458.308 Process Control & Design

Lecture 6: Dynamic Behavior and Stability of Closed-Loop Control Systems (Part 2)

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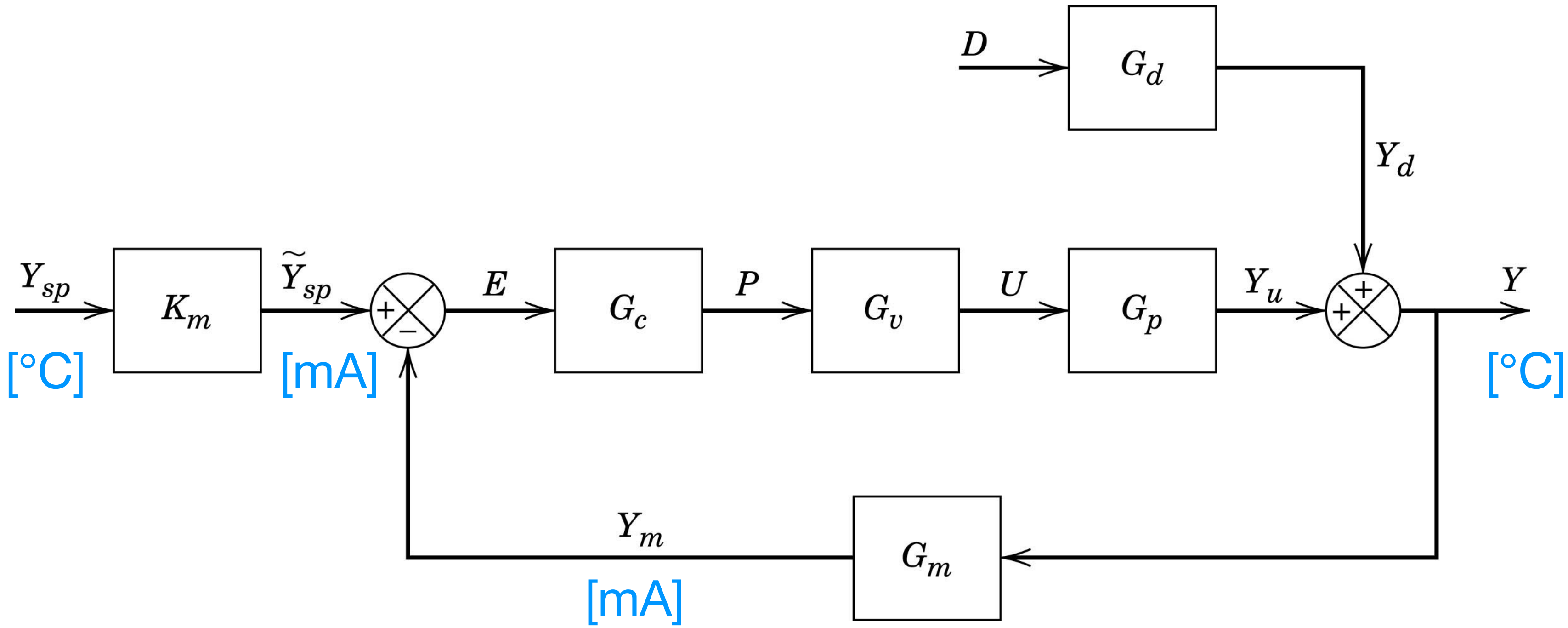
General Expression of Feedback Control System

Ex) Heated Tank System



K_m: Steady-state gain of ____

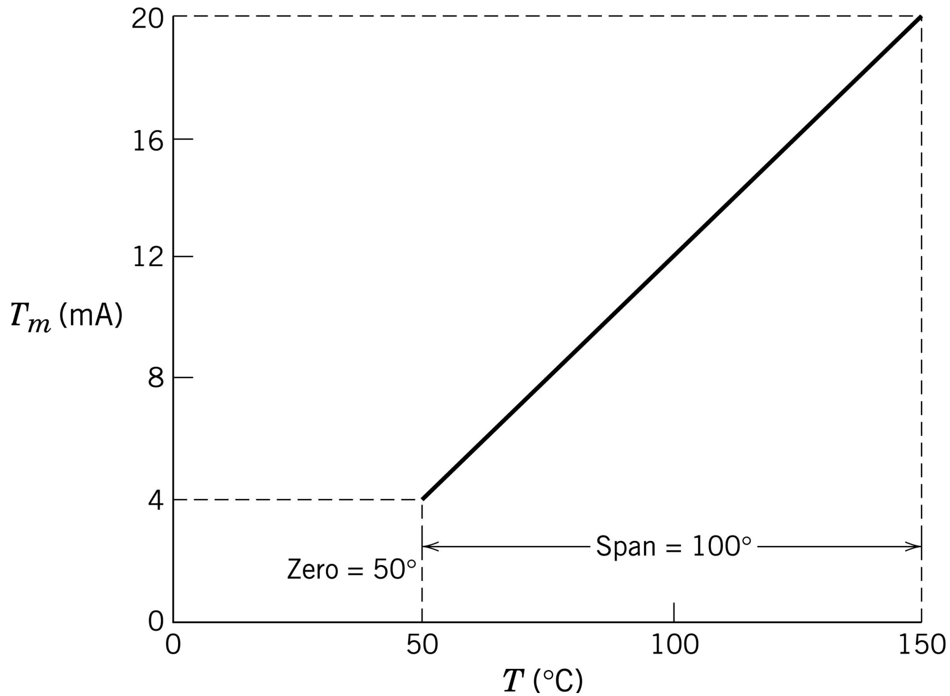
Y_u: Change in Y due to ____

Y_d: Change in Y due to ____

To simplify the notation, the primes and s dependence have been omitted; thus, Y is used rather than Y'(s)

