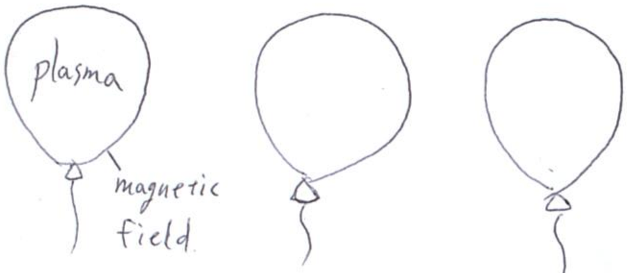


1. equilibrium — force balance
2. Stability
3. transport (기정들을 막는다)

equilibrium (멈춤)

non-stable (perturbation 시 되돌아 갈 수 없다.)

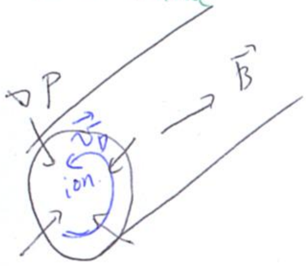
$$w = w_r + i w_i$$

$n = n_0 + n_1$, $n_1 = \bar{n}_1 \exp i(kx - \omega t)$

$\propto \exp w_i t$

$w_i > 0 \to$ unstable

$w_i < 0 \to$ stable



$$\vec{v}_D = \frac{\vec{B}\times\nabla P}{nqB^2}$$

$$\vec{j}_D = n_i q_i \vec{u}_D^{\,i} + n_e q_e \vec{u}_D^{\,e}$$

$n_i = n_e$ $\quad (\nabla P = \nabla P_i + \nabla P_e)$

$$\vec{j}_D = \frac{\vec{B}\times\nabla P_i}{B^2} + \frac{\vec{B}\times\nabla P_e}{B^2} = \frac{\vec{B}\times\nabla P}{B^2}$$

MHD Eq.

$$\cancel{\frac{\partial \rho}{\partial t}}^{\,0} + \cancel{\nabla\cdot(\rho\vec{v})}^{\,0} = 0$$

$$\cancel{\frac{\partial \sigma}{\partial t}}^{\,0} + \nabla\cdot\vec{j} = 0$$

$$\cancel{\rho\frac{d\vec{v}}{dt}}^{\,0} = \cancel{\sigma\vec{E}}^{\text{order 낮음}} + \vec{j}\times\vec{B} - \nabla P$$

$$\vec{E} + \cancel{\vec{v}\times\vec{B}}^{\,0} = \eta\vec{j} + \cancel{\frac{\vec{j}\times\vec{B} - \nabla P_e}{ne}}^{\text{order 낮음}}$$

$$\varepsilon_0 \nabla\cdot\vec{E} = \cancel{\sigma}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = \cancel{-\frac{\partial\vec{B}}{\partial t}}^{\,0}$$

$$\nabla\times\vec{B} = \mu_0\vec{j} + \cancel{\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}^{\,0}$$

Assuption $\frac{\partial}{\partial t} = 0$

$\vec{v} = 0$

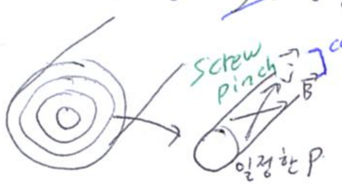
$\Rightarrow$

$$\boxed{\vec{j}\times\vec{B} = \nabla P}$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{B} = \mu_0\vec{j}$$

$$\vec{E} = \eta\vec{j}$$



$\nabla P \perp \vec{j}$

$\nabla P \perp \vec{B}$

$$\vec{B}\times\boxed{(\vec{j}\times\vec{B})} = \vec{B}\times\boxed{\nabla P}$$

$$\parallel$$

$$\vec{j}(\vec{B}\cdot\vec{B}) - \vec{B}(\vec{B}\cdot\vec{j}) \Rightarrow \vec{j}_\perp B^2 = \vec{B}\times\nabla P \quad \therefore \vec{j}_\perp = \frac{\vec{B}\times\nabla P}{B^2} = \vec{j}_D$$

