understand the physics of phenomena through a systematic identification of coupling terms. Also, these simple models have often been proven very useful for rotor system design and development.

A better representation for a rotor blade is to treat it as an elastic beam undergoing flap bending, lead-lag bending and torsion deflections. Several authors have investigated the aeroelastic stability of an elastic blade (see, general review papers 4-7). The equations of motion are given in 3.11 for a uniform blade and the generic aerodynamic forces are defined in 4.6. The common approach for calculating the trim deflections, as well as the aeroelastic stability, is the model method (Galerkin or Rayleigh-Ritz) using either coupled natural modes (rotating) or uncoupled beam modes (nonrotating). In general, the trim deflections are assumed to be large and are obtained by solving nonlinear steady-state equation; and the flutter equations of motion are linearized about the trim state. With the modal approach, it becomes increasingly difficult to handle geometric complexities. For example, it is difficult to effectively model the multibeam flexure of a bearingless blade. The finite element method is perhaps an ideal choice for complex blade configurations. The blade is divided into a number of beam elements and the application of energy principles or the method of weighted residuals yields approximate expressions for forces (inertial, elastic, etc.) over each elements. The global equations for motion are obtained by assembling the elements. Nonuniform properties can be easily accommodated. The finite element method is very flexible and the formulation can be adapted to different rotor blade configurations with a few modifications. Multibeams of a bearingless blade can be modeled individually (Refs. 10-11).

Most analyses apply quasisteady strip theory to obtain aerodynamic forces. Forces of noncirculatory origin from unsteady thin airfoil theory are also included. Normally, linearized two-dimensional airfoil lift, drag, and pitch moment coefficients are used. Typically,

$$c_l = a\alpha$$

$$c_d = c_{d_0}$$

$$c_m = 0$$

The correlation of theoretical and experimental results from scaled models has shown that nonlinear airfoil section characteristics may significantly influence low-frequency flap-lag-torsion stability. For example,

$$c_l = c_0 + c_1\alpha + c_2\alpha|\alpha|$$

$$c_d = d_0 + d_1\alpha^2|\alpha|$$

$$c_m = f_0 + f_1\alpha$$

appears quite adequate representation below stall condition. Some analyses have used data tables to obtain airfoil characteristics. Linearization of airfoil lift, drag and pitch moment coefficients about a trim angle of attack provides a simple way of treating these effects in a linear stability analysis.

$$c_l(\alpha) = c_l(\alpha_0) + \frac{\delta c_l}{\delta\alpha}(\alpha_0)\Delta\alpha$$

$$c_d(\alpha) = c_d(\alpha_0) + \frac{\delta c_d}{\delta\alpha}(\alpha_0)\Delta\alpha$$

$$c_m(\alpha) = c_m(\alpha_0) + \frac{\delta c_m}{\delta\alpha}(\alpha_0)\Delta\alpha$$

and

$$\alpha = \alpha_0 + \Delta\alpha$$

where a_0 is the trim angle of attack and $\delta\alpha$ is the perturbation in the angle of attack.

The induced inflow is calculated using simple momentum theory. The assumption of uniform inflow is widely used, though it is strictly true for ideally twisted blades. It is, however, observed that a small variation in inflow distribution has negligible influence on blade stability.

Quasisteady assumption appears satisfactory for low frequency modes. For high frequency pitch-flap flutter, one needs to include unsteady aerodynamic effects. The influence of unsteady aerodynamics can be introduced through a suitable modification of the circulatory lift with a suitable lift deficiency function. The airfoil characteristics become

$$c_l = c_l(\alpha_0) + \frac{\delta c_l}{\delta \alpha}(\alpha_0)\Delta\alpha$$

$$c_d = c_d(\alpha_0) + \frac{\delta c_d}{\delta \alpha}(\alpha_0)\Delta\alpha$$

$$c_m = c_m(\alpha_0) + \frac{\delta c_m}{\delta \alpha}(\alpha_0)C(k)\Delta\alpha$$

where C(k) is lift deficiency function and k is the reduced frequency, $\frac{\omega c}{2U}$. The ω is the frequency of oscillation, c is the chord, and U is the free-stream velocity. For hover, U becomes equal to Ωr, where Ω is rotational speed (rad/sec) and r is the radial position. The value of reduced frequency k varies along the length of the blade; the smaller value at the tip and the larger value near the root end of the blade. For hover, it is appropriate to use the Loewy function $C(k)$ and for forward flight Theodorsen function $C(k)$ is perhaps a better choice. There are two problems with this approach. First, the reduced frequency k varies radially and the second, $C(k)$ is a complex number. The first problem can be covered approximately through finite element formulation. The blade is divided into a number of elements and for each element an average value of reduced frequency corresponding to the mid-point is used. With the inclusion of complex numbers for lift deficiency functions, the equations become complex and there is no easy way to solve these equations. Thus,

$$C(k) = F(k) + iG(k)$$

One possible way is to arbitrarily neglect the complex component from the lift deficiency function ($G(k) = 0$) and retain the real component. There is a little justification with this assumption; more so, with higher frequencies.

Another simple way to include unsteady aerodynamic effects is to use dynamic inflow modeling. As discussed in art. 4.6, the dynamic inflow components are related to perturbation rotor loads (thrust, roll moment and pitch moment). For hovering flight, the dynamic inflow model is quite simple and is given as

$$\dot{\tilde{\lambda}} + \tilde{\lambda} = \text{sign } (c_T)k_p^2\frac{\Delta c_T}{4\lambda_0}$$

where $\tilde{\lambda}$ is a perturbation to the induced inflow from its steady value λ_0.

The blade motion is assumed to be small perturbation about steady deflected shape. The steady blade equilibrium position has an important influence on blade stability. The steady-state equations are obtained for hovering flight after dropping the time dependent terms. These nonlinear equations are solved iteratively using the Newton-Raphson procedure. The next step is to obtain the natural vibration characteristics of the rotating blade about its equilibrium position. This is done removing all aerodynamic terms and also neglecting damping matrix. This gives real eigenvalues. The last step is to calculate the flutter stability. Typically, this is done through the normal mode equations using few (about 6) natural vibration modes. This results in a complex eigenvalue problem. The condition of negative damping for a mode results in dynamic instability.

