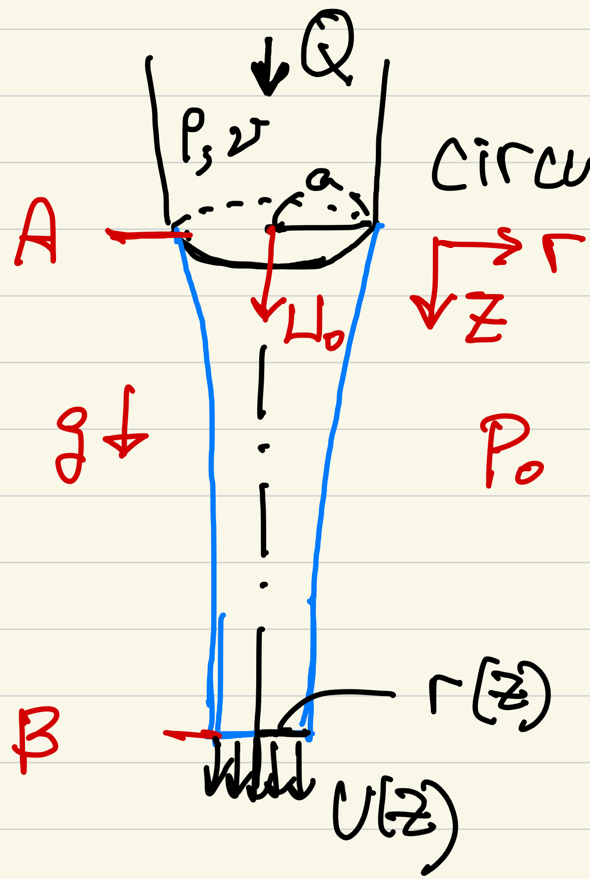
VII. LIQUID JET Falling Under the Gravity.

⊙ Shape of a falling fluid (liquid) jet.



$Re = Q/(a\nu) \gg 1$

(viscosity effect is negligible)

⇓

Bernoulli eq to A and B.

$$\frac{1}{2}\rho U_0^2 + \rho g z + P_A = \frac{1}{2}\rho U(z)^2 + P_B.$$

- local curvature of slender threads
  → two principal radii of curvature

$$\Rightarrow P_A \simeq P_0 + \frac{\sigma}{a}, \quad P_B \simeq P_0 + \frac{\sigma}{r}$$

$$\Rightarrow \frac{1}{2}\rho U_0^2 + \rho g z + P_0 + \frac{\sigma}{a} = \frac{1}{2}\rho U(z)^2 + P_0 + \frac{a}{r}.$$

$$\therefore \frac{U(z)}{U_0} = \boxed{\left[1 + \frac{2}{Fr}\cdot\frac{z}{a} + \frac{2}{We}\left(1 - \frac{a}{r}\right)\right]^{1/2}} \quad \text{—} \; ②$$

$Fr \rightarrow U_0^2/ga$

$We \rightarrow \rho U_0^2 a/\sigma.$

· Conservation of Q.

$$Q = 2\pi\int_0^r U(z)\, r(z)\, dr = \left[\underset{(\text{at } A)}{\pi a^2 U_0} = \underset{(\text{at } B)}{\pi r^2 U(z)}\right]$$

$$\rightarrow \frac{r(z)}{a} = \left[\frac{U_0}{U(z)}\right]^{1/2} = \left[1 + \frac{2}{Fr}\cdot\frac{z}{a} + \frac{2}{We}\left(1 - \frac{a}{r}\right)\right]^{-1/4} \quad \text{—} \; ①$$

