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Dept. of Civil and Environmental Engineering

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457.646 Topics in Structural Reliability

In-Class Material: Class 19

“ ” (cf.)

Ⅴ. Structural Reliability under Model & Stastical Uncertainties

(Ref.: “Analysis of Structural Reliability under Model and Statistical Uncertainties: A Bayesian Approach” ~ eTL)

◎ Formulation of Reliability Problems under Epistemic Uncertainties

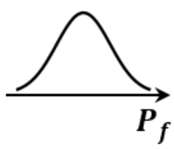
① Reliability Problem with Aleatoric uncertainties (only)

$P_f = \int f_{\mathbf{x}}(\mathbf{x})d\mathbf{x}$ $\mathbf{x}$: r.v’s representing aleatoric uncertainties in the problem

→ Use component and/or system reliability method

② Reliability Problem under Aleatoric & Epistemic certainties

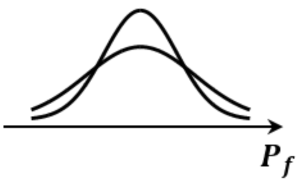
$$P_f(\boldsymbol{\theta}) = \int_{\cup\cap g(\mathbf{x};\quad)\le 0} f_{\mathbf{x}}(\mathbf{x};\quad)d\mathbf{x} \Rightarrow \beta(\boldsymbol{\theta}) = \Phi^{-1}[\qquad]$$

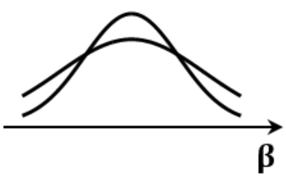


$\boldsymbol{\theta} = [\boldsymbol{\theta}_f \quad \boldsymbol{\theta}_g]$

uncertain parameters

$\Rightarrow P_f$ & β become __________ due to uncertainty in $\boldsymbol{\theta}_f$ and/or $\boldsymbol{\theta}_g$

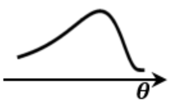




cf. $\beta(\boldsymbol{\theta}),\ P_f(\boldsymbol{\theta}) \Rightarrow$ _________ reliability index given value of uncertain parameters

◎ Three approaches for estimating reliability under epistemic uncertainties

Suppose $f_{|\boldsymbol{\theta}|}(\boldsymbol{\theta})$ is available,



① Point estimate of Reliability: $P_f(\boldsymbol{\theta})$ at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$

$\hat{\boldsymbol{\theta}}$: point estimate (representative) of $\boldsymbol{\theta}$

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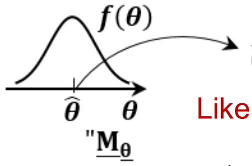
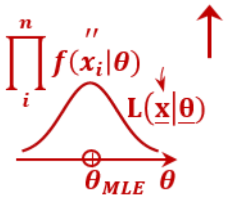
e.g. $\begin{cases} \hat{\boldsymbol{\theta}} = \mathbf{M}_{\boldsymbol{\theta}} = \int \boldsymbol{\theta} f_{\boldsymbol{\Theta}}(\boldsymbol{\theta})\, d\boldsymbol{\theta} & \text{Bayesian} \\ \hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_{MLE} = \arg\max_{\boldsymbol{\theta}} L(\mathbf{x};\boldsymbol{\theta}) & \text{Non-Bayesian} \end{cases}$



Likelihood function

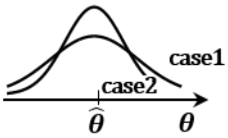
$L(\mathbf{x};\boldsymbol{\theta}) \propto P(x,\cdots,x_n|\boldsymbol{\theta})$



$\Rightarrow$ $P_f(\hat{\boldsymbol{\theta}})$, $\beta(\hat{\boldsymbol{\theta}})$: Perform reliability analysis with $\boldsymbol{\theta} =$ fixed

Note i) $P_f(\mathbf{M}_{\boldsymbol{\theta}}) = \mathbf{M}^{FO}_{P_f(\boldsymbol{\theta})}$

ii) Variability in $\boldsymbol{\theta}$ not considered



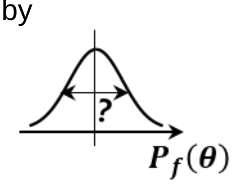
② "Predictive" Reliability

$$\tilde{P}_f = E_{\boldsymbol{\theta}}\left[P_f(\boldsymbol{\theta})\right]$$
$$= \int P_f(\boldsymbol{\theta}) \cdot f_{\boldsymbol{\Theta}}(\boldsymbol{\theta})\, d\boldsymbol{\theta}$$

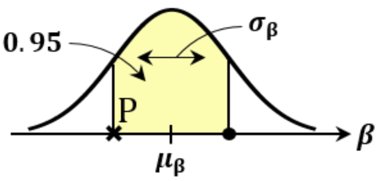
$$\tilde{\beta} = \Phi^{-1}[\qquad\qquad]$$

$\rightarrow$ incorporates variability in $\boldsymbol{\theta}$

$\rightarrow$ but still point estimate, i.e. does not measure variability in $P_f(\boldsymbol{\theta})$ caused by that in $\boldsymbol{\theta}$



③ Bounds on Reliability (Confidence Intervals)



$100 \times p(\%)$ confident that β is b/w x and o

First, find mean and variance of $\beta(\boldsymbol{\theta})$

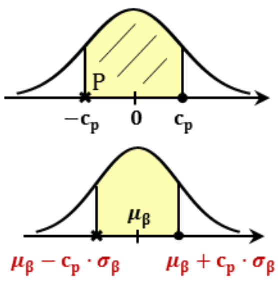
reliability analysis $\qquad f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$

$$\begin{cases} \mu_\beta \cong \mu_\beta^{FO} = \beta(\mathbf{M}_{\boldsymbol{\theta}}) \\ \sigma_\beta^{\,2} \cong \left(\sigma_\beta^{\,2}\right)^{FO} = \nabla_{\boldsymbol{\theta}}\beta(\mathbf{M}_{\boldsymbol{\theta}})\, \boldsymbol{\Sigma}_{\boldsymbol{\theta\theta}} \nabla_{\boldsymbol{\theta}}\beta(\mathbf{M}_{\boldsymbol{\theta}})^T \end{cases}$$

Parameter sensitivity (e.g. FORM)

Second, assume $\beta \sim N(\mu_\beta, \sigma_\beta)$

Std. normal



$$u_\beta = \frac{\beta - \mu_\beta}{\sigma_\beta}$$

P	c_p
0.70	1.04
0.80	1.28
0.90	1.64
0.95	1.96
0.99	2.58

$$\langle \beta \rangle_{100\times p(\%)} = \mu_\beta \pm c_p \sigma_\beta$$

(if $\tilde{\beta}$ available, $\tilde{\beta} \pm c_p \sigma_\beta$)

$$\langle P_f \rangle_{100\times p(\%)} = \Phi\left[-\left(\tilde{\beta} \pm c_p \sigma_\beta\right)\right] \qquad P_f = \Phi(-\beta)$$

Then, $f_{\boldsymbol{\theta}_f}(\boldsymbol{\theta}_f)$, $f_{\boldsymbol{\theta}_g}(\boldsymbol{\theta}_g)$??

(Review) Rel. Analysis under Epistemic Uncertainties (Model or Statistical)

① Point Estimate $\quad P_f(\hat{\theta})$, $\beta(\hat{\theta})$

② Predictive Reliability $\quad \tilde{P}_f = E_\theta\left[P_f(\theta)\right]$

③ Bounds $\quad \langle \beta \rangle_{100\times p(\%)} = \mu_\beta \pm c_p \sigma_\beta$

$f_{\theta_f}(\theta_f)$? $f_{\theta_g}(\theta_g)$?

◎ Bayesian Parameter Estimation

cf. Bayes rule

$$\underset{f}{P(A|B)} = \underset{c}{\frac{1}{P(B)}} \cdot \underset{L}{P(B|A)} \cdot \underset{p}{P(A)}$$

$$f(\boldsymbol{\theta}) = c \cdot L(\boldsymbol{\theta}) \cdot p(\boldsymbol{\theta})$$

① $P(\boldsymbol{\theta})$: (   ) distribution

- represents state of our knowledge (   ) making observations (objective information)
- may incorporate (   ) info. such as "engineering judgment"

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② $L(\boldsymbol{\theta})$: (   ) function

- represents (   ) information gained from the observation
- function (   ) to conditional prob. of the observation given $\boldsymbol{\theta}$

$$L(\boldsymbol{\theta}) \propto P(E_{obs}|\boldsymbol{\theta})$$

③ c: (   ) factor

- makes $c \cdot L(\boldsymbol{\theta}) \cdot p(\boldsymbol{\theta})$ a valid PDF

i.e. $\int_{\boldsymbol{\theta}} f(\boldsymbol{\theta}) = \int_{\boldsymbol{\theta}} c \cdot L(\boldsymbol{\theta}) \cdot P(\boldsymbol{\theta}) d\boldsymbol{\theta} =$

$\therefore \ c =$

④ $f(\boldsymbol{\theta})$: (   ) distribution

- represents updated knowledge about $\boldsymbol{\theta}$
- subjective + objective
-

; rare observation available

; as more observations are made

◎ Computation of c and posterior statistics

$$\left.\begin{aligned} c &= \left[\int L(\boldsymbol{\theta}) \cdot p(\boldsymbol{\theta}) \cdot d\boldsymbol{\theta}\right]^{-1} \\ \mathbf{M}(\boldsymbol{\theta}) &= \int \boldsymbol{\theta} \cdot f(\boldsymbol{\theta}) d\boldsymbol{\theta} = \int \boldsymbol{\theta} \cdot c \cdot L(\boldsymbol{\theta}) \cdot p(\boldsymbol{\theta}) \cdot d\boldsymbol{\theta} \\ \boldsymbol{\Sigma}_{\boldsymbol{\theta\theta}} &= \int \boldsymbol{\theta\theta}^T f(\boldsymbol{\theta}) d\boldsymbol{\theta} - \mathbf{M}(\boldsymbol{\theta})\mathbf{M}(\boldsymbol{\theta})^T \end{aligned}\right\} \text{ multi-fold integrals}$$

How?

- Convenient forms for special distribution (directly update statistics “conjugate”)
- Special numerical algorithms (Geyskens et al. 1993)
- Sampling methods: MCS, importance sampling, Markov Chain Monte Carlo (MCMC)

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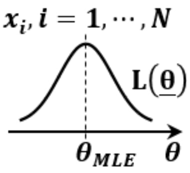
◎ Likelihood function $L(\boldsymbol{\theta})$ for distribution (statistical) parameters $\boldsymbol{\theta}_f$ (e.g. $\mu,\ \sigma,\ \lambda,\ \xi\ldots$)

① Measured value are available, $\mathbf{x}_i,\ i=1,\cdots,N$

Assuming the observations are s.i.

$$L(\boldsymbol{\theta}_f) \propto P\left(\bigcap_{i=1}^{N} \mathbf{X}=\mathbf{x}_i \,\middle|\, \boldsymbol{\Theta}_f=\boldsymbol{\theta}_f\right)$$
$$=\prod_{i=1}^{N} P(\mathbf{X}=\mathbf{x}_i \,|\, \boldsymbol{\Theta}_f=\boldsymbol{\theta}_f) \qquad (\because s.i.)$$
$$\propto \prod_{i=1}^{N} f_{\mathbf{x}}(\mathbf{x}_i \,|\, \boldsymbol{\theta}_f)$$



e.g. $\mathbf{x}=\{x\}$ uni-variate normal $N(\mu,\sigma^2)$

Two samples observed: 12.3($\leftarrow x_1$), 13.5($\leftarrow x_2$) $\quad f(\boldsymbol{\theta})=cL(\boldsymbol{\theta})\cdot P(\boldsymbol{\theta})$

$$L(\boldsymbol{\theta}_f) \propto \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{12.3-\mu}{\sigma}\right)^2\right)\times\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{13.5-\mu}{\sigma}\right)^2\right)$$

$$※\ L(\boldsymbol{\theta})\begin{cases}\text{MLE} \qquad \boldsymbol{\theta}_{\text{MLE}}=\arg\max L(\boldsymbol{\theta}) \quad \begin{matrix}\nearrow \dfrac{\partial L}{\partial \theta}=0 \\ \searrow \text{prefer } \dfrac{\partial \ln L}{\partial \theta}=0\end{matrix} \\ \text{Bayesian Parameter Extimation}\end{cases}$$

$$f(\boldsymbol{\theta})=c\cdot L(\boldsymbol{\theta})\cdot p(\boldsymbol{\theta})$$

② No direct measurement $\mathbf{x}$ of available, but a set of events that involve $\mathbf{x}$ are available

e.g. no measurement for compressive strength of concrete f_c' ($\leftarrow \mu,\ \sigma,\ \lambda\ldots$) available but spalling observed under a certain condition

Inequality events : $h_i(\mathbf{x})\le 0,\ i=1,\cdots,N$

Equality events : $h_i(\mathbf{x})=0$

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a) Inequality

e.g. $h_i(\mathbf{x}) = -C(\mathbf{x}) + D(\mathbf{x}) \le 0$ no failure observed