The correlation of experimental data with analytical results obtained using different codes for blade and rotor stability has been presented in ITR Methodology Workshop (1983). One particular example of hingeless blade in hovering flight is worked out here using finite element analysis. The blade is divided into seven elements. Each element consists of two end nodes and three internal nodes, with a total of fifteen degrees of freedom. Each of the end nodes has six degrees of freedom, namely, axial deflection u, lead-lag deflection v, v', flap deflection w, w', and elastic twist ϕ. The input data is given below.

Configuration Hingeless Rotor Isolated Stability
(Task IIA) Soft Flexure case

$$\begin{array}{lcll} \text{Rotor RPM} & & = & 1000 \\ \text{Lock Number} & \gamma & = & 5.3 \\ \text{Solidity ratio} & \sigma & = & .057 \\ \text{chord/radius} & \frac{c}{R} & = & .09 \\ \text{reference life curve slope} & a_r & = & 6.0 \end{array}$$

zero precone, zero pretwist, zero offsets of aerodynamic center, cg and tension center from elastic axis

$$c_l = 6\alpha - 10\alpha^2 \ \ (\text{sign } \alpha)$$

$$c_d = .01 + 11.1(\alpha)^3(\text{sign } \alpha)$$

$$c_m = 0$$

7 Finite Elements (No. 1 from tip)
lengths in terms of radius .1, .1, .1, .1, .25, .255, .095.
structural properties same for elements 1-6
Flapwise $EI_y/m_0\Omega^2R^4 = .005239$
Chordwise $EI_z/m_0\Omega^2R^4 = .1067$
Torsion $GJ/m_0\Omega^2R^4 = .00157$
Torsion Inertia $k_m^2/R^2 = .000647 = k_a^2/R^2$
m/m_0=1
For root element 7

$$EI_y/m_0\Omega^2R^4 = .1477 \qquad EI_z/m_0\Omega^2R^4 = .1866$$

$$GJ/m_0\Omega^2R^4 = .00116 \qquad k_m^2/R^2 = k_a^2/R^2 = .0131$$

$$m/m_0 = 12.1$$

Given nonrotating frequencies
flap frequency = .311/rev
lag frequency = 1.32/rev
torsion freq. = 2.30/rev
lag structural damping $2\zeta_L\frac{\omega_{\zeta 0}}{\Omega} = .0196$

For unsteady aerodynamics

Reduced frequencies k corresponding to lag mode for elements 1-7 are .066, .074, .084, .097, .133, .283 and 1.326 and Theodoresen's lift deficiency function C(k) are .8825, .8699, .8363, .7922, .6746 and .5262.
Calculated rotating frequencies ($\theta = 0$)
flap frequency = 1.17/rev
lag frequency = 1.33/rev
torsion freq. = 2.97/rev
The following reports discuss aeroelastic stability in hover for different types of rotors.

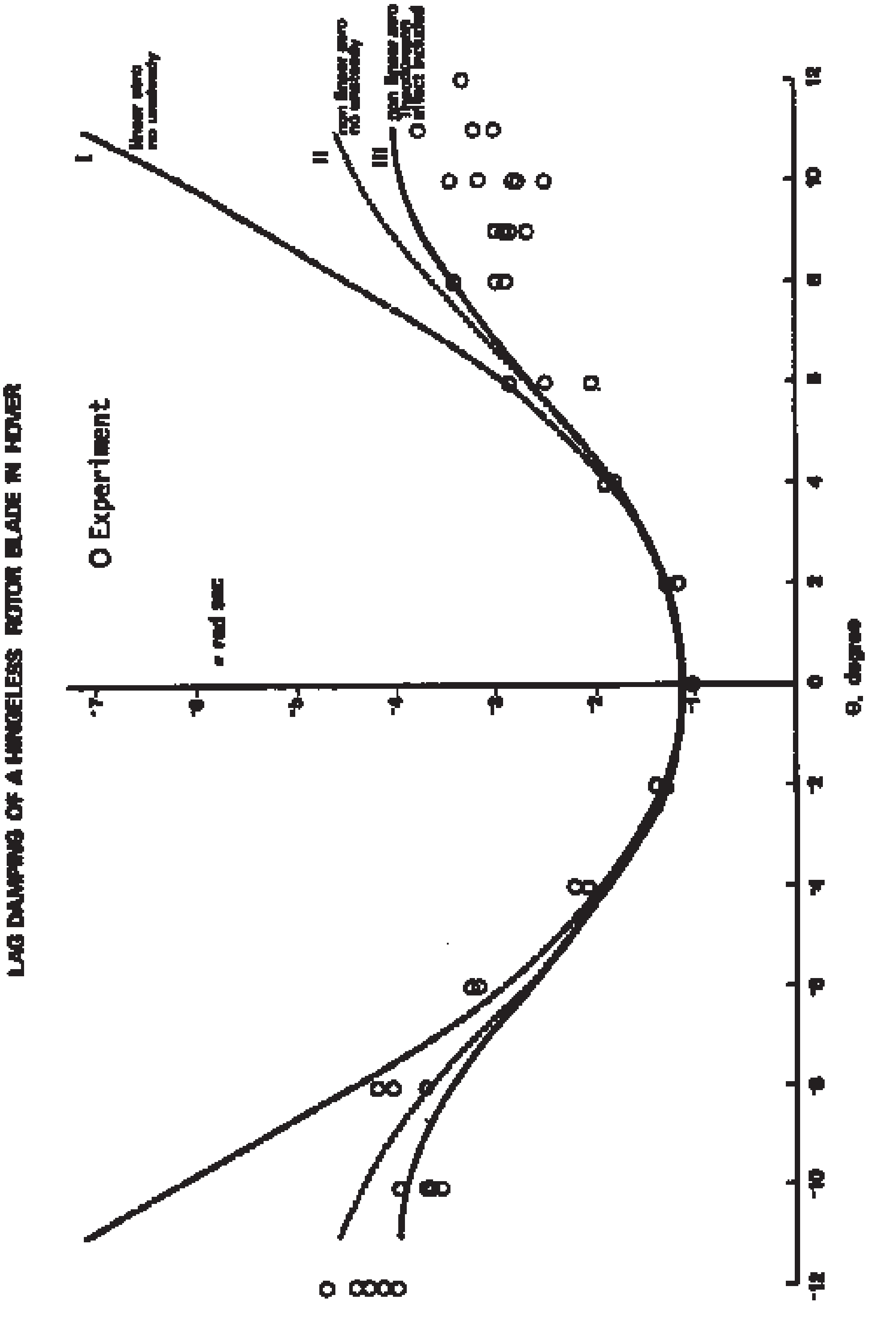
(a) Hingeless rotors - Hodges and Ormiston (1976).

