

## Precision Machine Design-Linear power transmission

### Linear power transmission

Power transmission from rotary motion to linear motion is essential for most precision application, and the frequently types are rack and pinion, friction drive, lead screw, ball screw; thus will be discussed in more detail.

#### 1) Rack and pinion

Rack and pinion is one of very low cost transmission methods from rotary to linear transmission, commonly used in large machines such as gantry robots and large machine tools whose travel length is over 3-5 m typically. The motor and pinions are often mounted on the carriage running on the rack.

It has drawbacks; there is no elastic averaging effect, and tooth form errors, backlash can be transmitted directly to the linear motion, and the varying sinusoidal load may generate varying straightness error along the rack. Thus fine motion stage can be required for higher positioning accuracy. But the rack pinion mechanism has advantages in manufacturing cost with relatively simple design, thus it is one of essential alternatives to consider in practical mechanical design.



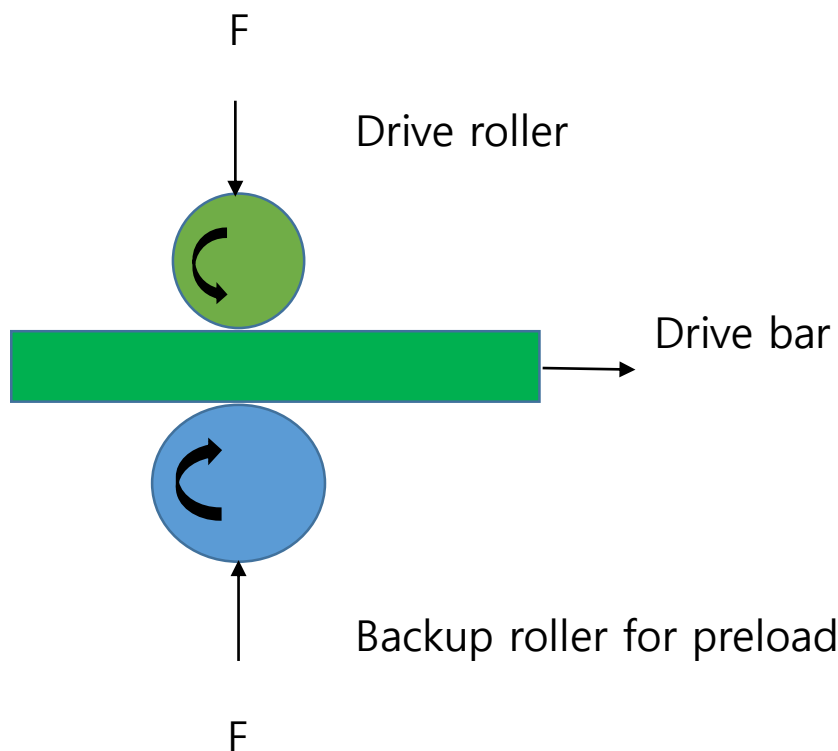
Rack and pinion (source:Wikipedia)

## 2)Friction drives

Friction drives provides very simple and precision linear transmission element consisting of: a wheel (capstan), flat bar, and backup roller, as in fig, and it is commonly used in linear actuators in precision machines.

Advantages are minimum backlash, or dead band due to elastic deformation, low drive friction, and simple design.

Drawbacks are, however, low driving force, relatively low stiffness and damping, limited transmission gain, and high sensitivity to the drive bar cleanliness, etc.



The preload,  $F$ , is usually about 10 times higher than the driving force, and the friction coefficient is 0.1 typically for most friction drive cases.

Stiffness in driving direction

=Tangential stiffness,  $K_{\text{tan}}$  is

$$K_{\text{tan}} = \partial F_{\text{tan}} / \partial \delta_{\text{tan}} = 4aE(1 - F_{\text{tan}} / \mu F)^{1/3} / [(2 - \nu)(1 + \nu)] \text{ with slip}$$

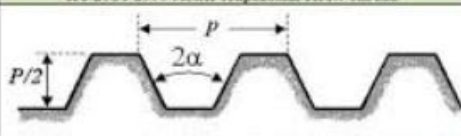
(If  $F_{\text{tan}} = 0$ , there is no slip between the interface)

