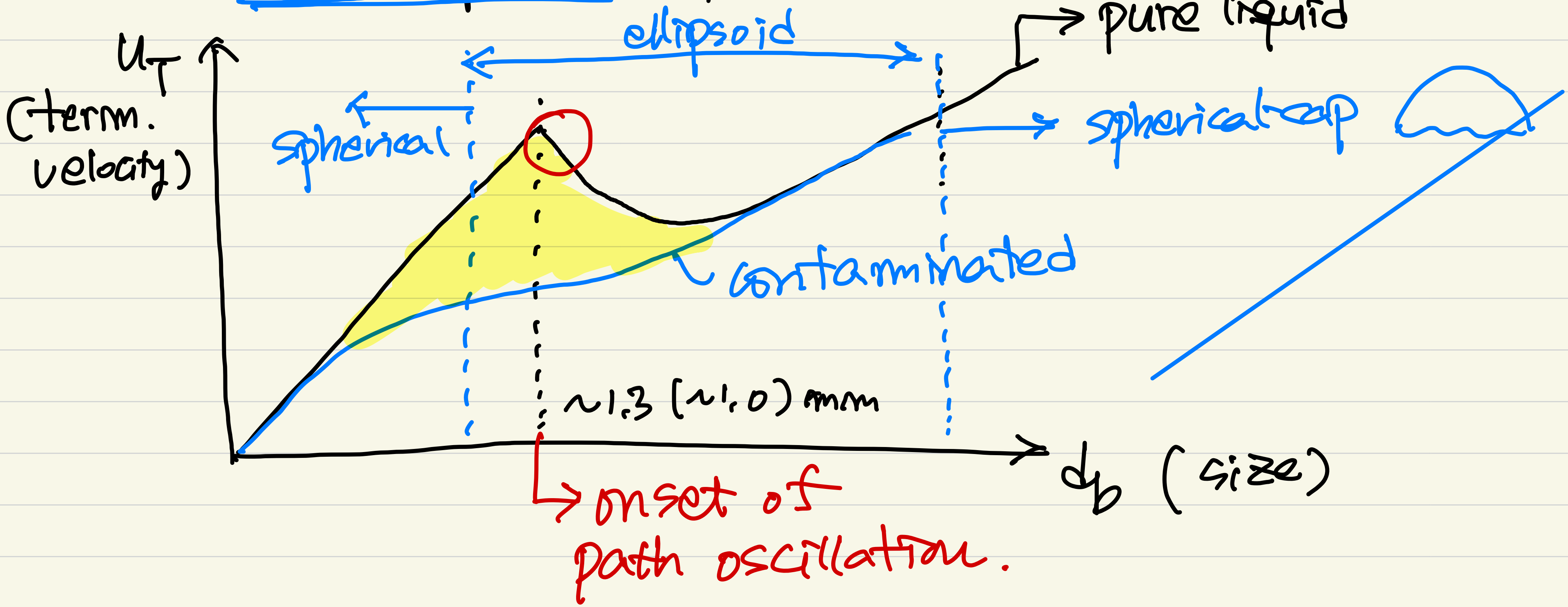
◎ Rise dynamics of a bubble.

: in many practical systems, bubbles and drops lie in the intermediate size range (mm scale) → approximated as oblate spheroid w/ vertical axis of symmetry.



- Path (and shape) instability.
  - At low Re, a bubble rises along a straight path
    → transition to oscillatory (zigzagging or helical) motions at higher Re.
  - the bubble shape is stable for low We
    → it becomes unstable at higher We ($\gtrsim 3.0$)

(Clift et al. 1978)

d_b [mm]	Re	Eo	path
<1.3	<565	>0.8	straight.
1.3-2.0	565-880	0.5-0.8	Helical
2.0-3.6	880-1350	0.36-0.5	zigzag-helical.
3.6-4.2	1350-1510	0.28-0.36	zigzag
→ 4.2-17	1510-4700	0.23-0.28	rocking.

- threshold of the instability is strongly dependent on the purity of the liquid.

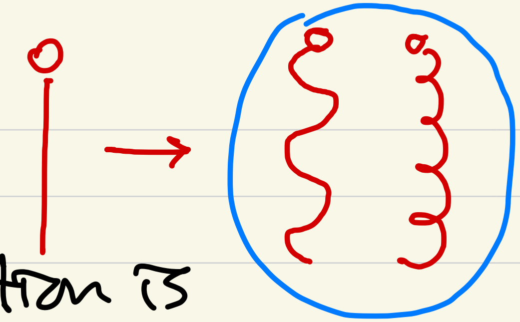
→ in a pure water, Re_{crit} extends up to 660.