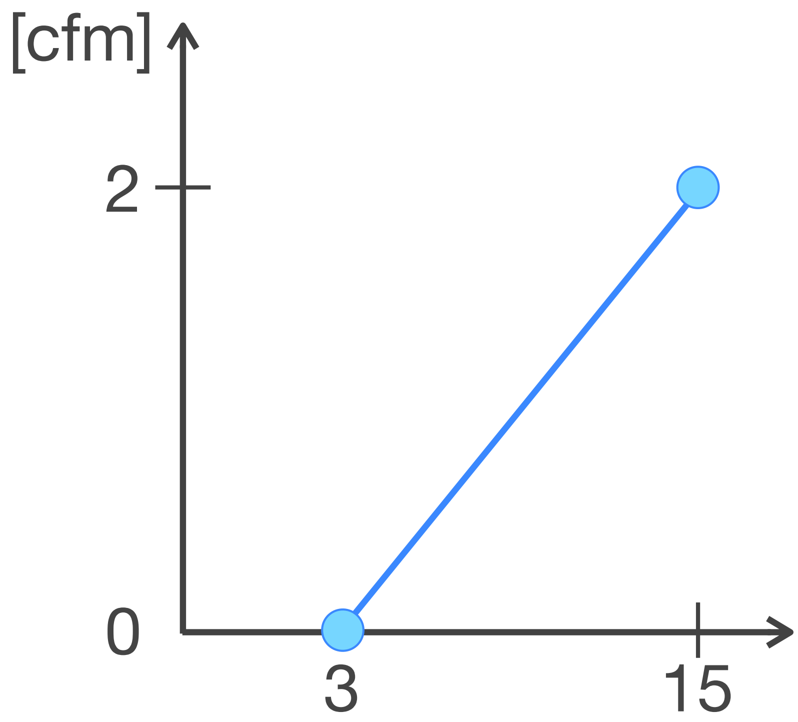
If the input range of a platinum resistance element (temp. sensor) is 50°C - 150°C is used with a standard transmitter, what is K_m?

The standard transmitter output is 4 - 20mA

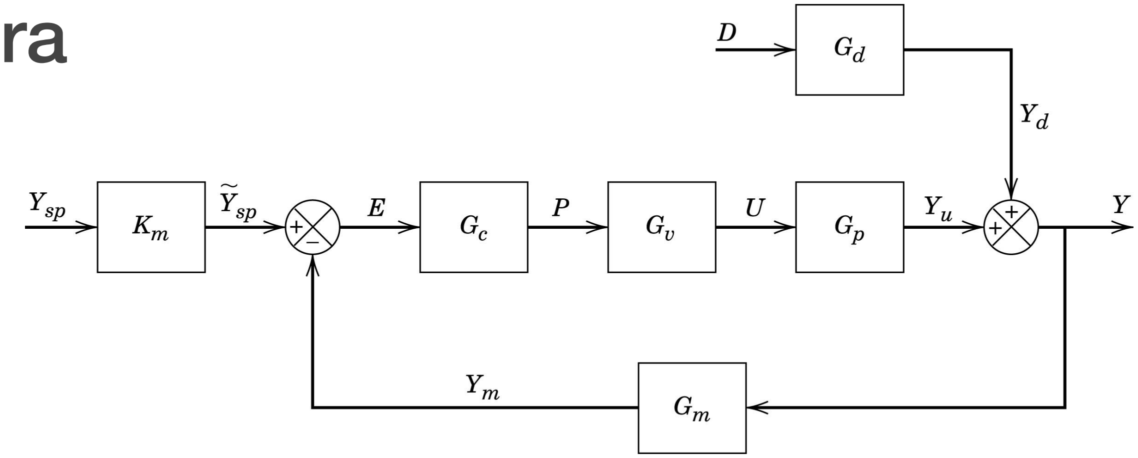


The flow rate through the valve ranges from 0 to 2 [cfm]. If the installed A-O valve exhibits linear characteristic with negligible dynamics what is G_v?

The input to the control valve ranges from 3 to 15 psi.



Transfer Function Algebra



$$Y = Y_u + Y_d = G_p U + G_d D$$

$$= G_p G_v G_c E + G_d D = G_p G_v G_c \left\{ \tilde{Y}_{sp} - Y_m \right\} + G_d D$$

$$= G_p G_v G_c \left\{ K_m Y_{sp} - G_m Y \right\} + G_d D$$

$$Y = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m} Y_{sp} + \frac{G_d}{1 + G_c G_v G_p G_m} D$$

$G_{OL} \triangleq G_c G_v G_p G_m$: __________ transfer function (relates $\tilde{Y}_{sp}$ to Y_m)

Denominator = 1 + G_{OL}

Analysis and Design Problem

- Analysis: Given particular G's and Gc
  - Are the closed-loop dynamics stable?
  - Speed of response?
  - Damping?
  - Gains for Y/Y_{sp} and Y/D
- Design: Given particular G's, choose ("design") Gc so that
  - the closed-loop dynamics are stable
  - Y/Y_{sp} has a gain of ~ (   ) and Y/D has a gain of ~ (   )
  - the dynamics are sufficiently fast (but not too fast) and smooth (without excessive oscillation).

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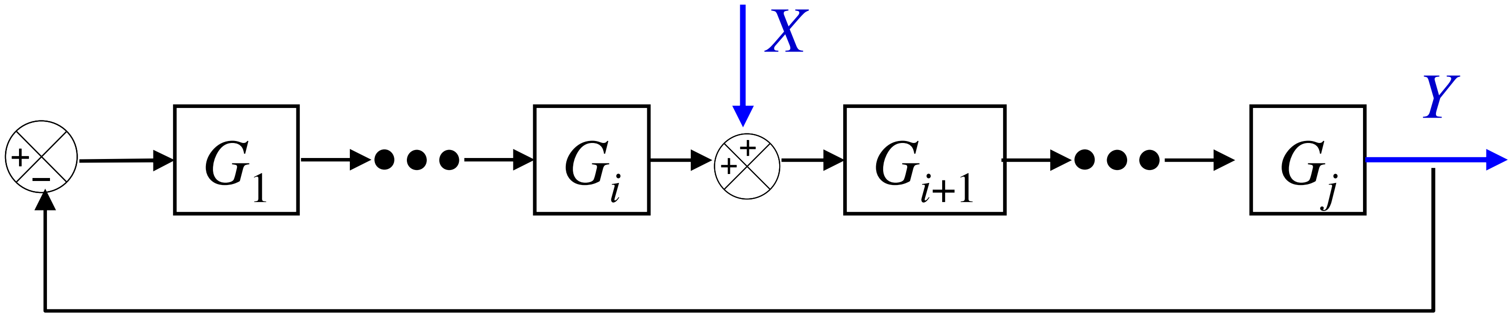
Model Used for Analysis and Design

