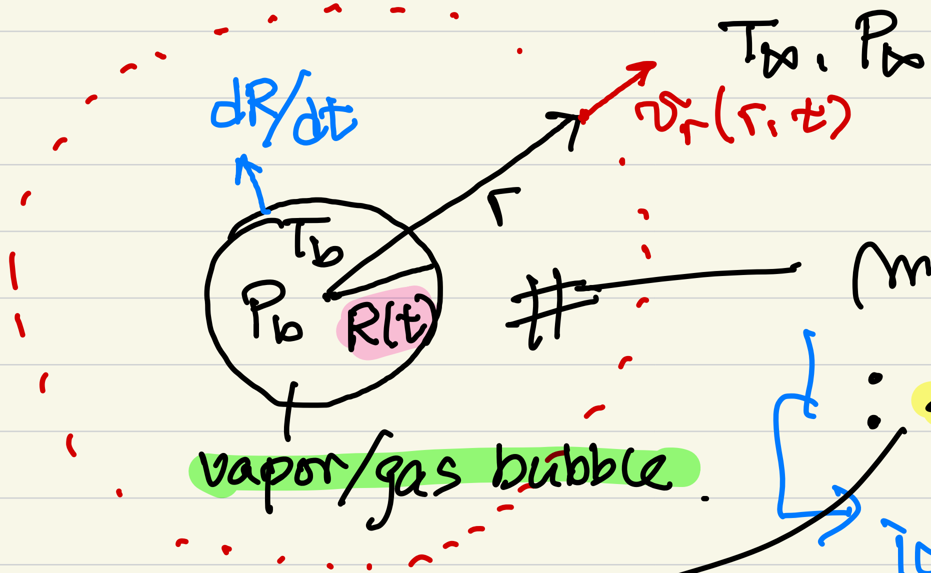
- while there are some popular flow maps, there are still challenging issues
  - non-dimensionalization.
  - developing vs fully-developed.
  - initial condition.
  - lack of fundamental understandings.

only available for simple geometry like vertical/horizontal pipes.

VI BUBBLE GROWTH/COLLAPSE and CAVITATION

- Rayleigh-Plesset eq. to describe bubble oscillation
  - single spherical bubble in unbounded cond.

- constant properties.
- uniform temperature (T_b) and pressure (P_b) $\underbrace{\phantom{T_b \text{ and pressure } P_b}}_{\text{bubble.}}$
- constant vapor pressure.



mass conservation ✓

$$\therefore \; 4\pi R^2 \cdot \frac{dR}{dt} = 4\pi r^2 \cdot v_r(r,t)$$

ideal condition
(zero mass transport across the bubble interface).

valid for many evaporation and condensation proplems.

$$\therefore v_r(r,t) = \frac{R^2 \cdot \frac{dR}{dt}}{r^2} = \frac{F(t)}{r^2}$$

N-S eq in r-direction:

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} = -\frac{1}{\rho_l}\frac{\partial p}{\partial r} + \cancel{\nu_l \left\{ \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_r}{\partial r}\right) - \frac{2 v_r}{r^2}\right\}}$$

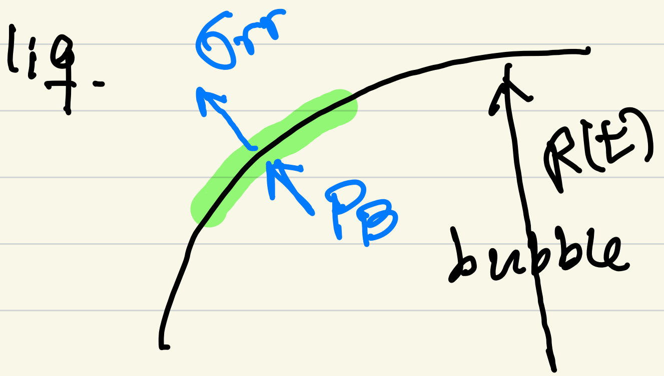
viscous term vanishes.

$$-\frac{1}{\rho_l}\frac{\partial p}{\partial r} = \frac{1}{r^2}\frac{dF}{dt} - \frac{2F^2}{r^5}$$

↓ integrate $\int_r^\infty dr$

$$\boxed{\frac{P-P_\infty}{\rho_l} = \frac{1}{r}\frac{dF}{dt} - \frac{1}{2}\frac{F^2}{r^4}}$$

← to solve this, we need a dynamic BC at the bubble interface.



