Waves in plasma.

electron waves

$$\omega^2 = \omega_p^2 + \frac{\gamma k T_e}{m} k^2$$

ion waves

$$\omega^2 = k^2 \frac{kT_e + \gamma k T_i}{M}$$

electromagnetic waves

$B_0 = 0$, $B_1 \neq 0$ $\quad \omega^2 = c^2 k^2$

$$\nabla \times \vec{E}_1 = -\dot{\vec{B}}_1$$

$$c^2 \nabla \times \vec{B}_1 = +\dot{\vec{E}}_1$$

$\Downarrow$

변수3개

$$\nabla \times \vec{E}_1 = -\dot{B}_1$$

$$\nabla \times \vec{B}_1 = \mu_0 \vec{J}_1 + \frac{1}{c^2}\dot{E}_1$$

$$\begin{pmatrix} n_e = n_0 + n_1 \\ \vec{u}_e = \cancel{\vec{u}_0} + \vec{u}_1 \\ \vec{E} = \cancel{\vec{E}_0} + \vec{E}_1 \\ \vec{B} = \cancel{\vec{B}_0} + \vec{B}_1 \end{pmatrix}$$

$$\vec{J}_1 = (n_i e \vec{u}_i - n_e e \vec{u}_e)_1 = -(n_0 + n_1) e \vec{u}_1 \approx -n_0 e \vec{u}_1$$

무시

$$m n_e \left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e \cdot \nabla)\vec{u}_e\right] = -\cancel{\nabla P_e} - n_e e (\vec{E} + \vec{u}_e \times \vec{B})$$

$$\Rightarrow m n_0 \frac{\partial \vec{u}_1}{\partial t} = -n_0 e \vec{E}_1$$

$$-i\omega m n_0 u_1 = -n_0 e E_1$$

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\mu_0 \dot{\vec{J}} - \frac{1}{c^2}\ddot{\vec{E}}$$

$$\nabla(\nabla \cdot \vec{E}_1) - \nabla^2 \vec{E}_1 = \mu_0 n_0 e \dot{\vec{u}}_1 - \frac{1}{c^2}\ddot{\vec{E}}_1$$

$$i\vec{k}(i\vec{k} \cdot \vec{E}_1) - (ik \, ik)\vec{E}_1 = -i\omega \mu_0 n_0 e \vec{u}_1 - \frac{1}{c^2}(-i\omega)(-i\omega)\vec{E}_1$$

$\rightarrow \vec{k} \parallel \vec{E}_1 \rightarrow ik(ik\vec{E}_1) - (ik\,ik\,E_1) = 0$

$$0 = -i\omega \mu_0 n_0 e \vec{u}_1 + \frac{1}{c^2}\omega^2 \vec{E}_1$$

$$\cancel{i\omega} \mu_0 n_0 e \frac{e}{\cancel{i\omega} m} \cancel{\vec{E}_1} = \frac{1}{c^2}\omega^2 \cancel{\vec{E}_1}$$

$$\therefore \omega^2 = c^2 \mu_0 \frac{n_0 e^2}{m} = \frac{n_0 e^2}{m \varepsilon_0} = \omega_p^2$$

$\rightarrow \vec{k} \perp \vec{E} \rightarrow i\vec{k} \cdot \vec{E} = 0$

$$k^2 \cancel{\vec{E}_1} = \frac{-\mu_0 n_0 e^2}{m} \cancel{\vec{E}_1} + \frac{\omega^2}{c^2}\cancel{\vec{E}_1} \qquad \left(c^2 = \frac{1}{\mu_0 \varepsilon_0}\right)$$

$$\therefore \omega^2 = \omega_p^2 + c^2 k^2$$

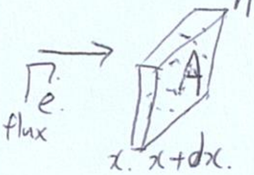
$k = 0 \Rightarrow$ cutoff ($\omega = \omega_p$)

$k \to \infty \Rightarrow$ resonance. ($\vec{k} \parallel \vec{B}$ 하게 넣어줄 때)