- Case 1 (Less Frequent)
  - From a fundamental model, perform linearization and Laplace transform of the linearized ODEs to find $G_p(s)$ and $G_d(s)$
  - Find actuator and measurement dynamics G_v and G_m
- Case 2 (More Frequent)
  - The composite model $G(= G_m G_p G_v)$ is fitted to data of y_m obtained by perturbing *p* (e.g., by making a step change)

Calculation of Closed-Loop Functions: Generalization



$$\frac{Y(s)}{X(s)} = \frac{G_{i+1}G_{i+2}\cdots G_j}{1+G_1G_2\cdots G_j} = \frac{\prod_f}{1+\prod_e}$$

Assume negative feedback

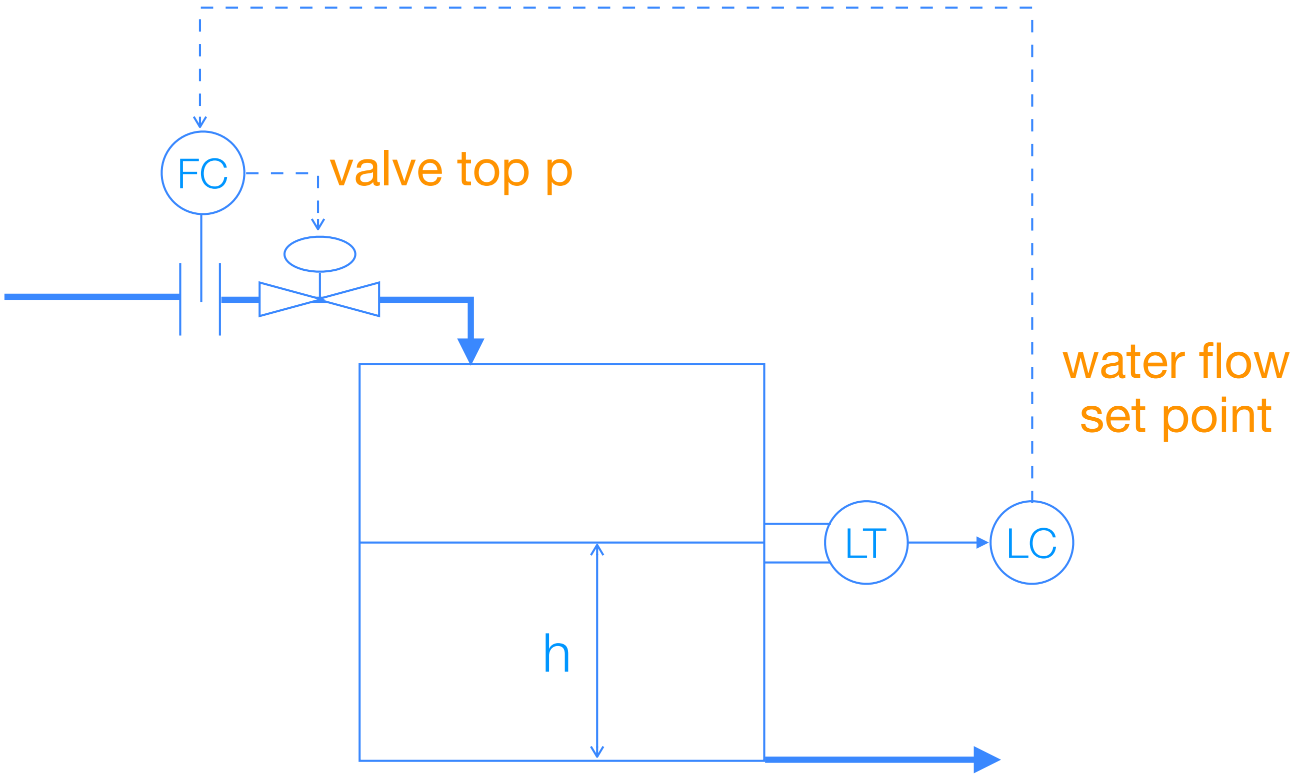
$\prod_f$: Product of the transfer functions in the forward path from X to Y