← force balance along the normal (radial) direction

$$\therefore \left.\sigma_{rr}\right|_{r=R} + P_B - \frac{2\sigma}{R} = 0$$

$$\sigma_{rr} = -p + 2\mu_l \frac{\partial u}{\partial r}$$

dynamic BC at bubble interface.

$$\therefore P_B - \left.p\right|_{r=R} - \frac{4\mu_l}{R}\frac{dR}{dt} - \frac{2\sigma}{R} = 0.$$

$$\therefore \frac{P_B - P_\infty}{\rho_l} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\nu_l}{R}\frac{dR}{dt} + \frac{2\sigma}{\rho_l R}$$

: Generalized Rayleigh-Plesset eq.

w/ P_∞ as an external excitation (forcing),

$R(t)$ is obtained when P_B is known.

function of bubble contents.

General Rayleigh-Plesset eq.

kinematic viscosity

$$\frac{P_B - P_\infty}{\rho_l} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\nu_l}{R}\frac{dR}{dt} + \frac{2\sigma}{\rho_l R}.$$

? Reasonably, it is assumed that

① bubble contains some contaminant gas (like vapor), of which the partial pressure is P_{G0} @ R_0 and T_∞.

initial

② no mass transport across the interface (in equilibrium)

should be determined!

$$P_B(t) = P_V(T_B) + P_{G0}\left(\frac{T_B}{T_\infty}\right)\left(\frac{R_0}{R}\right)^3 \rightarrow \text{into R-P eq.}$$

vapor pressure

$$\frac{P_v(T_\infty) - P_\infty(t)}{\rho_l} + \cancel{\frac{P_v(T_B) - P_v(T_\infty)}{\rho_l}} + \frac{P_{G0}}{\rho_l}\left(\frac{T_B}{T_\infty}\right)\left(\frac{R_0}{R}\right)^3$$

$$= R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right) + \frac{4\nu_l}{R}\frac{dR}{dt} + \frac{2\sigma}{\rho_l R}$$

instantaneous driving term for tension or compression, far from the bubble.

thermal term : for small ΔT

$$\Delta T \simeq A(T_B - T_\infty)$$

$$A \equiv \frac{1}{\rho_l}\frac{dP_v}{dT} = \frac{\rho_v(T_\infty)L(T_\infty)}{\rho_l T_\infty}$$

- w/o thermal effect. (inertially controlled case)
  - thermal term is zero.
  - $P_G = P_{G0}\left(\frac{R_0}{R}\right)^{3k}$: polytropic gas $\begin{cases} k=1, \text{ isothermal (const. temp.)} \\ k=\gamma, \text{ adiabatic} \end{cases}$