Effect of bubble surf. contamination.

- reduction of rise velocity (drag increases)
- change in the lateral migration (lift decreases) in the presence of shear. (Takagi & Matsumoto, 2010, ARFM)
- if a bubble size is large (> 2.5 mm) enough (Duineveld, 1994) and the traveling time is not too long ($\sim 50 \cdot r_b$) (Bachhuber & Sanford, 1974), the bubble dynamics in

tap water can be approximated as that in pure water.

- Origin of the path instability?



· Saffman (1956): any zigzag deviation is amplified when $\chi > 1.2$, and is coupled w/ the oscillation of the wake.
aspect ratio

· Hartunian & Sears (1957): shape oscillation
→ path instability
when $We > We_{crit}$.

· El sawi (1974), Benjamin (1987): bifurcation from axisymmetric to asymmetric bubble shape at We_{crit} → zigzag motion.

· Meiron (1989): whatever We, no path instability
→ flow needs to be rotational

Somewhere around the bubble.

the role of bubble wake has been cleared only recently.

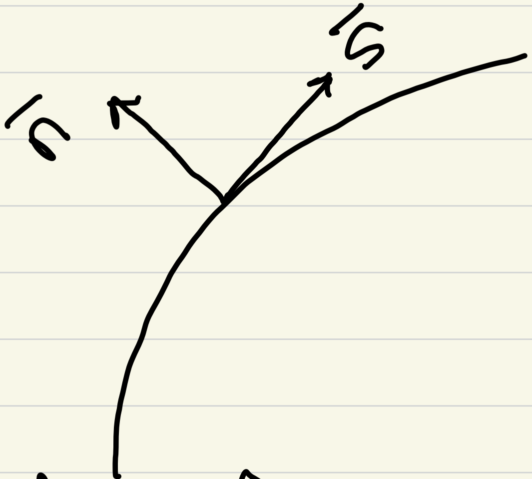
A) Asymmetry of the vertical structure in the wake (i.e, wake instability), caused by the excess of accumulated vorticity on the bubble surface, which is shed into vortices, is responsible for the path instability.

surf. vorticity generation on the shear-free surface.

: Vorticity generation on curved shear-free surface due to zero tangential stress →

relation between tangential velocity and its gradient in normal direction (Batchelor, 1967).

Magnaudet & Mougin (2007, JFM)



shear-free surface

at the surface

$$\begin{cases} \bar{v} = \bar{v}_s + v_n \bar{n}, & (v_n = \bar{v}\cdot\bar{n}) \\ \bar{\omega} = \bar{\omega}_s + \omega_n \bar{n}, & (\omega_n = \bar{\omega}\cdot\bar{n}) \end{cases}$$

let's define, $\nabla_s = \nabla - \frac{\partial}{\partial n}\bar{n}$, $\frac{\partial}{\partial n} = \bar{n}\cdot\nabla$

(surf. grad. operator).

Following Wu (1995), shear-free condition at surface shear stress

$$\Rightarrow \bar{\omega}_s = 2\bar{n}\times(\bar{v}_s\cdot\nabla_s\bar{n} - \nabla_s v_n)$$

surface curvature tensor.

$$\frac{\partial \bar{v}}{\partial t} + \bar{v}\cdot\nabla\bar{v} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\bar{v}$$

$$\Downarrow$$

$$\begin{cases} \nu\dfrac{\partial \bar{\omega}_s}{\partial n} = \bar{n}\times\left(\dfrac{D\bar{v}}{Dt} + \nabla\dfrac{p}{\rho}\right) + \nu\left(\nabla_s\omega_n - \bar{\omega}_s\cdot\nabla_s\bar{n}\right) \\ \nu\dfrac{\partial \bar{\omega}_n}{\partial n} = -\nu\left(\omega_n\underbrace{\nabla_s\cdot\bar{n}}_{\text{mean surface curvature}} + \nabla_s\bar{\omega}_s\right). \end{cases}$$

at the surface BC : $v_n = 0$.

$$\hookrightarrow \underbrace{\bar{\omega}_s}_{\text{surf. vorticity}} = 2\bar{n}\times(\bar{v}_s\cdot\nabla_s\bar{n}). \quad \sim O(r_b Re^{-1/2}).$$