The bar can be of channel shape, thus the centre of driving force can be located beneath the driving roll to prevent any bending moment due to the friction force. Two back rollers can help to prevent the pitch motion of drive bar during

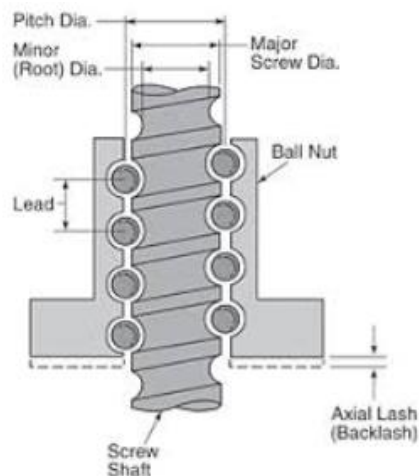
transmission. Round bar with V grooved roller configuration is also desirable, based on the kinematic design, minimizing any misalignment issues. Modular design for the friction drives are commercially available.

## 2) Lead screws

Lead screw is one of the simplest transmission elements, and it has been traditionally used for more than few thousand years such as in Archimedes' screw pump for irrigation from water reservoir, Lathe for cutting wooden screw in 15C, Henry Maudslay's lead screw cutting machine of 1/1000 inch (=25.4um) accuracy in 1800, Joseph Whitworth's lead screw driven machine tool of 1 micro inch (25.4nm) resolving capacity in 1855.

Lead Screw Calculation (Metric/ISO) v2.0	
ISO 2904-1977 Metric Trapezoidal Screw Thread	
	
Major Diameter of Lead Screw, $d_{major}$ (mm)	∞
Pitch, $p$ (mm)	*
Load, $F$ (N)	1000
Coeffint of Friction, $\mu$	0.1
Thread Angle, $2\alpha$	30
Mean Diameter of Lead Screw, $d_{mean}$	18
Lead, $l$	4
Lead angle	4.05
<b>Torque To Raise The Load (N-mm)</b>	<b>1579.94</b>
<b>Torque To Lower The Load (N-mm)</b>	<b>292.98</b>

A MEADInfo Product



Lead screw (source:meadinfo.org, designworldonline.com)

The travel length (L) of linear transmission can be connected through the screw rotation angle( $\varphi$ ) and lead( $\lambda$ ) such as,

$$L = \lambda\varphi / 2\pi$$

The lead angle ( $\theta$ ), or helix angle, is

$$\theta = \arctan (\lambda / 2\pi R)$$

where R is the pitch radius of screw.

Also, detail moment analysis can be given

(source: Slocum's precision machine design);

Let  $M_z$ ,  $F_z$  be the torque and force applied to the lead screw, respectively, and R is the radius of lead screw, and  $M_x$ ,  $M_y$  are the noise moments, respectively.

In case of lifting the load,

$$M_z = F_z R [\lambda \cos \alpha + 2\pi R \mu] / [2\pi R \cos \alpha - \mu \lambda] \quad \text{eq(1)}$$

and

$$M_x = F_z [C_{RR} (\cos \varphi - \sin \varphi / \varphi) + C_{\theta R} \{ (1 - \cos \varphi) / \varphi - \sin \varphi \}] \lambda / 2\pi \\ + R (1 - \cos \varphi) / \varphi]$$

$$M_y = F_z [C_{\theta R} (\cos \varphi - \sin \varphi / \varphi) + C_{RR} \{ \sin \varphi + (\cos \varphi - 1) / \varphi - R \sin \varphi / \varphi \}]$$

$$C_{\theta R} = (\lambda \cos \alpha + 2\pi R \mu) / (2\pi R \cos \alpha - \mu \lambda)$$

$$C_{RR} = -\sin\alpha / (\cos\alpha \sin\theta - \mu \sin\theta)$$

Where  $2\alpha$  = thread angle of screw, 45 deg. for ball screw thread ;

29 deg. for the standard ACME thread

$\mu$  = friction coefficient, 0.005~0.1

In case of lowering the load,

$$M_z = F_z R [2\pi R \mu - \lambda \cos\alpha] / [2\pi R \cos\alpha + \mu \lambda] \quad \text{eq(2)}$$

Observations from the analysis are;

(1) Back-drivability

From eq(2),

If  $\lambda \leq 2\pi R \mu / \cos\alpha$ , then  $M_z \geq 0$ .