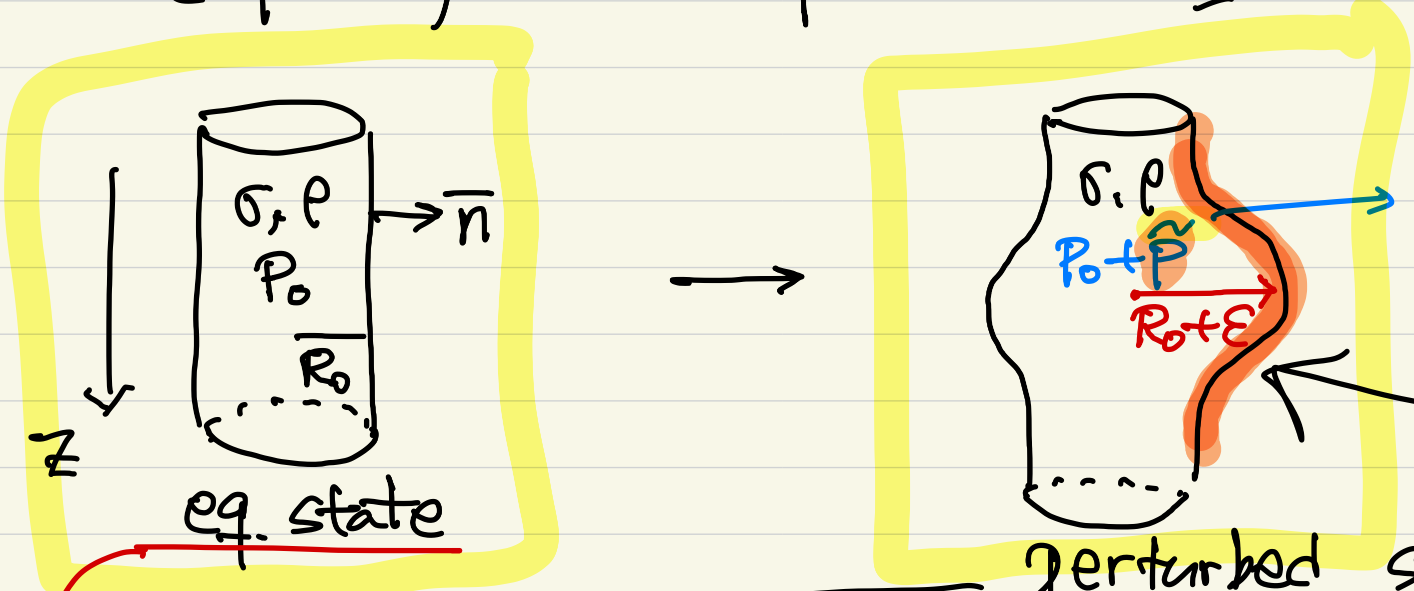
By solving ①, we get $r(z)/a$. → We solve ② to get profile of $U(z)$.

· In the limit of $We \to \infty$ (inertia ≫ surf. tension)

$$\frac{r}{a} \to \left(1 + \frac{2gz}{U_0^2}\right)^{-1/4} \text{ and } \frac{U(z)}{U_0} \to \left(1 + \frac{2gz}{U_0^2}\right)^{1/2}.$$

⊙ Rayleigh-Plateau instability

: capillary driven pinch-off of thin water column.



$\sim$: perturbed

perturbed state

inviscid, infinitely long liquid column, no gravity

$$P_0 = \underset{\text{surf. tension}}{\sigma}(\underbrace{\nabla\cdot\bar{n}}_{\text{curvature.}}) \longrightarrow \boxed{P_0 = \frac{\sigma}{R_0}}$$

→ consider the evolution of infinitesimal varicose perturbations on the interface.

$$\Rightarrow \tilde{R} = R_0 + \varepsilon e^{\omega t + i\hat{k}z} \quad (\varepsilon \ll R_0)$$

given as a disturbance

ω: growth rate of the instability

$\hat{k}$: wave number of the disturbance

(wave length $= 2\pi/\hat{k}$)

$\tilde{u}_r$, $\tilde{u}_z$ and $\tilde{p}$ $\longrightarrow$ N-S equation. and only keep terms to the order of ε.

$$\therefore \frac{\partial \tilde{u}_r}{\partial t} = -\frac{1}{\rho}\cdot\frac{\partial \tilde{p}}{\partial r}, \quad \frac{\partial \tilde{u}_z}{\partial t} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial z} \quad : \text{N-S eq.}$$

$$\frac{\partial \tilde{u}_r}{\partial r} + \frac{\tilde{u}_r}{r} + \tilde{u}_z = 0 \quad \text{(linearized continuity)}$$

$(*)$

$$\left\{\begin{array}{l}\tilde{u}_r = R(r)\, e^{\omega t + ikz}, \quad \tilde{u}_z = Z(r)\, e^{\omega t + ikz} \\ \tilde{p} = P(r)\, e^{\omega t + ikz}\end{array}\right. \longrightarrow (*)$$

$$\therefore \omega R = -\frac{1}{\rho}\frac{dP}{dr}, \quad \omega Z = -\frac{ik}{\rho} P, \quad \frac{dR}{dr} + \frac{R}{r} + ikZ = 0.$$

$$\hookrightarrow \underline{r^2 \frac{d^2R}{dr^2} + r\frac{dR}{dr} - (1 + (kr)^2) R = 0.} \Leftarrow \text{eq. of "R" only.}$$

modified Bessel eq. of order 1.