$\prod_e$: Product of every transfer function in the feedback loop

Check with slide p. 5

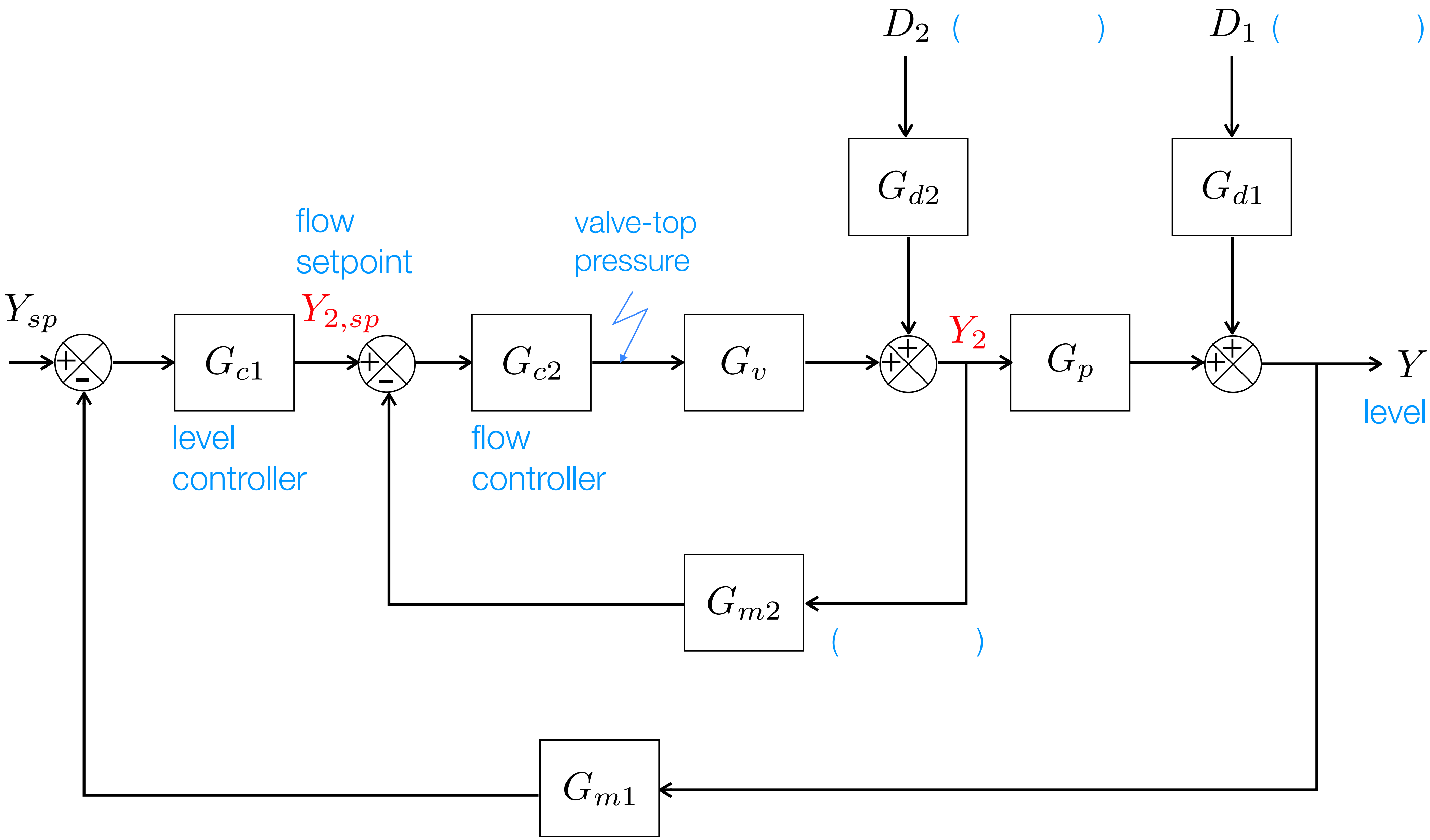
Cascaded Loop

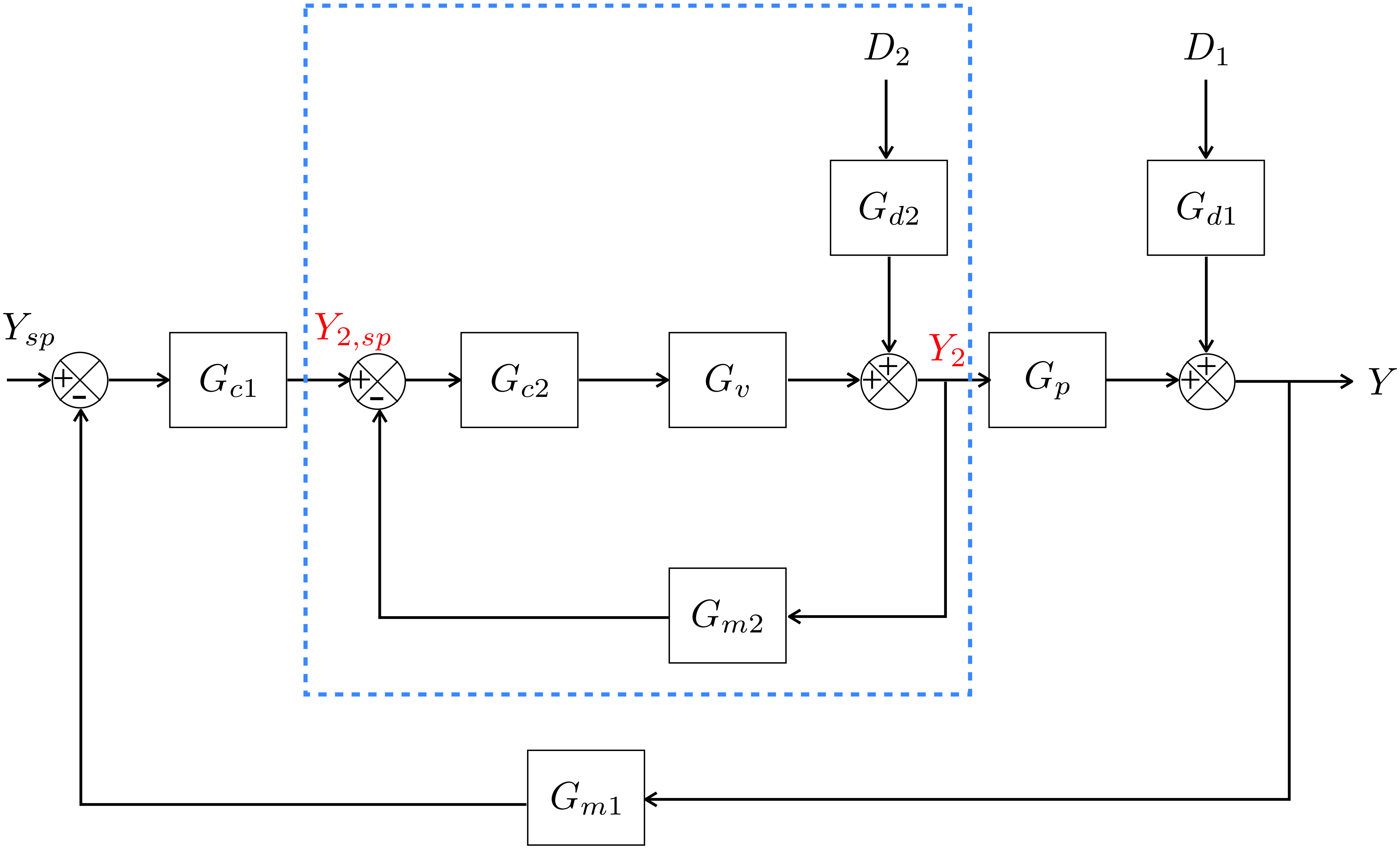
Multiple output measurements & a single manipulated input