Thus further torque is required to lower the load. This condition  $M_z \geq 0$  is the non-back-drivability, or self-locking condition. This can be useful for brake sizing or holding power calculation.

## (2) Efficiency

The efficiency,  $e$ , can be defined as the actual work divided by the ideal work ( $\mu=0$  case);

$e = \cos\alpha(\pi\beta\cos\alpha - \mu) / [\pi\beta\cos\alpha(\cos\alpha + \pi\beta\mu)]$  when lifting a load

where  $\beta = \text{Diameter to lead ratio} = 2R/\lambda$

When  $\mu$  decrease,  $\beta$  decreases; then  $e$  increase

## (3) Noise moment, $M_x$ and $M_y$

When lead screw is driven along Z axis, Moments  $M_x$  and  $M_y$  are also generated, and they are moments along the perpendicular directions, and it is due to unwanted resultant moment. These moments are called as the noise moments, they often contribute to noise generation during screw driving.

①  $M_x, M_y$  can be large depending on location and angles, thus care should be taken in design

②  $M_x, M_y$  are changing sinusoidally even in the small range of motion, that means there can be existing pitch and yaw errors in the small range of motion.

③  $M_x, M_y$  decrease as friction coefficient increase; it

explains why lead screw based on sliding friction gives less noise than the ball screw based on rolling friction

④  $M_x, M_y$  decrease, as  $\beta$  (diameter to lead ratio) increase; Lead screw of smaller helix angle gives quieter motion

⑤  $M_x, M_y$  increase when thread angle  $\alpha$  increase; lead screw of Acme thread ( $2\alpha=29\text{deg}$ ) gives quieter than ball screw thread ( $2\alpha=45\text{deg}$ )

⑥  $M_x, M_y$  are significantly meaningful when the submicron resolution is required.

#### (4) Axial stiffness for lead screw

From work balance;

$$F_z \lambda = 2\pi M_z \therefore F_z = 2\pi M_z / \lambda$$

Divide by  $\theta$ ;  $F_z / \theta = 2\pi K_\theta / \lambda$  as  $K_\theta = M_z / \theta$

Remembering  $\theta / 2\pi = x / \lambda$ , thus

$$F_z / \theta = F_z \lambda / 2\pi x = K_x \lambda / 2\pi \quad (\because K_x = F_z / x)$$

$$\therefore F_z / \theta = K_x \lambda / 2\pi = 2\pi K_\theta / \lambda$$

Thus  $K_x = 4\pi^2 K_\theta / \lambda^2$  and  $K_x / K_\theta = 4\pi^2 / \lambda^2$ ; eq(3)



Eq(3) shows that the equivalent axial stiffness( $K_x$ ) can be derived from the rotational stiffness( $K_\theta$ ) of lead screw.

The axial stiffness also can be related with torsional stiffness;

For a torsion bar, that is a cylinder of length( $L$ ) and cross sectional area( $A$ )



$$x = FL/EA$$

$$\theta = ML/GJ$$

$$K_x = F/x = EA/L$$

$$K_\theta = M/\theta = GJ/L$$

Where  $E$ =Young's modulus,  $G$ =Shear modulus,

$J$ =polar moment of inertia

Thus ratio of axial stiffness to the rotational stiffness is

$$K_x/K_\theta = EA/GJ = 16(1+\nu)/d^2 \quad \text{eq(4)}$$

For a typical lead screw of  $d=0.05\text{m}$ ,  $\lambda=0.01\text{m}$

Lead screw(Eq(3)):  $K_x/K_\theta = 4\pi^2/\lambda^2 = 4(3.14)^2/0.01^2 = 394,384$

