Diffusion in Plasmas

$$0 = \pm ne(\vec{E} + \vec{u}\times\vec{B}) - k_B T \nabla n - mn\nu\vec{u}$$

i) $B=0$, weakly ionized $\qquad \vec{\Gamma} = n\vec{u}$

$$\vec{u} = \pm\mu\vec{E} - D\frac{\nabla n}{n}, \quad \mu = \frac{e}{m\nu}, \quad D = \frac{k_B T}{m\nu}, \sim \frac{(\Delta x)^2}{\Delta t} = \nu\lambda^2$$

ii) $B\neq 0$, weakly ionized

$$\vec{u}_\perp = \pm\frac{\mu}{1+\omega_c^2/\nu^2}\vec{E} - \frac{D}{1+\omega_c^2/\nu^2}\frac{\nabla n}{n} + \frac{(\vec{u}_E + \vec{u}_D)\,\omega_c^2/\nu^2}{1+\omega_c^2/\nu^2}$$

① $\omega_c^2/\nu^2 \ll 1 \longrightarrow \mu_\perp \sim \mu, \quad D_\perp \sim D$ $\quad$ (질량 $m\uparrow \rightarrow r_L = \frac{mv_\perp}{B|q|}\uparrow$ → 이동이 더 많다)

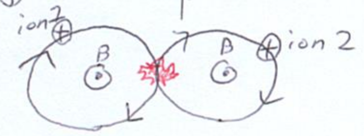
② $\omega_c^2/\nu^2 \gg 1 \rightarrow \mu_\perp \simeq \frac{\mu}{\omega_c^2/\nu^2}, \quad D_\perp \sim \frac{D}{\omega_c^2/\nu^2} = \frac{k_B T\nu}{\omega_c^2 m} \propto \nu$

$$\left(\omega_c = \frac{Be}{m} = \frac{v_\perp}{\frac{m v_\perp}{Be}} = \frac{v_\perp}{r_L}\right)$$

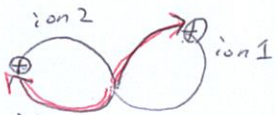
$$\frac{k_B T\nu}{\omega_c^2 m} = \frac{m v_\perp^2}{\left(\frac{v_\perp}{r_L}\right)^2 m} = \nu r_L^2 = \frac{(\Delta x)^2}{\Delta t}$$

iii) $B\neq 0$, fully ionized

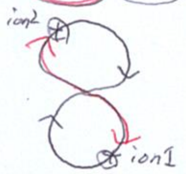
① like-particle collision



→ 180° collision

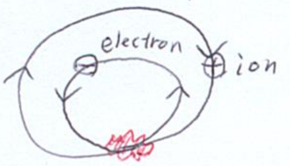


→ 90° collision

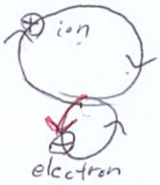


⇒ 거의 영향 X

② Unlike-particle collision



→



$(\because M \gg m)$

$B\uparrow \Rightarrow D\downarrow$

$T\downarrow \Rightarrow \nu_{ei}\uparrow \Rightarrow D\uparrow$

$n\uparrow \Rightarrow D\uparrow$

$$\left(D_\perp = \frac{k_B T}{m\nu}\frac{\nu^2}{\omega_c^2} \propto T\right)$$ 다른 경향 (ν_D)

* Set of MHD equations

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

$$\frac{\partial\sigma}{\partial t} + \nabla\cdot\vec{J} = 0$$

$$\boxed{\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{j}\times\vec{B} - \nabla P}$$ (0, steady state, 작다.)

$$\boxed{\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j} + \frac{\cancel{\vec{j}\times\vec{B} - \nabla P}}{ne}}$$ (작다.)

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0$$

$$\varepsilon_0\nabla\cdot\vec{E} = \sigma$$

steady state plasma

둘이 같다. → $\vec{j}\times\vec{B} = \nabla P = k_B T\nabla n$

$$(\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j})\times\vec{B}$$

$$\vec{E}\times\vec{B} + \vec{B}\times(\vec{B}\times\vec{v}) = \eta\vec{j}\times\vec{B}$$

$$\vec{E}\times\vec{B} - B^2 v_\perp = \eta_\perp k_B T\nabla n$$

$$\vec{v_\perp} = \frac{\vec{E}\times\vec{B}}{B^2} - \frac{\eta_\perp k_B T\nabla n}{B^2}$$

$$\vec{\Gamma}_\perp = n\vec{v}_\perp = -\frac{n\eta_\perp k_B T}{B^2}\nabla n = -D_\perp\nabla n$$