$\vec{B} \leftarrow \vec{E} \leftarrow \vec{k}$

Diffusion and mobility in weakly ionized gases

collision parameters



n_n 개

flux

target atoms

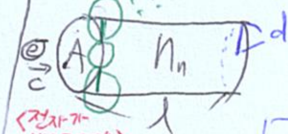
단위면적당 충돌확률

$$\Gamma(x+dx) = \Gamma(x) - \Gamma(x)\, n_n A\, dx \,\frac{\sigma}{A}$$

$$\frac{\Gamma(x+dx) - \Gamma(x)}{dx} = -\Gamma(x)\, n_n \sigma$$

$$\Gamma(x) = \Gamma_0 e^{-n_n \sigma x} = \Gamma_0 e^{-\frac{x}{\lambda}}$$

$\lambda = \frac{1}{n_n \sigma}$: (Mean Free Path)



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$$\frac{ct}{n_n \pi d^2 ct}$$

$$MFP = \boxed{\frac{l}{\text{충돌 횟수}}} = \frac{l}{n_n A l} = \frac{1}{n_n A} = \frac{1}{n_n \sigma}$$

$$\tau = \frac{\lambda}{c} = \frac{1}{n_n \sigma c}$$

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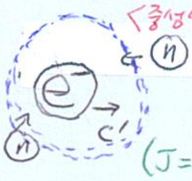


$$MFP = \frac{c' t}{\frac{1}{4} n \bar{c}\, 4\pi d^2 t} = \frac{c'}{n(\pi d^2)\bar{c}} = \frac{c'}{n \sigma \bar{c}}$$

$(J = \frac{1}{4} n \bar{c} A)$ $\quad (\bar{c} \sim \sqrt{2} c') \sim \frac{1}{\sqrt{2}\, n \sigma}$

$$\tau = \frac{\lambda}{c'} = \frac{1}{\sqrt{2}\, n \sigma c'} = \frac{kT}{\sqrt{2}\, n k T \sigma c'} = \frac{kT}{\sqrt{2}\, P \sigma c'}$$

(1atm, 20°C, $\tau \sim 100$ps)

$$\cancel{nm\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right]} = \pm ne\vec{E} - \nabla P - mn\nu\vec{u}$$

0가정

$$mn\nu\vec{u} = \pm ne\vec{E} - \nabla P \qquad (P = nkT, \text{isothermal})$$

$$\vec{u} = \pm\frac{\cancel{n}e\vec{E}}{m\cancel{n}\nu} - \frac{kT\nabla n}{mn\nu} = \pm\frac{e\vec{E}}{m\nu} - \frac{kT}{m\nu}\frac{\nabla n}{n}$$

$$= \pm \mu \vec{E} - D\frac{\nabla n}{n}$$

mobility $\quad$ diffusion coefficient

$\therefore \vec{\Gamma} = n\vec{u} = \pm \mu n \vec{E} - D\nabla n$ $\quad$ Fick's Law

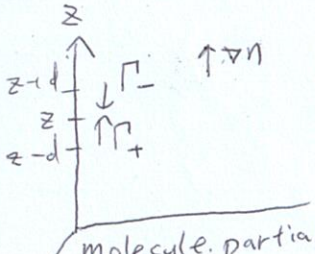
$\mu = \frac{De}{kT}$ Einstein relation

i) $\vec{E} = 0$, $\vec{\Gamma} = -D\nabla n$

ii) $\nabla n = 0$, $\vec{\Gamma} = \pm \mu n \vec{E}$

iii) $\nu = 0$, Boltzmann relation

Plasma Diffusion



$$\Gamma_{net} = \Gamma_+ - \Gamma_- = \frac{J_+ - J_-}{A} = \left\{ \frac{1}{4} n(z-d)\bar{c}A - \frac{1}{4} n(z+d)\bar{c}A \right\} / A$$

$$= \frac{1}{4}\bar{c}\left\{ \left[\cancel{n(z)} - d\frac{dn}{dz}\right] - \left[\cancel{n(z)} + d\frac{dn}{dz}\right] \right\} = -\frac{1}{2}\bar{c}d\frac{dn}{dz} = -\underbrace{\boxed{\frac{1}{2}\bar{c}d}}_{D}\nabla n$$