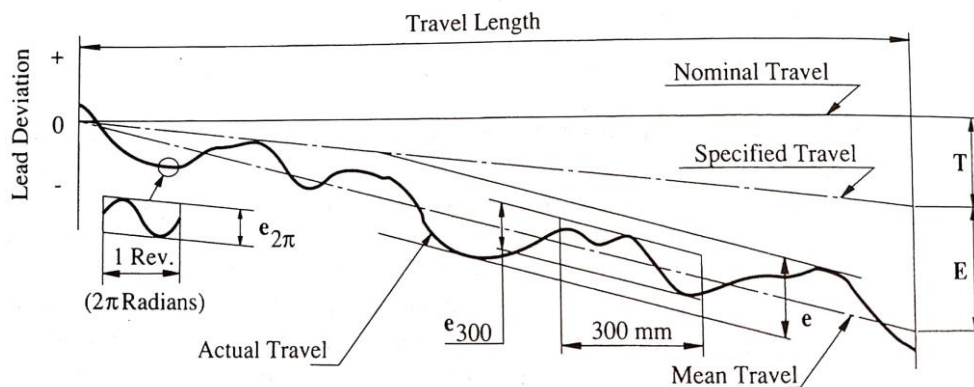
Torsion bar(Eq(4)):  $K_x/K_\theta = 16(1.3)/0.05^2 = 8,320$

Thus very high axial stiffness can be obtained from the lead screw drive, and it is a great benefit and strong advantage in view of actuator stiffness to use the lead screw as the actuator.

### 3) Ball screws

It is the most common type of leadscrew and thus widely used in precision motion. It can achieve about few  $\mu\text{m}$  order repeatability without any difficulty, sometimes it can be around or under the sub micro-inch (25.4nm) repeatability under special conditions. Ball screws can achieve very high efficiency with very low friction, because rolling steel balls and smaller spacer balls are transferring load from the screw threads to the nut threads.

Ball screws are classified according the accuracy grade.



Definition of Lead accuracy (source: Slocum's precision machine

design,NSK)

Definitions;

*Nominal travel:* nominal travel of nut

*Actual travel:* actual travel of nut

*Actual mean travel:* Best fit line of actual travel

*Specified travel:* Pre-specified travel to give allowance for the thermal expansion. Negative T is for compensation of positive thermal expansion

*Mean travel deviation, E:* Difference between the specified travel and actual mean travel

*Travel variation:* Distance between the two lines envelope the actual travel line, where,

e: total variation over the total travel length

$e_{300}$ : variation over any 300mm travel length

$e_{2\pi}$ : variation over one revolution

Deviation and variation are key factors for grading lead screw accuracy, and detail specifications are shown in fig.

The ball screws have variety of application according to the table shown.

Mean Travel Deviation (E) and Travel Deviation (e)											
Travel length		C0		C1		C2		C3		C5	
Over	Incl.	± E	± e	± E	± e	± E	± e	± E	± e	± E	± e
-	100	3	3	3.5	5	5	7	8	8	18	18
100	200	3.5	3	4.5	5	7	7	10	8	20	18
200	315	4	3.5	6	5	8	7	12	8	23	18
315	400	5	3.5	7	5	9	7	13	10	25	20
400	500	6	4	8	5	10	7	15	10	27	20
500	630	6	4	9	6	11	8	16	12	30	23
630	800	7	5	10	7	13	9	18	13	35	25
800	1000	8	6	11	8	15	10	21	15	40	27
1000	1250	9	6	13	9	18	11	24	16	46	30
1250	1600	11	7	15	10	21	13	29	18	54	35
1600	2000	-	-	18	11	25	15	35	21	65	40
2000	2500	-	-	22	13	30	18	41	24	77	46
2500	3150	-	-	26	15	36	21	50	29	93	54
3150	4000	-	-	30	18	44	25	60	35	115	65
4000	5000	-	-	-	-	52	30	72	41	140	77
5000	6300	-	-	-	-	65	36	90	50	170	93
6300	8000	-	-	-	-	-	-	110	60	210	115
8000	10000	-	-	-	-	-	-	-	-	260	140
10000	12500	-	-	-	-	-	-	-	-	320	170

Variation per 300 mm (e <sub>300</sub> ) and Wobble error (e <sub>2π</sub> )					
	C0	C1	C2	C3	C5
<b>e<sub>300</sub></b>	3.5	5	7	8	18
<b>e<sub>2π</sub></b>	2.5	4	5	6	8