$h_i(\mathbf{x}) = C(\mathbf{x}) - D(\mathbf{x}) \le 0$ failure observed

$$L(\boldsymbol{\theta}_f) \propto \prod_{i=1}^{N} P(h_i(\mathbf{x}) \le 0 | \boldsymbol{\theta}_f)$$

$$= \prod_{i=1}^{N} \int_{h_i(\mathbf{x})\le 0} f_{\mathbf{x}}(\mathbf{x};\boldsymbol{\theta}_f) d\mathbf{x} \Rightarrow \text{structural reliability analysis}$$

b) Equality

e.g. $h_i(\mathbf{x}) = a(\mathbf{x}) - a_o = 0$

$a(\mathbf{x})$: fatigue crack growth model, e.g. Paris law

a_o : measured crack size

$$L(\boldsymbol{\theta}_f) \propto \prod_{i=1}^{N} \lim_{\delta\to 0} P[0 < h_i(\mathbf{x}) \le \delta]$$

$$= \prod_{i=1}^{N} \frac{\partial}{\partial \delta} P[h_i(\mathbf{x}) - \delta \le 0]\Big|_{\delta=0}$$

Proof

$$\lim_{\Delta\delta\to 0} \frac{P[h_i(\underline{\mathbf{x}}) - \delta - \Delta\delta \le 0] - P[h_i(\underline{\mathbf{x}}) - \delta \le 0]}{\Delta\delta}\Bigg|_{\delta=0}$$

$$= \lim_{\Delta\delta\to 0} \frac{P[h_i(\underline{\mathbf{x}}) - \Delta\delta \le 0] - P[h_i(\underline{\mathbf{x}}) \le 0]}{\Delta\delta}$$

$$\propto \lim_{\Delta\delta\to 0} P[0 \le h_i(\underline{\mathbf{x}}) \le \Delta\delta]$$

$\nabla_{\boldsymbol{\delta}} P_f |_{\delta=0}$: can be considered as parameter sensitivity of P_f w.r.t δ (model parameter)

FORM-based (Madsen, 1987)

Good review & new development (Straub, 2011)

↘ a trick to transform equality constraint to ________ constraint

◎ **Likelihood function for limit-state model parameters,** $L(\boldsymbol{\theta}_g)$

e.g. $g(\mathbf{x};\boldsymbol{\theta}_g) = \underline{V_c(\mathbf{x};\boldsymbol{\theta}_g)} - V_d(\mathbf{x};\boldsymbol{\theta}_g) \le 0$

$\frac{1}{6}\sqrt{f_c'} b_w d$ (ACI 11-3)

① Statistical model (using original deterministic model)

$y = \underline{\hat{g}(\mathbf{x};\boldsymbol{\theta}_g)} + \sigma\varepsilon$ ~ submodel or limit state function

e.g. $\theta_1 f_c'^{\theta_2} b_w d$ (ACI 11-3) $\boldsymbol{\theta}_g = \{\theta_1, \cdots, \theta_n, \sigma\}$

$\mathbf{x}$: observable input parameters ($f_c^{'}$, b_w, d, $\cdots$)

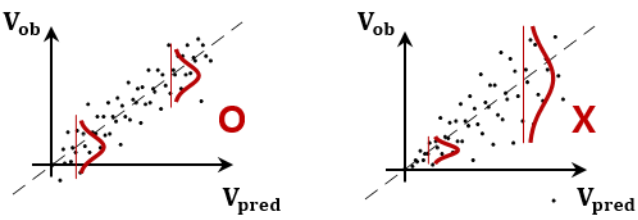
$\mathbf{y}$: observable output parameters (V_c)

$\boldsymbol{\theta}_g$: uncertain model parameters (θ_1, $\theta_2 \cdots$)

$\sigma\varepsilon$: uncertainty due to missing variables and/or inexact mathematical form

- ε : std. normal r.v “ ” assumption
- σ : magnitude of model error (uncertain parameter)

  → constant over $\mathbf{x}$ “ ” assumption
- $\mu_\varepsilon = 0$: unbiased model



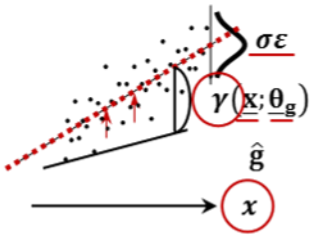
May achieve H_______________ by a proper nonlinear transformation

e.g. $\ln y = \ln \hat{g}(\mathbf{x}, \boldsymbol{\theta}_g) + \sigma\varepsilon$

①’ Statistical model (based on deterministic model, Gardoni et al. 2002)

$$y = \hat{g}(\mathbf{x}) + \gamma(\mathbf{x}; \boldsymbol{\theta}_g) + \sigma\varepsilon$$

$\hat{g}(\mathbf{x})$: original deterministic model (e.g. $\frac{1}{6}\sqrt{f_c^{'}} b_w d$)



$\gamma(\mathbf{x}; \boldsymbol{\theta}_g)$: corrects the bias

$\sigma\varepsilon$: remaining scatter

e.g. RC beam w/o stirrups shear capacity
(Song et al. 2010, Structural Eng & Mechanics)

$$\ln V = \ln \hat{v}(\mathbf{x}) + \Sigma\theta_g \ln h_i(\mathbf{x}) + \sigma\varepsilon$$

$\hat{v}(\mathbf{x})$: 8 models from codes & papers

$h_i(\mathbf{x})$: explanatory terms from the shear transfer mechanism

Find $\mathbf{M}_{\boldsymbol{\theta}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\theta\theta}}$ $\quad \frac{\sigma_\theta}{\mu_\theta} = \delta_\theta$, $\rho_{\theta_i \theta_j}$
using
Bayesian
Parameter
Estimation

② Likelihood function $L(\boldsymbol{\theta}_g)$?

Observed event Equality: $y = y_i,\ i = 1,\cdots,m$ know v_c when failed

Inequality: $\begin{cases} y > a_i & i = m+1,\cdots,m+n \\ y > b_i & i = m+1,\cdots,m+n+N \end{cases}$ No failure up to Vc

Failed but do not know when

Model $Y = \hat{g} + \gamma + \sigma\varepsilon$

a) $P(Y = y_i) = P(\sigma\varepsilon = y_i - \hat{g}(\mathbf{x}) - \gamma(\mathbf{x}, \theta_g))$

$$P(Y = y_i) \propto f_Y(y_i)$$

$$= f_Q(q_i) \cdot \frac{dq}{dy}$$

$$= f_\varepsilon(\varepsilon_i) \cdot \frac{d\varepsilon}{dq}$$

$$= \frac{1}{\sigma}\varphi\left(\frac{y_i - \hat{g} - \gamma}{\sigma}\right)$$

$$f_Y(y_i) = f_Q(q) \cdot \frac{dq}{dy_i}$$

$$f_Q(q) = f_\varepsilon(\varepsilon) \cdot \frac{d\varepsilon}{dq}$$

$$q = \sigma \cdot \varepsilon$$

b) $P(Y > a_i) = P(\hat{g} + \gamma + \sigma\varepsilon > a_i)$

$$= P(\sigma\varepsilon > a_i - \hat{g} - \gamma)$$

$$= \Phi\left(-\frac{a_i - \hat{g} - \gamma}{\sigma}\right)$$

c) $P(Y < b_i) = P(\hat{g} + \gamma + \sigma\varepsilon < b_i)$

$$= P(\sigma\varepsilon < b_i - g - \gamma)$$

$$= \Phi\left(\frac{b_i - g - \gamma}{\sigma}\right)$$

$$\therefore L(\theta_g) = \prod_{i=1}^{m}\frac{1}{\sigma}\varphi\left(\frac{y_i - \hat{g} - \gamma}{\sigma}\right) \times \prod_{i=m+1}^{m+n}\Phi\left(-\frac{a_i - \hat{g} - \gamma}{\sigma}\right) \times \prod_{i=m+n+1}^{m+n+N}\Phi\left(\frac{b_i - \hat{g} - \gamma}{\sigma}\right)$$

※ Matlab codes for “Model Development by Bayesian method”

→ MDB (by Prof. S.Y. Ok at Hankyoung Univ. for educational purpose)

Probabilistic Shear Strength Models for RC Beams by Bayesian Updating Based on Experimental Observations

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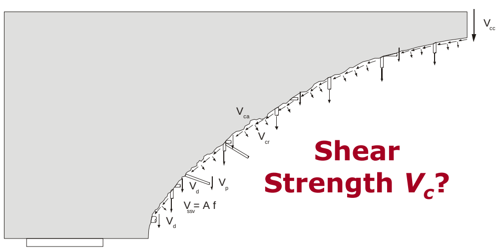
Probabilistic shear strength models

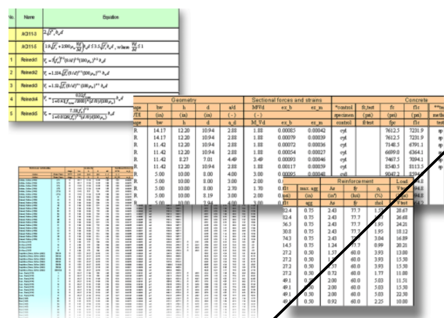


Shear Strength V_c?

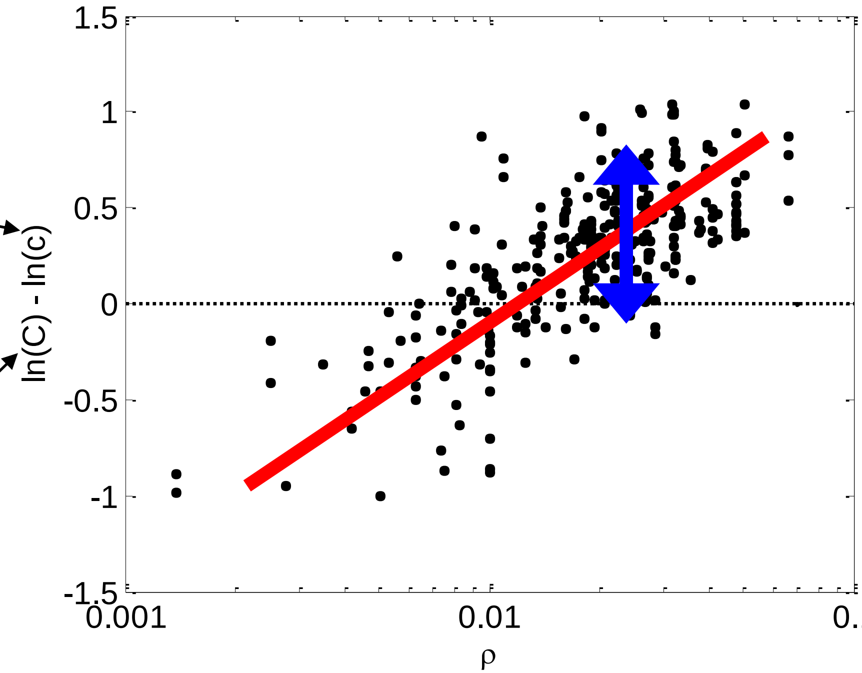
Reinforced Concrete Beam w/o Shear Reinforcement

$V_c = (1/6)\, f_c'^{0.5} b_w d$

Empirical Formulas



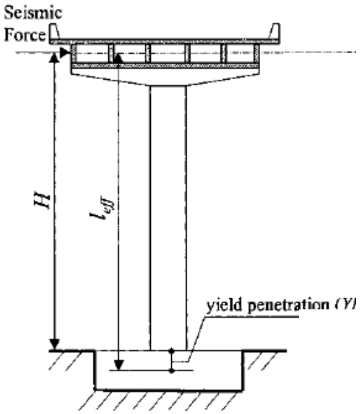
Database of Experimental Observations



- Empirical formulas are widely used for code provisions and designs
  ~ based on simplified mechanics rules and limited amount of experimental observations.
- Inaccurate description of physics & missing variables → **biases** and **scatters**
- Need probabilistic shear strength models that **correct the biases** **and** **quantify the uncertainties** based on comprehensive database of **experimental observations**

Probabilistic models by Bayesian updating*

* Gardoni, P., Der Kiureghian, A., and Mosalam, K.M. (2002)
"**Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations**"
Journal of Engineering Mechanics, Vol. 128(10)



Bridge Column

$$C(\mathbf{x}, \boldsymbol{\Theta}) = c(\mathbf{x}) + \gamma(\mathbf{x}, \boldsymbol{\theta}) + \sigma\varepsilon$$

Capacity | **Prediction by deterministic Model** | **Bias-correction** | **Remaining errors**

Assumptions:

- $\sigma\varepsilon$ is independent of input variables ~ "**Homoskedasticity**"
- ε has the normal distribution ~ "**Normality**"

Probabilistic models by Bayesian updating*

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Journal of Engineering Mechanics, Vol. 128(10)

Explanatory functions

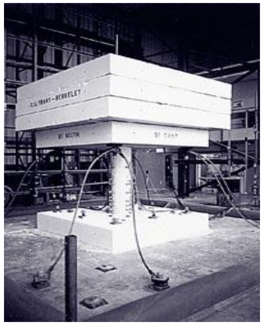
$$\ln[C(\mathbf{x},\mathbf{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \boxed{\theta_i} h_i(\mathbf{x}) + \boxed{\sigma\varepsilon}$$

? ?

