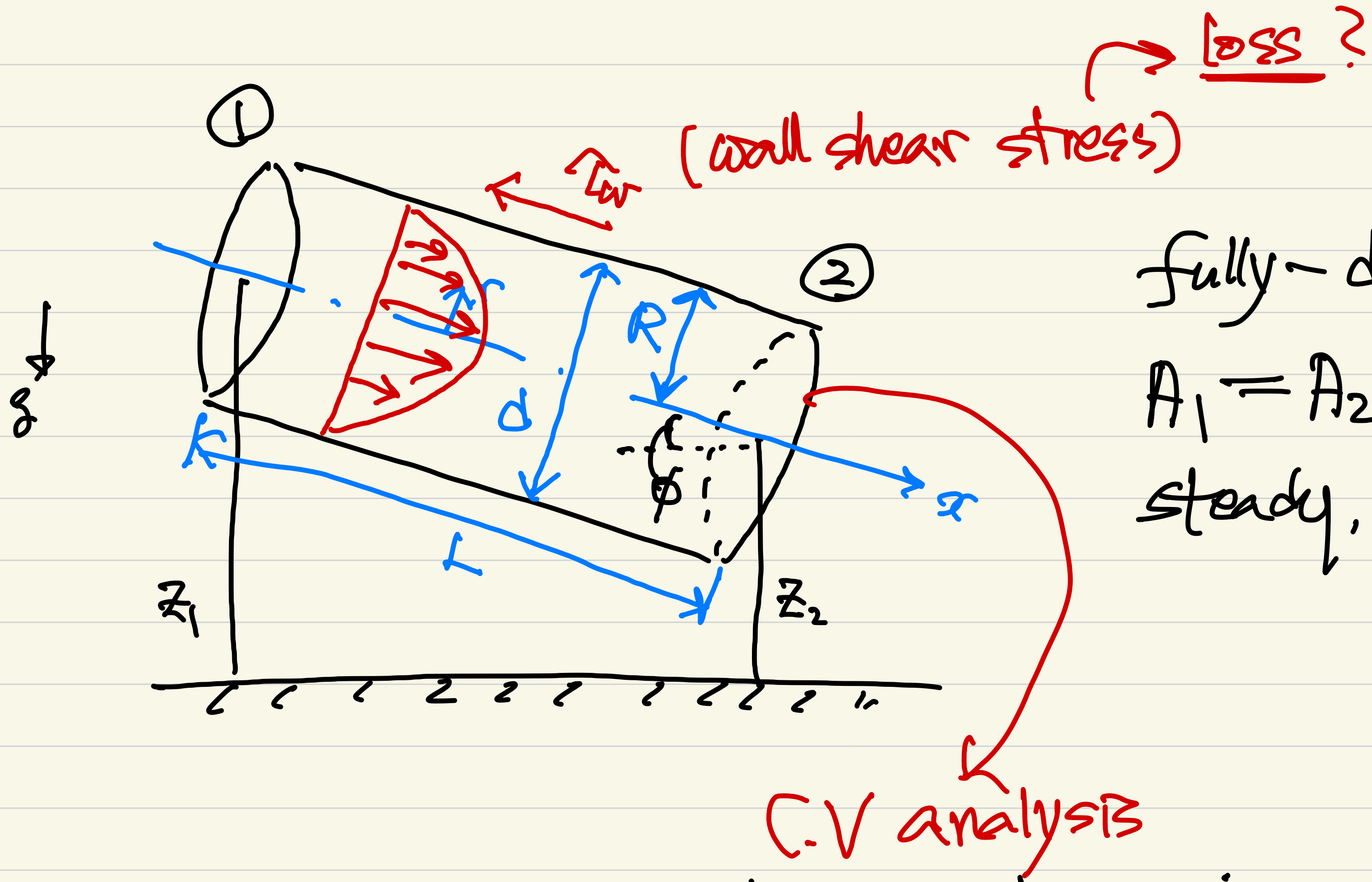


fully-developed ($\partial/\partial x \simeq 0$)

$A_1 = A_2$.

steady, incompressible ($\rho = \text{const}$)

- mass conservation : $\dot{m}_1 = \dot{m}_2 \Rightarrow Q_1 = Q_2$

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \underline{\underline{V_1 = V_2}} .$$