## Accuracy grade of Ball Screw

(source: Slocums' precision machine design, NSK)

	C0	C1	C2	C3	C5	C7	C10
Boring machine							
CMMs	•	•	•	•	•		
Drilling machine				•	•	•	
EDM		•	•	•	•		
Grinding machine	•	•	•	•			
Jig borer	•	•					
Lathe	•	•	•	•	•		
Laser cutting machine				•	•		
Milling machine		•	•	•	•		
Machining center	•	•	•	•	•		
Punching press				•	•		
Robots:							
Cartesian-assembly		•	•	•	•		
Cartesian-material handling					•	•	•
Revolute-assembly		•	•	•	•		
Revolute-material handling				•	•	•	
Semiconductor equipment							
Insertion machine			•	•	•	•	
PCB driller		•	•	•	•	•	
Prober	•	•	•				
Steppers	•	•					
Wire bonder		•	•				
Wood working machines					•	•	•

## Ball screw applications and accuracy

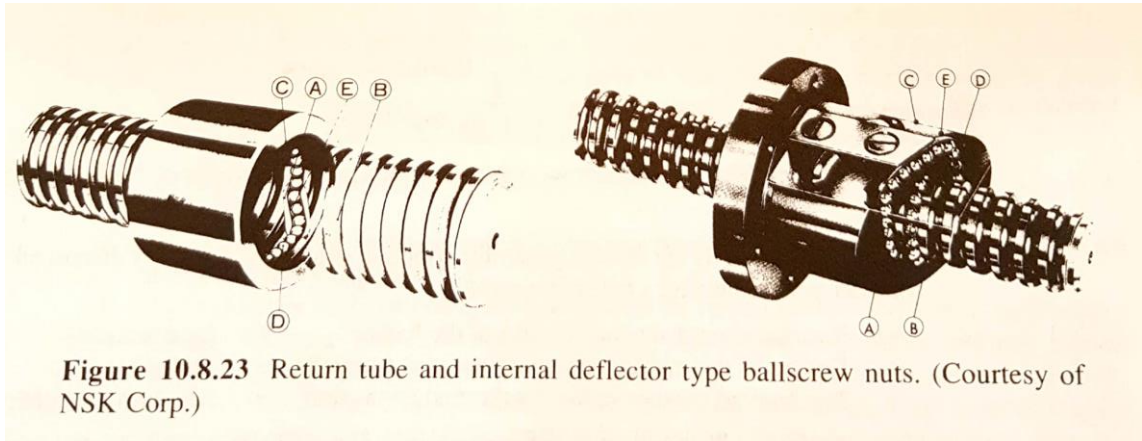
(source:precision machine design,NSK)

### Nut Design

There are two type of nut designs;

*Tube type:* Tube to gather the balls when they exit the nut's thread helix, then back to the beginning of the thread helix

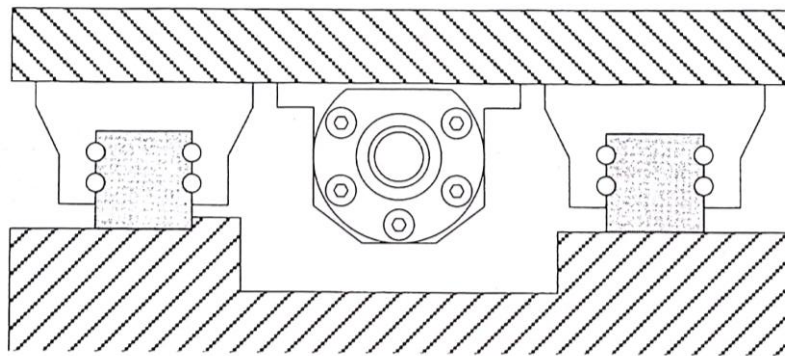
*Internal deflector type:* Internal tube to get the balls and back them to the nut's thread's helix, with small and moderate leads, making quieter ball screw easily mountable.



(source:precision machine design,NSK)

## Nut flange

Standard ball screw nut has circular flange with or without large flat. With the large flat, the screw shaft is placed to the carriage with accuracy. The mounting surface should be perpendicular to the moving direction of carriage, preventing moments on the ball screw, that could generate carriage motion errors and reduce the life of ball screw. Non-perpendicularity of 200 $\mu$ rad or 20 $\mu$ m are typical tolerances. Fig. shows a typical nut mounting.