$$D_\perp = \frac{n\eta_\perp k_B T}{B^2} \propto T_e^{-\frac{1}{2}}$$

$$\eta_\perp = \frac{m\nu_{ei}}{ne^2} \propto T_e^{-\frac{3}{2}}$$

$B\uparrow \Rightarrow D\downarrow$

$\nu\uparrow \Rightarrow D\uparrow$

$n\uparrow \Rightarrow D\uparrow$

실험식

$$D_\perp = \frac{1}{16}\frac{k_B T}{eB}$$

* Diffusion in Plasma

gas $\quad \vec{\Gamma} = -D\nabla n, \quad D = \frac{(\Delta x)^2}{\Delta t} = \lambda^2 \nu$ (a)

weakly ionized gas $B=0$ $\quad \vec{\Gamma} = \pm n\mu\vec{E} - D\nabla n, \quad D = \frac{k_B T}{m\nu}$ (b)

weakly ionized gas $B \neq 0$ $\quad \vec{\Gamma}_\perp = \pm n\mu_\perp \vec{E} - D_\perp \nabla n + \frac{n\omega_c^2/\nu^2}{1+\omega_c^2/\nu^2}(\vec{u}_E + \vec{u}_D), \quad D_\perp = \frac{k_B T/m\nu}{1+\omega_c^2/\nu^2}$ (c)

fully ionized plasma $B \neq 0$ $\quad \vec{\Gamma}_\perp = \frac{\vec{E}\times\vec{B}}{B^2} - D_\perp \nabla n; \quad D_\perp = \frac{n\eta\Sigma k_B T}{B^2}$ (d) $\quad \eta = \frac{m\nu_{ei}}{ne^2}$

① ν

(a). $D \sim \frac{v^2}{\nu^2}\nu = \frac{v^2}{\nu} \propto \frac{1}{\nu}$

(b) $D = \frac{k_B T}{m\nu} \propto \frac{1}{\nu}$

(c) $D_\perp \sim \frac{k_B T/m\nu}{\omega_c^2/\nu^2} \propto \nu$

(d) $D_\perp = \frac{n \frac{m\nu_{ei}}{ne^2} k_B T}{B^2} \propto \nu_{ei}$

② n. $\quad (\nu = n\sigma v)$ → cross section $= \pi d^2$

(a) $D \sim \frac{v^2}{n\sigma v} = \frac{v}{n\sigma} \propto \frac{1}{n}$

(b) $D \propto \frac{1}{\nu} \propto \frac{1}{n}$

(c) $D_\perp \propto \nu \propto n$

(d) $D_\perp \propto n\eta = n\frac{m\nu_{ei}}{ne^2} \propto \nu_{ei} = \frac{ne^4}{16\pi\varepsilon_0^2 m^2 v^3} \propto \frac{n}{v^3} \propto nT^{-\frac{3}{2}}$

③ T

(a) $D \sim \frac{v}{n\sigma} \propto v \propto \sqrt{T}$

(b) $D = \frac{k_B T}{mn\sigma v} \propto \frac{T}{v} \propto \sqrt{T}$

(c) $D_\perp \propto k_B T\nu \propto Tv \propto T^{\frac{3}{2}}$

(d) $D_\perp \propto \eta T = \frac{m\nu_{ei}}{ne^2}T \propto T^{-\frac{3}{2}}T \propto T^{-\frac{1}{2}}$

④ m $\quad (k_B T \doteqdot \frac{1}{2}mv^2 = \text{const})$

(a) $D \propto v \propto \frac{1}{\sqrt{m}}$

(b) $D \propto v \propto \frac{1}{\sqrt{m}}$

(c) $D_\perp \sim \frac{mv^2/mn\sigma v}{B^2e^2/m^2n^2\sigma^2v^2} \propto m^2v^3 \propto \sqrt{m}$ $\quad r_L$ 때문

(d) $D_\perp \propto \eta T \propto m\nu_{ei} mv^2 \propto \frac{m^2v^2}{m^2 v^3} \propto \sqrt{m}$

⑤ B

(a) X

(b) X

(c) $D_\perp \propto \frac{1}{\omega_c^2} \propto \frac{1}{B^2}$

(d) $D_\perp \propto \frac{1}{B^2}$

예측

$D_i \sim 40 D_e$

$D \sim 10^{-4}\, m^2/s$

$D_\perp \sim \frac{T^{-\frac{1}{2}}}{B^2}$

실험

$D_i \sim D_e$

$D \sim 1\, m^2/s$

$D_B = \frac{1}{16}\frac{k_B T}{eB}$