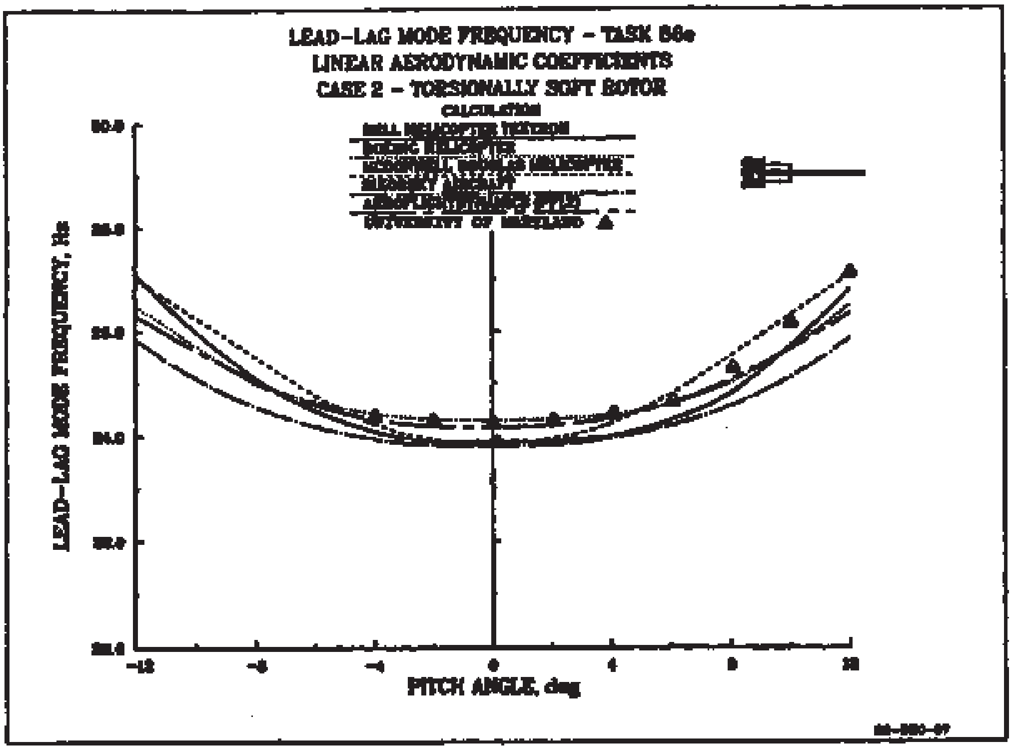
(b) Circulation control rotors Chopra and Johnson (1979) Chopra (1984).

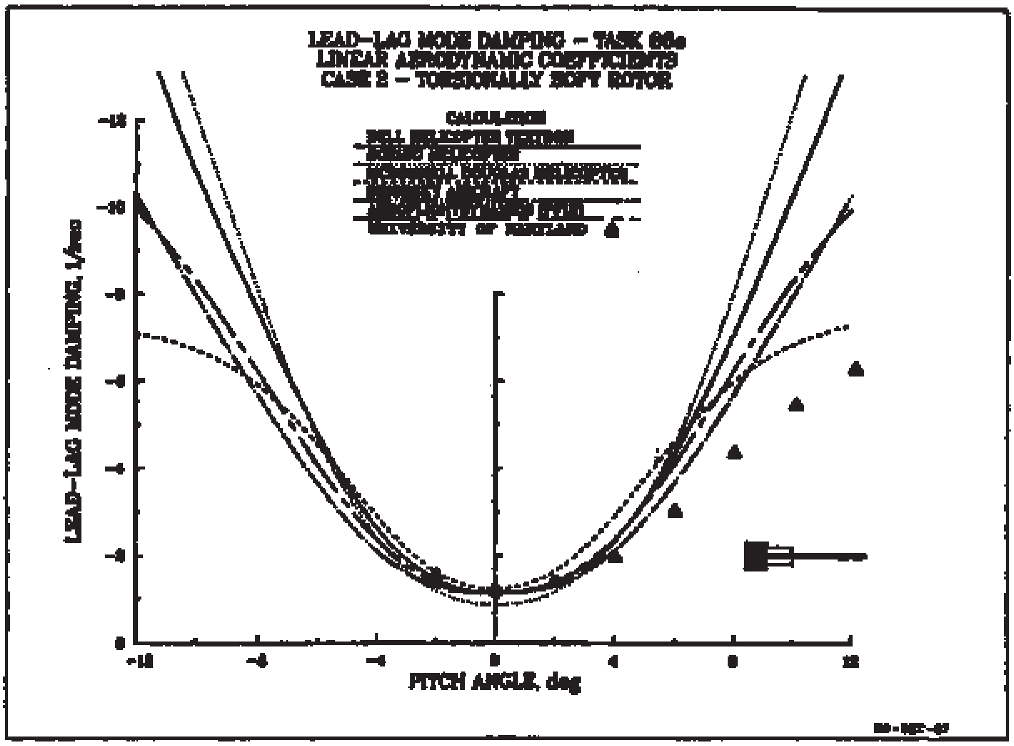
(c) Composite Blades - Hong and Chopra (1985).