Nonlinear transformation to achieve "homoskedasticity"

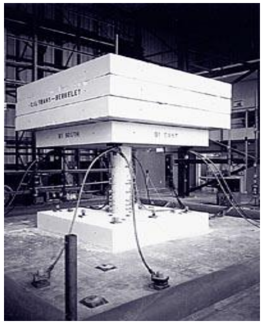
$$f(\mathbf{\Theta}) = \kappa L(\mathbf{\Theta})\, p(\mathbf{\Theta})$$

Bayesian parameter estimation



Database of 106 columns

Step-wise removal process



$$f(\Theta) = \kappa L(\Theta)\, p(\Theta)$$

Bayesian parameter estimation

$$\mu_\theta, \sigma_\theta, \delta_\theta, \rho_{\theta_i \theta_j}$$

- Remove an explanatory terms with the highest c.o.v. (most uncertain)
- Continue until the mean of σ starts increasing significantly

Table 2. Explanatory removing process for joint shear strength, equations (1) and (8)

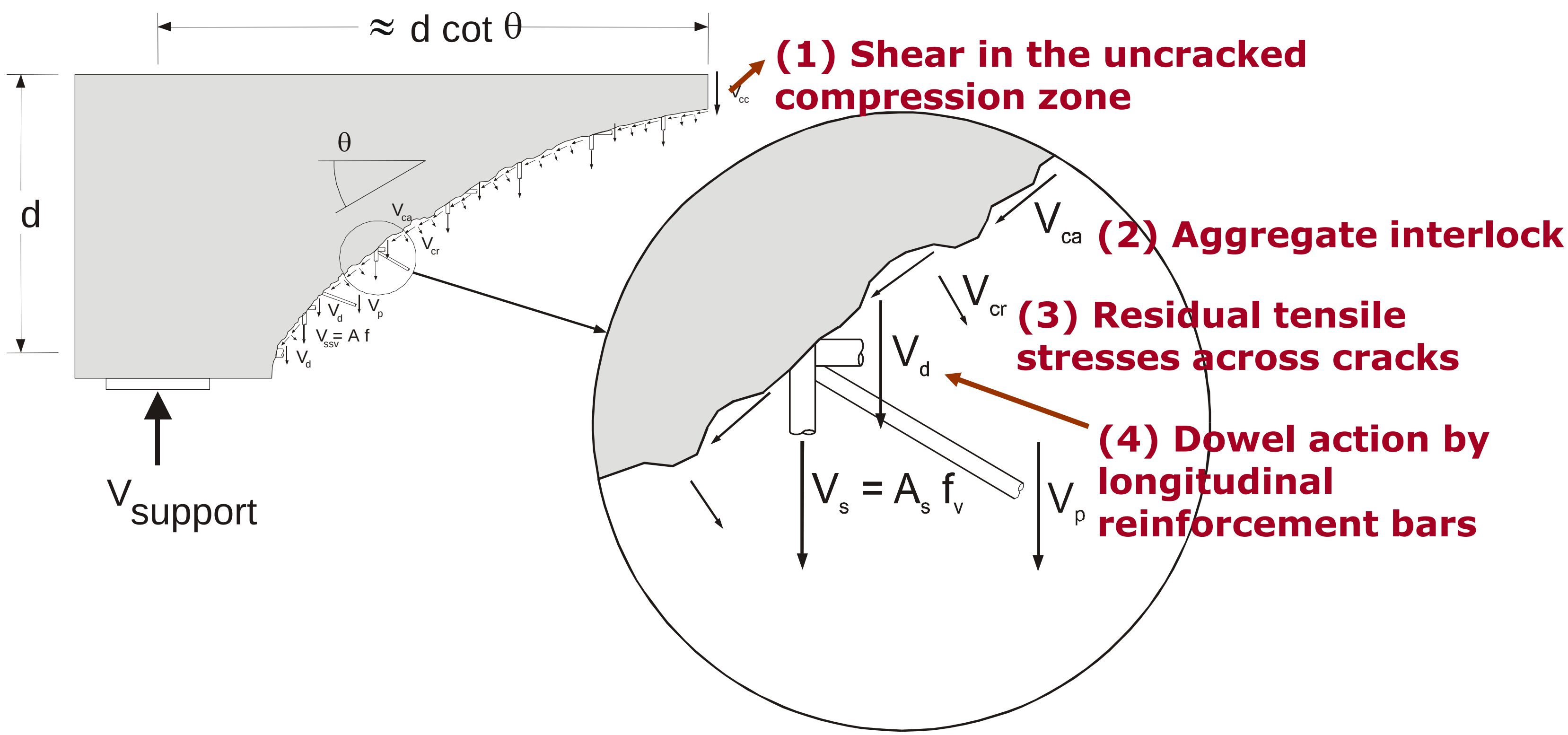
Step	1	2	3	4	5	6	7	8	9	10
f_c'	O	O	O	O	O	O	O	O	O	O
JP	O	O	O	O	O	O	O	O	O	X
BI	O	O	O	O	O	O	O	O	X	X
JI	O	O	O	O	O	O	O	X	X	X
$1 - e/b_c$	O	O	O	O	O	O	X	X	X	X
TB	O	O	O	O	O	X	X	X	X	X
$A_{sh,pro}/A_{sh,req}$	O	O	O	O	X	X	X	X	X	X
h_b/h_c	O	O	O	X	X	X	X	X	X	X
b_b/b_c	O	O	X	X	X	X	X	X	X	X
s_{pro}/s_{req}	O	X	X	X	X	X	X	X	X	X
Mean of σ	0·150	0·150	0·150	0·150	0·151	0·156	0·165	0·186	0·231	0·359

O: Included explanatory term
X: Not-included explanatory term

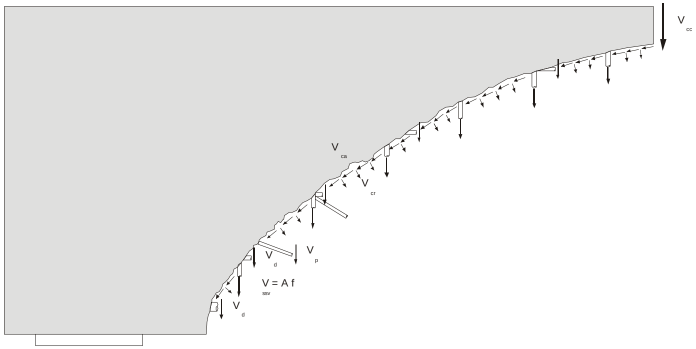
Kim, J., LaFave, J., and Song, J. (2009)
"**Joint Shear Behavior of Reinforced Concrete Beam-Column Connections**"
Magazine of Concrete Research, Vol. 61(**2**), 119-132.

Shear transfer mechanism

Joint ASCE-ACI Committee 426 (1973) & 445 (1998)



Variables affecting shear strengths



$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon$$

$$\mathbf{x} = (f_c', d, a, \rho, \ldots)$$

(1) **Concrete compressive strength:** f_c'

~ tensile strength increases the shear strength (approximated in terms of compressive strength)

(2) **Member depth:** d

~ shear strength decreases as the member depth increases ("size effect")

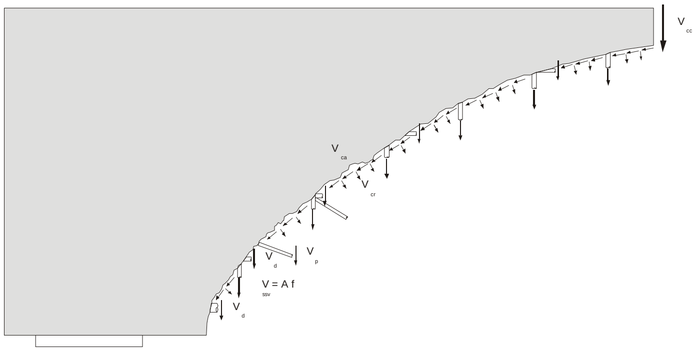
(3) **Shear span-to-depth ratio:** a/d

~ shear strength increases as the ratio decreases ("arch action" of "deep" beam)

(4) **Amount of longitudinal reinforcement:** ρ

~ shear strength increases as the reinforcement increases ("dowel action")

Empirical shear strength models



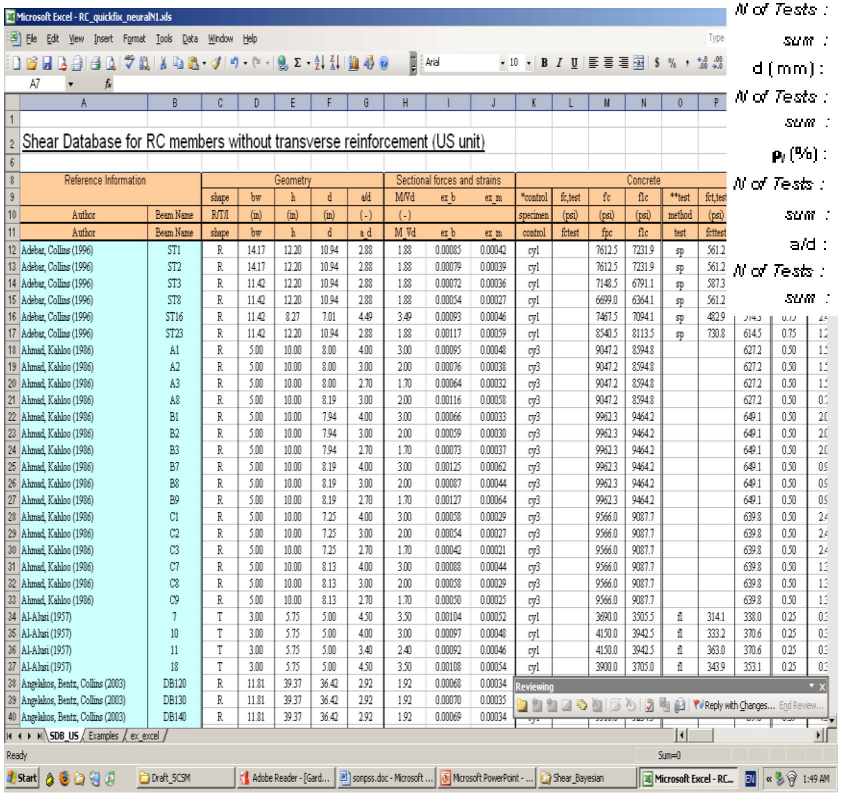
$$\ln[C(\mathbf{x},\mathbf{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon$$

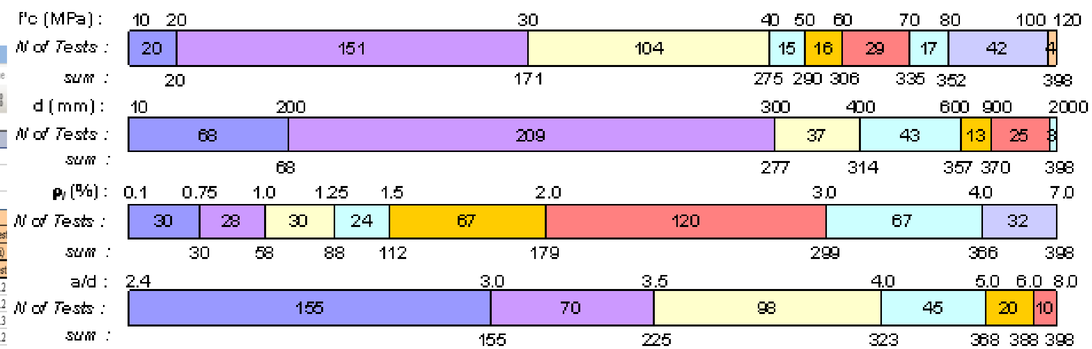
$$\mathbf{x} = (f_c', d, a, \rho, \ldots)$$

Model	Formula	characteristics
ACI 11-3	$V_c = \frac{1}{6}\sqrt{f_c'}\,b_w d$	accounts for compressive strength only
ACI 11-5	$V_c = \left(0.158\sqrt{f_c'} + 17\rho\frac{V_u d}{M_u}\right) b_w d$	compressive strength + ρ
Zsutty	$V_c = 2.2\left(f_c'\rho\frac{d}{a}\right)^{1/3} b_w d$	more accurate than ACI models
Eurocode Draft	$V_c = 0.12k(100\rho f_c')^{1/3} b_w d$	tends to underestimate (conservative)
Okamura & Higai	$V_c = 0.2\frac{(100\rho)^{1/3}}{(d/1000)^{1/4}}(f_c')^{1/3}\left(0.75 + \frac{1.40}{a/d}\right)^{1/3} b_w d$	good without severe biases
Tureyen & Frosch	$V_c = \frac{5}{12}\sqrt{f_c'}\,b_w c$	tends to overestimate for deep beams
Bazant & Yu	$V_c = 1.1044\cdot\rho^{3/8} b_w\left(1+\frac{d}{a}\right)\sqrt{\frac{f_c' d_0 d}{1+d_0/d}}$	mechanics-based, semi-empirical, accurate
Russo et al.	$V_c = 0.72\,\xi\left[\rho^{0.4}(f_c')^{0.39} + 0.5\rho^{0.83} f_y^{0.89}\left(\frac{a}{d}\right)^{-1.2-0.45(a/d)}\right] b_w d$	semi-empirical, large database

Shear strength database

* Reineck, K.H., Kuchma, D.A., **Kim, K.S.**, and Marx, S. (2003)
"**Shear database for reinforced concrete members without shear reinforcement**"
ACI Structural Journal, Vol. 100(2)



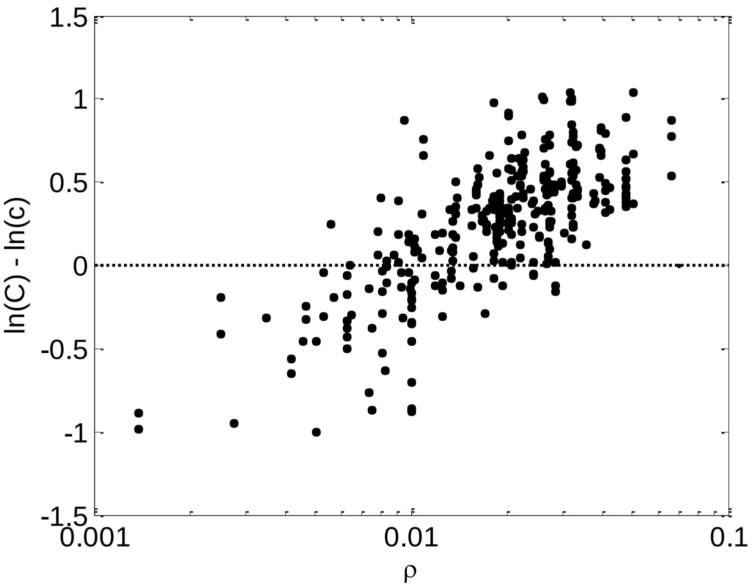


- Checked by various selection criteria discussed by ACI-ASCE Committee 445
- **398** shear strength test data
- Used **341** test data for this study (**57** data: missing aggregate sizes)