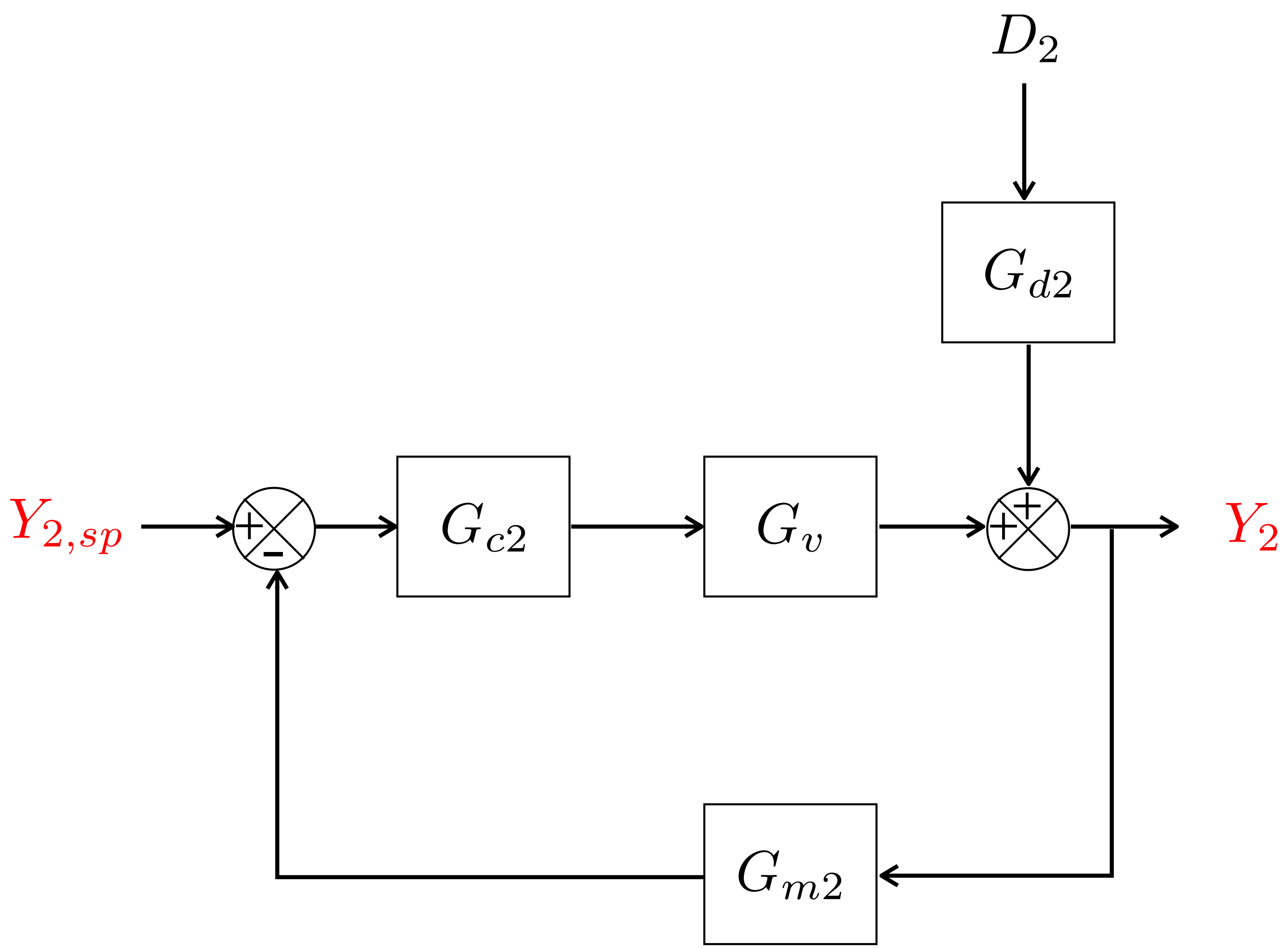
cascades the level controller to a water flow controller

- popularly used for flow control
- compensates for disturbances directly affecting a manipulated flow rate
- disturbance: water header pressure


D_2 (   )
D_1 (   )
G_{d2}
G_{d1}
flow setpoint
valve-top pressure
Y_{sp}
$Y_{2,sp}$
Y_2
G_{c1}
G_{c2}
G_v
G_p
Y
level
level controller
flow controller
G_{m2}
(   )
G_{m1}



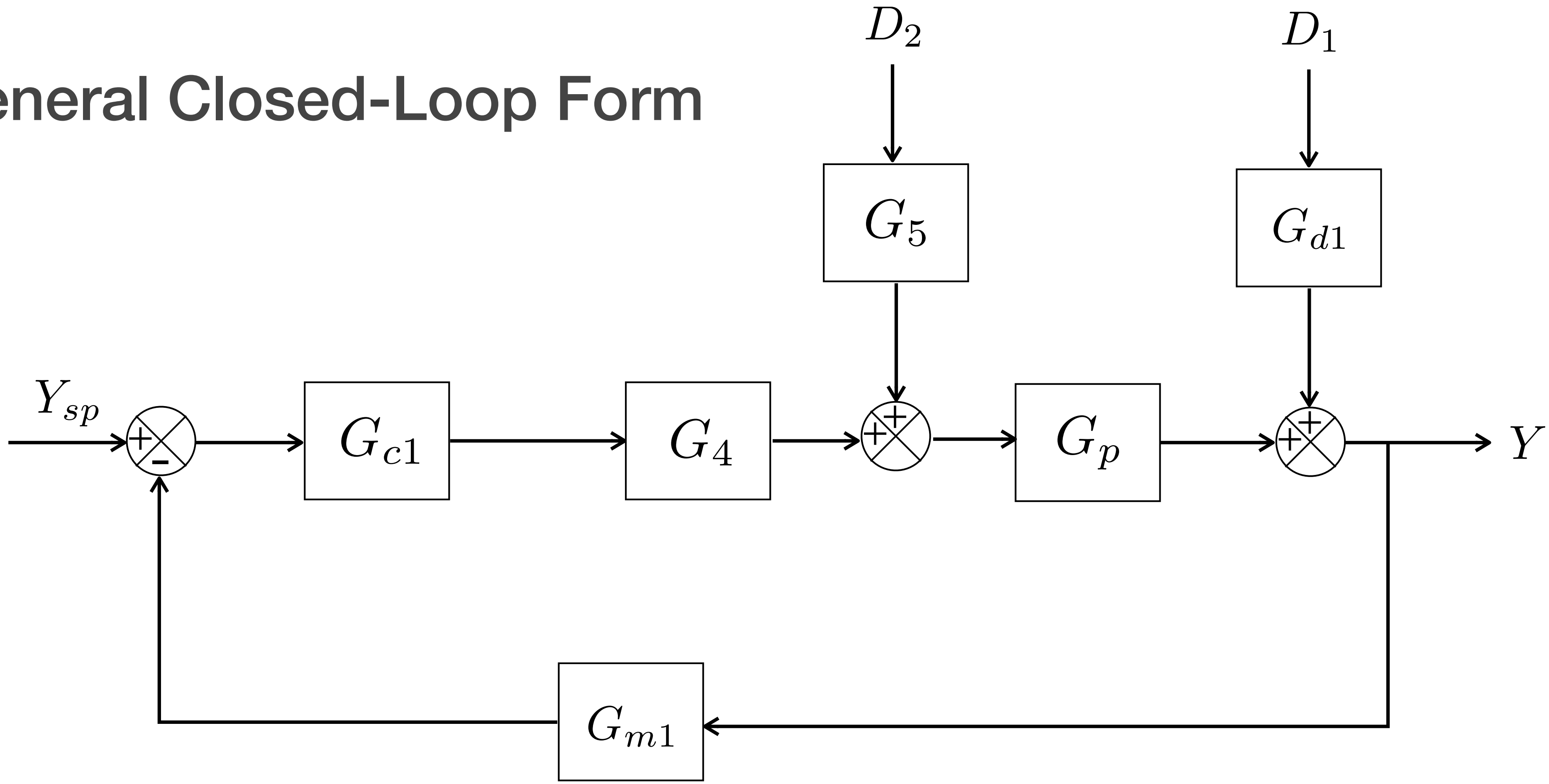
Y_{sp}
G_{c1}
$Y_{2,sp}$
G_{c2}
G_v
D_2
G_{d2}
Y_2
G_p
D_1
G_{d1}
Y
G_{m2}
G_{m1}