$\left(v_{th} = \frac{\lambda}{\tau} = \lambda\nu\right)$

molecule partial current $J = \frac{n\bar{c}A}{4}$

$$D = \frac{kT}{m\nu} \cong \frac{\frac{1}{2}m v_{th}^2}{m\frac{v_{th}}{\lambda}} = \frac{v_{th}\boxed{\lambda}}{2} \sim \frac{1}{2}\bar{c}\boxed{d} \sim \frac{(\Delta x)^2}{\Delta t} \sim \frac{\lambda^2}{\tau} = \lambda^2\nu$$

* ambipolar diffusion

$\vec{\Gamma}_e = \vec{\Gamma}_i = \vec{\Gamma}$

$n = n_i = n_e$

$\mu = \frac{e}{m\nu} \rightarrow \mu_i \ll \mu_e$

$$\left.\begin{aligned} \vec{\Gamma}_i &= \mu_i n\vec{E} - D_i\nabla n \\ \vec{\Gamma}_e &= -\mu_e n\vec{E} - D_e\nabla n \end{aligned}\right] =$$

$$\mu_i n\vec{E} - D_i\nabla n = -\mu_e n\vec{E} - D_e\nabla n$$

$$(\mu_i n + \mu_e n)\vec{E} = (D_i - D_e)\nabla n$$

$$\vec{E} = \frac{D_i - D_e}{\mu_i + \mu_e}\frac{\nabla n}{n}$$

$$\vec{\Gamma}_i = \mu_i \cancel{n}\frac{D_i - D_e}{\mu_i + \mu_e}\frac{\nabla n}{\cancel{n}} - D_i\nabla n = \left(\frac{\mu_i D_i - \mu_i D_e}{\mu_i + \mu_e} - D_i\right)\nabla n$$

$$= \left(\frac{\cancel{\mu_i D_i} - \mu_i D_e - \cancel{\mu_i D_i} - \mu_e D_i}{\mu_i + \mu_e}\right)\nabla n = -\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}\nabla n = -D_a\nabla n$$

$\left(D_a = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}\right)$

$$D_a \equiv \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \sim D_i + \frac{\mu_i}{\mu_e}D_e$$

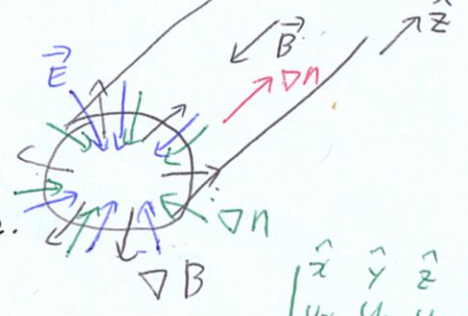
$$= D_i + \frac{\frac{D_i \cancel{e}}{kT_i}}{\frac{D_e \cancel{e}}{kT_e}}\cancel{D_e}$$

$$= D_i + \frac{T_e}{T_i}D_i \sim 2D_i \quad (T_i = T_e)$$

$$\frac{\partial n}{\partial t} + \nabla\cdot\underbrace{(n\vec{u})}_{\vec{\Gamma}} = S_{source} - S_{sink}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot\underbrace{(-D_a\nabla n)}_{\vec{\Gamma}} = \frac{\partial n}{\partial t} - D_a\nabla^2 n = 0$$

$\vec{B} \neq 0$ Diffusion across $\vec{B}$



Assume the magnetic field is uniform (no gradient) unlike the figure.