Overall errors of the existing models

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \theta + \sigma\varepsilon$$

- μ_θ : overall bias of the existing model
- μ_σ : overall scatter of the existing model



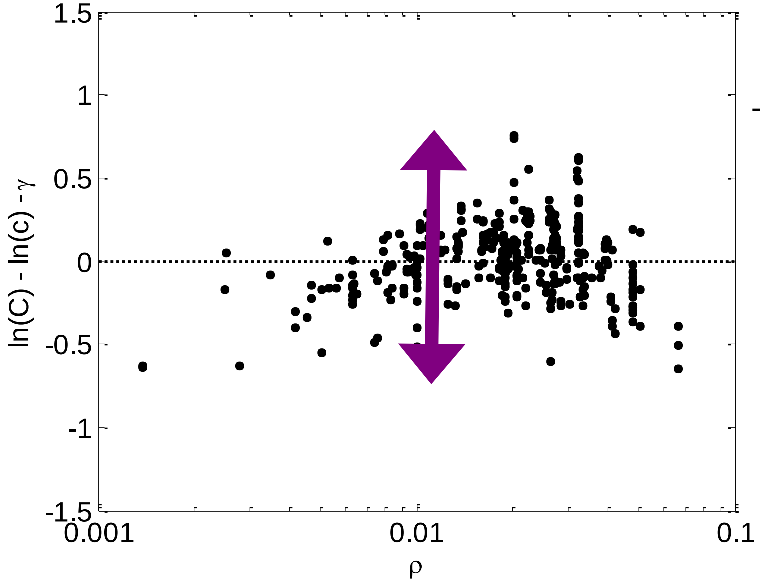
ACI 11-3

Model	Posterior means	
	θ (bias)	σ (scatter)
ACI 11-3	0.257	0.382
ACI 11-5	0.165	0.335
Eurocode Draft	0.456	0.223
Tureyen & Frosch	0.287	0.245
Zsutty	0.0261	0.244
Okamura & Higai	0.116	0.176
Bazant & Yu	0.0142	0.166
Russo et al.	0.00120	0.156

Bayesian updating with bias-correction (H1)

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon$$

- μ_σ : approximately represents the **uncertainties after the bias correction** (scatter)
- $h_i(\mathbf{x}): 2, \rho, \frac{a}{d}, \frac{E_c}{E_s}, \frac{d_a}{d}, \frac{d}{h}, \frac{b_w}{h}$ dimensionless explanatory terms



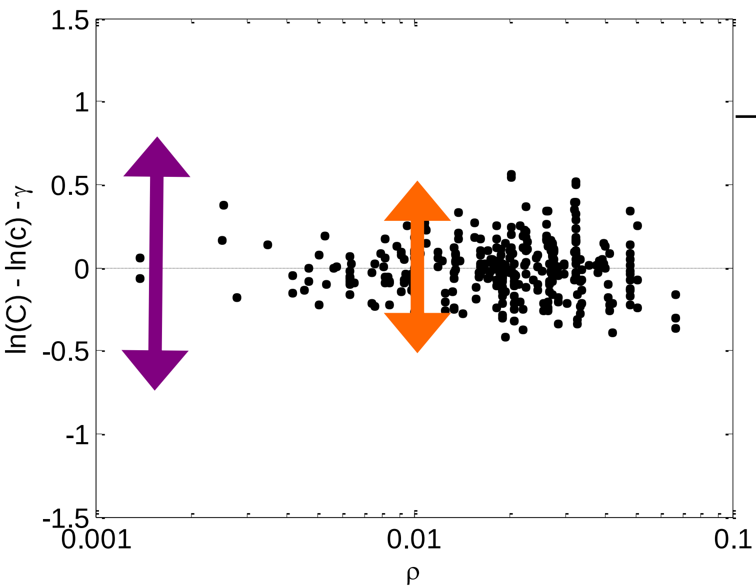
ACI 11-3

Model	Posterior means of σ		
	Constant bias	H_1	H_2
ACI 11-3	0.382	0.222	0.165
ACI 11-5	0.335	0.218	0.177
Eurocode Draft	0.223	0.172	0.165
Tureyen & Frosch	0.245	0.178	0.167
Zsutty	0.244	0.185	0.168
Okamura & Higai	0.176	0.159	0.157
Bazant and Yu	0.166	0.156	0.154
Russo et al.	0.156	0.146	0.146

Bayesian updating with bias-correction (H2)

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$$

- Logarithms are applied to the explanatory functions.
- Consistent with the product forms of the deterministic formulas $C(\mathbf{x},\boldsymbol{\Theta}) = c(\mathbf{x}) h_1(\mathbf{x})^{\theta_1} \cdots h_p(\mathbf{x})^{\theta_p} \exp(\sigma\varepsilon)$



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Calibration of existing models

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \cancel{\ln[c(\mathbf{x})]} + \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$$

- Use the fractions of the empirical formulas as the explanatory functions

  e.g. Zsutty's model $$V_c = 2.2\left(f_c'\rho\frac{d}{a}\right)^{1/3} b_w d$$

  $$h_i(\mathbf{x}) : 2, f_c', \rho, \frac{a}{d}, b_w, d$$

- Do not drop explanatory terms with large c.o.v.'s

- Explanatory functions **do not have to be dimensionless**
  ~ may be **more effective** in representing the physics than the dimensionless terms

$$\mu_\sigma = 0.166 \cong 0.168$$ (posterior mean by $\ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$)

Construction of new models

- Select some dimensional terms to make the same dimension as quantity and add more non-dimensional terms. Perform the Bayesian parameter estimation by models such as

$$\ln[C(\mathbf{x},\mathbf{\Theta})] = \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$$

Product

$$\ln[C(\mathbf{x},\mathbf{\Theta})] = \sum_{i=1}^{l} \theta_i \ln[h_i(\mathbf{x})] + \ln\left[\prod_{i=l+1}^{m} h_i^{\theta_i} + \prod_{i=m+1}^{n} h_i^{\theta_i}\right] + \sigma\varepsilon$$

Product of Sums

- Do not drop "dimensional" explanatory terms

- Useful when

  (1) there exist no empirical models that can be used as a base model.
  (2) the effects of explanatory terms are not well known.

- Shear strength example: tried 17 explanatory terms

  → Similar forms & parameter values with the two best formulas (with smaller μ_σ)

  Zsutty's $\quad V_c = 2.2\left(f_c'\rho\dfrac{d}{a}\right)^{1/3} b_w d$

  Okamura & Higai $\quad V_c = 0.2\dfrac{(100\rho)^{1/3}}{(d/1000)^{1/4}}(f_c')^{1/3}\left(0.75+\dfrac{1.40}{a/d}\right)^{1/3} b_w d$

"Probabilistic" Models

- General form

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[\hat{C}(\mathbf{x},\boldsymbol{\theta})] + \sigma\varepsilon \longrightarrow C(\mathbf{x},\boldsymbol{\Theta}) = \hat{C}(\mathbf{x},\boldsymbol{\theta}) \cdot \exp(\sigma\varepsilon)$$

- Capacity ~ follows the **lognormal** distribution
- Mean and c.o.v. are derived as

$$\mu_C(\mathbf{x}) = \hat{C}(\mathbf{x},\boldsymbol{\mu}_\theta) \cdot \exp(\mu_\sigma \varepsilon) \cong \hat{C}(\mathbf{x},\boldsymbol{\theta}) \text{ for } \mu_\sigma << 1$$

$$\delta_C(\mathbf{x}) = \delta_C = [\exp(\mu_\sigma^2) - 1]^{1/2} \cong \mu_\sigma \text{ for } \mu_\sigma << 1$$

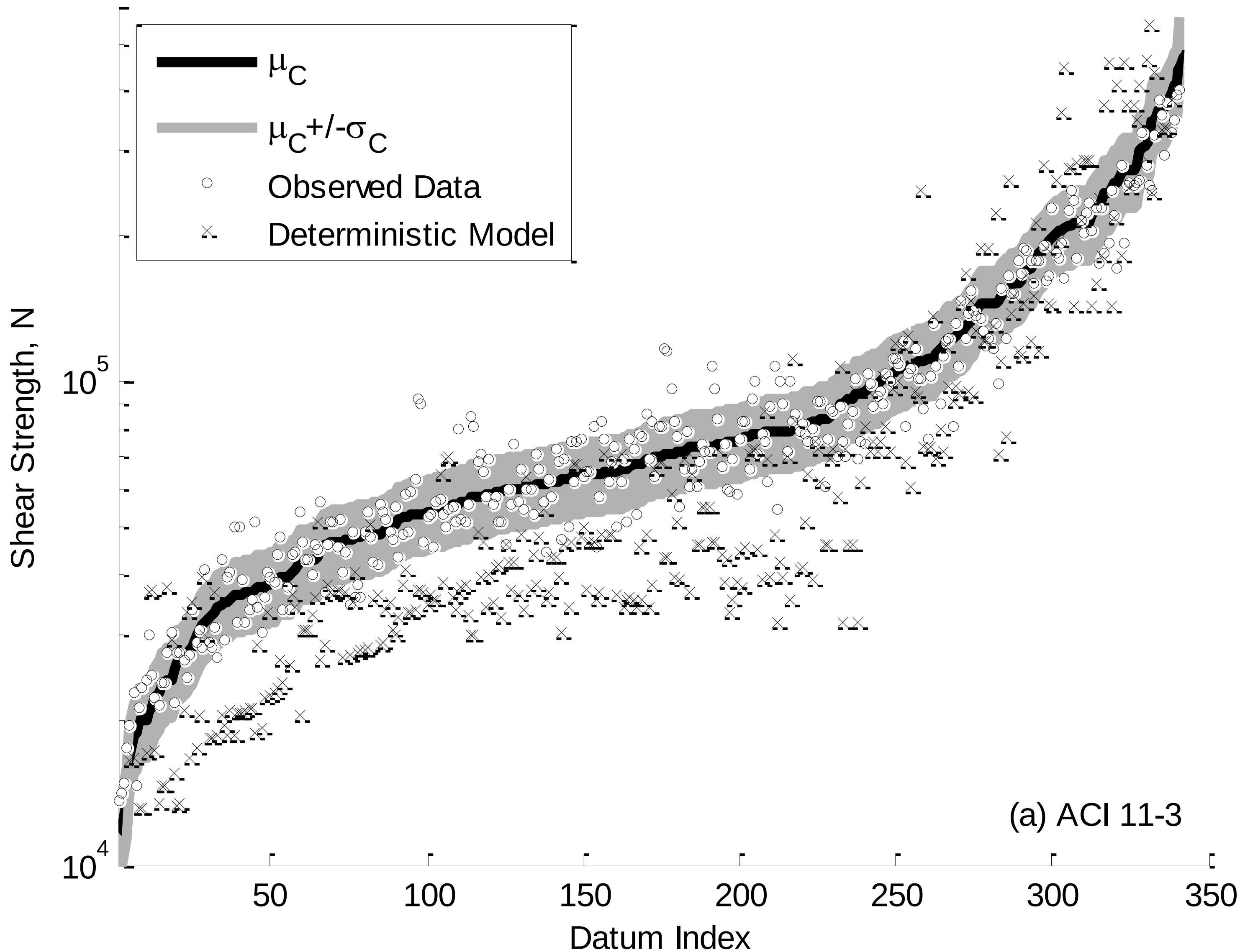
- Conditional pdf of capacity for given $\mathbf{x}$

$$f_C(c \mid \mathbf{x}) = \frac{1}{\sqrt{2\pi}\mu_\sigma c} \exp\left[-\frac{1}{2} \left(\frac{\ln c - \ln \hat{C}(\mathbf{x},\boldsymbol{\mu}_\theta)}{\mu_\sigma} \right)^2 \right]$$