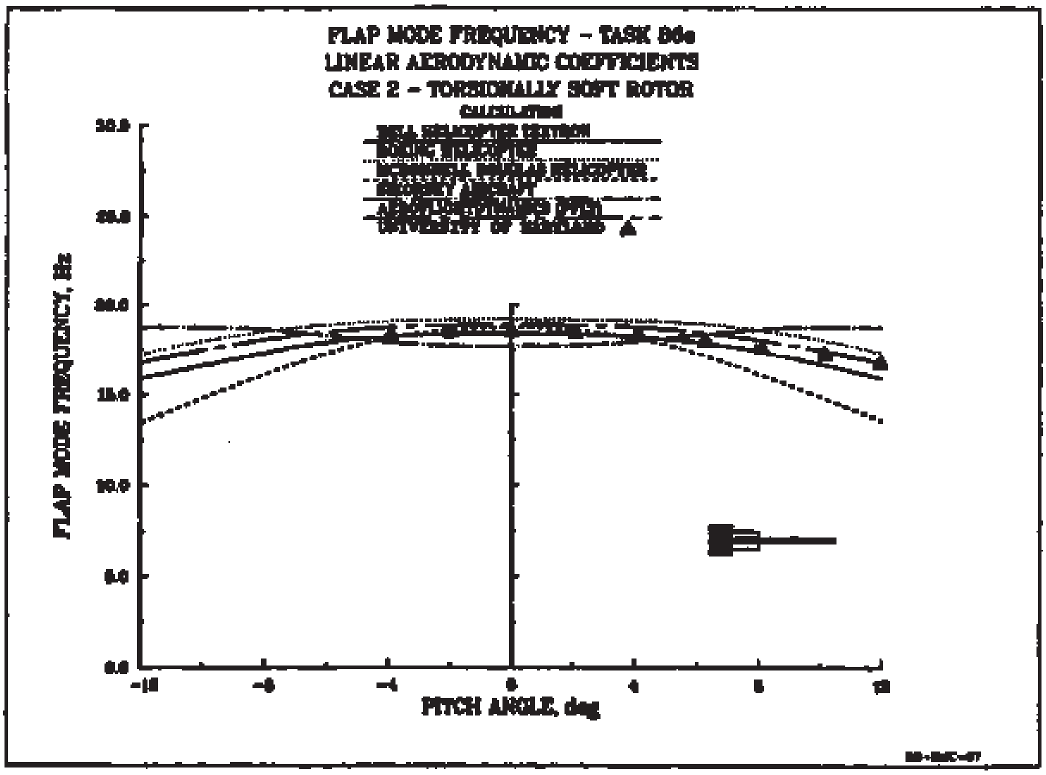
(d) Bearingless Rotors - Sivaneri and Chopra (1984), Chopra (1984), Hong and Chopra (1985), Hodges (1979).

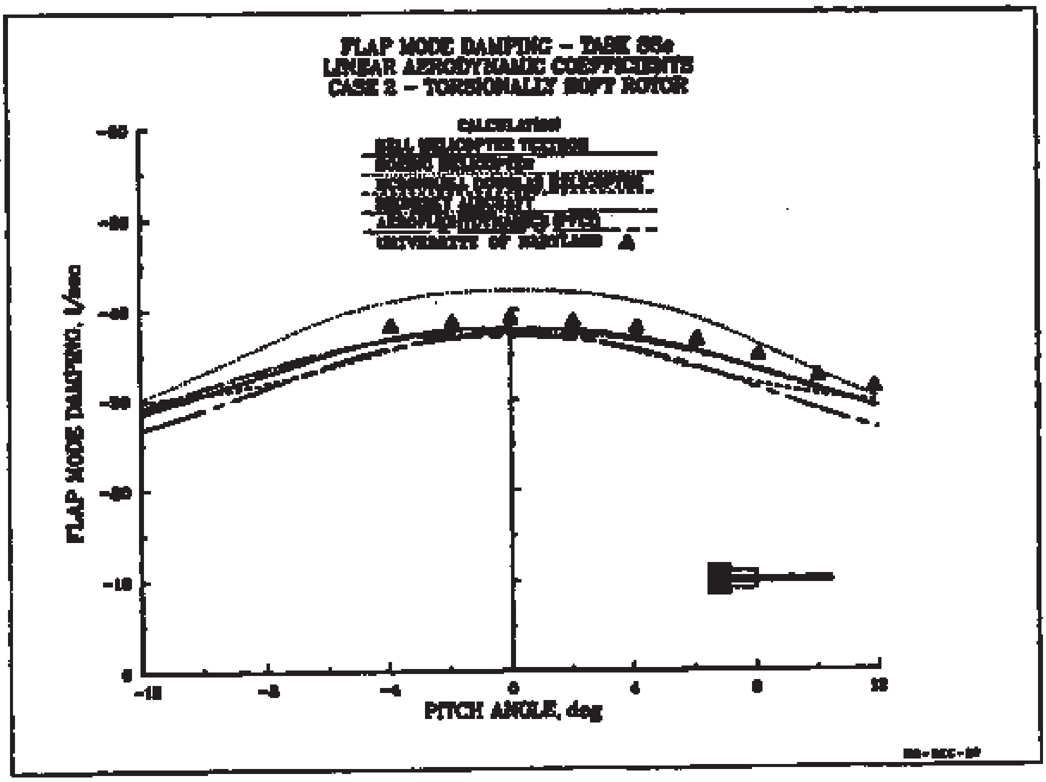
(e) Tilting Proprotor (JVX, XV-15) - Johnson (1975).