$$0 = \pm ne\left(\vec{E} + \vec{u}\times\vec{B}\right) - kT\nabla n - mn\nu\vec{u}$$

$$\vec{u} = \pm\frac{e}{m\nu}\vec{E} \pm \frac{ne}{mn\nu}\vec{u}\times\vec{B} - \frac{kT}{m\nu}\frac{\nabla n}{n} = \pm\mu\vec{E} \pm \frac{e}{m\nu}\vec{u}\times\vec{B} - D\frac{\nabla n}{n}$$

$$\vec{u}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ u_x & u_y & u_z \\ 0 & 0 & B \end{vmatrix}$$

$$\left.\begin{aligned} u_x &= \pm\mu E_x \pm \frac{e}{m\nu}u_y B - \frac{D}{n}\frac{\partial n}{\partial x} \\ u_y &= \pm\mu E_y \mp \frac{e}{m\nu}u_x B - \frac{D}{n}\frac{\partial n}{\partial y} \end{aligned}\right) \Rightarrow u_x = \pm\mu E_x \pm \frac{\overset{\omega_c}{eB}}{m\nu}\left(\pm\mu E_y \mp \frac{e}{m\nu}u_x B - \frac{D}{n}\frac{\partial n}{\partial y}\right) - \frac{D}{n}\frac{\partial n}{\partial x}$$

$$= \pm\mu E_x + \frac{\omega_c}{\nu}\mu E_y - \frac{\omega_c^2}{\nu^2}u_x \mp \frac{\omega_c}{\nu}\frac{D}{n}\frac{\partial n}{\partial y} - \frac{D}{n}\frac{\partial n}{\partial x}$$

$$\mu = \frac{e}{m\nu} = \frac{eB}{m\nu B} = \frac{\omega_c}{\nu B}$$

$$D = \frac{kT}{m\nu} = \frac{eB\,kT}{m\nu\,Be} = \frac{\omega_c kT}{\nu eB}$$

$$\Rightarrow \left(1 + \frac{\omega_c^2}{\nu^2}\right)u_x \doteq \pm\mu E_x - D\frac{1}{n}\frac{\partial n}{\partial x} + \frac{\omega_c^2}{\nu^2}\left(\frac{E_y}{B} \mp \frac{kT}{neB}\frac{\partial n}{\partial y}\right)$$

$$\therefore u_x = \underbrace{\frac{\pm\mu}{1 + \frac{\omega_c^2}{\nu^2}}}_{\pm\mu_\perp}E_x - \underbrace{\frac{D}{1 + \frac{\omega_c^2}{\nu^2}}}_{D_\perp}\frac{1}{n}\frac{\partial n}{\partial x} + \frac{\omega_c^2/\nu^2}{1 + \frac{\omega_c^2}{\nu^2}}\left(\underbrace{\frac{E_y}{B}}_{\frac{E_y B}{B^2}\,\Rightarrow\,\vec{u}_E} \underbrace{\mp\frac{kT}{neB}\frac{\partial n}{\partial y}}_{\frac{B\times\nabla P}{nqB^2}\,=\,\vec{u}_D}\right)$$

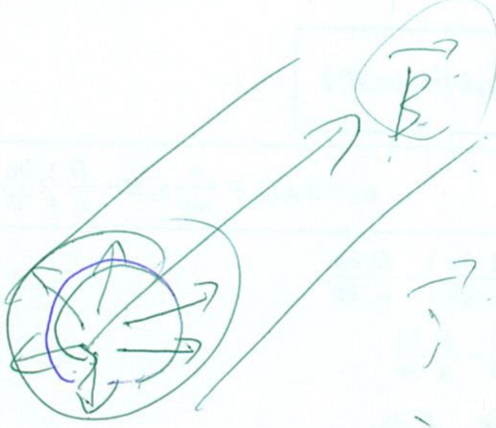
$\vec{u}_E$: E×B drift

$\vec{u}_D$: diamagnetic drift

$\oplus \longleftrightarrow \ominus$ → 계적 → diffusion → $\vec{E}$ → 전자 붙어

$\oplus$ → 그대로

↳ $\vec{E}$ → diffusion → $\vec{E} \times \vec{B}$ → 회전 → diff



$\vec{j}$ ion.

$$j \times B = \nabla P$$

Solving the Equation with Cylindrical Coordination.