Mounting of nut having partial circular flange

(source Slocums' precision machine design)

### Nut Preloading

There are several methods for preloading the nut;

Tensile preloading is to use oversized spacer between two nuts, then clamp two nuts tightly (back to back mounting). When the shaft is more heated than nut (this is common case, as the shaft does not have much heat sinks) and thermally expanded, the ball contacts points become loosened, lowering the preload, but the shaft diameter expands thus making the preload constant. It is suitable design for thermal expansion.

Compressive preloading is to use undersized spacer between the two nuts (face to face mounting), it is thermally stable when nut has higher temperature.

*P type* preloading is to use oversized balls for preloading.

Appropriate for light preloading, less expensive due to using only one nut.

*Z type* preloading is achieved with one nut by shifting the lead between the ball circuit, making two points contact between the balls and groove, medium preloading.

*J type* preloading uses a spring (disc spring) between nuts to give constant preload, thus most uniform drive torque is obtained. But the carriage is mounted to one nut, thus load capacity and stiffness may change according to the direction including hysteresis effect on motion reversal. It is a good method to give the uniform preload with time.

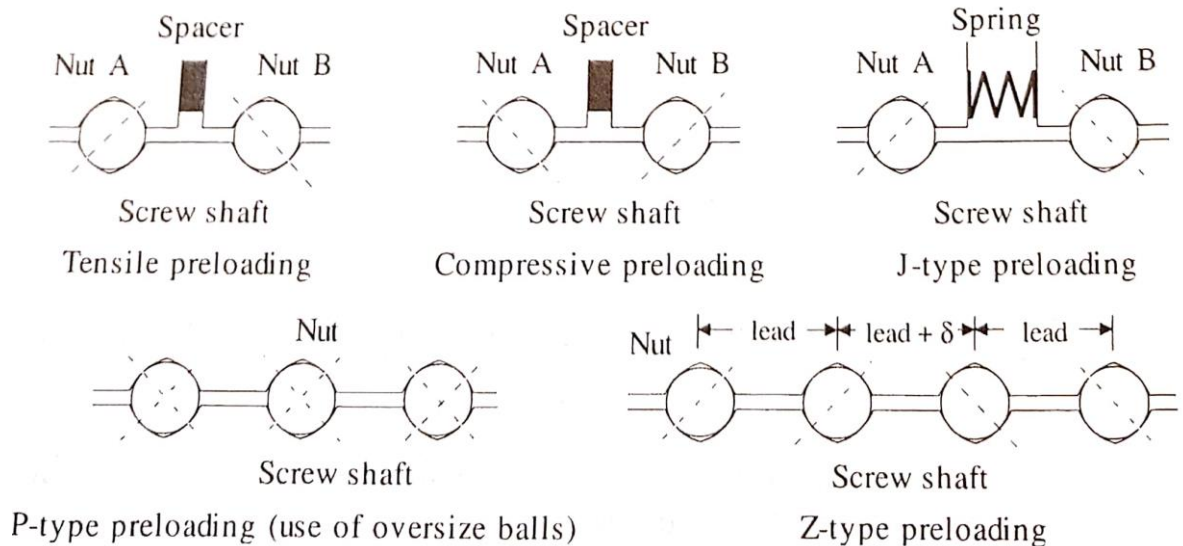
Space balls are used in high speed or high precision ball screws to reduce the balls rubbing against each other, typically under low load and 1 $\mu$ m order resolution actuation. For high load or shock load application, the space balls are not used in order to increase the load capacity and the stiffness.

The friction coefficients are varying according to the preloaded condition. A ball screw with space balls gives smoother and lower friction, and P type preloading with oversized balls gives higher friction coefficient due to four points contact, and preloading with two nuts will give lower friction coefficient due



to the two points contact of balls on the grooves.

Higher preload will give higher stiffness, but generate higher heat generation and wear. Thus optimum preloading is desirable. Typically about 10% of maximum load is recommended, and the friction coefficient of 0.005 is assumed for precision ball screws.