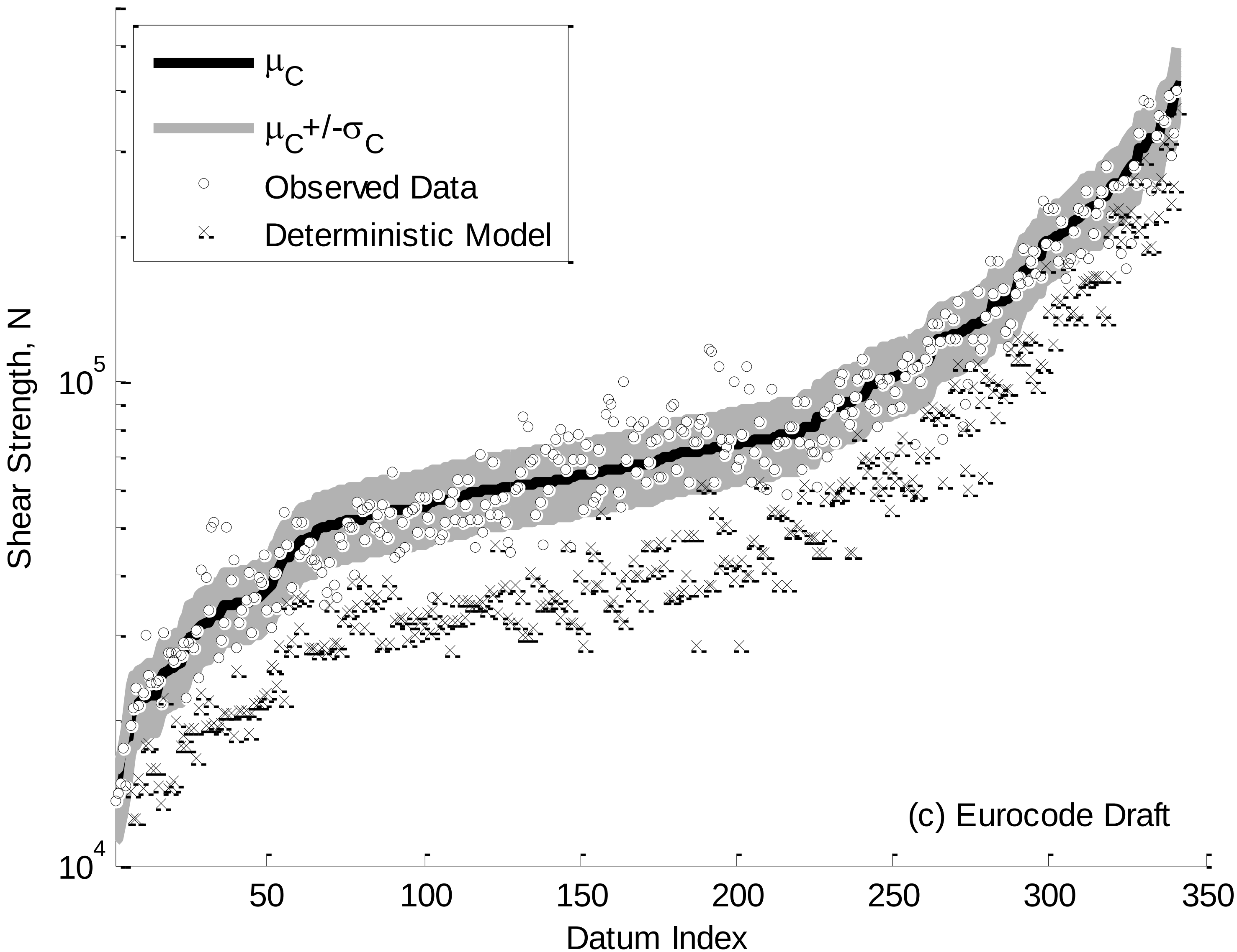
- Predictive pdf of capacity for unknown $\mathbf{x}$

$$f_C(c) = \int_{-\infty}^{\infty} f_C(c \mid \mathbf{x}) \cdot f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}$$

Performance of probabilistic models

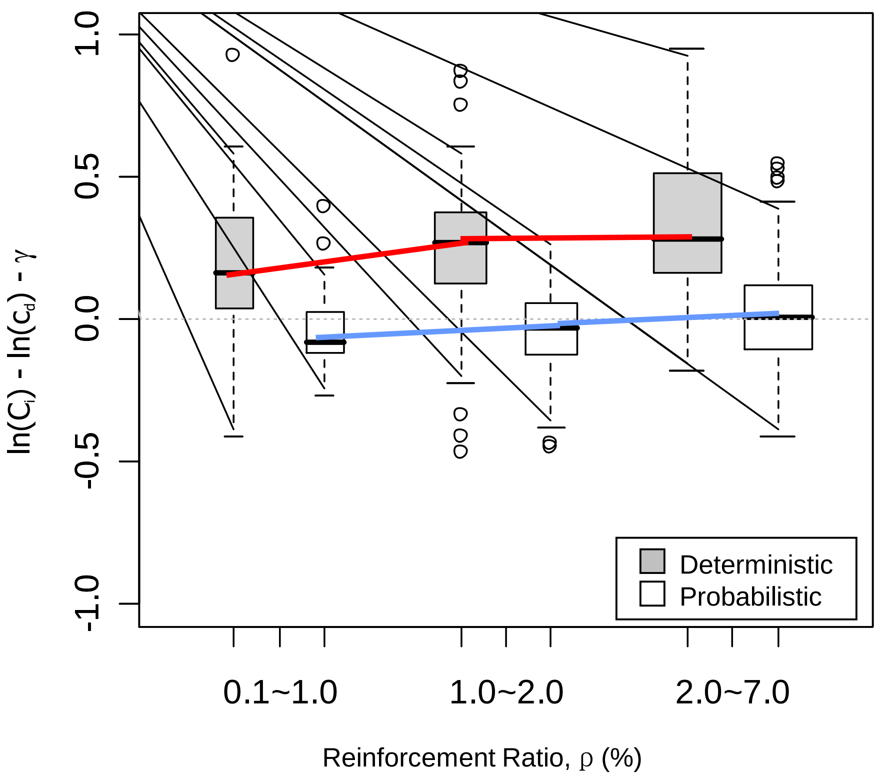


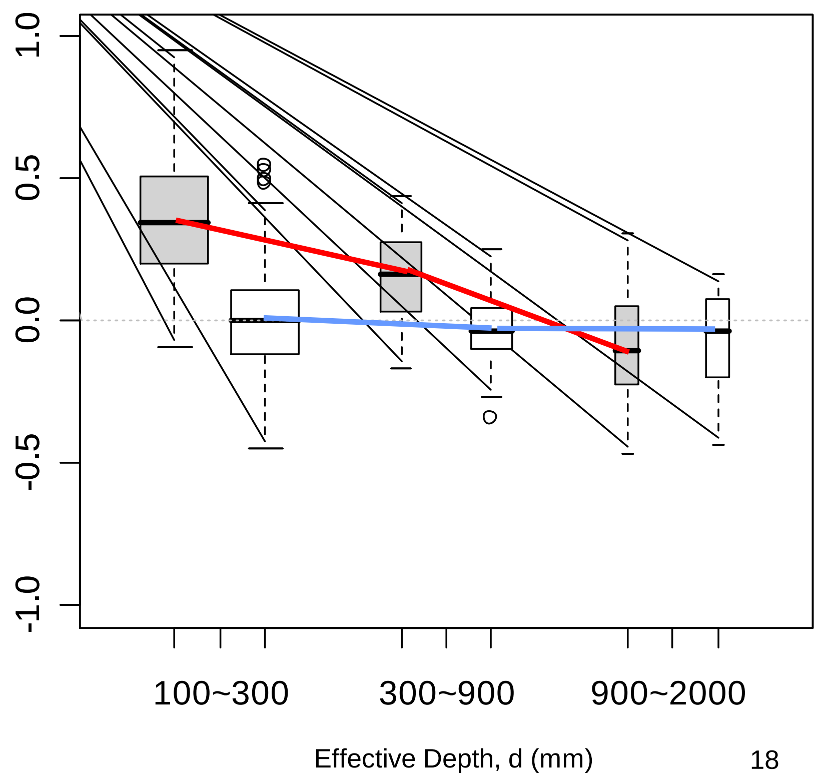
Performance of probabilistic models



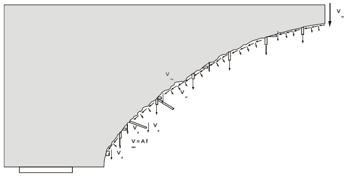
Performance of probabilistic models

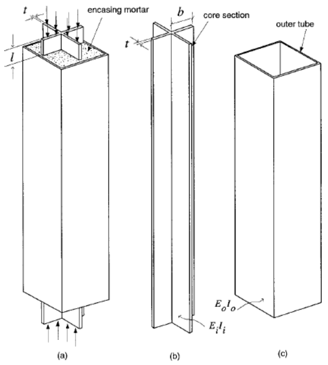
- e.g. Tureyen & Frosch (2003) and a probabilistic strength model developed by this study
- Box plots of errors ~ show that the developed models are unbiased and have consistently good performance for the whole ranges of the parameters.



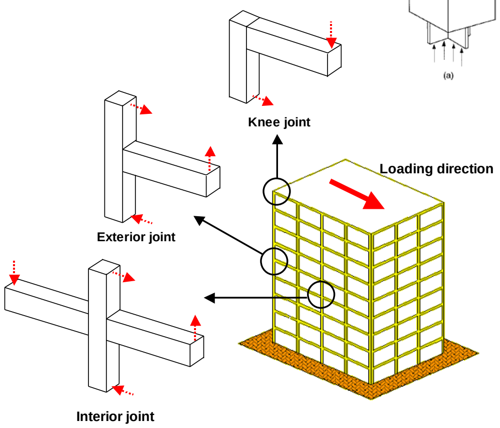


Other Applications

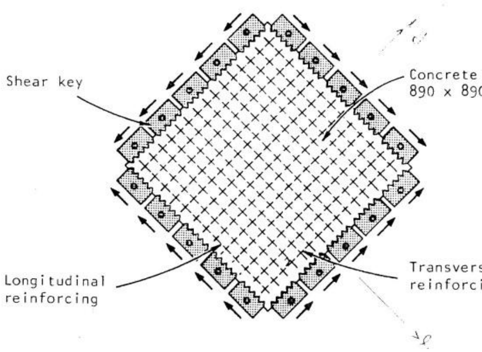




- Shear strengths of RC beams with shear reinforcements (W.-H. Kang, J. Song, and K.S. Kim)
- Seismic strengths of buckling-restrained bracings (B.M. Andrews, J. Song, and L.A. Fahnestock) (Andrews et al. 2009a, 2009b)
- Strengths/ of RC beam-column connections (J. Kim, J.M. LaFave, and J. Song)



- Statistical validation/verification of concrete FEM (H.H. Lee and D.A. Kuchma)
- Shear strengths of RC “deep” beams (strut-and-tie models) (Chetchotisak, P., J. Teerawong, S. Yindeesuk, and J. Song, 2014)



- Course term projects
  - Strengths of concrete-filled tubes (Mark Denavit)
  - Fracture toughness (Tam H. Nguyen)

References

- Gardoni, P., A. Der Kiureghian, and K.M. Mosalam (2002). "Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations." *Journal of Engineering Mechanics*, ASCE, 128(10): 1024-1038 [→ Seismic capacity of RC columns]
- Song, J., W.-H. Kang, K.S. Kim, and S. Jung. "Probabilistic shear strength models for reinforced concrete beams without shear reinforcement based on experimental observations," under review [→ Shear strengths of RC beams without stirrups]
- Kang, W.-H., J. Song, and K.S. Kim (2007). "Probabilistic shear strength models for reinforced concrete beams with shear reinforcements by Bayeisan updating." *Proc. 18th Engineering Mechanics Division Conference of ASCE (ASCE EMD 2007)*, June 3-6, Blacksburg, VA. [→ Shear strengths of RC beams with stirrups]
- Kim, J., J.M. LaFave, and J. Song (2007). "A new statistical approach for joint shear strength determination of RC beam-column connections subjected to lateral earthquake loading." *Structural Engineering and Mechanics*, Vol. 27(4), 439-456 [→ Strength of RC beam-column connection ~ strength only]
- Kim, J., J.M. LaFave, and J. Song. Joint shear behavior of RC beam-column connections, under review [→ RC beam-column connections ~ strength & corresponding strain]
- Chetchotisak, P., J. Teerawong, S. Yindeesuk, and J. Song (2014). "New strut-and-tie-models for shear strength prediction and design of RC deep beams." *Computers and Concrete*, 14(1): 19-40 [→ Shear strength of deep RC beams]

Probabilistic Shear Strength Models for RC Beams by Bayesian Updating Based on Experimental Observations

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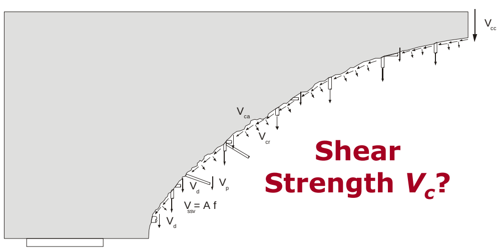
Seoul National University, Korea

Won Hee Kang University of Western Sydney, Australia
Kang Su Kim University of Seoul, Korea
Sungmoon Jung Florida A&M-Florida State University, USA