$$Y_2 = \boxed{}\, Y_{2,sp} + \boxed{}\, D_2$$

G_4 $\qquad$ G_5

General Closed-Loop Form



$$\frac{Y}{Y_{sp}} = \frac{G_{c1}G_4G_p}{1+G_{c1}G_4G_pG_{m1}} = \frac{G_{c1}\frac{G_{c2}G_v}{1+G_{c2}G_vG_{m2}}G_p}{1+G_{c1}\frac{G_{c2}G_v}{1+G_{c2}G_vG_{m2}}G_pG_{m1}}$$

$$= \frac{G_{c1}G_{c2}G_vG_p}{1+G_{c2}G_vG_{m2}+G_{c1}G_{c2}G_vG_pG_{m1}}$$

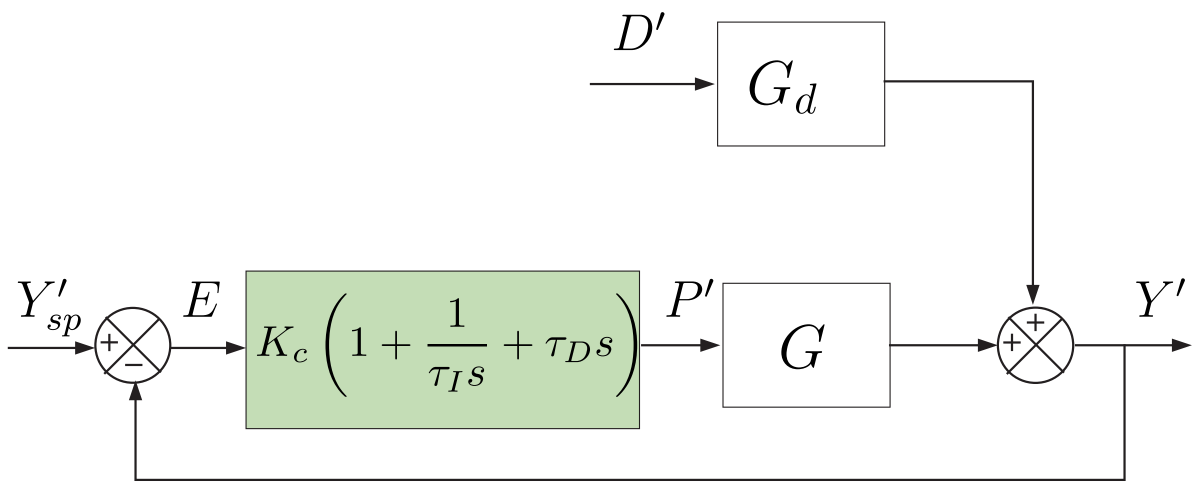
$\frac{Y}{D_2}, \frac{Y}{D_1}$?

TF of PID Controller

$$p(t) = \bar{p} + K_c \left(e(t) + \frac{1}{\tau_I} \int_0^t e(t^*) dt^* + \tau_D \frac{de}{dt} \right)$$

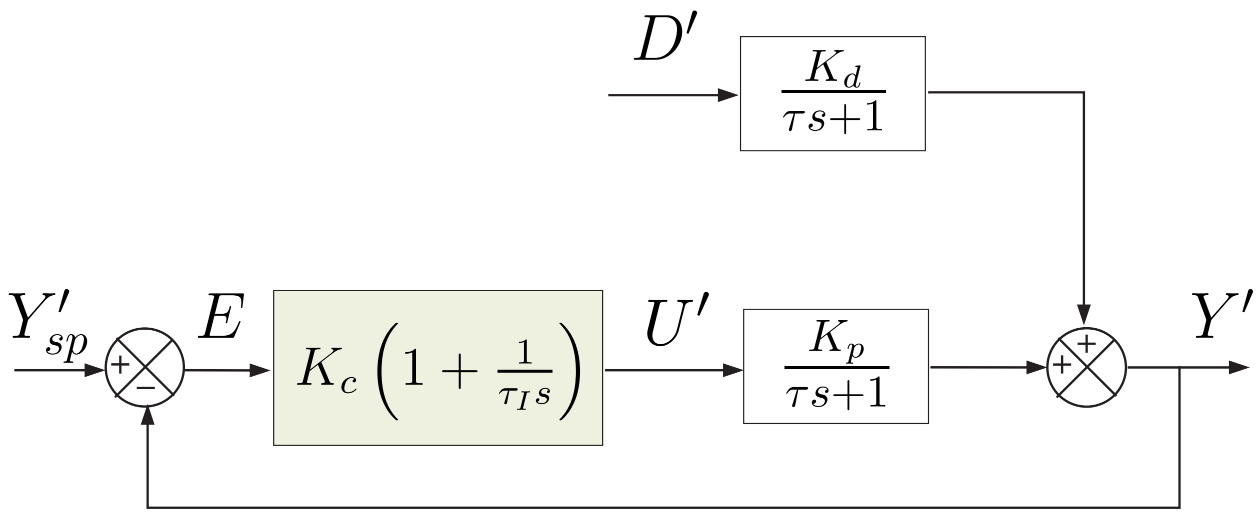
$$\Rightarrow \quad P'(s) = K_c \left(E(s) + \frac{1}{\tau_I s} E(s) + \tau_D s E(s) \right)$$