Concept of β

$$\nabla P = \vec{j}\times\vec{B} = \frac{1}{\mu_0}(\nabla\times\vec{B})\times\vec{B} = \frac{1}{\mu_0}\left[(\vec{B}\cdot\nabla)\vec{B} - \frac{1}{2}\nabla B^2\right] \Rightarrow \nabla\left(P + \frac{B^2}{2\mu_0}\right) = \cancel{\frac{1}{\mu_0}(\vec{B}\cdot\nabla)\vec{B}}$$

if $\nabla B = 0$

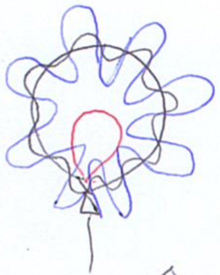
actually order 낮음

$$\nabla\left(P + \frac{B^2}{2\mu_0}\right) = 0 \quad \to \quad P + \boxed{\frac{B^2}{2\mu_0}} = \text{constant}$$

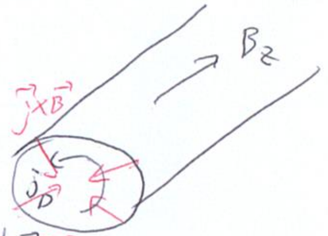
magnetic field pressure

$$\beta \equiv \frac{P}{B^2/2\mu_0} = \frac{\text{Plasma particle } P}{\text{magnetic field } P}$$

Plasma Equilibrium



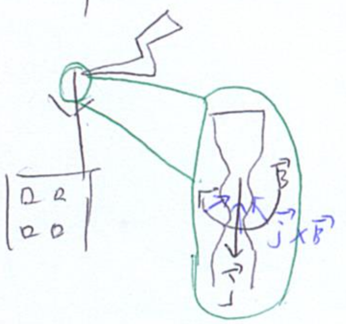
force balance = equilibrium   perturbation ↓ = stable   transport = diffusion



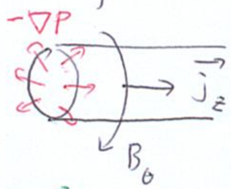
$$\cancel{\rho\frac{d\vec{v}}{dt}}^{\,0} = \vec{j}\times\vec{B} - \nabla P$$

$$\vec{v}_D = \frac{\vec{B}\times\nabla P}{n q B^2}$$

$$\vec{j}_D = \frac{\vec{B}\times\nabla P}{B^2}$$



* z - pinch



$$\nabla\times\vec{B} = \mu_0\vec{j}$$
$$\vec{j}\times\vec{B} = \nabla P$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu_0 j_z$$
$$-j_z B_\theta = \frac{\partial P}{\partial r}$$

$$\nabla\times\vec{B} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ 0 & rB_\theta & 0 \end{vmatrix}$$

$$\vec{j}\times\vec{B} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ 0 & 0 & j_z \\ 0 & B_\theta & 0 \end{vmatrix}$$

$$-\frac{B_\theta}{r\mu_0}\frac{\partial}{\partial r}(rB_\theta) = \frac{\partial P}{\partial r}$$

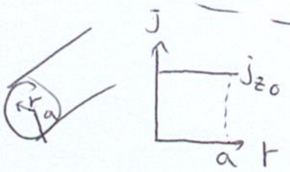
$$-\frac{B_\theta}{r\mu_0}\left(B_\theta + r\frac{\partial B_\theta}{\partial r}\right) = \frac{\partial P}{\partial r}$$

$$-\frac{B_\theta^2}{\mu_0 r} - \frac{B_\theta}{\mu_0}\frac{\partial B_\theta}{\partial r} = \frac{\partial P}{\partial r}$$

$$-\frac{B_\theta^2}{\mu_0 r} - \frac{1}{2\mu_0}\frac{\partial B_\theta^2}{\partial r} = \frac{\partial P}{\partial r}$$

$$[N/m^3] \quad \frac{\partial}{\partial r}\left(P + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

magnetic tension force



$$I = \pi a^2 j_{z0} \qquad P(a) = 0$$

$$B_{\theta a} = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0}{2} j_{z0}\, a$$