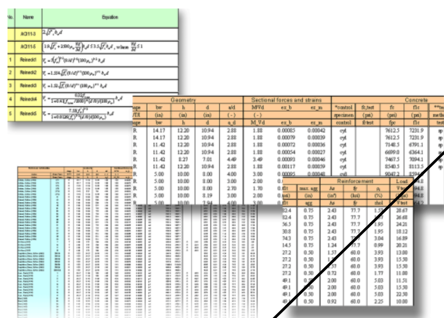
Probabilistic shear strength models



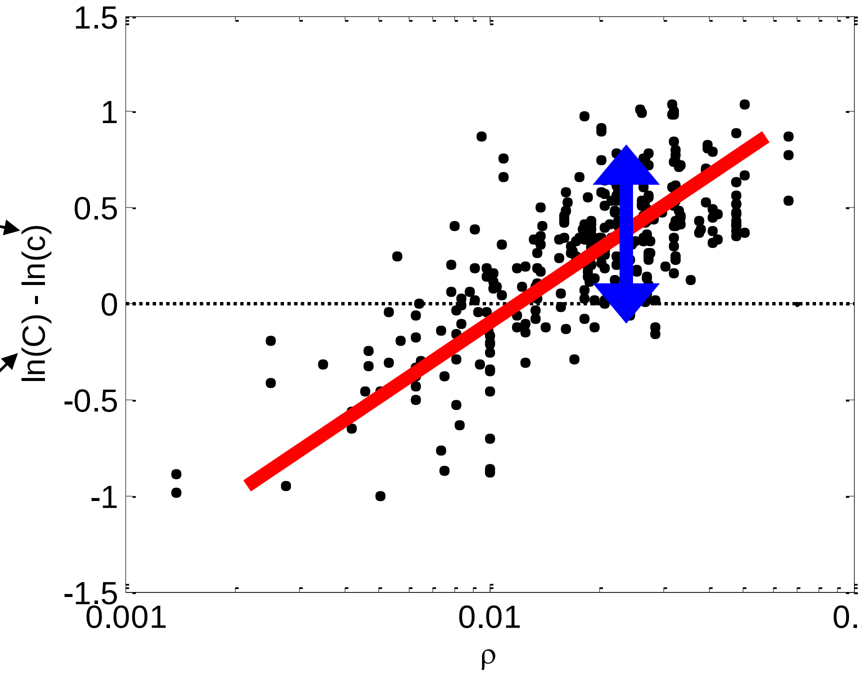
Reinforced Concrete Beam w/o Shear Reinforcement

$$V_c = (1/6)\, f_c'^{0.5} b_w d$$

Empirical Formulas



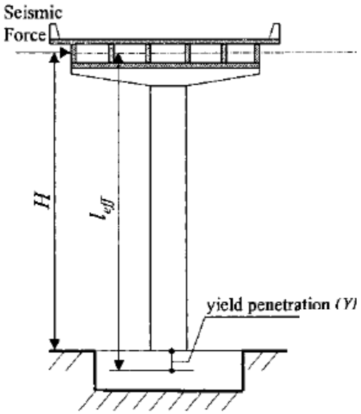
Database of Experimental Observations



- Empirical formulas are widely used for code provisions and designs ~ based on simplified mechanics rules and limited amount of experimental observations.
- Inaccurate description of physics & missing variables → **biases** and **scatters**
- Need probabilistic shear strength models that **correct the biases and quantify the uncertainties** based on comprehensive database of **experimental observations**

Probabilistic models by Bayesian updating*

* Gardoni, P., Der Kiureghian, A., and Mosalam, K.M. (2002)
"**Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations**"
Journal of Engineering Mechanics, Vol. 128(10)



Bridge Column

$$C(\mathbf{x},\boldsymbol{\Theta}) = c(\mathbf{x}) + \gamma(\mathbf{x},\boldsymbol{\theta}) + \sigma\varepsilon$$

Capacity | **Prediction by deterministic Model** | **Bias-correction** | **Remaining errors**

Assumptions:

- $\sigma\varepsilon$ is independent of input variables ~ "**Homoskedasticity**"
- ε has the normal distribution ~ "**Normality**"

Probabilistic models by Bayesian updating*

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Journal of Engineering Mechanics, Vol. 128(10)

Explanatory functions

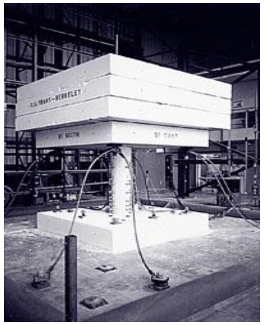
$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon$$

? **?**

Nonlinear transformation to achieve "homoskedasticity"

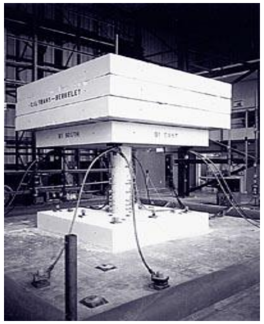
$$f(\boldsymbol{\Theta}) = \kappa L(\boldsymbol{\Theta})\, p(\boldsymbol{\Theta})$$

Bayesian parameter estimation



Database of 106 columns

Step-wise removal process



$$f(\Theta) = \kappa L(\Theta)\, p(\Theta)$$

Bayesian parameter estimation

$$\mu_\theta, \sigma_\theta, \delta_\theta, \rho_{\theta_i \theta_j}$$

- Remove an explanatory terms with the highest c.o.v. (most uncertain)
- Continue until the mean of σ starts increasing significantly

Table 2. Explanatory removing process for joint shear strength, equations (1) and (8)

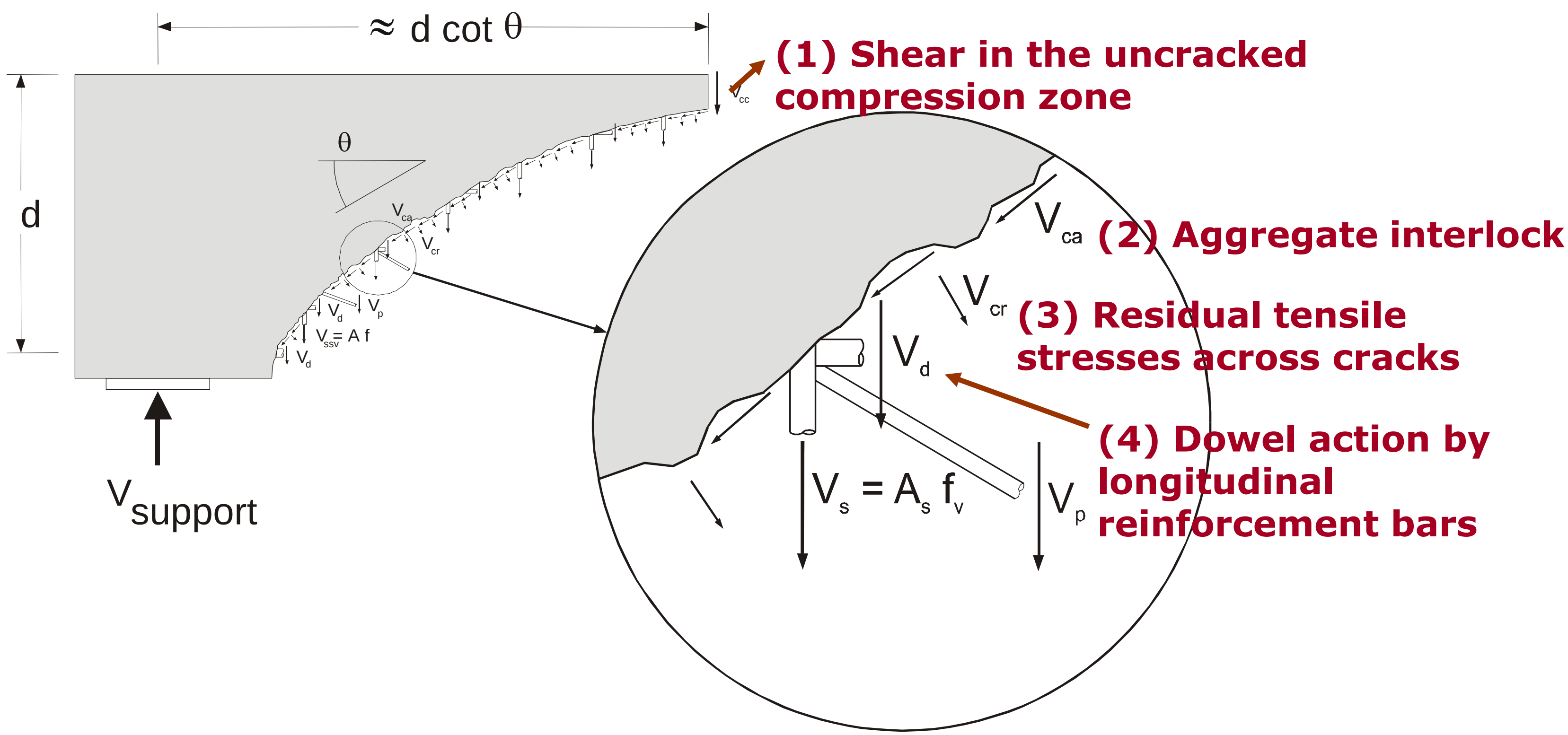
Step	1	2	3	4	5	6	7	8	9	10
f'_c	O	O	O	O	O	O	O	O	O	O
JP	O	O	O	O	O	O	O	O	O	X
BI	O	O	O	O	O	O	O	O	X	X
JI	O	O	O	O	O	O	O	X	X	X
$1 - e/b_c$	O	O	O	O	O	O	X	X	X	X
TB	O	O	O	O	O	X	X	X	X	X
$A_{sh,pro}/A_{sh,req}$	O	O	O	O	X	X	X	X	X	X
h_b/h_c	O	O	O	X	X	X	X	X	X	X
b_b/b_c	O	O	X	X	X	X	X	X	X	X
s_{pro}/s_{req}	O	X	X	X	X	X	X	X	X	X
Mean of σ	0·150	0·150	0·150	0·150	0·151	0·156	0·165	0·186	0·231	0·359

O: Included explanatory term
X: Not-included explanatory term

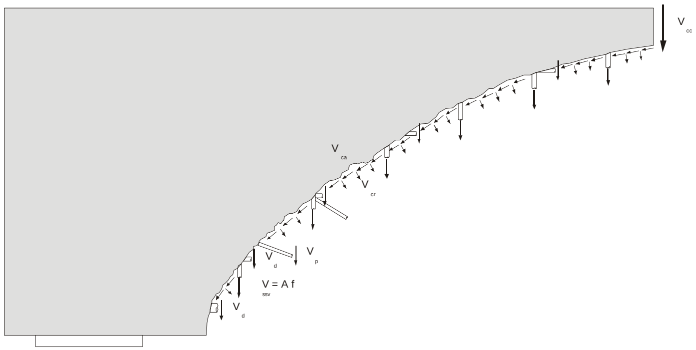
Kim, J., LaFave, J., and Song, J. (2009)
"**Joint Shear Behavior of Reinforced Concrete Beam-Column Connections**"
Magazine of Concrete Research, Vol. 61(**2**), 119-132.

Shear transfer mechanism

Joint ASCE-ACI Committee 426 (1973) & 445 (1998)



Variables affecting shear strengths



$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon$$

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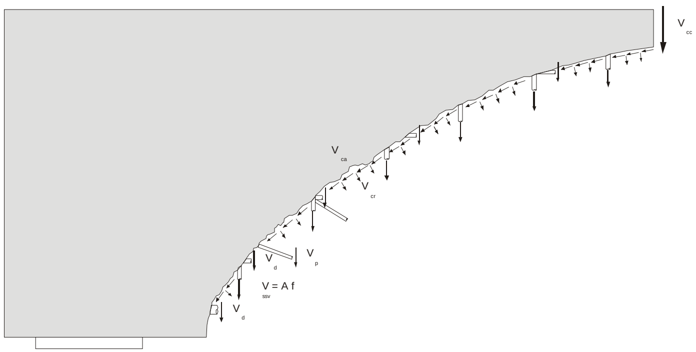
Empirical shear strength models



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Okamura & Higai	$V_c = 0.2\frac{(100\rho)^{1/3}}{(d/1000)^{1/4}}(f_c')^{1/3}\left(0.75 + \frac{1.40}{a/d}\right)^{1/3} b_w d$	good without severe biases
Tureyen & Frosch	$V_c = \frac{5}{12}\sqrt{f_c'}\, b_w c$	tends to overestimate for deep beams
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Shear strength database

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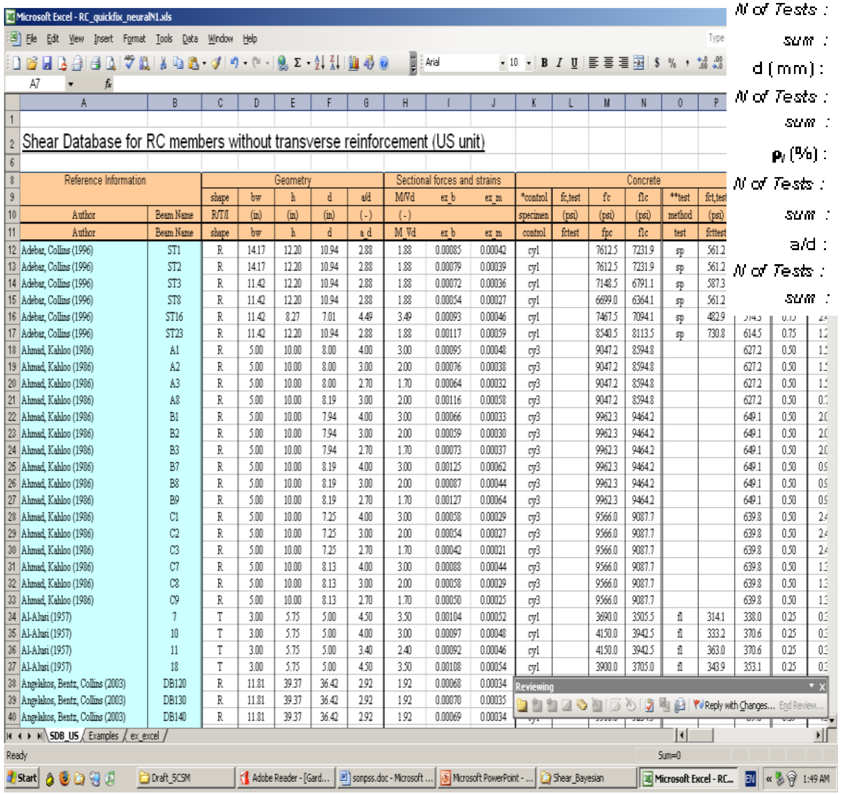
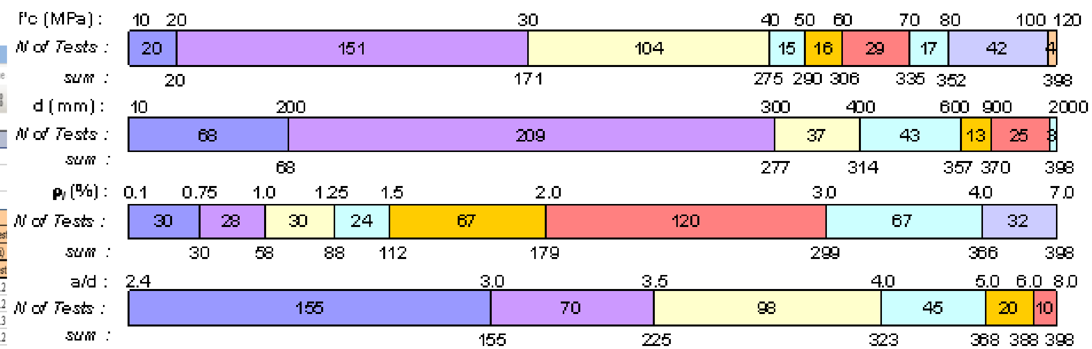
 ACI Structural Journal, Vol. 100(2)



