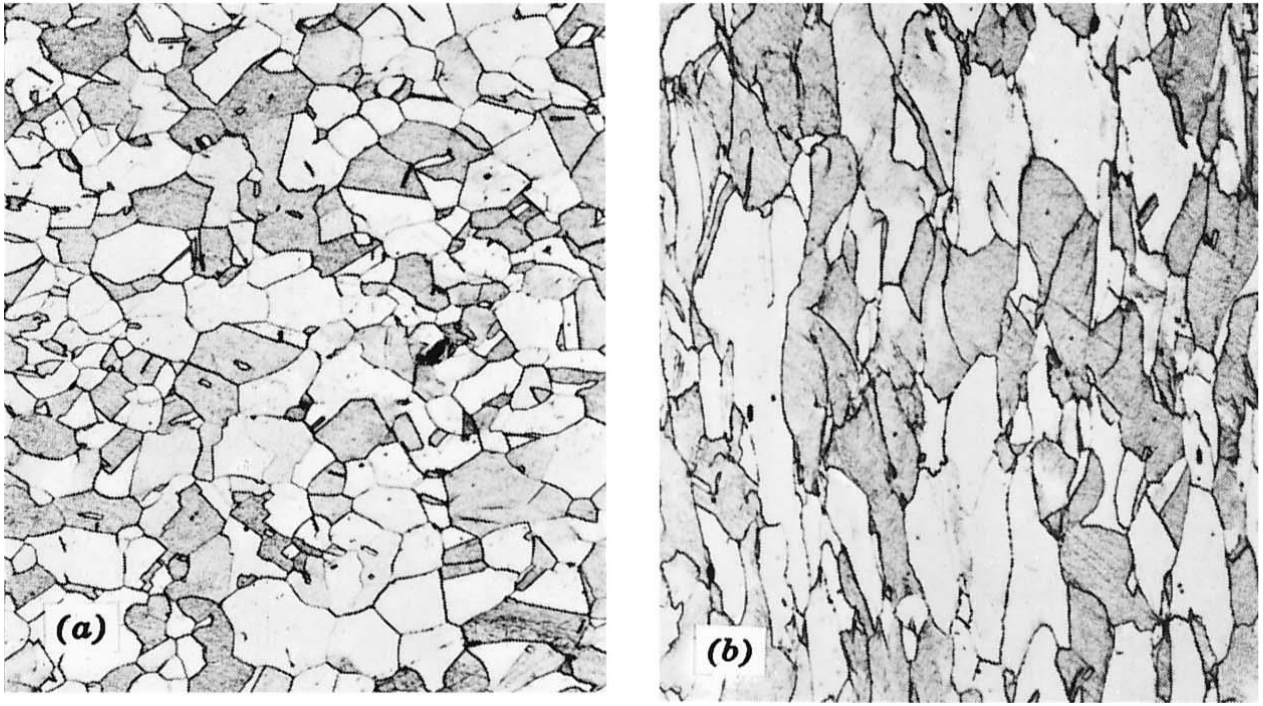
Niobium (bcc)

Implications for polycrystalline materials

Slip lines on surface of polycrystalline Cu (173 x)



unstressed **stressed**



Grains rotate and elongate

Implications for polycrystalline materials