- Bernoulli eq w/ loss term

$$\frac{P_1}{\rho} + \cancel{\frac{1}{2}V_1^2} + gz_1 = \frac{P_2}{\rho} + \cancel{\frac{1}{2}V_2^2} + gz_2 + g h_f$$

friction loss

$$h_f = \left(z + \frac{P}{\rho g}\right)_1 - \left(z + \frac{P}{\rho g}\right)_2$$

[L]

$$= \Delta z + \frac{\Delta P}{\rho g}$$

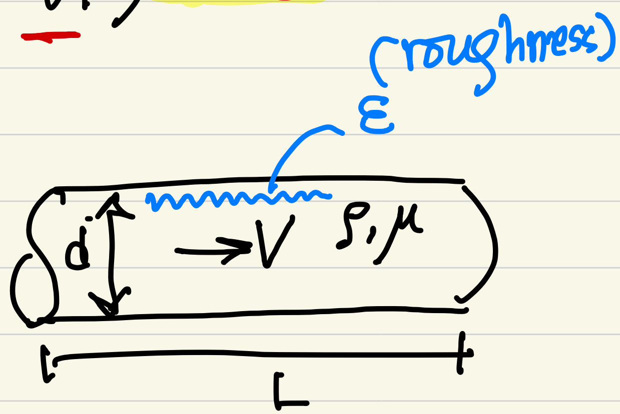
change in HGL (hydraulic grade line)

mtm conservation

$$\sum F_x = \Delta P \cdot \pi R^2 + \rho g (\pi R^2) L \cdot \sin\phi - \tau_w \cdot 2\pi R \cdot L$$

$$= \dot{m}(V_2 - V_1) = 0$$

$$\Delta z + \frac{\Delta P}{\rho g} = \boxed{h_f = \frac{2\tau_w}{\rho g} \cdot \frac{L}{R} = \frac{4\tau_w}{\rho g} \cdot \frac{L}{d}}$$



$$\tau_w = f(\rho, V, \mu, d, \varepsilon)$$

ϕ dimensional analysis

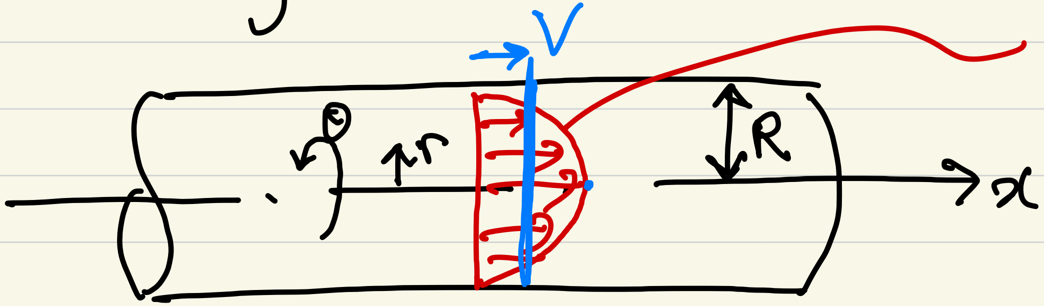
$$\frac{\hat{\tau}_w}{\rho V^2} = g\left(Re_d, \frac{\varepsilon}{d}\right) \Rightarrow \boxed{\frac{8\hat{\tau}_w}{\rho V^2} \equiv f}$$

(darcy friction factor)

$$\therefore \underline{h_f} \equiv \frac{4\hat{\tau}_w}{\rho g}\cdot\frac{L}{d} = f\cdot\frac{L}{d}\cdot\frac{V^2}{2g}.$$

(Darcy – Weisbach eq.)