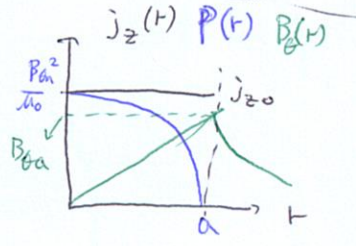
$$B_{\theta r} = \frac{\mu_0 I_r}{2\pi r} = \frac{\mu_0}{2} j_{z0}\, r = \frac{r}{a}B_{\theta a}$$

$$I_r = \pi r^2 j_{z0}$$

$$P_r = \frac{B_{\theta a}^2}{\mu_0}\left(1 - \frac{r^2}{a^2}\right) \qquad \therefore \beta = 2\left(\frac{a^2}{r^2} - 1\right)$$

$$\langle\beta_\theta\rangle = \frac{\langle P\rangle}{B_{\theta a}^2/2\mu_0}, \qquad \langle P\rangle = \frac{\int_0^a 2\pi r P(r)\,dr}{\pi a^2} = \frac{B_{\theta a}^2}{2\mu_0}$$

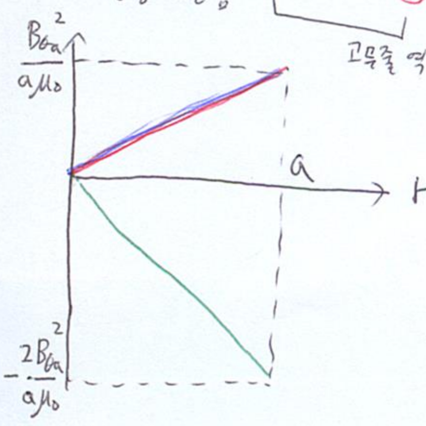
$$= 1$$



$$\frac{\partial}{\partial r}\left(P + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

① 팽창하려는 힘 ② ③

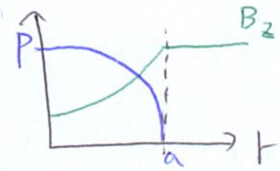
고무줄 역할



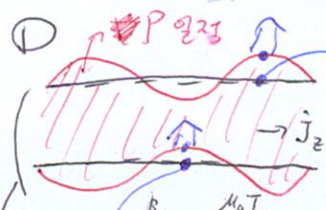
$$\rho\frac{d\vec{v}}{dt} = \vec{j}\times\vec{B} - \nabla p$$

$$\frac{d\vec{v}}{dt} = \cancel{\frac{\partial\vec{v}}{\partial t}} + \cancel{(\vec{v}\cdot\nabla)\vec{v}}$$

steady state, 속도가 느릴 때만



①



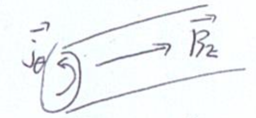
$\frac{\partial}{\partial r}\frac{B_\theta^2}{2\mu_0}$ 작고 $\frac{\partial P}{\partial r}$는 크다 → 더 늘린다.

$$B_{\theta a} = \frac{\mu_0 I}{2\pi a}$$

sausage instability

$\frac{\partial}{\partial r}\frac{B_\theta^2}{2\mu_0}$ 크고, $\frac{\partial P}{\partial r}$는 작다. → 더 조인다.