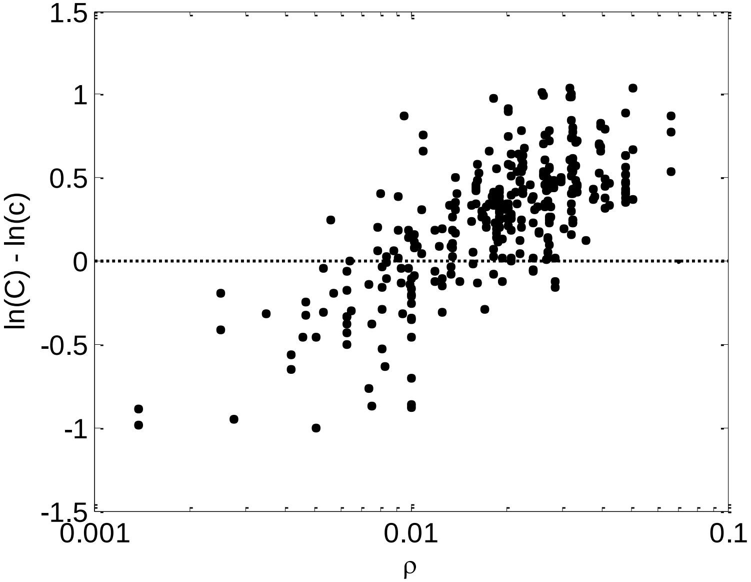


- Checked by various selection criteria discussed by ACI-ASCE Committee 445
- **398** shear strength test data
- Used **341** test data for this study (**57** data: missing aggregate sizes)

Overall errors of the existing models

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \theta + \sigma\varepsilon$$

- μ_θ : overall bias of the existing model
- μ_σ : overall scatter of the existing model



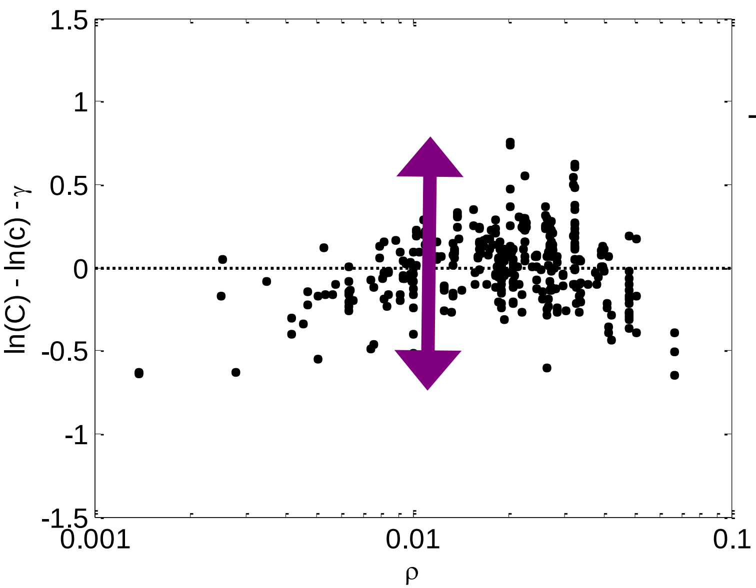
ACI 11-3

Model	Posterior means	
	θ (bias)	σ (scatter)
ACI 11-3	0.257	0.382
ACI 11-5	0.165	0.335
Eurocode Draft	0.456	0.223
Tureyen & Frosch	0.287	0.245
Zsutty	0.0261	0.244
Okamura & Higai	0.116	0.176
Bazant & Yu	0.0142	0.166
Russo et al.	0.00120	0.156

Bayesian updating with bias-correction (H1)

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) + \sigma\varepsilon$$

- μ_σ : approximately represents the **uncertainties after the bias correction** (scatter)
- $h_i(\mathbf{x}) : 2, \rho, \frac{a}{d}, \frac{E_c}{E_s}, \frac{d_a}{d}, \frac{d}{h}, \frac{b_w}{h}$ dimensionless explanatory terms



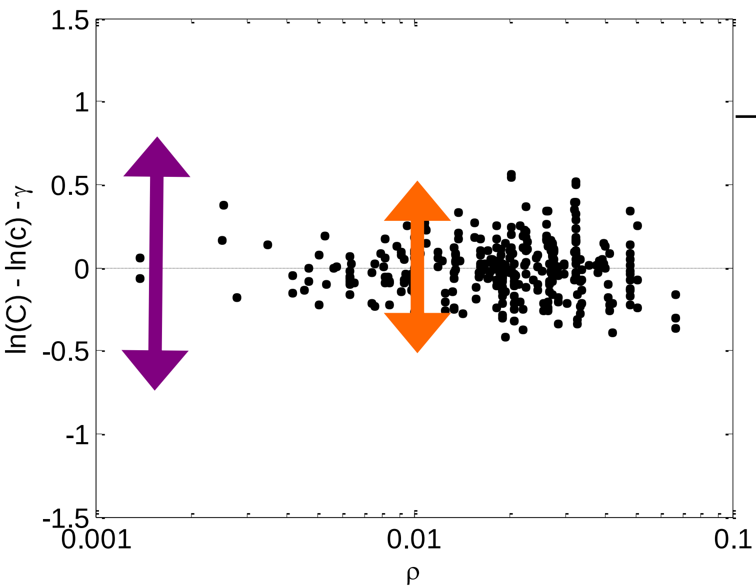
ACI 11-3

Model	Posterior means of σ		
	Constant bias	H_1	H_2
ACI 11-3	0.382	0.222	0.165
ACI 11-5	0.335	0.218	0.177
Eurocode Draft	0.223	0.172	0.165
Tureyen & Frosch	0.245	0.178	0.167
Zsutty	0.244	0.185	0.168
Okamura & Higai	0.176	0.159	0.157
Bazant and Yu	0.166	0.156	0.154
Russo et al.	0.156	0.146	0.146

Bayesian updating with bias-correction (H2)

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$$

- Logarithms are applied to the explanatory functions.
- Consistent with the product forms of the deterministic formulas $C(\mathbf{x},\boldsymbol{\Theta}) = c(\mathbf{x}) h_1(\mathbf{x})^{\theta_1} \cdots h_p(\mathbf{x})^{\theta_p} \exp(\sigma\varepsilon)$



ACI 11-3

Model	Posterior means of σ		
	Constant bias	H_1	H_2
ACI 11-3	0.382	0.222	0.165
ACI 11-5	0.335	0.218	0.177
Eurocode Draft	0.223	0.172	0.165
Tureyen & Frosch	0.245	0.178	0.167
Zsutty	0.244	0.185	0.168
Okamura & Higai	0.176	0.159	0.157
Bazant and Yu	0.166	0.156	0.154
Russo et al.	0.156	0.146	0.146

Calibration of existing models

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \cancel{\ln[c(\mathbf{x})]} + \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$$

- Use the fractions of the empirical formulas as the explanatory functions

  e.g. Zsutty's model $$V_c = 2.2\left(f_c'\rho \frac{d}{a} \right)^{1/3} b_w d$$

  $$h_i(\mathbf{x}) : 2, f_c', \rho, \frac{a}{d}, b_w, d$$

- Do not drop explanatory terms with large c.o.v.'s

- Explanatory functions **do not have to be dimensionless**
  ~ may be **more effective** in representing the physics than the dimensionless terms

$\mu_\sigma = 0.166 \cong 0.168$ (posterior mean by $\ln[c(\mathbf{x})] + \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$)

Construction of new models

- Select some dimensional terms to make the same dimension as quantity and add more non-dimensional terms. Perform the Bayesian parameter estimation by models such as

$$\ln[C(\mathbf{x},\mathbf{\Theta})] = \sum_{i=1}^{p} \theta_i \ln[h_i(\mathbf{x})] + \sigma\varepsilon$$ **Product**

$$\ln[C(\mathbf{x},\mathbf{\Theta})] = \sum_{i=1}^{l} \theta_i \ln[h_i(\mathbf{x})] + \ln\left[\prod_{i=l+1}^{m} h_i^{\theta_i} + \prod_{i=m+1}^{n} h_i^{\theta_i}\right] + \sigma\varepsilon$$ **Product of Sums**

- Do not drop "dimensional" explanatory terms
- Useful when

  (1) there exist no empirical models that can be used as a base model.
  (2) the effects of explanatory terms are not well known.

- Shear strength example: tried 17 explanatory terms

  → Similar forms & parameter values with the two best formulas (with smaller μ_σ)

  - Zsutty's $\quad V_c = 2.2\left(f_c'\rho\frac{d}{a}\right)^{1/3} b_w d$
  - Okamura & Higai $\quad V_c = 0.2\frac{(100\rho)^{1/3}}{(d/1000)^{1/4}}(f_c')^{1/3}\left(0.75+\frac{1.40}{a/d}\right)^{1/3} b_w d$

“Probabilistic” Models

- General form

$$\ln[C(\mathbf{x},\boldsymbol{\Theta})] = \ln[\hat{C}(\mathbf{x},\boldsymbol{\theta})] + \sigma\varepsilon \longrightarrow C(\mathbf{x},\boldsymbol{\Theta}) = \hat{C}(\mathbf{x},\boldsymbol{\theta}) \cdot \exp(\sigma\varepsilon)$$

- Capacity ~ follows the **lognormal** distribution
- Mean and c.o.v. are derived as

$$\mu_C(\mathbf{x}) = \hat{C}(\mathbf{x},\boldsymbol{\mu}_\theta) \cdot \exp(\mu_\sigma \varepsilon) \cong \hat{C}(\mathbf{x},\boldsymbol{\theta}) \text{ for } \mu_\sigma << 1$$

$$\delta_C(\mathbf{x}) = \delta_C = [\exp(\mu_\sigma^2) - 1]^{1/2} \cong \mu_\sigma \text{ for } \mu_\sigma << 1$$

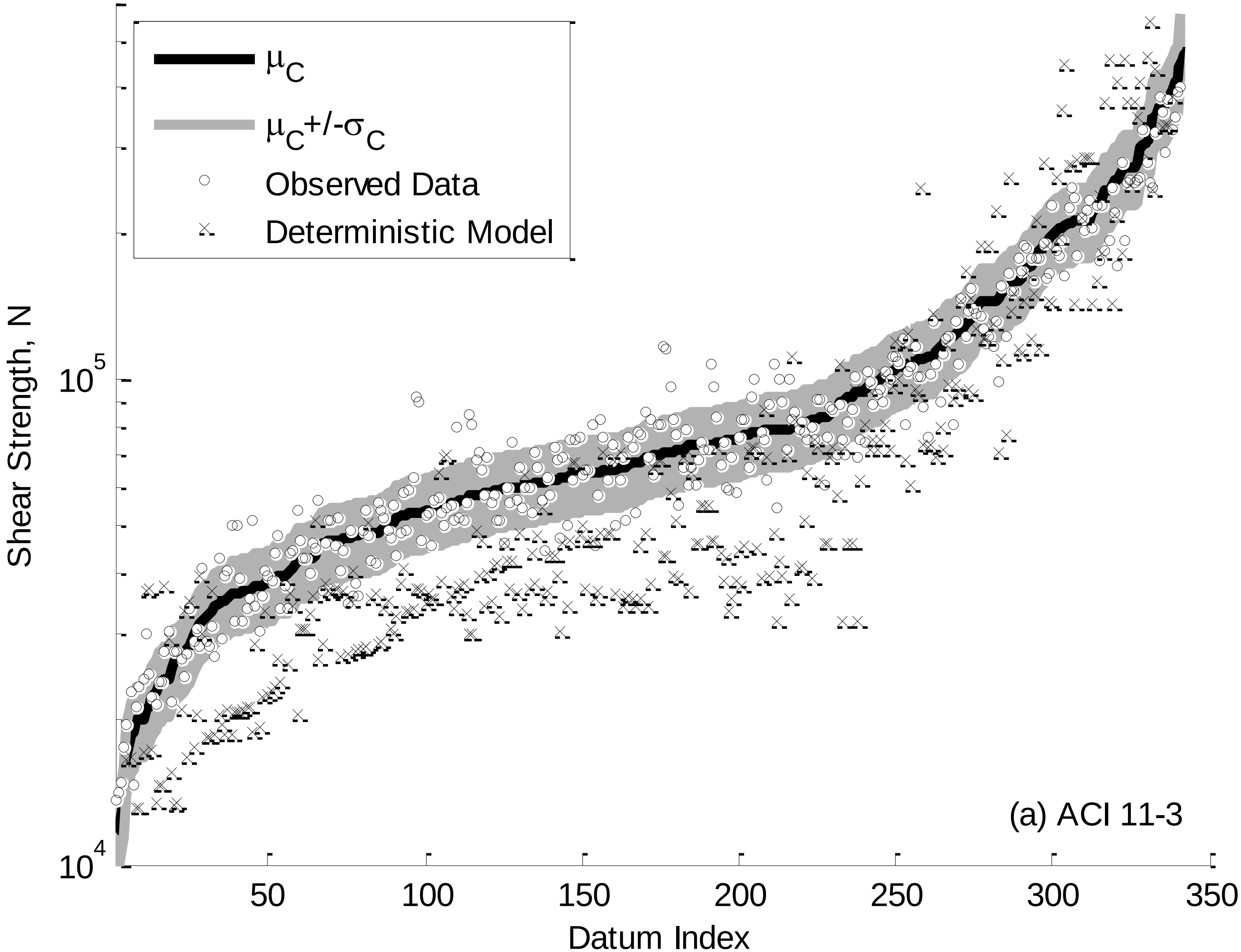
- Conditional pdf of capacity for given $\mathbf{x}$

$$f_C(c\,|\,\mathbf{x}) = \frac{1}{\sqrt{2\pi}\mu_\sigma c} \exp\left[-\frac{1}{2} \left(\frac{\ln c - \ln \hat{C}(\mathbf{x},\boldsymbol{\mu}_\theta)}{\mu_\sigma} \right)^2 \right]$$