6.4. Laminar fully-developed pipe flow
(Hagen-Poiseuille flow).



. continuity eq : $\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta} v_\theta + \frac{\partial u}{\partial x} = 0$

$\cancel{v_\theta = 0}$, $\frac{\partial}{\partial \theta}(\cdot)$, $\frac{\partial}{\partial x}(\cdot) = 0$. $\boxed{u = u(r).}$

$$\Rightarrow \frac{\partial}{\partial r}(r v_r) = 0 \qquad \left.\begin{array}{l} r v_r = \text{constant.} \\ v_r = 0 \ @ \ r = R \end{array}\right) \Rightarrow \cancel{\boxed{v_r = 0}}$$

· N-S eq (x-dir)

viscous stress

$$\cancel{\rho \frac{\partial u}{\partial t}} + \cancel{\rho u \frac{\partial u}{\partial x}} + \cancel{\cdots} = \boxed{-\frac{dp}{dx} + \rho g_x} + \boxed{\frac{1}{r}\frac{\partial}{\partial r}(r\tau)}$$

$$\left(\tau = \mu \frac{\partial u}{\partial r}\right)$$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(r\tau) = \boxed{\frac{d}{dx}(P - \rho g x \sin\phi)} = \text{constant.}$$

$[= c_1]$

fcn of r

fcn of x

indep.

$$\therefore \frac{1}{r}\frac{\partial}{\partial r}(r\tau) = c_1 \rightarrow \frac{\partial}{\partial r}(r\tau) = c_1 r \rightarrow r\tau = \frac{1}{2}c_1 r^2 + c_2$$

$$\tau = \frac{1}{2}C_1 r + \frac{C_2}{r} \qquad \tau = 0 \text{ @ } r = 0 \rightarrow C_2 = 0$$

$$\therefore \tau = \frac{1}{2}C_1 r = \frac{1}{2} r \frac{d}{dx}(p + \rho g z)$$

$$\tau_w = \tau\Big)_{r=R} = \frac{1}{2} R \frac{d}{dx}(p + \rho g z) \doteqdot \frac{1}{2} R \frac{\Delta p + \rho g \Delta z}{L}$$

(wall shear stress)

for laminar flow, $\tau = \boxed{\mu \frac{du}{dr} = \frac{c}{2} r} = \frac{1}{2} r \underbrace{\frac{d}{dx}(p + \rho g z)}_{= c}$

$$u(r) = \frac{1}{4} r^2 \frac{c}{\mu} + C_1 \,, \qquad u = 0 \text{ @ } r = R.$$

$$= \frac{1}{4\mu}(-c)(R^2 - r^2) = \frac{1}{4\mu}\underbrace{\left[-\frac{d}{dx}(p + \rho g z)\right](R^2 - r^2)}_{\text{parabolic velocity profile}}$$

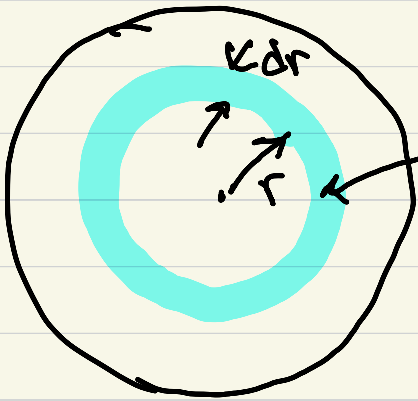
(Hagen-Poiseuille flow)

· $u_{max} = u)_{r=0} = \frac{1}{4\mu}\left[-\frac{d}{dx}(p+\rho g z)\right]R^2$

$$u(r) = u_{max}\left(1-\frac{r^2}{R^2}\right).$$

· bulk velocity (area-averaged), $V = \frac{Q}{A}$.

$$= \frac{1}{\pi R^2}\int u\,dA = \frac{R^2}{8\mu}\left[-\frac{d}{dx}(p+\rho g z)\right] = \frac{1}{2}u_{max}.$$



$$= \frac{1}{\pi R^2}\int_0^R u\cdot 2\pi r\,dr$$

· for a horizontal pipe ($\Delta z = 0$, only $\frac{dp}{dx} \simeq \frac{\Delta p}{L}$)

$$\rightarrow \Delta p = \frac{8\mu L Q}{\pi R^2} \Rightarrow \boxed{\Delta p \sim Q.}$$

wall shear stress, $\tau_w = \mu \frac{\partial u}{\partial r}\Big)_{r=R} = \frac{2\mu u_{max}}{R}$