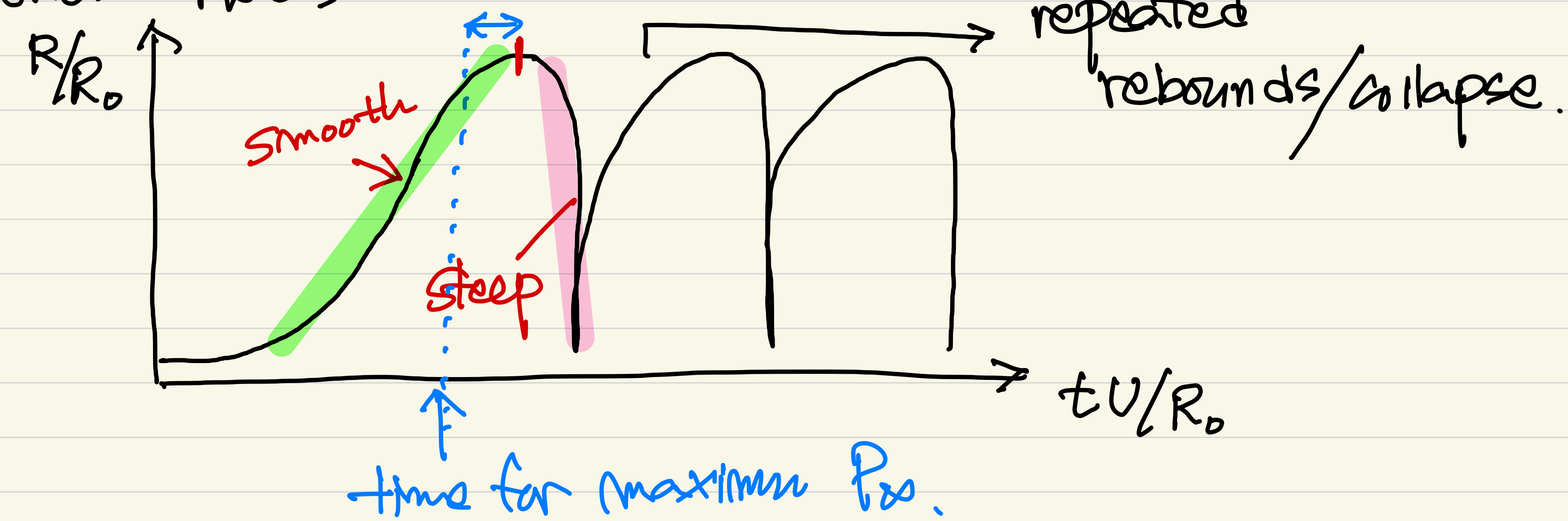
$$\Rightarrow \frac{P_V(T_\infty) - P_\infty(t)}{\rho_l} + \frac{P_{G0}}{\rho_l}\left(\frac{R_0}{R}\right)^{3k} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \underline{\frac{4\nu_l}{R}\frac{dR}{dt}} + \frac{2\sigma}{\rho_l R}.$$

↳ numerically integrated to get $R(t)$

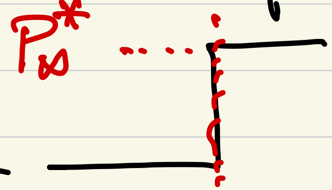
w/ assuming the equilibrium at $t=0$

$$\boxed{P_V(T_\infty) + P_{G0} = \underline{P_\infty(0)} + \underline{\frac{2\sigma}{R_0}}}$$

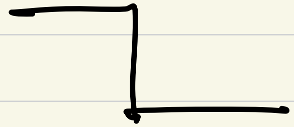
· when $P_\infty(t)$ is sinusoidal.



• Analytical solution (for step function change in P_∞)



or



for $P_\infty(t>0) = P_\infty^*$ (constant), $\uparrow t=0$

inviscid assumption.

$\rightarrow$ (R-P eq.) $\times\ 2R^2 \dfrac{dR}{dt} \rightarrow$ integrate w/ $\dfrac{dR}{dt}(0)=0$

$$\left(\frac{dR}{dt}\right)^2 = \frac{2(P_V - P_\infty^*)}{3\rho_l}\left(1-\frac{R_0^3}{R^3}\right) + \frac{2P_{G0}}{3\rho_l(1-k)}\left(\frac{R_0^{3k}}{R^{3k}} - \frac{R_0^3}{R^3}\right) - \frac{2\sigma}{\rho_l R}\left(1-\frac{R_0^2}{R^2}\right)$$

(for adiabatic)

$\rightarrow\ 2\dfrac{P_{G0}}{\rho_l}\dfrac{R_0^3}{R^3}\ln\left(\dfrac{R_0}{R}\right)$

(for isothermal)

$\downarrow$

$$t = R_0 \int_1^{R/R_0} \left[\frac{2(P_V - P_\infty^*)(1 - x^{-3})}{3\rho_l} + \frac{2P_{G0}(x^{-3k} - x^{-3})}{3(1-k)\rho_l} - \frac{2\sigma(1-x^{-2})}{\rho_l R_0 x} \right]^{-1/2} dx$$

(for adiabatic)

$$\rightarrow \frac{2P_{G0}}{x^3} \ln x$$ (for isothermal)

if $P_\infty^* < P_\infty(0)$, the bubble growth rate is

asymptotically ($R \gg R_0$)

$$\rightarrow \frac{dR}{dt} \rightarrow \left[\frac{2}{3} \frac{P_V - P_\infty^*}{\rho_l} \right]^{1/2}$$

if $P_\infty \rightarrow P_\infty^*$ ($R \ll R_0$), the bubble collapse rate

$$\frac{dR}{dt} \rightarrow -\left(\frac{R_0}{R}\right)^{3/2} \left[\frac{2}{3} \frac{P_\infty^* - P_V}{\rho_l} + \frac{2\sigma}{\rho_l R_0} - \frac{2}{3} \frac{P_{G0}}{(k-1)\rho_l} \left(\frac{R_0}{R}\right)^{3(k-1)} \right]^{1/2}$$