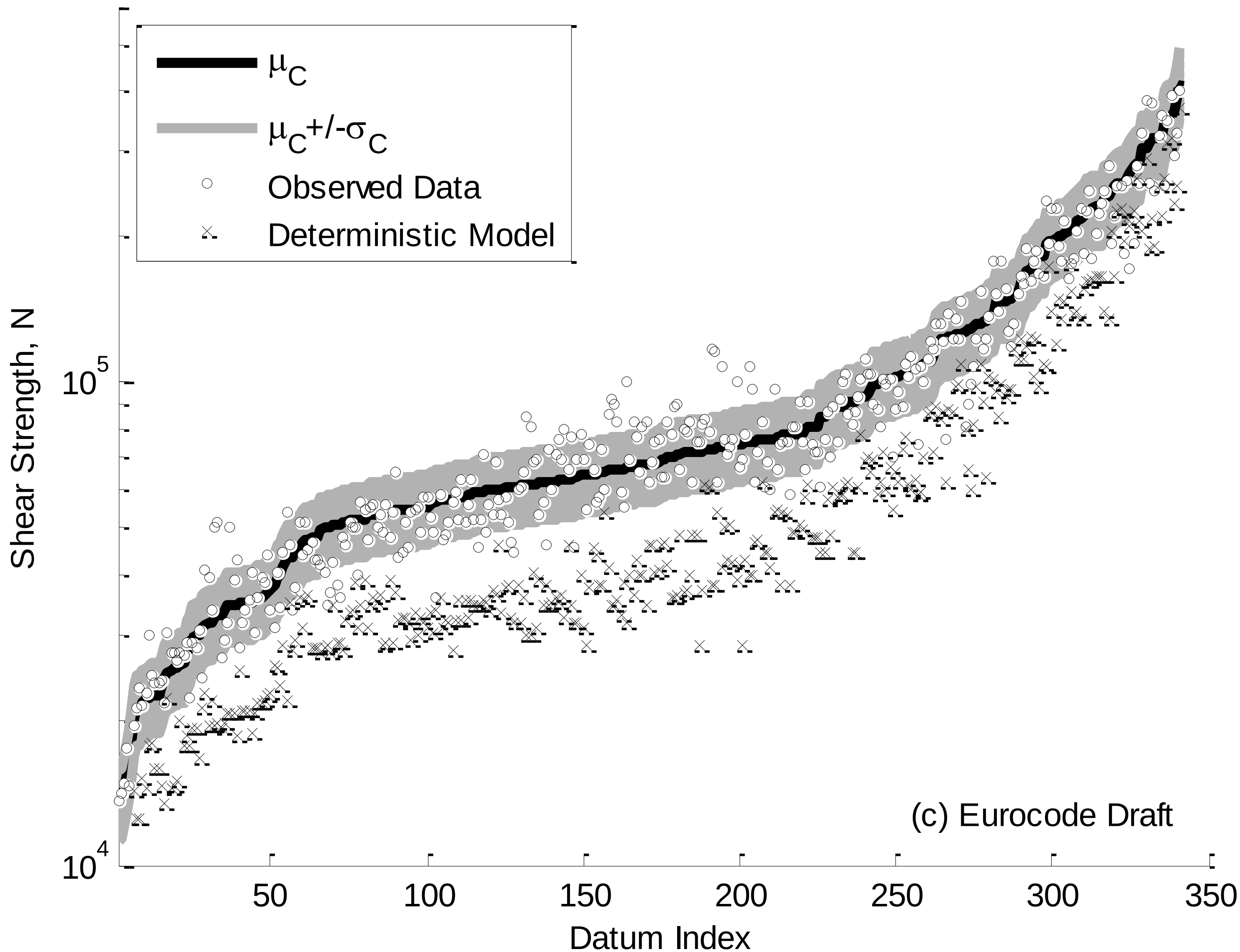
- Predictive pdf of capacity for unknown $\mathbf{x}$

$$f_C(c) = \int_{-\infty}^{\infty} f_C(c\,|\,\mathbf{x}) \cdot f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}$$

Performance of probabilistic models

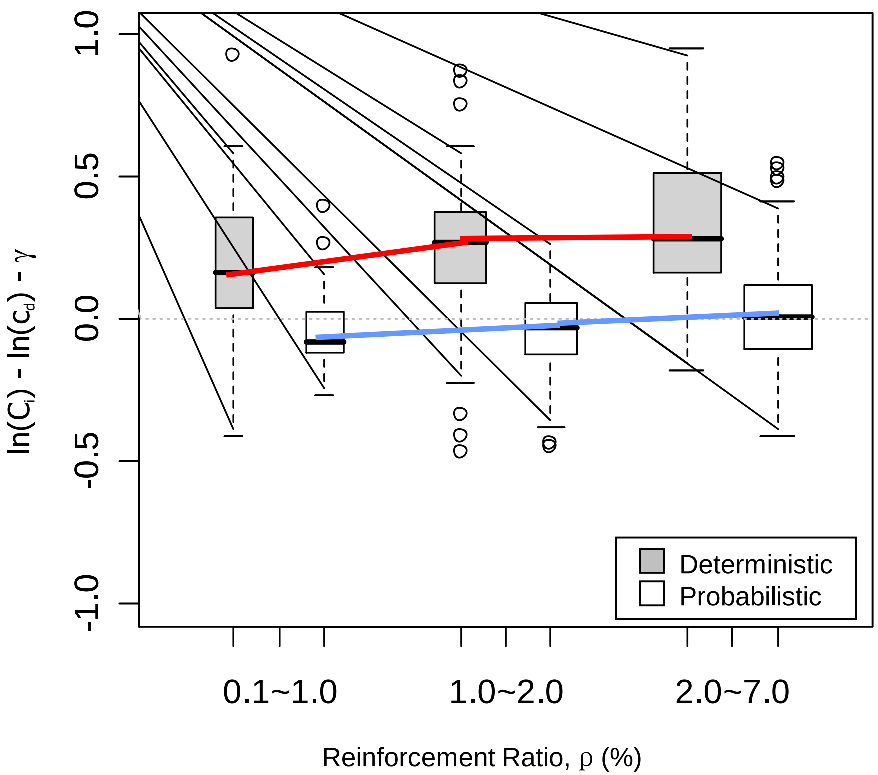


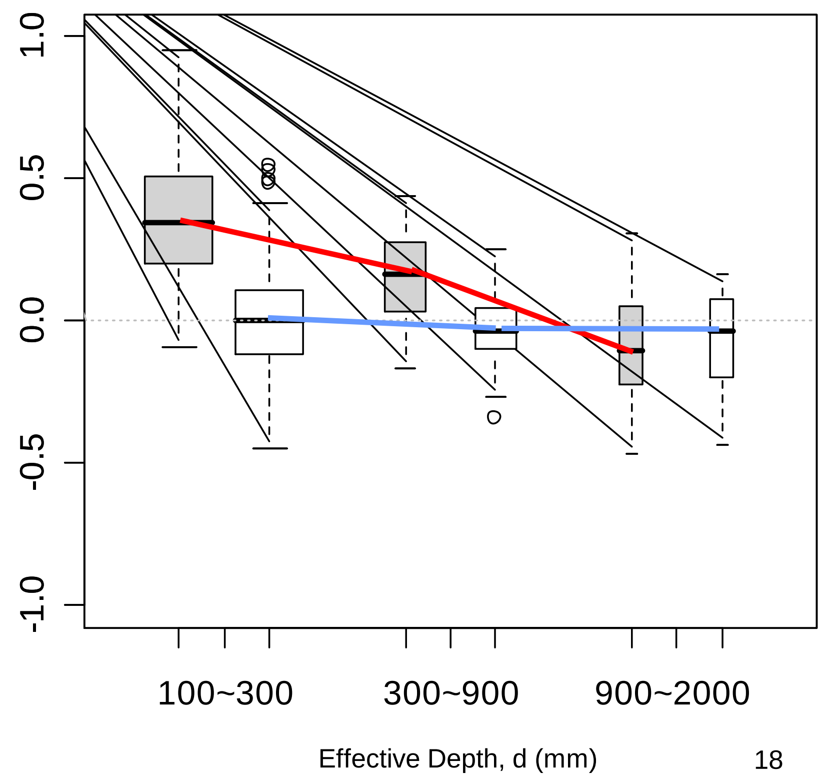
Performance of probabilistic models



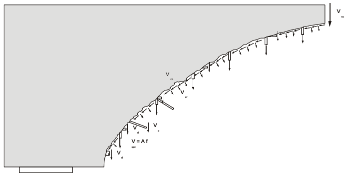
Performance of probabilistic models

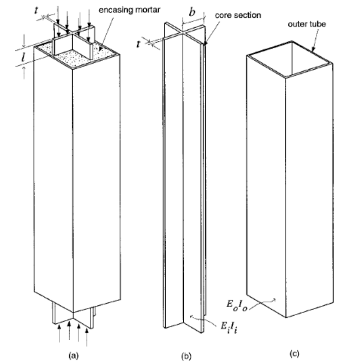
- e.g. Tureyen & Frosch (2003) and a probabilistic strength model developed by this study
- Box plots of errors ~ show that the developed models are unbiased and have consistently good performance for the whole ranges of the parameters.





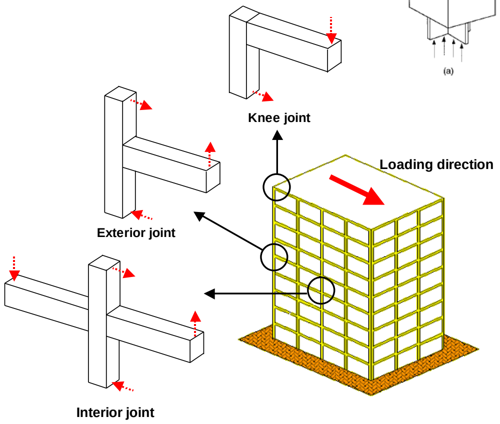
Other Applications





- Shear strengths of RC beams with shear reinforcements
(W.-H. Kang, J. Song, and K.S. Kim)

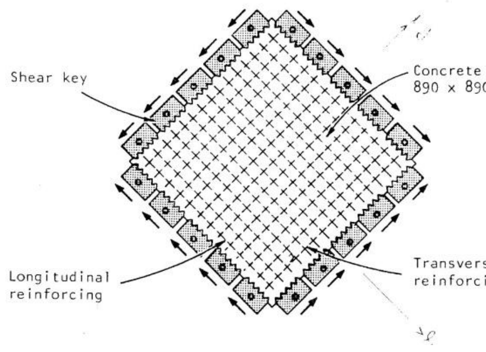
- Seismic strengths of buckling-restrained bracings
(B.M. Andrews, J. Song, and L.A. Fahnestock)
(Andrews et al. 2009a, 2009b)



- Strengths/ of RC beam-column connections
(J. Kim, J.M. LaFave, and J. Song)

- Statistical validation/verification of concrete FEM
(H.H. Lee and D.A. Kuchma)

- Shear strengths of RC "deep" beams (strut-and-tie models)
(Chetchotisak, P., J. Teerawong, S. Yindeesuk, and J. Song, 2014)



- Course term projects
- Strengths of concrete-filled tubes (Mark Denavit)
- Fracture toughness (Tam H. Nguyen)

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- Song, J., W.-H. Kang, K.S. Kim, and S. Jung. "Probabilistic shear strength models for reinforced concrete beams without shear reinforcement based on experimental observations," under review [→ Shear strengths of RC beams without stirrups]
- Kang, W.-H., J. Song, and K.S. Kim (2007). "Probabilistic shear strength models for reinforced concrete beams with shear reinforcements by Bayeisan updating." *Proc. 18th Engineering Mechanics Division Conference of ASCE (ASCE EMD 2007)*, June 3-6, Blacksburg, VA. [→ Shear strengths of RC beams with stirrups]
- Kim, J., J.M. LaFave, and J. Song (2007). "A new statistical approach for joint shear strength determination of RC beam-column connections subjected to lateral earthquake loading." *Structural Engineering and Mechanics*, Vol. 27(4), 439-456 [→ Strength of RC beam-column connection ~ strength only]
- Kim, J., J.M. LaFave, and J. Song. Joint shear behavior of RC beam-column connections, under review [→ RC beam-column connections ~ strength & corresponding strain]
- Chetchotisak, P., J. Teerawong, S. Yindeesuk, and J. Song (2014). "New strut-and-tie-models for shear strength prediction and design of RC deep beams." *Computers and Concrete*, 14(1): 19-40 [→ Shear strength of deep RC beams]