Darcy friction factor

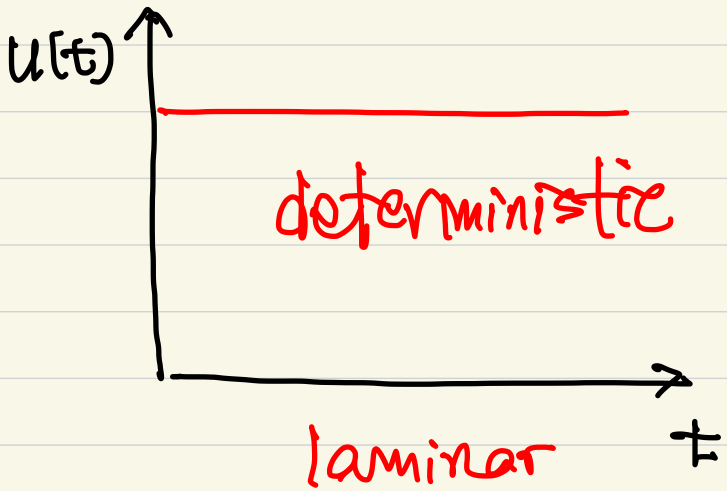
$$\underline{f} \equiv \frac{8\tau_w}{\rho V^2} = \boxed{\frac{64\mu}{\rho V d}} = \frac{64}{Re_d} \quad \left(Re_d \equiv \frac{\rho V d}{\mu} \right)$$

(laminar)

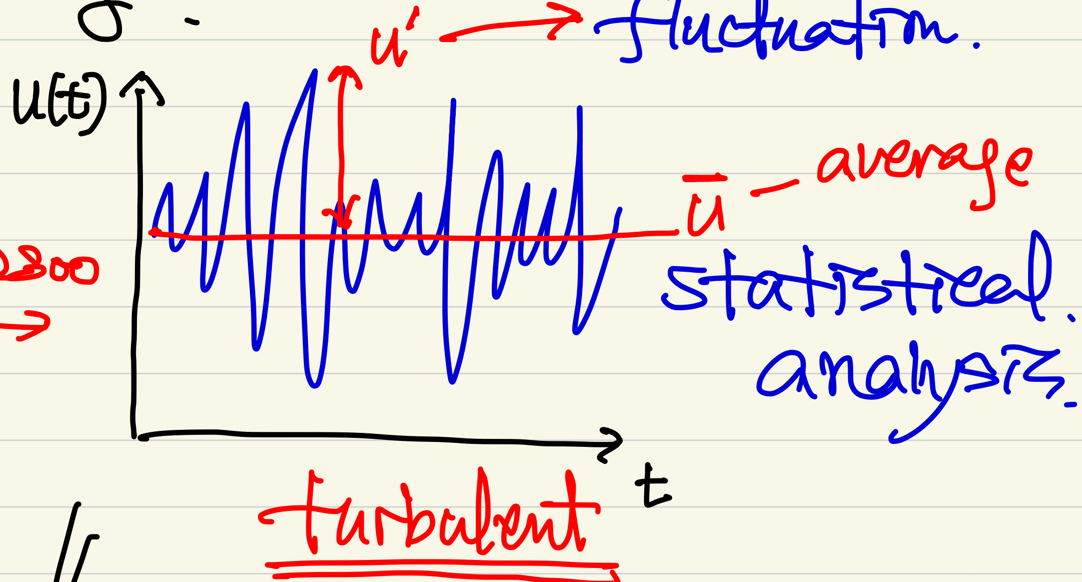
- Head loss,

$$h_{f,lam} = \frac{2\tau_w}{\rho g} \cdot \frac{L}{R} = \frac{128 \mu L Q}{\pi \rho g d^4}.$$

6.4. Turbulence modeling.



$Re > 2300$



$$\bar{u} = \frac{1}{T}\int_0^T u \cdot dt$$: mean velocity.

$$\boxed{\underline{u(t)} = \bar{u} + u'}$$

instantaneous velocity

fluctuating velocity.

⇓

Reynolds decomposition.