$\Downarrow$

Solution is written in terms of

$\underline{I_1}(kr)$ and $\cancel{\underline{K_1}(kr)}$

: modified Bessel function of first and second kind. ($K_1(kr) \to \underline{\infty}$ as $r \to 0$)

$$\therefore R(r) = \underline{C} \cdot I_1(kr)$$

constant to be determined by the BC.

$\Rightarrow$ pressure : $P(r) = -\dfrac{\omega \rho C}{k} I_0(kr)$ $(**)$

BC's

① kinematic condition at the free surface.

$$\frac{\partial \tilde{R}}{\partial t} \simeq \tilde{u}_r \rightarrow C = \frac{\varepsilon \omega}{I_1(kR_0)} \quad (**)$$

② dynamic condition : normal stress balance at the interface.

$$P_0 + \tilde{p} = \sigma(\nabla \cdot \bar{n}) = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_1} = \frac{1}{R_0 + \varepsilon e^{\omega t + ikz}} \simeq \frac{1}{R_0} - \frac{\varepsilon}{R_0^2} e^{\omega t + ikz}$$

$$\frac{1}{R_2} = \varepsilon k^2 e^{\omega t + ikz}$$

$$\therefore \underbrace{P_0}_{=\sigma/R_0} + \tilde{p} = \frac{\sigma}{R_0} - \frac{\varepsilon\sigma}{R_0^2}\left(1-k^2R_0^2\right)e^{\omega t+ikz}$$

$$\Rightarrow \tilde{p} = -\frac{\varepsilon\sigma}{R_0^2}\left(1-k^2R_0^2\right)e^{\omega t+ikz} \qquad (**)$$

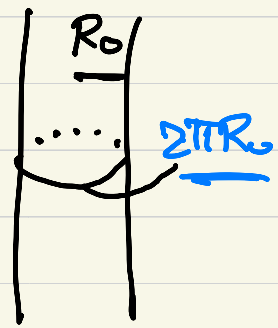
$(**)$: dispersion relation

$$\underline{\omega^2} = \underbrace{\frac{\sigma}{\rho R_0^3}\, kR_0 \,\frac{I_1(kR_0)}{I_0(kR_0)}}_{>0} \underline{\left(1-k^2R_0^2\right)}.$$

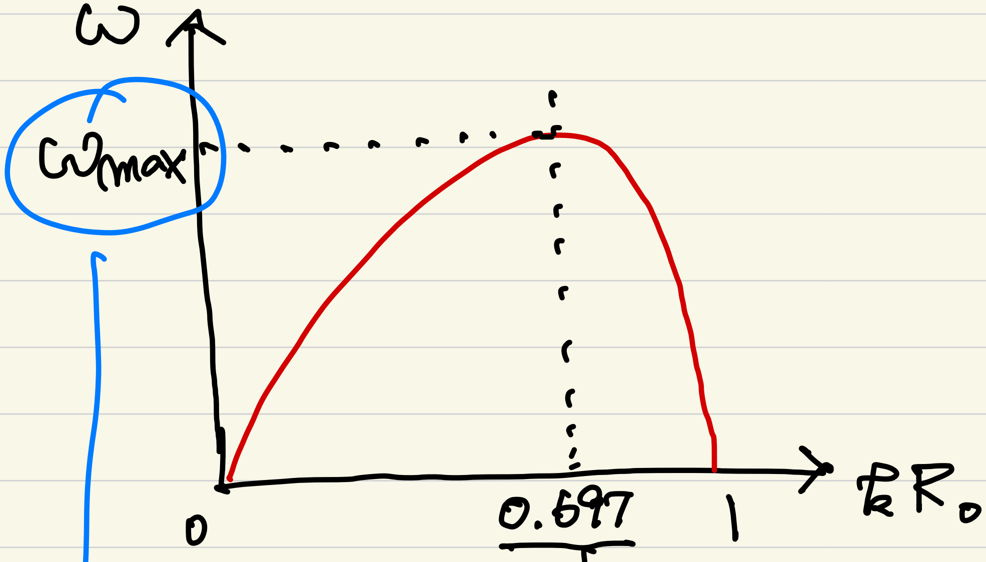
$\Rightarrow$ unstable modes are only possible when $\underline{1-k^2R_0^2 > 0} \rightarrow kR_0 < 1$

$$\boxed{\lambda > 2\pi R_0} \Leftarrow \frac{2\pi}{\lambda}R_0 < 1$$

(wave length)



Liquid column is thus unstable to the disturbance whose wave length exceed the circumference of the column.



fastest growing mode.