## Methods of Nut Preloading

(source:Slocum's precision machine design,NSK)

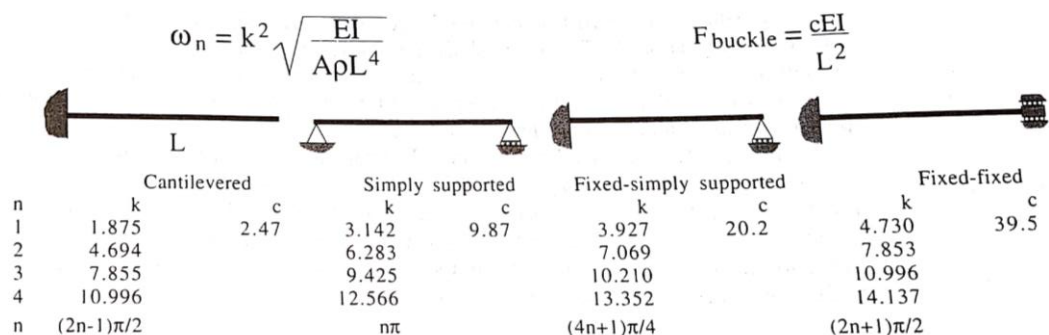
## Critical speed and Buckling in screw shaft

When the shaft is rotating, the critical speed of rotation should be considered. If the rotational frequency equals to the natural

frequency of shaft bending, then unstable condition 'shaft whip' occurs, and it may lead to failure of shaft system. When the screw is very long (e.g.  $L/d \geq 70$ , typically ) or rotation speed is very high, care must be taken. Thus it is very important to keep the rotating speed below the critical speed.

When the screw is very long or under very high axial compressive load such as due to thermal expansion with fixed constraint, then the screw may buckle, or may lead to catastrophic failure of shaft system. Thus it is also important to keep the compressive load far below under the buckling load, or preferably to keep the screw in tension rather than in compression.

The following figure shows the critical speed for the various lead screw mounting configurations.



Natural frequencies for bending for various screw mounting

(source:Slocum's precision machine design)

## Leadscrew mounting

There are two methods for screw mounting: Nut rotating and Shaft rotating.

Nut rotating method, in which shaft is stationary and nut is rotating, is preferably used when the maximum desired rotation speed is over the critical speed, especially in long screw shaft. The rotating nut is supported by bearings and can be rotated with use of gears, timing belts, or DD motors.

Shaft rotating method, in which shaft is rotating while the nut is stationary, is widely used due to simple mechanism of shaft rotation.

Fig shows typical lead screw mounting methods, and the unsupported length is shown with various supporting conditions, which is a key parameter for buckling load and critical speed.

For the fixed-fixed condition, the lead screw is usually stretched in assembly for compensation under thermal expansion, that can increase the load in journal bearings, generating more heat. Stretching can help the screw from the compressive load, thus can increase the buckling load, which is desirable.

For the simply supported screw, the lateral force and the moment on the nut can be minimum, thus it can give minimum errors in deformation, with increasing the life of lead screw unit.