- $\overline{u'} = (\overline{u - \bar{u}}) = \bar{u} - \bar{\bar{u}} = \bar{u} - \bar{u} = 0.$

- $\overline{(u')^2} \neq 0 \Rightarrow (u')^2, (v')^2, (w')^2$: turbulence intensity (난류강도)

- $\overline{u'v'} \neq 0$, $\overline{u'p'} \neq 0$. $\overline{u'\bar{v}} = \overline{(u-\bar{u})\bar{v}} = 0$

$$\cancel{\overline{\rho \frac{\partial u}{\partial t}}} + \overline{\rho \frac{\partial}{\partial x}(uu)} + \overline{\rho \frac{\partial}{\partial y}(uv)} + \overline{\rho \frac{\partial}{\partial z}(uw)}$$

$$= -\overline{\frac{\partial p}{\partial x}} + \overline{\frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right)} + \overline{\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)} + \overline{\frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right)}.$$

0

$$\overline{\rho \frac{\partial}{\partial x}(uu)} = \overline{\rho \frac{\partial}{\partial x}(\bar{u}+u')(\bar{u}+u')} = \overline{\rho \frac{\partial}{\partial x}(\bar{u}\bar{u} + 2u'\bar{u} + u'u')}$$

$$= \rho \frac{\partial}{\partial x}\left(\bar{u}\bar{u} + \cancel{2\overline{u'\bar{u}}} + \overline{u'u'}\right)$$

$$= \rho \frac{\partial}{\partial x}(\bar{u}\bar{u}) + \frac{\partial}{\partial x}(\rho \overline{u'u'})$$

$$\overline{\rho \frac{\partial}{\partial y}(uv)} = \rho \frac{\partial}{\partial y}(\bar{u}\bar{v}) + \frac{\partial}{\partial y}(\rho \overline{u'v'})$$

$$\rho \frac{\partial}{\partial z}(uw) = \rho \frac{\partial}{\partial z}(\bar{u}\bar{w}) + \frac{\partial}{\partial z}(\rho \overline{u'w'})$$

RANS eq.

$$\Rightarrow \boxed{\begin{aligned} &\rho \frac{\partial}{\partial x}(\bar{u}\bar{u}) + \rho \frac{\partial}{\partial y}(\bar{u}\bar{v}) + \rho \frac{\partial}{\partial z}(\bar{u}\bar{w}) \\ &= -\frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial \bar{u}}{\partial x} - \rho \overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'}\right) \\ &\quad + \frac{\partial}{\partial z}\left(\mu \frac{\partial \bar{u}}{\partial z} - \rho \overline{u'w'}\right) \end{aligned}}$$

→ look like stress

: turbulent stress

Closure problem → modeling

$$\cdot \text{cont}: \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0.$$

* Total shear stress (turbulent flow)

$$\bar{\tau} = \underbrace{\mu \frac{\partial \bar{u}}{\partial y}}_{\text{viscous stress}} \underbrace{- \rho \overline{u'v'}}_{\text{turbulent stress}} \equiv \tau_{vis} + \tau_{turb}.$$

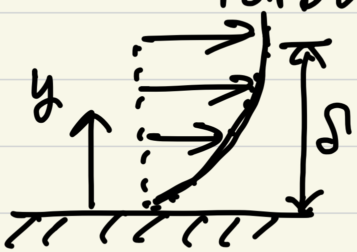
modeling.

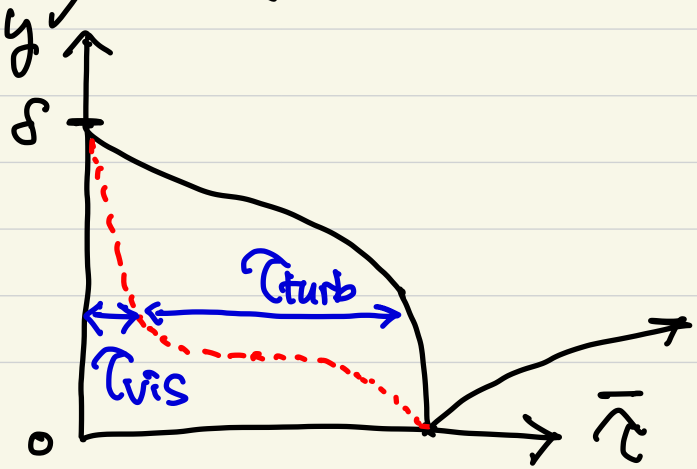
$$-\rho \overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y} \quad \text{(Prandtl)}$$

μ_t: eddy (turbulent) viscosity

TBL (boundary layer).

turbulent.