(for adiabatic)

$$\frac{2P_{G0}\cdot \ln(R_0/R)}{\rho_l}$$ for isothermal ($k=1$)

[for most engineering cases, the bubble collapse is so fast! ⇒ adiabatic.

w/ substantial gas contents, this dR/dt is not reached, but oscillate at a smaller size.

w/ little gas content

$$R_{min} = R_0\left[\frac{1}{k-1}\cdot\frac{P_{G0}}{P_\infty^* - P_v + 2\sigma/R_0}\right]^{\frac{1}{3(k-1)}}$$

is achieved after several rebounds.

• Stability of vapor/gas bubble.

$$P_V - P_\infty + P_{GE} - \frac{2\sigma}{R_E} = 0$$ in Equilibrium.

$\uparrow$ $\uparrow$ $\uparrow$

$\Rightarrow$ does not always represent the 'stable' condition.

• perturbation analysis, $\boxed{R = R_E(1+\varepsilon)}$, $\varepsilon \ll 1$

slow gas diffusion

↳ (i) P_{GE} is maintained to be the same

(ii) mass of gas and T_B are maintained to be the same.

fast gas diffusion.

into R-P eq.

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + 4\nu_l \frac{1}{R}\frac{dR}{dt} = \frac{\varepsilon}{\rho_l}\left[\overbrace{\frac{2\sigma}{R_E} - \boxed{3nkP_{GE}}}^{>0}\right]$$

$n = 0$ for (i)

$n = 1$ for (ii)

for stable

1.0

if $\boxed{\dfrac{2\sigma}{R_E} > 3nk\,\underline{P_{GE}}}$

→ dR/dt, and/or d^2R/dt^2 has the same sign w/ ε. ⇒ unstable.

– for (i), $n=0$ $\left(\dfrac{2\sigma}{R_E} > 0\right)$ → always unstable.

(time for the mass diffusion is guaranteed!)

– for (ii) : most engineering cases
$\underline{\quad\quad}$
$(n=1)$ (pressure changing time scale is shorter).

⇒ stable equilibrium

: $P_{GE} \left(= \dfrac{m_G T_B k_G}{\frac{4}{3}\pi R_E^3} \right) > \dfrac{2\sigma}{3kR_E}$.

gas constant.

for given m_G,

$$R_c = \left[\frac{9k m_G T_B k_G}{8\pi\sigma} \right]^{1/2}$$

Blake critical radius (1949?).

if $R_E < R_c$ – stable equilibrium

($R_E > R_c$ – unstable ")

· Blake critical radius (1949)

$$R_c = \left[\frac{9 k m_G T_B k_G}{8 \pi \sigma} \right]^{1/2}$$

$R_E < R_c$: stable Equilibrium

$R_E > R_c$: unstable 〃

↓

Blake threshold pressure.

$$P_{\infty c} = P_V - \frac{4\sigma}{3} \left[\frac{8\pi\sigma}{9 k m_G T_B k_G} \right]^{1/2}$$

[critical R_c can be reached by decreasing the ambient pressure from P_∞ to $P_{\infty c}$).

· for cavitation nuclei, $R_c \simeq 4\sigma / 3(P_V - P_\infty)^*$, (* measured at certain condition)

larger than R_c will grow explosively and cavitate!

⓪ Cavitation

- inception, growth and implosions of cavities in the body of a flowing or a quiescent liquid due to changes in the fluid pressure.

$$\sigma \text{ (cavitation number)} \equiv \frac{P_\infty - P_v(T_\infty)}{\frac{1}{2}\rho_\ell U_\infty^2}$$

vapor pressure.

↳ potential of cavitation.

- damage to solid surface.
  noise, vibration, ...
- cleaning, lubrication, drug delivery. ←

• inception.