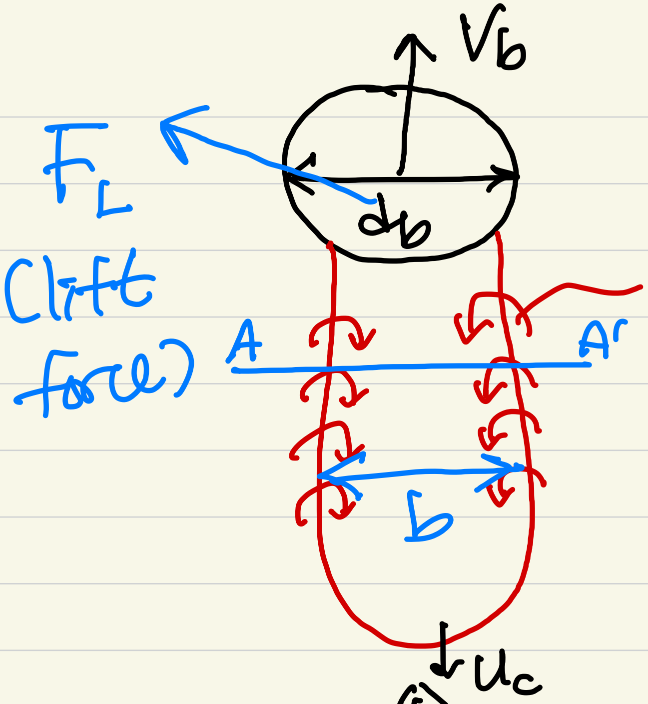
∴ surface vorticity flux (injected toward flow)

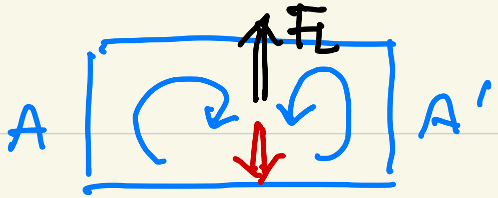
$$\frac{\partial \bar{\omega}_s}{\partial n} \sim Re^{-1/2}\cdot \underbrace{\chi^{7/2}}_{\text{aspect ratio}}.$$

, and the maximum surf. vorticity

$$\frac{\omega_{max} \cdot r_b}{U_\infty} \sim \chi^{8/3}$$

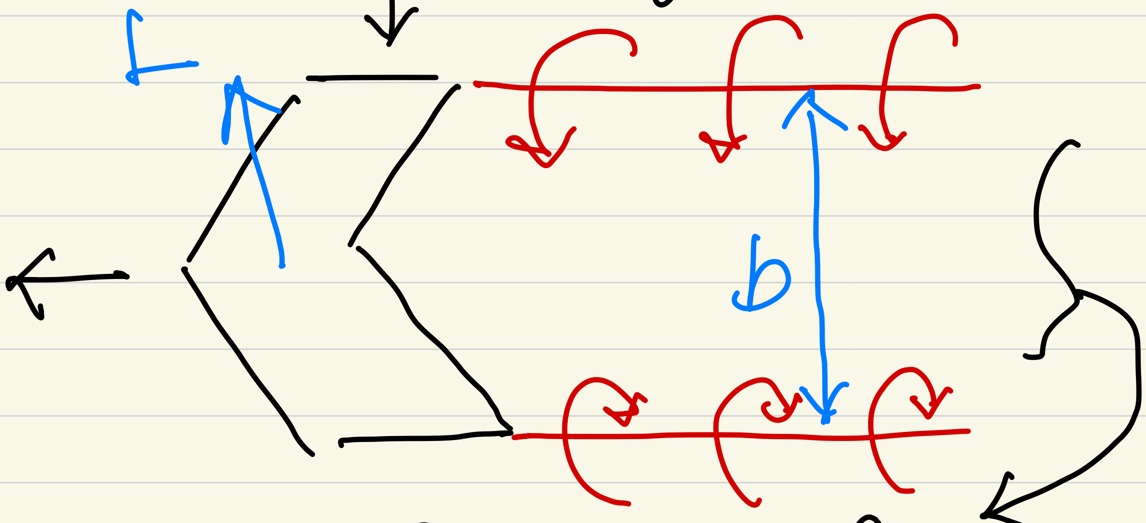
- Oscillating (zigzag, helical (or spiral) path.
  - w/ the appearance of two counter-rotating vortices (w/ streamwise vorticity component) behind the bubble.
  - change the sign to induce the lateral motion.
  - the trailing vortex pair is wrapped up for the spiral path.