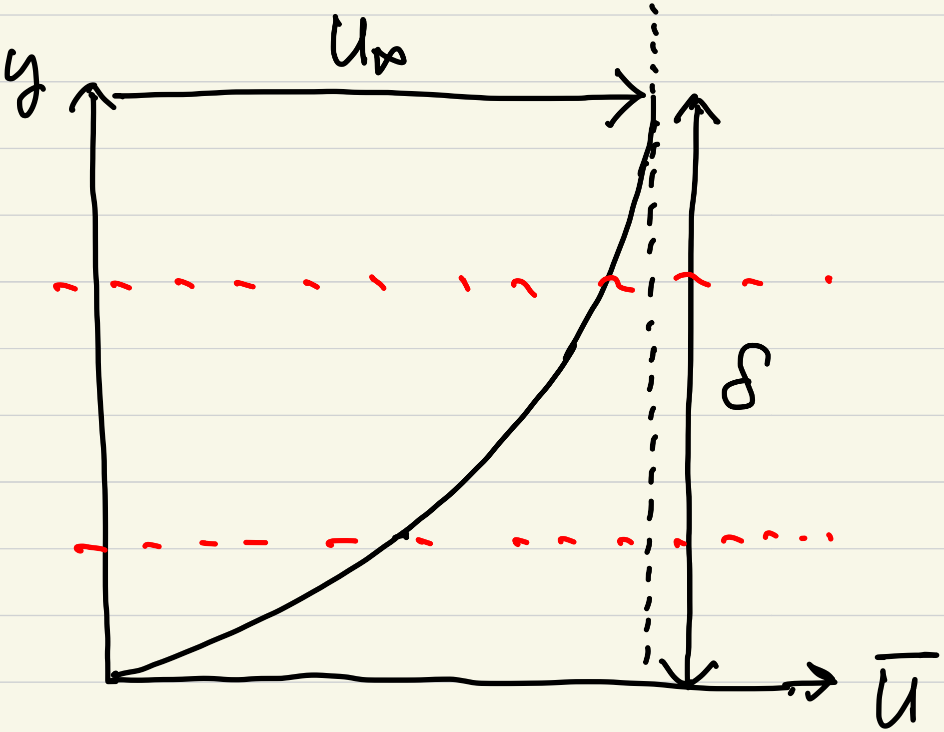




at wall: $u = v = w = 0$

$\rightarrow \bar{u} = \bar{v} = \bar{w} = 0$

· TBL



$\Rightarrow u' = v' = w' = 0$

$\rightarrow \overline{u'v'} = 0$

$\tau_{vis} \ll \tau_{turb}$: outer layer

$\tau_{vis} \simeq \tau_{turb}$: overlap layer (log)

$\tau_{vis} \gg \tau_{turb}$: viscous wall layer

· Viscous wall layer (Prandtl, 1930)

$u = f(\mu, \rho, y, \tau_w)$ wall shear stress

$u^* \equiv \sqrt{\frac{\tau_w}{\rho}}$ friction velocity (wall-shear velocity, turbulent velocity)

$\xrightarrow{\text{non-dimensionalization}}$ $u^+ = \frac{u}{u^*} = F\left(\frac{y u^*}{\nu}\right) = F(y^+)$

$\therefore\ u^+ = F(y^+)$: law of the wall (벽법칙)

- Outer layer (Karman, 1933)

$$\hookrightarrow\ U_\infty - u = g(\delta, \tau_w, \rho, y).$$

$$\rightarrow\ \frac{U_\infty - u}{u^*} = G\left(\frac{y}{\delta}\right)$$: Velocity defect law.