* θ-pinch



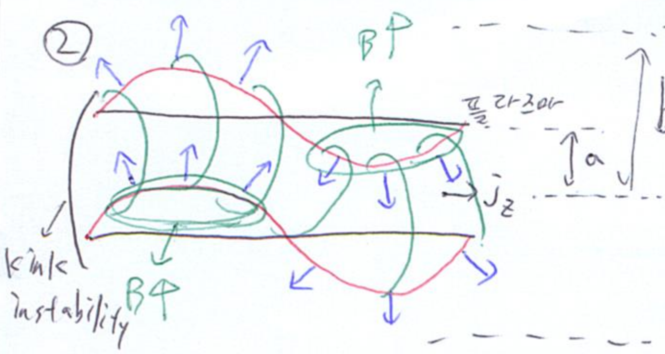
$$\nabla\times\vec{B} = \mu_0\vec{j}$$
$$\vec{j}\times\vec{B} = \nabla P$$

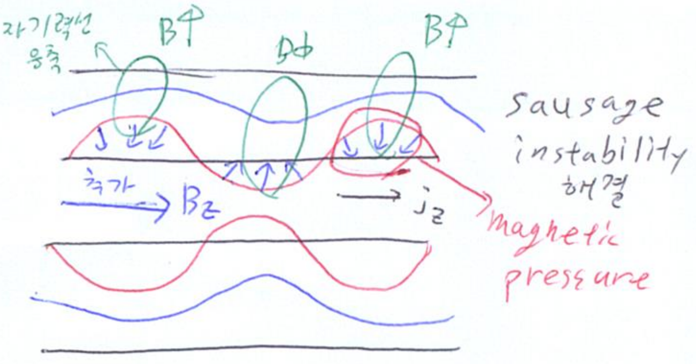
$$-\frac{\partial B_z}{\partial r} = \mu_0 j_\theta$$
$$j_\theta B_z = \frac{\partial P}{\partial r}$$

$$-\frac{B_z}{\mu_0}\frac{\partial B_z}{\partial r} = \frac{\partial P}{\partial r}$$

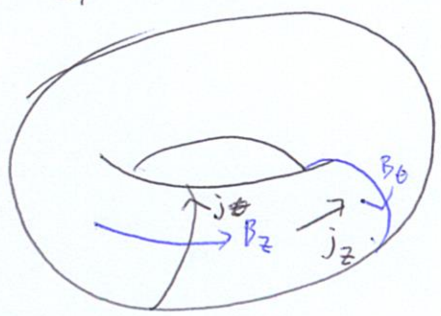
$$-\frac{\partial}{\partial r}\left(\frac{B_z^2}{2\mu_0}\right) = \frac{\partial P}{\partial r}$$

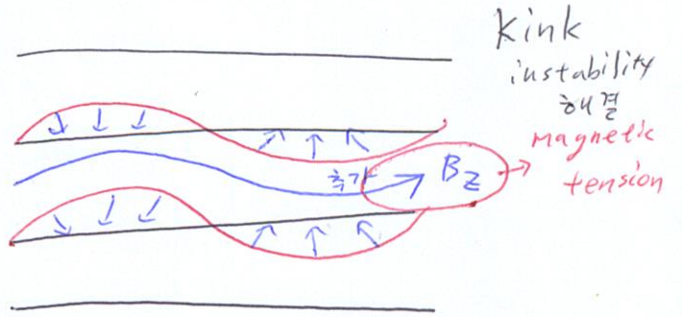
$$\frac{\partial}{\partial r}\left(P + \frac{B_z^2}{2\mu_0}\right) = 0$$