streamwise vortex pair, aligned w/ the free-stream direction (or bubble rise direction)

analogy to the aircraft trailing vortex



(Govardhan & Williamson, 2005)

$$\Gamma = \oint \bar{u} \cdot d\bar{l} = \int_A (\boxed{\nabla \times \bar{u}}) \cdot \bar{n}\, dA = \int_A \bar{\omega} \cdot \bar{n}\, dA$$

stokes theorem.

from Kutta-Joukowski Law : $F_L = -\rho \underline{u_c} \Gamma b$.

density

convection vel.

$\Downarrow$

$$C_L = \frac{F_L}{\frac{1}{2}\rho V_b^2 \cdot \frac{\pi}{4} d_b^2} = \frac{\rho}{\pi} \cdot \frac{u_c}{V_b} \cdot \frac{\Gamma}{V_b d_b} \cdot \frac{b}{d_b}.$$

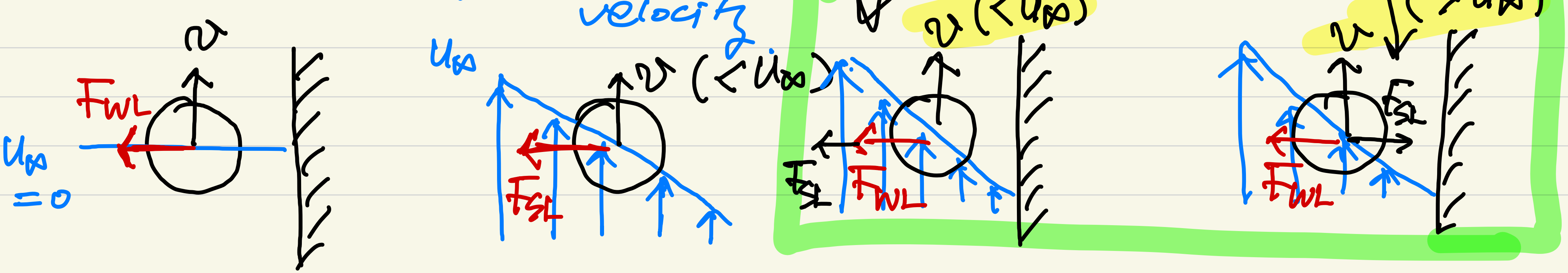
⓪ Bubble rise in a uniform shear.
(linear)

→ primary source of lift in bubbly flows.
(Shi & Rzehak, 2020)

① shear lift and ② wall (repulsive) lift

(in unbounded flow)
⇓
directed to the direction of
$\bar{\omega} \times \bar{u}_r$
vorticity
relative velocity

(zero in unbounded flow)
⇓
directed away from the wall



· Saffman lift (uniform shear, viscous, unbounded)

$$F_L = 6.46\,\mu_l u_r \left(\frac{d_b}{2}\right)^2 \left|\frac{du_l}{dr}\right|^{0.5} \Big/ \nu^{0.5}$$

liquid

relative ($u_r = v - u$)

└ slip velocity

local fluid shear rate.

→ the most popular ones

· Auton lift (rotational inviscid flow)

└→ net lift force due to SL and WL

$$F_L = -C_L \rho_l \cdot \frac{\pi d_b^3}{6} \cdot u_r \times \frac{\partial u_l}{\partial r}.$$

← no viscosity term.

C_L → 0.5 (analytical solution by Auton (1987))

or

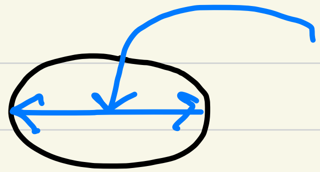
take the minimum of

$\min[0.288 \tanh(0.121 Re), f(E_o)]$, $E_o < 4.0$

$f(E_o)$ $\quad 4 \le E_o \le 10.7$

$$f(E_0) = 0.00105 E_0^3 - 0.0159 E_0^2 - 0.0204 E_0 + 0.474$$

$$\left(Re = \rho_l U_r d_b / \mu_l , \quad E_0 = g \Delta\rho d_H^2 / \sigma \right)$$



d_H : max. horizontal length of the bubble.