P_{min} (minimum pressure in the flow) $\lesssim P_v$

$$\underbrace{\frac{P_{min} - P_\infty}{\frac{1}{2}\rho_l U_\infty^2}}_{C_{p,min}} \leq \underbrace{\frac{P_v - P_\infty}{\frac{1}{2}\rho_l U_\infty^2}}_{-\sigma} \Rightarrow \sigma \leq -C_{p,min}$$

pressure coeff.

press. coeff. $C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho U^2}$

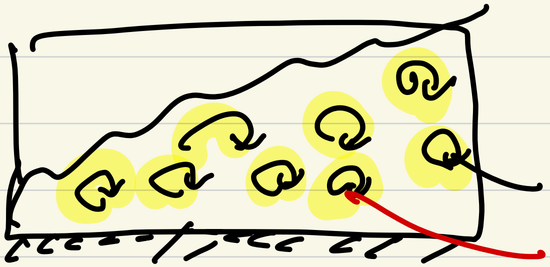
or, $\sigma_i = -C_{p,min}$ (ideal)

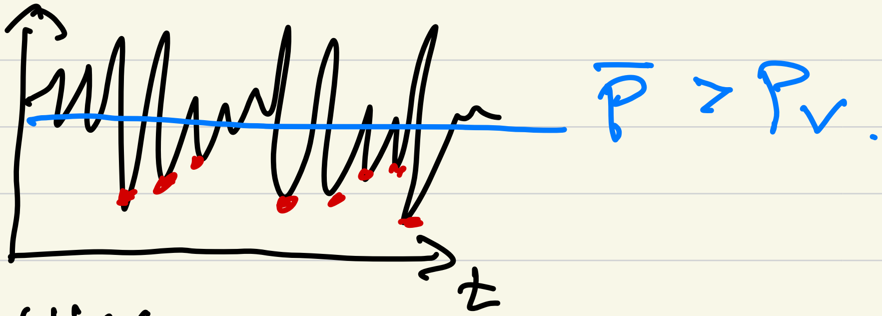
inception

practically, there are many factors affecting σ_i

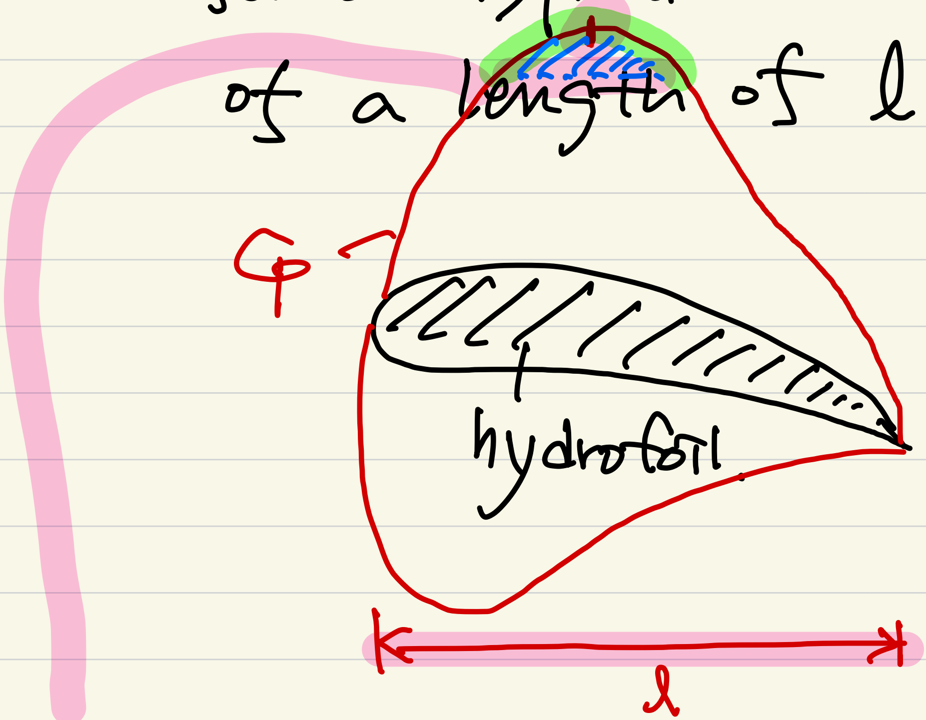
- ability of a liquid to sustain the tension.
  - ~ contamination of the liquid, i.e, existence of cav. nuclei (e.g. microscopic bubbles, particles, dissolved gas)
- residence time requirement.
- turbulent fluctuation : nuclei in the

middle of turb. eddy experiences pressures below P_v, even when the mean pressure is above it.