$$0=\pm ne(\vec{E}+\vec{u}\times\vec{B})-\nabla P-mn\nu\vec{u}$$

$$\vec{u}=\pm\frac{e}{m\nu}\vec{E}\pm\frac{ne}{mn\nu}\vec{u}\times\vec{B}-\frac{k_BT}{m\nu}\frac{\nabla n}{n}$$

$$\boxed{\text{cross product}\quad \vec{u}\times\vec{B}=\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{z}\\ u_r & u_\theta & u_z\\ 0 & 0 & B\end{vmatrix}=(u_\theta B)\hat{r}-(u_rB)\hat{\theta}}$$

$$\boxed{u_r=\pm\mu E_r\pm\frac{e}{m\nu}u_\theta B-\frac{D}{n}\frac{\partial n}{\partial r}\qquad u_\theta=\pm\mu E_\theta\mp\frac{e}{m\nu}u_rB-\frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial\theta}}$$

$$u_r=\pm\mu E_r\pm\frac{eB}{m\nu}\left\{\pm\mu E_\theta\mp\frac{e}{m\nu}u_rB-\frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial\theta}\right\}-\frac{D}{n}\frac{\partial n}{\partial r}$$

$$u_r=\pm\mu E_r+\frac{\omega_c}{\nu}\left\{\mu E_\theta-\frac{\omega_c}{\nu}u_r\mp\frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial\theta}\right\}-\frac{D}{n}\frac{\partial n}{\partial r}$$

$$u_r=\pm\mu E_r+\frac{\omega_c}{\nu}\mu E_\theta-\frac{\omega_c^2}{\nu^2}u_r\mp\frac{\omega_c}{\nu}\frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial\theta}-\frac{D}{n}\frac{\partial n}{\partial r}$$

$$\left(1+\frac{\omega_c^2}{\nu^2}\right)u_r=\pm\mu E_r+\frac{\omega_c}{\nu}\mu E_\theta\mp\frac{\omega_c}{\nu}\frac{D}{n}\frac{1}{r}\frac{\partial n}{\partial\theta}-\frac{D}{n}\frac{\partial n}{\partial r}$$

$$\boxed{\mu=\frac{e}{m\nu}=\frac{eB}{m\nu B}=\frac{\omega_c}{B\nu}\qquad D=\frac{k_BT}{m\nu}=\frac{k_BTeB}{m\nu eB}=\frac{k_BT}{eB}\frac{\omega_c}{\nu}}$$

$$u_r=\frac{\pm\mu}{(1+\omega_c^2/\nu^2)}E_r-\frac{D}{(1+\omega_c^2/\nu^2)}\frac{1}{n}\frac{\partial n}{\partial r}+\frac{\omega_c^2/\nu^2}{(1+\omega_c^2/\nu^2)}\left(\frac{E_\theta}{B}\mp\frac{k_BT}{neB}\frac{1}{r}\frac{\partial n}{\partial\theta}\right)$$

$$u_\theta=\frac{\pm\mu}{(1+\omega_c^2/\nu^2)}E_\theta-\frac{D}{(1+\omega_c^2/\nu^2)}\frac{1}{n}\frac{1}{r}\frac{\partial n}{\partial\theta}-\frac{\omega_c^2/\nu^2}{(1+\omega_c^2/\nu^2)}\left(\frac{E_r}{B}\mp\frac{k_BT}{neB}\frac{\partial n}{\partial r}\right)$$

$$\boxed{\mu_\perp=\frac{\mu}{1+\omega_c^2/\nu^2}\qquad D_\perp=\frac{D}{1+\omega_c^2/\nu^2}}$$

$$\vec{u}=\pm\mu_\perp\vec{E}-D_\perp\frac{\nabla n}{n}+\frac{\omega_c^2/\nu^2}{(1+\omega_c^2/\nu^2)}\left(\frac{\vec{E}\times\vec{B}}{B^2}+\frac{\vec{B}\times\nabla P}{nqB^2}\right)$$

$$\therefore\vec{u}=\pm\mu_\perp\vec{E}-D_\perp\frac{\nabla n}{n}+\frac{\omega_c^2/\nu^2}{(1+\omega_c^2/\nu^2)}\left(\overrightarrow{u_E}+\overrightarrow{u_D}\right)$$