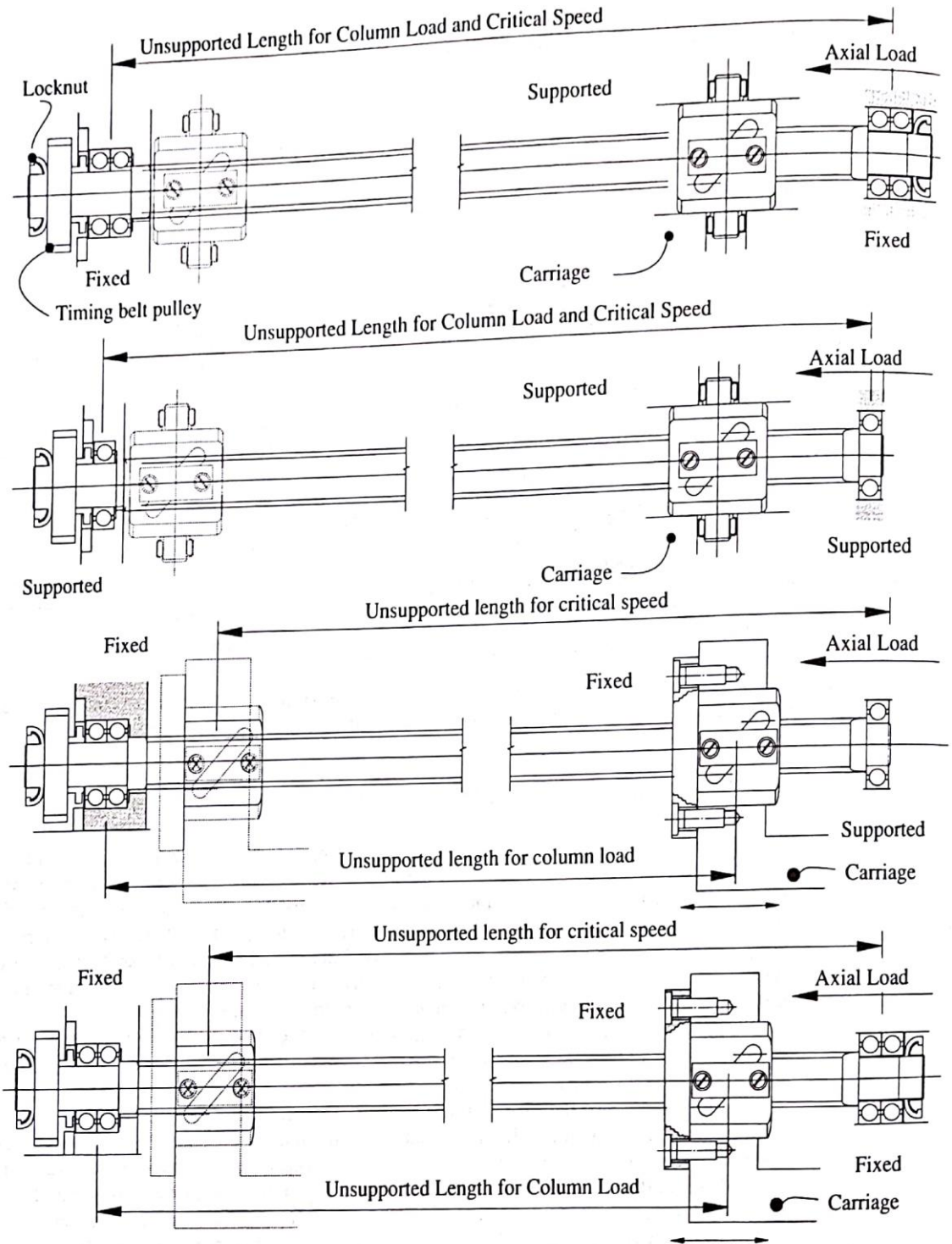


Figure 10.8.35 Examples of leadscrew mounting methods. (Courtesy of NSK Corp.)

Lead screw mounting method for various support conditions

(source: Slocum's precision machine design, NSK)

### Optimal Lead for screw

The optimal lead can be calculated, based on the optimum transmission ratio. But sometimes it may lead quite small lead, thus requiring very high rotation speed exceeding the critical speed of rotation. In this case, the lead can be chosen according to the critical speed criteria;

Thus, when  $V_{\max}$ , the maximum desirable linear velocity, is given,  $\omega_{\max}$ , the maximum angular velocity, is chosen below the critical speed, then the lead can be chosen as follows, although it is not in the optimal transmission ratio.

$$\text{Screw Lead, } \lambda = 2\pi V_{\max} / \omega_{\max} \text{ [m]}$$

### Load Life equation for Screw

The load life, or fatigue life, equation for screw is as follows;

$$L = [C_a / \{F_a f_w\}]^3 \times 10^6 \text{ [rev]; where}$$

$C_a$  = basic dynamic load rated [N]

$F_a$  = Axial load [N]

$f_w$  = load factor

= 1.0-1.2 for smooth without impact

=1.2-1.5 for normal operation

=1.5-3.0 for impact and vibration

Thus life in hour or in km are;

$L_t = L / \{60N\}$  [hour]; where N is rotary speed in rpm (rev per min)

$L_s = L\lambda / 10^6$  [km]; where  $\lambda$  is lead in mm