· in a pipe (channel) flow

i) upward flow

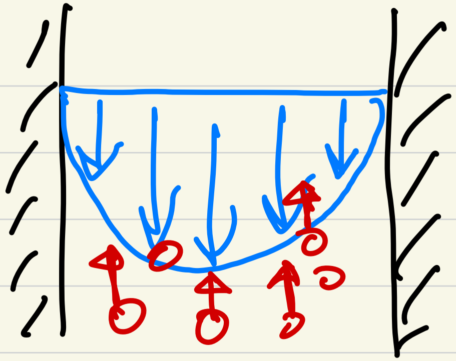
$U_r > 0$, $\nabla \times U_l > 0$.

→ small bubbles, $C_L > 0 \Rightarrow$ to wall.

large 〃 $\begin{cases} E_0 < 6.0 : C_L > 0 \text{ 〃} \\ E_0 \geq 6.0 : C_L < 0 \end{cases}$

to core.

ii) downward flow



$u_r > 0$. $\nabla \times u_\ell < 0$.

→ small bubbles, $C_L > 0$ → to core

large 〃 $\begin{cases} Eo < 6.0 : C_L > 0 \ 〃 \\ Eo > 6.0 : C_L < 0 \text{ to wall.} \end{cases}$

⇓

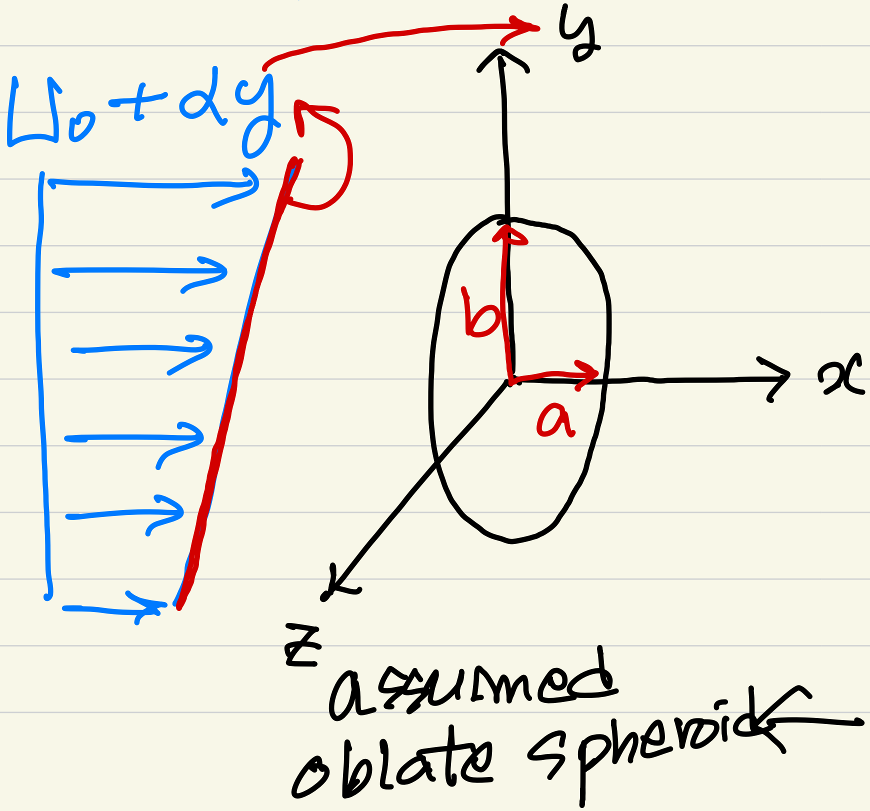
◎ Peaking conditions in bubbly flows.

: bubbly flow is characterized by gas-phase distribution, exhibiting lateral void fraction profiles, depending on flow rate of gas and liquid phases.

↳ wall peak, intermediate peak, core peak

⊙ Effect of aspect ratio (χ) on the lift coefficient.
(Adoua et al. 2009. JFM)

→ ~~un~~like the spherical bubble, larger bubble tends to reverse the direction of lateral migration when its size (aspect ratio) exceeds the critical value.



$$Re = U_0 (2b)/\nu = 50 - 4000$$

$$Sr\ (\text{shear rate}) = \alpha \cdot \frac{2b}{U_0}$$

$$= 1.0 \sim 27$$

$$\chi = b/a \ (\text{aspect ratio})$$

$$r_b = (ab^2)^{1/3} = b\chi^{-1/3}$$

$$We = \rho U_0^2 b/\sigma = O(1).$$

$(We \cdot Sr^2 \ll 1)$.