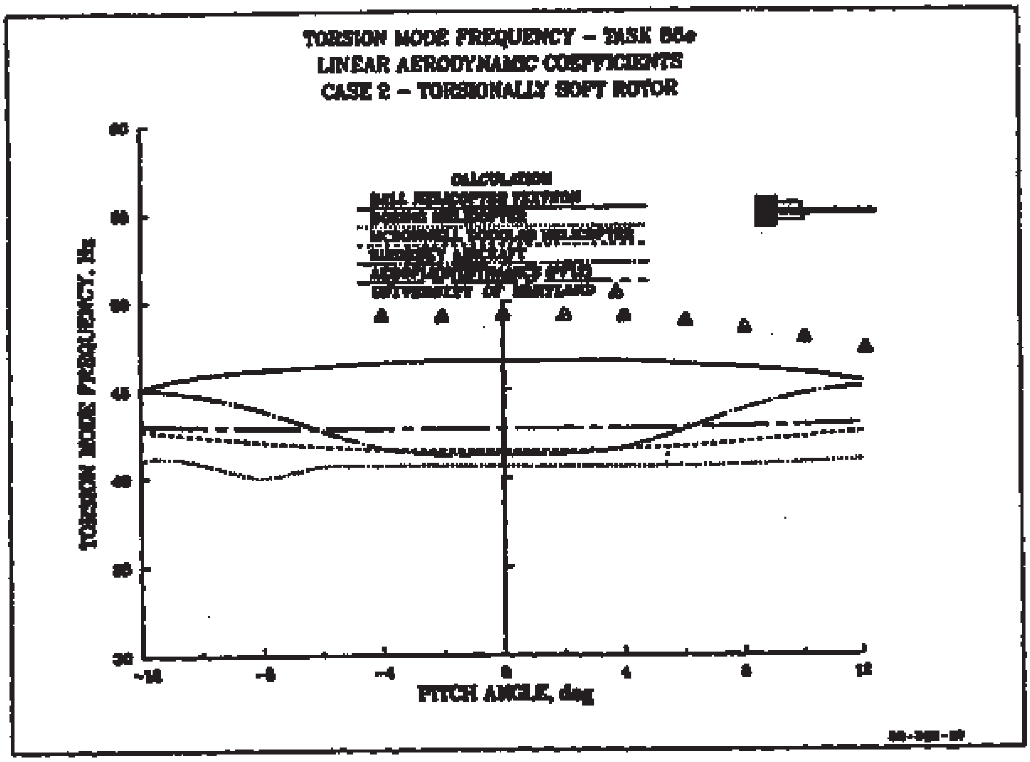

LAG DAMPING OF A HINGELESS ROTOR BLADE IN HOVER
O Experiment
θ, degree

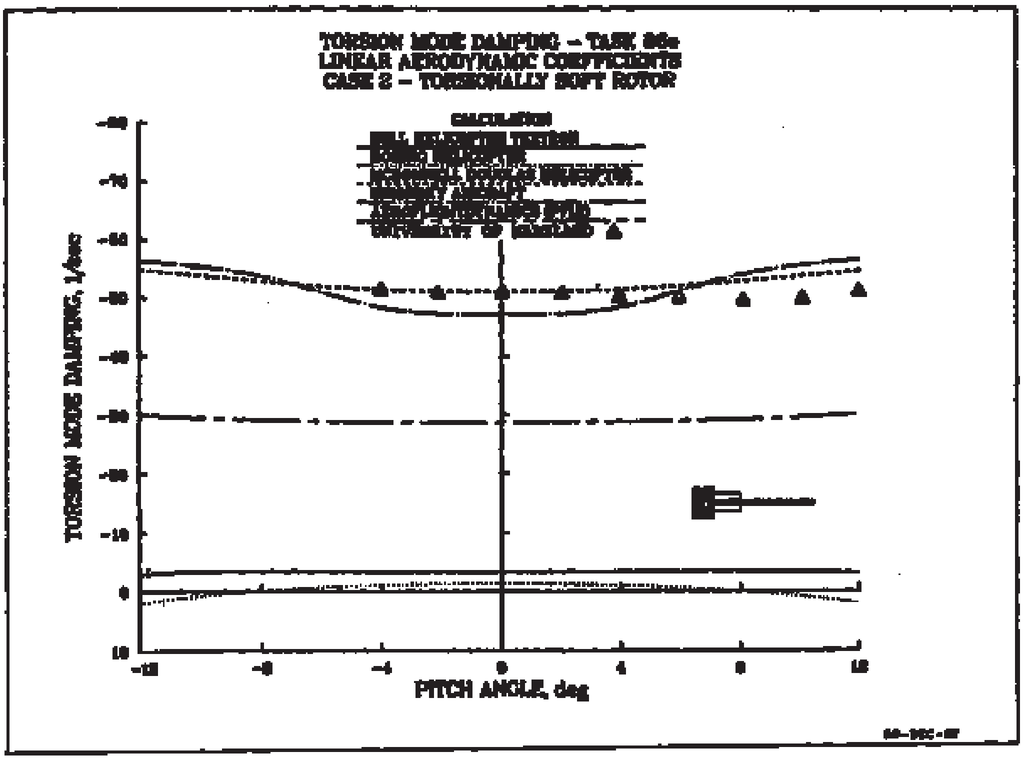

LINEAR AERODYNAMIC COEFFICIENTS
CASE 2 - TORSIONALLY SOFT ROTOR
CALCULATION
UNIVERSITY OF MARYLAND
LEAD-LAG MODE FREQUENCY, Hz
-12
-8
-4
0
4
8
12
PITCH ANGLE, deg