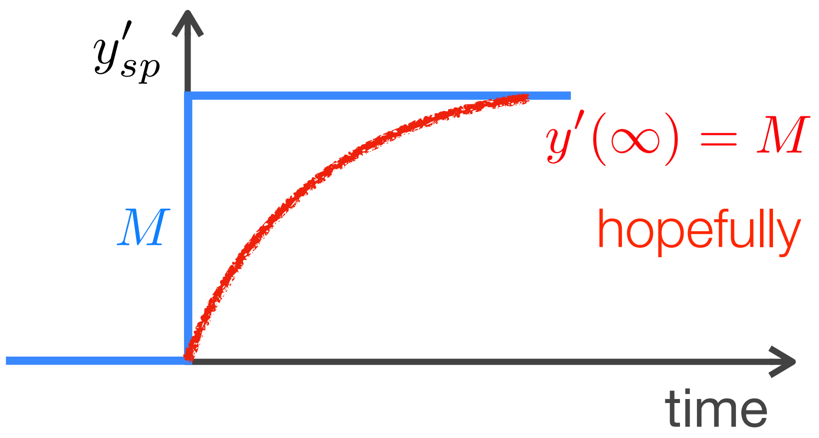
$$\Rightarrow \quad \frac{P'(s)}{E(s)} = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right)$$



Closed-Loop Responses of PI Control

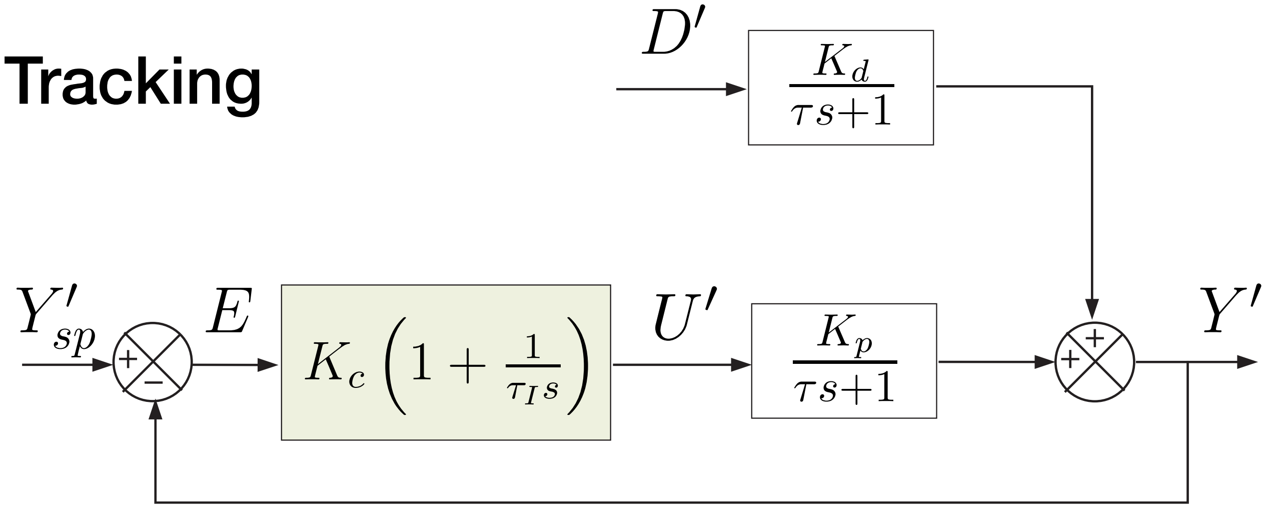
Integral mode removes the offset





$$Y'_{sp}(s) = \frac{M}{s}$$

Set point Tracking



$$\frac{Y'(s)}{Y'_{sp}(s)} = \frac{\frac{K_cK_p(\tau_I s+1)}{\tau_I s(\tau s+1)}}{1+\frac{K_cK_p(\tau_I s+1)}{\tau_I s(\tau s+1)}} = \frac{K_cK_p(\tau_I s+1)}{\tau_I s(\tau s+1)+K_cK_p(\tau_I s+1)}$$

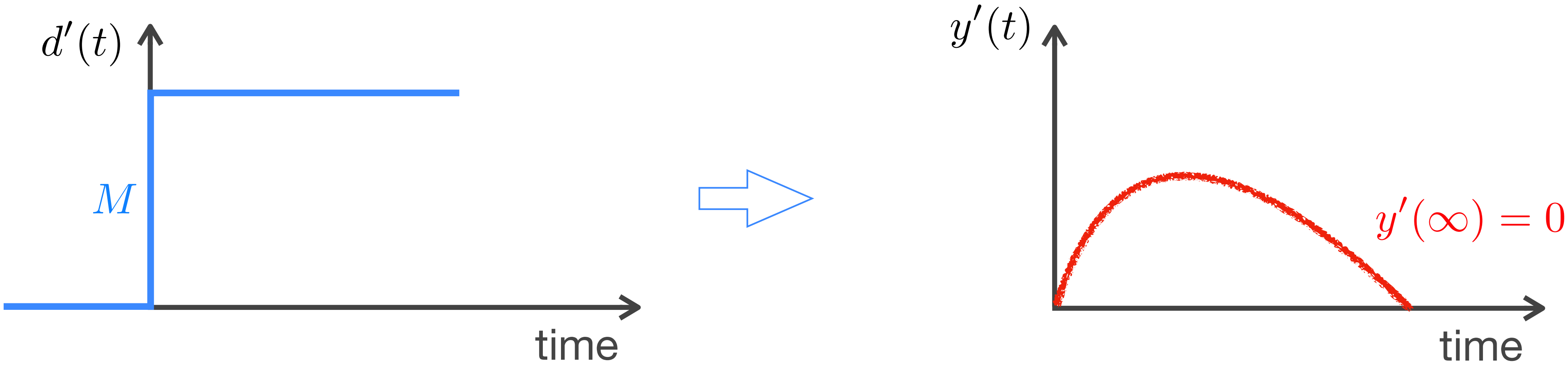
$$Y'(s) = \frac{K_cK_p(\tau_I s+1)}{\tau_I s(\tau s+1)+K_cK_p(\tau_I s+1)} \cdot \frac{M}{s}$$