· with a weak shear, and inviscid approximation.

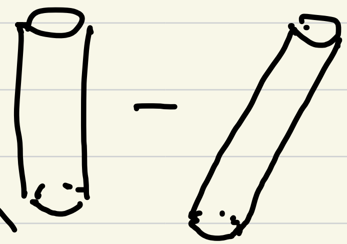
$$F_L \sim \rho \underset{\text{volume}}{V_b} \alpha \omega_0 = C_L \left(\frac{4}{3}\pi a b^3\right) \rho \alpha \omega_0 .$$

· explanation of reversed sign of C_L w/ χ.

Helmholtz vorticity equation w/o viscosity term.

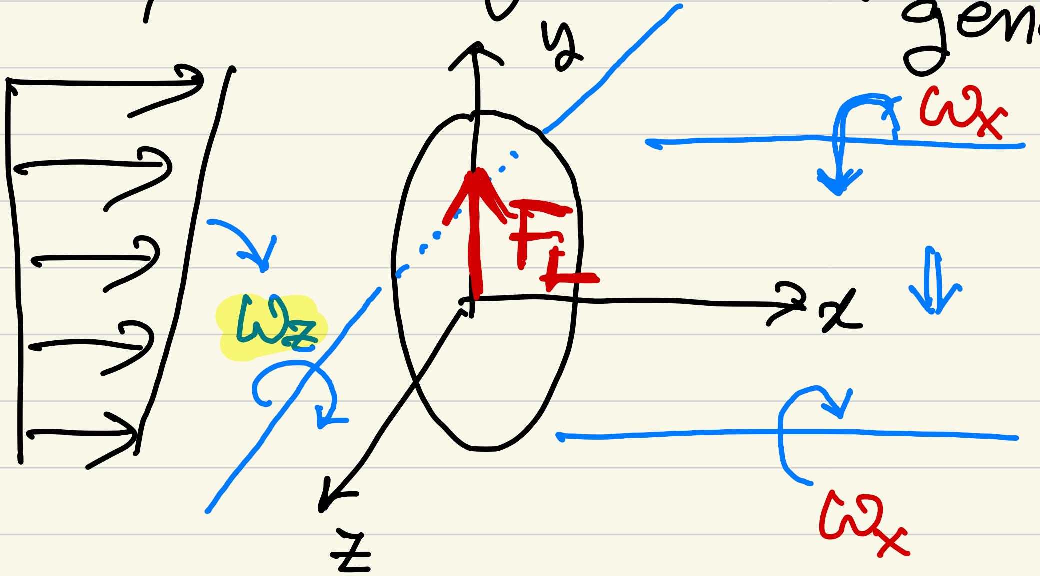
$$\nabla \times (\text{N-S eq}) \quad \nabla \times \left(\rho \frac{D\bar{u}}{Dt} = -\nabla p + \mu \nabla^2 \bar{u} \right)$$

$$\Rightarrow \frac{D\bar{\omega}}{Dt} = \frac{\partial \bar{\omega}}{\partial t} + (\bar{u}\cdot\nabla)\bar{\omega} = (\bar{\omega}\cdot\nabla)\bar{u}$$



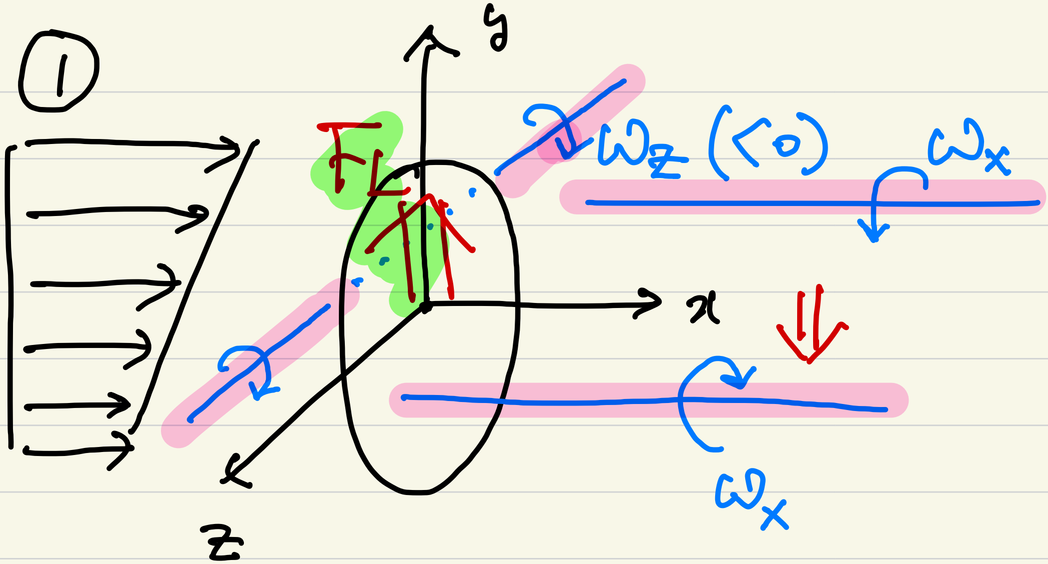
← stretching / tilting of the vortex line (tube).