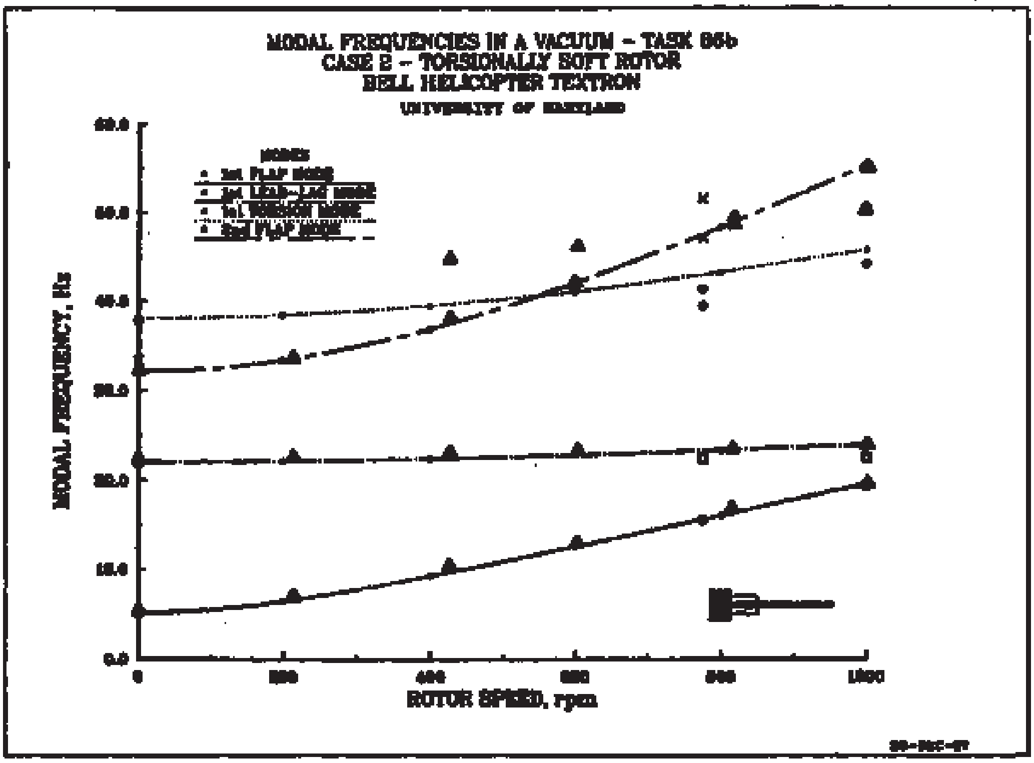

LINEAR AERODYNAMIC COEFFICIENTS
CASE 2 - TORSIONALLY SOFT ROTOR
CALCULATION
UNIVERSITY OF MARYLAND
LEAD-LAG MODE DAMPING, 1/sec
-12
-10
-8
-6
-4
-2
0
-12
-8
-4
0
4
8
12
PITCH ANGLE, deg

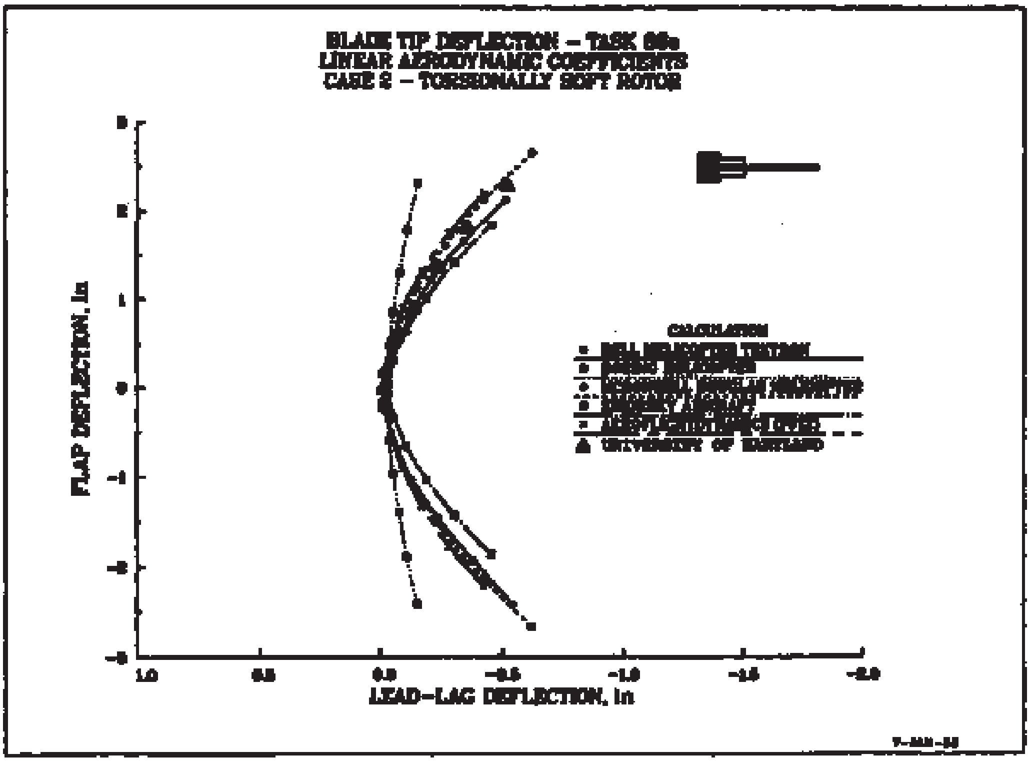

FLAP MODE FREQUENCY – TASK 86e
LINEAR AERODYNAMIC COEFFICIENTS
CASE 2 – TORSIONALLY SOFT ROTOR
CALCULATIONS
UNIVERSITY OF MARYLAND
FLAP MODE FREQUENCY, Hz
PITCH ANGLE, deg


FLAP MODE DAMPING – TASK 86e
LINEAR AERODYNAMIC COEFFICIENTS
CASE 2 – TORSIONALLY SOFT ROTOR
CALCULATIONS
UNIVERSITY OF MARYLAND
FLAP MODE DAMPING, 1/sec
PITCH ANGLE, deg


TORSION MODE FREQUENCY – TASK
LINEAR AERODYNAMIC COEFFICIENTS
CASE 2 – TORSIONALLY SOFT ROTOR
CALCULATION
UNIVERSITY OF MARYLAND
TORSION MODE FREQUENCY, Hz
PITCH ANGLE, deg
-12
-8
-4
0
4
8
12


TORSION MODE DAMPING – TASK
LINEAR AERODYNAMIC COEFFICIENTS
CASE 2 – TORSIONALLY SOFT ROTOR
CALCULATION
UNIVERSITY OF MARYLAND
TORSION MODE DAMPING, 1/sec
PITCH ANGLE, deg
-12
-8
-4
0
4
8
12





<u>Questions</u>

Justify the following:

- Flutter is different from forced vibrations.