②




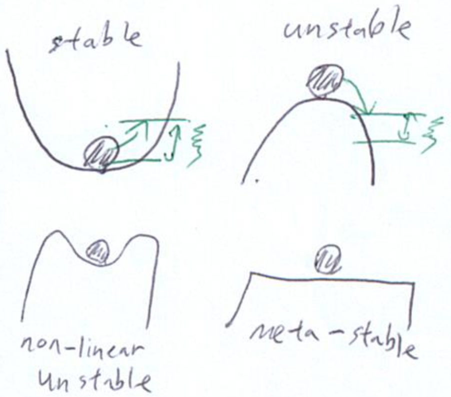
Grad-Shafranov Equation





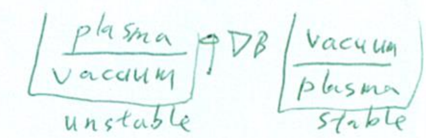
Plasma Instability

non-uniformity $\nabla P, j$ $\xrightarrow{}$ Instability
perturbation



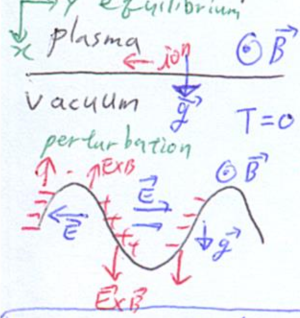
- streaming instability ; $\vec{j}$, fast particle
- Rayleigh-Taylor instability ; external force
  ↳ ∇p 에 의해 생김!

- Universal instability : ∇n, ∇p → drift wave
  압력 차이 → oscilliation → wave



- Velocity space instability : loss cone instability
  (kinetic instability)

Rayleigh-Taylor instability



$$\vec{u}_{i0} = \frac{M\vec{g}\times\vec{B}}{eB^2} \qquad \vec{u}_{e0} \simeq 0$$

equilibrium

$$n_0 M\left[\cancel{\frac{\partial \vec{u}_0}{\partial t}} + \cancel{(\vec{u}_0\cdot\nabla)\vec{u}_0}\right] = n_0 e(\cancel{\vec{E}_0} + \vec{u}_0\times\vec{B}) + n_0 M\vec{g}$$

초기속도, 공간일정, 초기엔 0

$$\Rightarrow e(\vec{u}_0\times\vec{B}) + M\vec{g} = 0$$

$$\boxed{(n_0+n_1)e(\vec{u}_0\times\vec{B}) + (n_0+n_1)M\vec{g} = 0}$$

$$nM\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right] = ne(\vec{E}+\vec{u}\times\vec{B}) + nM\vec{g}$$

$u_1 \propto \exp i(ky-\omega t)$

$$n = n_0 + n_1, \quad \vec{u} = \vec{u}_0 + \vec{u}_1, \quad \vec{E} = \cancel{\vec{E}_0} + \vec{E}_1$$

$$\nabla\cdot\vec{u}_0 = 0, \quad \frac{\partial \vec{u}_0}{\partial t} = 0$$

$$\Rightarrow n_0 M\frac{\partial \vec{u}_1}{\partial t} + n_0 M\vec{u}_0\cdot\nabla\vec{u}_1 = n_0 e\vec{E}_1 + n_0 e\vec{u}_1\times\vec{B}$$

Electron plasma wave

$$\frac{\partial n_0}{\partial t} + \nabla\cdot(n_e\vec{u}_e) = 0$$

$$n_e m\left[\frac{\partial \vec{u}_e}{\partial t} + (\vec{u}_e\cdot\nabla)\vec{u}_e\right] = -n_e e(\vec{E} + \cancel{\vec{u}_e\times\vec{B}}) - \gamma k_B T\nabla n_e$$

$$\varepsilon_0\nabla\cdot\vec{E} = e(n_i - n_e)$$

$$\rightarrow \omega^2 = \omega_p^2 + \frac{\gamma k_B T}{m}k^2 \quad \left(\omega_p^{-2} = \frac{m\varepsilon_0}{ne^2}\right)$$

$$n_1 = \bar{n}_1 \exp i(kx - \omega t)$$

$$= \bar{n}_1 \exp i(kx - \omega_r t - i\gamma t)$$

$$= \bar{n}_1 \exp i(kx - \omega_r t)\exp(\gamma t)$$

- $\gamma > 0$: unstable — growth rate
- $\gamma < 0$: stable