· for a typical external flow around a hydrofoil of a length of l.



bubble size

$$\Leftarrow \frac{dR}{dt} \simeq U_\infty \left(-\sigma - C_{p,min}\right)^{1/2} \quad \text{— ①}$$

Assuming the distribution around $C_{p,min}$ is parabolic (Brennen, 1995)

, then the region (below P_v)

$\Rightarrow \sim l(-\sigma - C_{p,min})^{1/2}$

; the residence time, $t_m \sim \dfrac{l(-\sigma - C_{p,min})^{1/2}}{U_\infty}$ — ②

① + ② $\Rightarrow$ maximum size

$$R_M \sim \underbrace{\cancel{U_\infty}(-\sigma - C_{p,min})^{1/2}}_{①} \cdot \underbrace{\frac{l(-\sigma - C_{p,min})^{1/2}}{\cancel{U_\infty}}}_{②}$$

$$\sim l(-\sigma - C_{p,min}).$$

◎ Nuclei

- phase change occurs at the phase boundary in the liquid $\rightarrow$ if not, the bulk liquid sustain the pressure below P_v $\Rightarrow$ homogeneous nucleation
- mostly, voids/bubbles appear as the liquid evaporates at the interface of small gas

↳ inclusions: heterogeneous nucleation.

↳ <u>nuclei</u>: small ($\mathcal{O}(1-10^2)\,\mu m$) gas bubbles.

  floating or trapped, dissolved gas

  ↳ actual amount of nuclei determines σ_i !!

⓪ Bubble growth (R-P eq.)

$$\frac{P_v(T_\infty)-P_\infty(t)}{\rho_l}+\frac{P_{G0}}{\rho_l}\left(\frac{R_0}{R}\right)^{3k}=R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2+\frac{4\nu_l}{R}\frac{dR}{dt}+\frac{2\sigma}{\rho_l R}$$

$$P_{G0}=\underline{P_\infty}(0)-P_v(T_\infty)+\frac{2\sigma}{R_0}$$ (initial condition)

$$\rho_l\left[R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2\right]+\frac{4\mu_l}{R}\frac{dR}{dt}$$

$$=\left(P_\infty(0)-P_v+\frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3k}+P_v(T_\infty)-P_\infty(t)-\frac{2\sigma}{R}.$$

(gas follows the polytropic relation

$k=1$, isothermal, $k=C_p/C_v$. adiabatic)