- Flap-lag flutter is a unique aeroelastic instability with rotor blades, and it has nothing to do with fixed wing.
- Flap-lag flutter is a weak instability and can be easily stabilized.
- Soft lag rotors get stabilized with a small elastic coupling.
- Pitch divergence of the blades does not depend on the elastic axis position.
- Through a simple analysis, the blade was found to be unstable from pitch-flap instability and the flutter frequency was calculated to be 18 Hz. During the hover test, the rotor model was found to be quite stable at the operating speed of 360 RPM. However, when the speed was slightly reduced, an instability appeared and the rotor started shaking violently.
- After the blade was built, the analysis showed the possibility of pitch-flap flutter. You would like to do some quick fix to the problem.
- How would you identify the wake excited flutter? Suggest ways to get rod of it.
- A great effort is made to keep the cg and the elastic axis at the quarter-chord position.
- Through a quasielastic torsion modelling, the important pitch-flap and pitch-lag terms are retained.
- The pitch divergence of the blades does not depend on the thrust level at which the rotor is being operated.

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Chapter 6

Ground and Air Resonance

6.1 Ground Resonance

Ground resonance is a dynamic instability caused by the coupling of the blade lag motion and the hub inplane motion. The word resonance is used because at the instability condition one of the lag frequencies in the fixed frame becomes equal to the support frequency. It is called ground resonance because this instability takes place when the helicopter is on the ground. That is the reason that the landing gear and the supporting structure characteristics are important for this instability. It is also called mechanical instability because the aerodynamic forces do not play an important role in causing this instability. The ground resonance is a problem of soft lag rotors.

Ground resonance is a violent instability and would result in a catastrophe. A major design consideration is to avoid this instability. The selection of the operating rotor speed is made with the consideration that there is no possibility of resonance at or near this speed. The inclusion of damping in lag mode is very beneficial for this instability. This is the reason that most of the flying rotors have mechanical lag dampers near the root of the blade. This type of instability is also possible when the helicopter is in flight, then it is called air resonance. This instability is more common with hingeless blades.

6.1.1 Blade Lag Motion in Fixed Coordinates

Let us examine the Fourier coordinate transformation of blade lag motion.

$\zeta^{(m)}$ = lag motion of the m^{th} blade in rotating frame

$\zeta_o, \zeta_{nc}, \zeta_{ns}, \zeta_{N/2}$ = lag motions in fixed frame

The FCT is a linear transformation of N degrees of motion in the rotating frame to N degrees of motion in the fixed frame.

For N bladed rotor

$$\zeta_o = \frac{1}{N}\sum_{m=1}^{N}\zeta^{(m)}$$

$$\zeta_{nc} = \frac{2}{N}\sum_{m=1}^{N}\zeta^{(m)}\cos n\psi_m$$

$$\zeta_{ns} = \frac{2}{N}\sum_{m=1}^{N}\zeta^{(m)}\sin n\psi_m$$

$$\zeta_{N/2} = \frac{1}{N}\sum_{m=1}^{N}\zeta^{(m)}(-1)^m$$

and

$$\zeta^{(m)} = \zeta_o + \sum_{n=1}(\zeta_{nc}\cos n\psi_m + \zeta_{ns}\sin n\psi_m) + \zeta_{N/2}(-1)^m$$

The summation

$$\sum_n \Rightarrow \begin{array}{ll} n = 1 \text{ to } \frac{N-1}{2} & \text{for N odd} \\ n = 1 \text{ to } \frac{N-2}{2} & \text{for N even} \end{array}$$

and

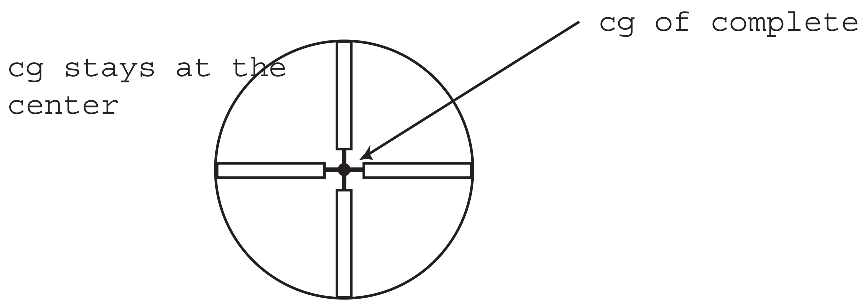
$$\begin{array}{ll} \psi_m = \psi + (m-1)\Delta\psi & \text{m=1,2,\ldots,N} \\ \Delta\psi = \frac{2\pi}{N} & \\ \psi = \Omega t & \text{where } \Omega \text{ is rotational speed.} \end{array}$$

Let us examine the fixed frame terms. First consider a four bladed rotor. The analysis is similar for three bladed rotors. Then consider a two bladed rotor. The analysis for two bladed rotors is distinctly different from three or higher blades.