- Overlap layer (Millikan, 1937)

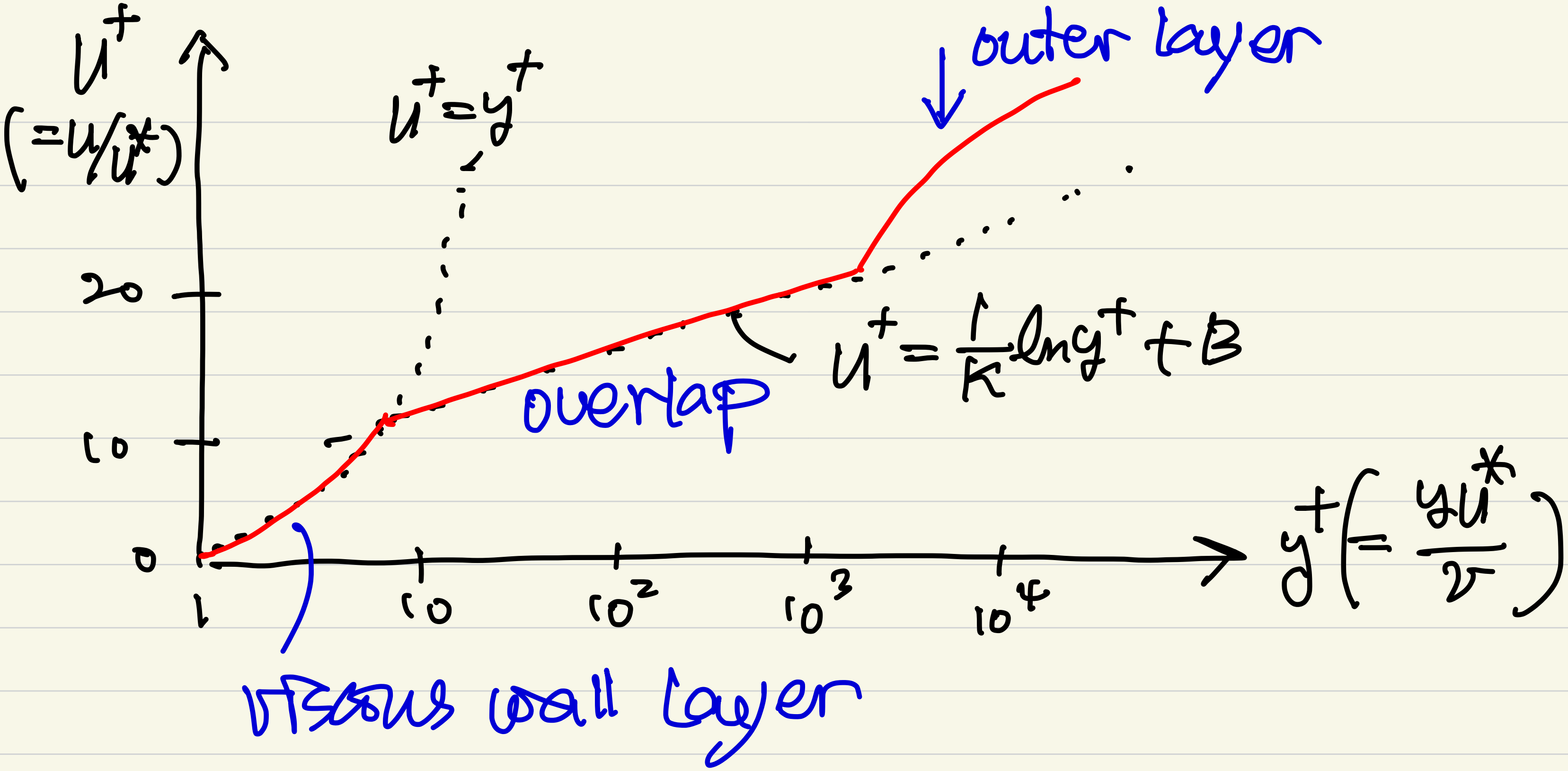
$$\frac{\bar{u}}{u^*} = \frac{1}{\kappa}\ln\left(\frac{y u^*}{\nu}\right) + B.$$

$$u^+ = \frac{1}{\kappa}\ln y^+ + B$$

$(\kappa \simeq 0.41,\ B = 5.0)$

$\boxed{\text{log law}}$

logarithmic overlap layer



u^+ $(= u/u^*)$
$u^+ = y^+$
outer layer
20
10
0
$u^+ = \frac{1}{\kappa} \ln y^+ + B$
overlap
$y^+ \left(= \frac{y u^*}{\nu}\right)$
1
10
10^2
10^3
10^4
viscous wall layer