If the closed-loop system is stable,

$$y'(\infty) = \lim_{s\to 0}\{s\cdot Y'(s)\} = \frac{K_cK_p}{K_cK_p}\cdot M = M \qquad \text{(Gain = 1)}$$

$$\text{offset} = y'_{sp}(\infty) - y'(\infty) = M - M = 0$$

Disturbance Rejection



$$\frac{Y'(s)}{D'(s)} = \frac{K_d/(\tau s + 1)}{1 + K_c\left(1 + \frac{1}{\tau_I s}\right)\left(\frac{K_p}{\tau s + 1}\right)}$$

$$Y'(s) = \frac{K_d \tau_I s}{\tau_I \tau s^2 + (\tau_I + K_p K_c \tau_I)s + K_c K_p} \cdot \frac{M}{s}$$ (Gain = 0)

If the closed-loop system is stable,

$$y'(\infty) = \lim_{s \to 0} \{sY'(s)\} = 0$$ No offset

Closed-Loop Stability

Characteristic Equation

$$1 + G_{OL} = 0$$

Roots of the above equation are the poles of the closed-loop functions (important information for analyzing closed-loop dynamics)

For stability, make sure all the roots are in the Left-Half-Plane (negative real parts)

- can be checked by Routh's test
- or by direct substitution

Note: Poles of the systems higher than 2^{nd}-order are difficult to find

Routh's Test

Determining whether any of the roots are positive (unstable) without calculating the roots

Setting up the Routh Array

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0 \quad (\Leftrightarrow 1 + G_{OL} = 0)$$

i) Make $a_n > 0$

ii) A necessary condition for stability

: All of the coefficients (a_n, ..., a_0) must be positive. If any coef. is negative or zero, at least one root lies to the right of, or on, the imaginary axis

If all the a_i's are positive, construct Routh Array

iii) Construct Routh Array

Row				
1	a_n	a_{n-2}	a_{n-4}	$\cdots$
2	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
3	b_1	b_2	b_3	
4	c_1	c_2	c_3	
$\vdots$				
$n+1$				

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$$

$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

$$c_1 = \frac{b_1 a_{n-3} - b_2 a_{n-1}}{b_1}$$

$$c_2 = \frac{b_1 a_{n-5} - b_3 a_{n-1}}{b_1}$$

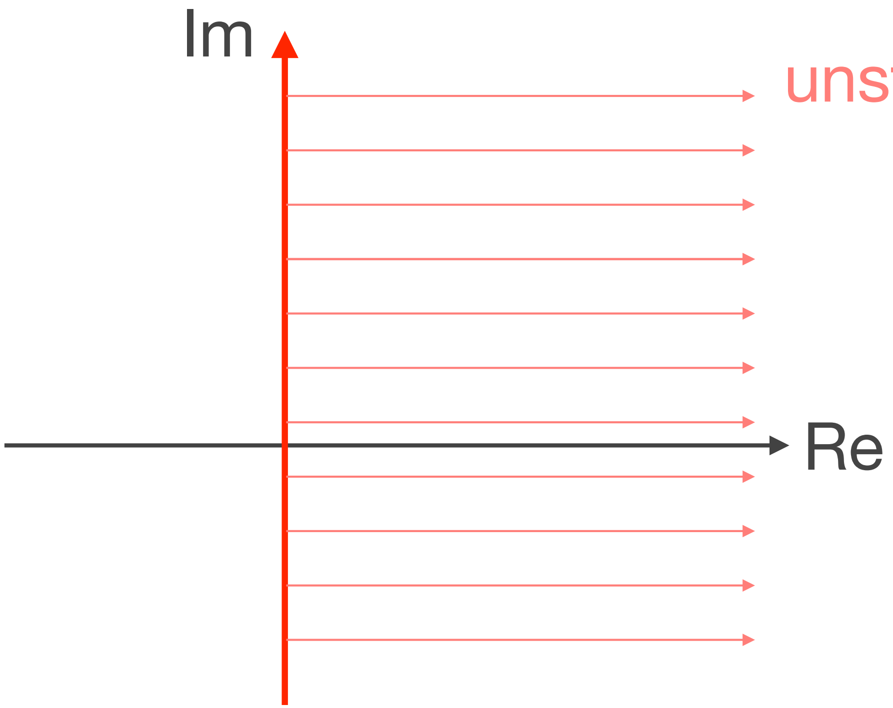
$\vdots$

All the coefficients in the first column are positive ⟶ all the poles are negative.

Find the upper bound on Kc (P-controller) for closed-loop stability of

$$G_p(s) = \frac{1}{(3s+1)(2s+1)(s+1)} = \frac{1}{6s^3 + 11s^2 + 6s + 1}$$

Direct Substitution



At the limits of instability, the closed-loop poles will be on the imaginary axis

$$1 + G_{OL} = 0 \qquad (s = j\omega)$$

This method works with a system with time delay. Routh's method does not

$$6s^3 + 11s^2 + 6s + 1 + K_c = 0$$

$$6(j\omega)^3 + 11(j\omega)^2 + 6(j\omega) + 1 + K_c = 0$$

$$(1 + K_c - 11\omega^2) + 6(-\omega^3 + \omega)j = 0$$