6.1.2 Three and Four bladed Rotors

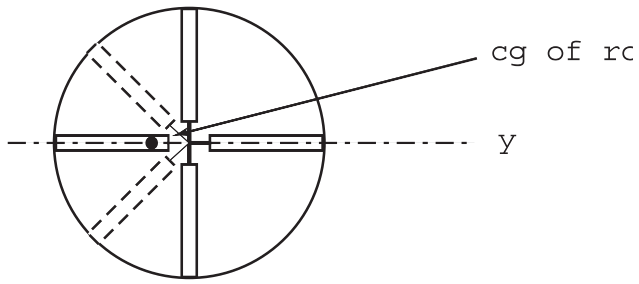
For 4-bladed rotors, N=4, there are four rigid lags in the rotating frame, one for each blade. This results in four degrees of motion in the fixed frame, i. e. , ζ_o, ζ_{lc}, ζ_{ls}, and ζ_2.

$$\zeta^{(m)} = \zeta_o$$



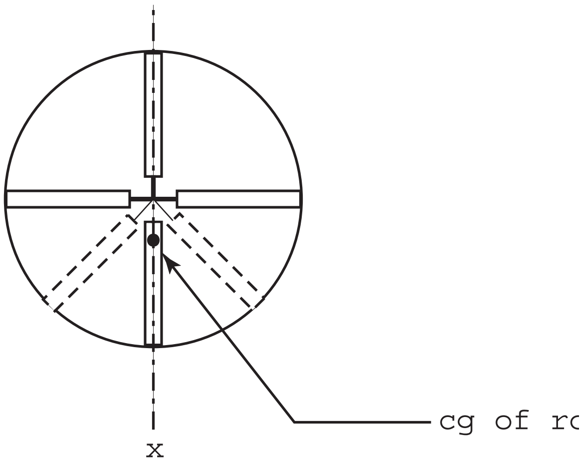
ζ_o is a collective lag motion

$$\zeta^{(m)} = \zeta_{lc}\cos\psi_m$$



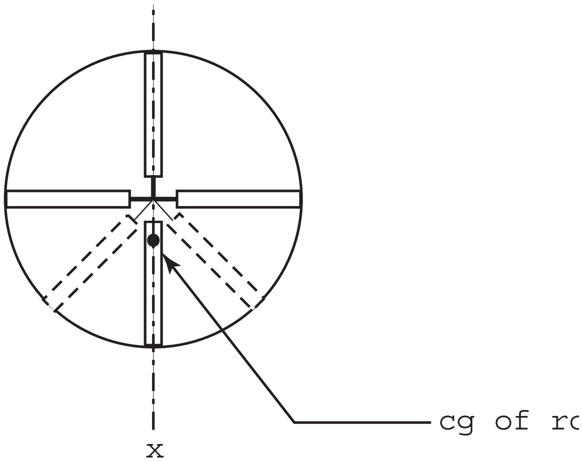
ζ_{lc} represents a lateral shift of rotor cg, in negative y directional.

$$\zeta^{(m)} = \zeta_{ls} \sin \psi_m$$



ζ_{ls} represents a longitudinal shift of rotor cg, in positive x direction.

$$\zeta^{(m)} = \zeta_2 (-1)^m$$



ζ_2 Scissoring motion, cg stays at the center.

The transformed lag motion in the fixed system can be coupled with the hub motion. The uncoupled lag equation for a blade with rigid lag is

$$\mathrm{I}_\zeta(\overset{**}{\zeta} + \nu_\zeta^2 \zeta + \mathrm{C}_\zeta^* \overset{*}{\zeta}) = \gamma \overline{\mathrm{M}}_\zeta$$

The right side is the aerodynamic force which is small and its effect can be taken care of through the damping term C_ζ^*.

$$\mathrm{I}_\zeta(\overset{**}{\zeta} + \nu_\zeta^2 \zeta + \mathrm{C}_\zeta^* \overset{*}{\zeta}) = 0$$

Using ‘FCT’ for a 4-bladed rotor

$$\mathrm{I}_\zeta(\overset{**}{\zeta}_o + \nu_\zeta^2\zeta_o + \mathrm{C}_\zeta^* \overset{*}{\zeta}_o) = 0$$

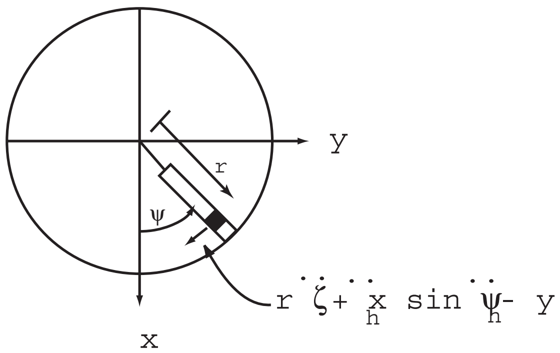
$$\mathrm{I}_\zeta(\overset{**}{\zeta}_2 + \nu_\zeta^2\zeta_2 + \mathrm{C}_\zeta^* \overset{*}{\zeta}_2) = 0$$

$$\mathrm{I}_\zeta\{\overset{**}{\zeta}_{lc} + 2\overset{*}{\zeta}_{ls} - \zeta_{lc} + \nu_\zeta^2\zeta_{lc} + \mathrm{C}_\zeta^*(\overset{*}{\zeta}_{lc} + \zeta_{ls})\} = 0$$

$$\mathrm{I}_\zeta\{\overset{**}{\zeta}_{ls} - 2\overset{*}{\zeta}_{lc} - \zeta_{ls} + \nu_\zeta^2\zeta_{ls} + \mathrm{C}_\zeta^*(\overset{*}{\zeta}_{ls} - \zeta_{lc})\} = 0$$

The last two equations are longitudinal and lateral inertial equations of the rotor. Now there is a hub motion of x_h and y_h superimposed on lag motion, then the blade lag equation becomes:

$$\mathrm{I}_\zeta(\overset{**}{\zeta} + \nu_\zeta^2\zeta + \mathrm{C}_\zeta^* \overset{*}{\zeta}) + \mathrm{S}_\zeta(\frac{\ddot{x}_h}{\Omega^2}\sin\psi - \frac{\ddot{y}_h}{\Omega^2}\cos\psi) = \gamma\overline{\mathrm{M}}_\zeta\mathrm{I}_b$$



6.1.3 Ground Resonance Equations

As we have seen earlier for a 4-bladed rotor that there is no shift of rotor cg with collective lag ζ_o and differential lag ζ_2. The important lag motions are ζ_{lc} and ζ_{ls} which cause lateral and longitudinal shift of rotor cg. These motions can couple with inplane longitudinal and lateral hub motions, x_h and y_h. Therefore, a four degree of freedom model is quite useful to explain the phenomenon of ground resonance.