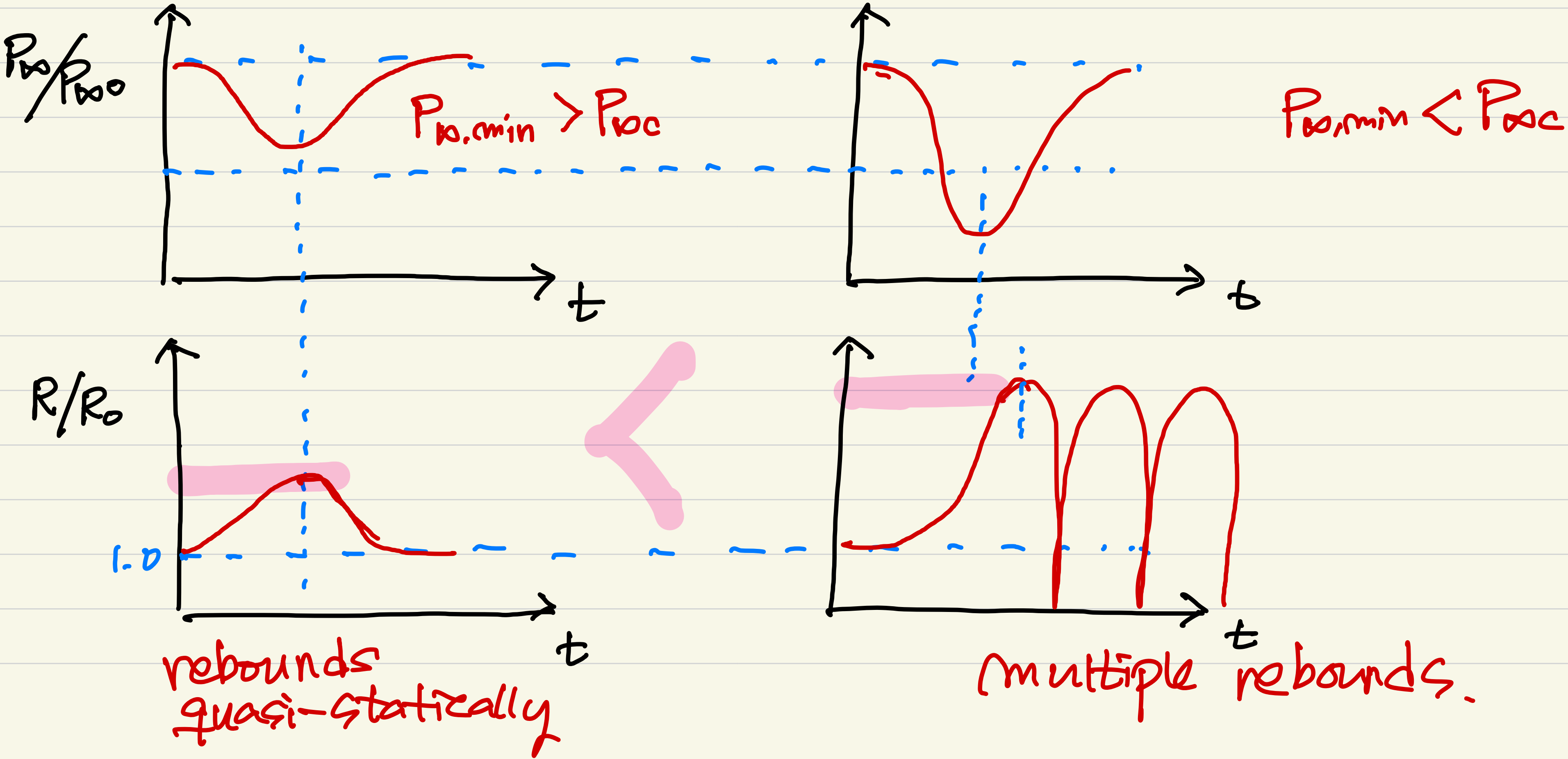
if we can assume the 「quasi-static equilibrium」 state:

$$0=\left(P_\infty(0)-P_v+\frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3k}+P_v(T_\infty)-P_\infty(t)-\frac{2\sigma}{R}$$

Consider a small bubble (i.e., nuclei) in

$$\underline{P_B} = P_G + P_V = \frac{2\sigma}{R} + P_\infty$$

as $P_\infty \downarrow \Rightarrow$ bubble grows quasi-statically from $\underline{R_0}$ to larger equilibrium radius.



⊙ Bubble collapse

- Practical importance owing to the noise and damage caused by the high velocities, pressure and temperature
- at final stage of implosion, interface speed reach or exceed the speed of sound

→ shock wave, liquid compressibility

→ modified R-P eq. (Keller & Kolodner, 1956
Prosperetti & Lezzi, 1986)

$$\left(1-\frac{1}{c_L}\frac{dR}{dt}\right)R\frac{d^2R}{dt^2}+\frac{3}{2}\left(1-\frac{1}{3c_L}\frac{dR}{dt}\right)\left(\frac{dR}{dt}\right)^2$$

$$=\left(1+\frac{1}{c_L}\frac{dR}{dt}\right)\frac{1}{\rho_\ell}\left\{P_B-P_\infty-P_c\left(t+\frac{R}{c_L}\right)\right\}+\frac{R}{\rho_\ell c_L}\frac{dP_B}{dt}.$$

+ speed of sound in the liquid

$P_c(t)$: variable pressure of the liquid at the bubble center w/o bubble.

⊙ Cavitating flows.

- vortex cavitation

in many high-Re flows, the region of concentrated vorticity (significantly low pressure at the core) ⇒ source of inception.

e.g.) tip vortices from ship propellor, pump impeller, or swirling flow of a turbine.

- Cloud cavitation

  periodic formation and shedding of a "cloud" of cavitation bubbles

  → more intense noise and damage than the non-fluctuating flows.

  → critical for pump and propeller.

- Attached or sheet cavitation

  when a wake or separated flow fills with vapor.

  (called blade cavitation for pumps).