(1) w/ incoming shear (stronger than vorticity generated on bubble surface)



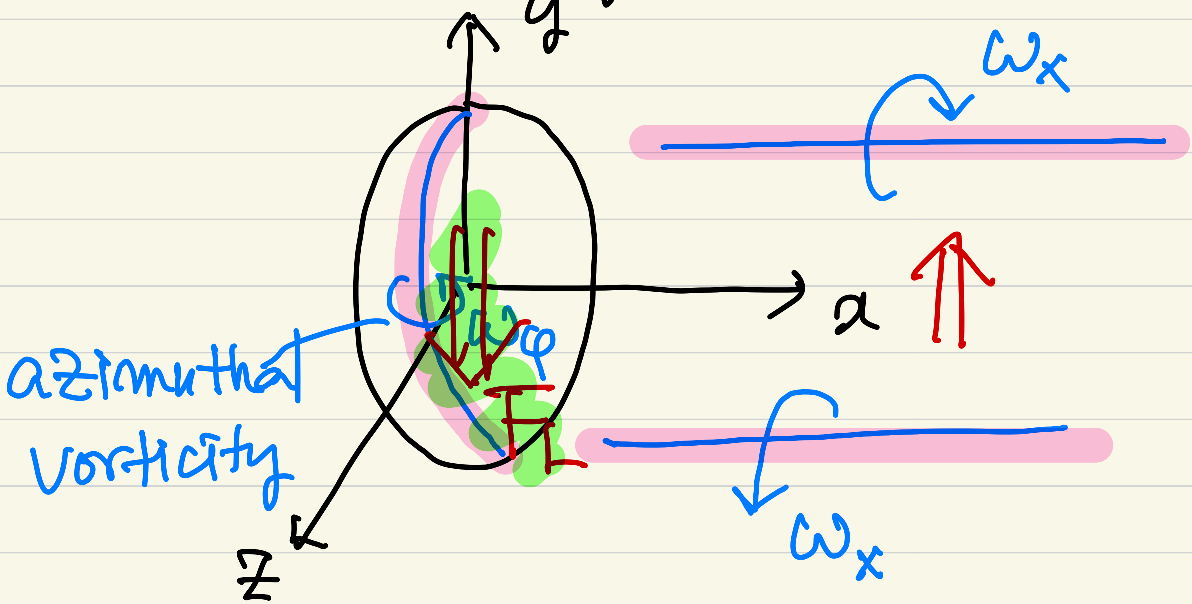
← classical Lighthill's mechanism (Lighthill, 1956).

①



classical Lighthill's mechanism.

② however, w/ stronger surf. vorticity with increasing the aspect ratio



← this will not happen if the wake is stable!

FLOW Patterns

- Geometric distribution or topology of components → mass, mtm, energy transport

↓ flow patterns (or regimes)

- visualization
- dependent on volume fluxes, volume fraction, density, viscosity, surf. tension...
- transition between regimes?

  instability, but it is mostly unpredictable.

→ boundaries are not distinctive and more poorly defined transition zone.

- while there are some popular flow maps, there are still challenging issues

only available for simple geometry like vertical/horizontal pipes.

- non-dimensionalization.
- developing vs fully-developed.
- initial condition.
- lack of fundamental understandings.

VI BUBBLE GROWTH/COLLAPSE and CAVITATION

- Rayleigh-Plesset eq. to describe bubble oscillation
  - single spherical bubble in unbounded cond.