$\lambda_{max} \simeq 9.02 R_0$

$$t_{breakup} \simeq 2.91 \sqrt{\frac{\rho R_0^3}{\sigma}}.$$

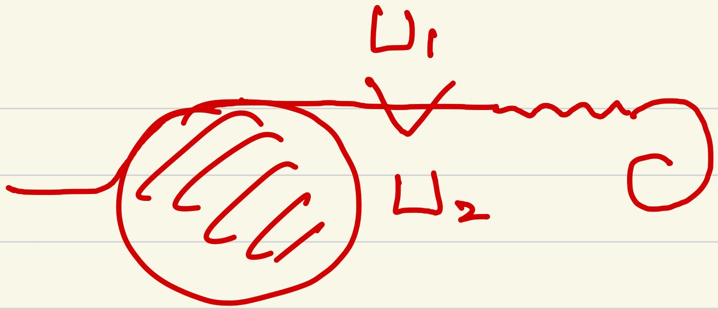
- Kelvin-Helmholtz instability : velocity gradient
- Rayleigh-Taylor " : density "

f
g

G
$\downarrow$
U_1
ρ_1

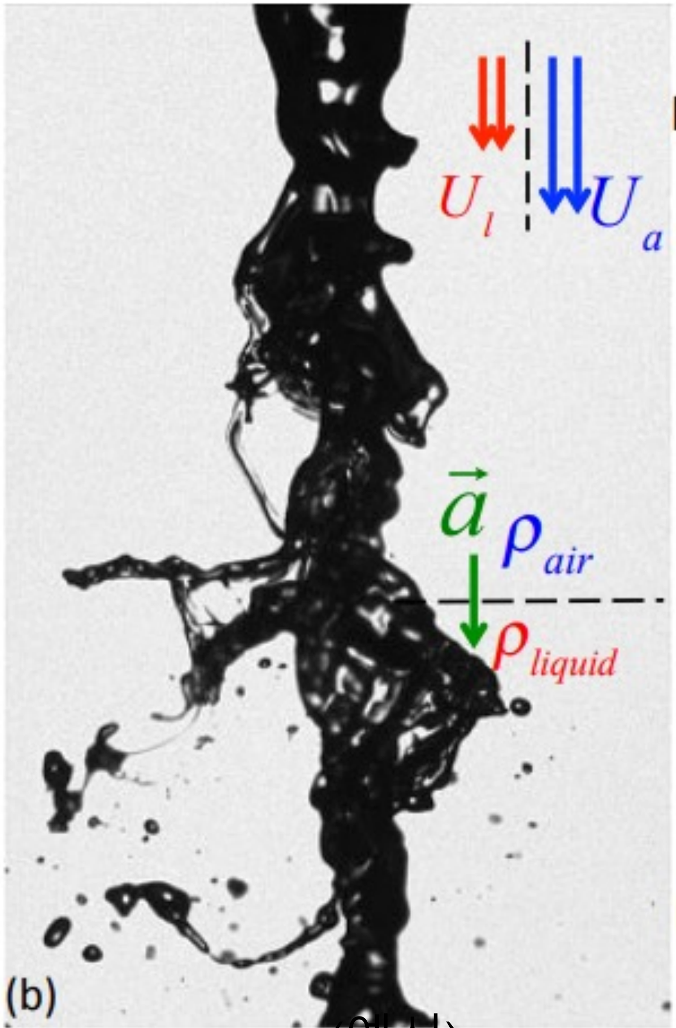
L
$\downarrow$
U_2
ρ_2

G
$\downarrow$
U_1
ρ_1



Breakup process of the falling liquid

prepared by
Mr. Daehyun Choi



Kelvin-Helmholtz Instabilities

Rayleigh-Taylor Instabilities

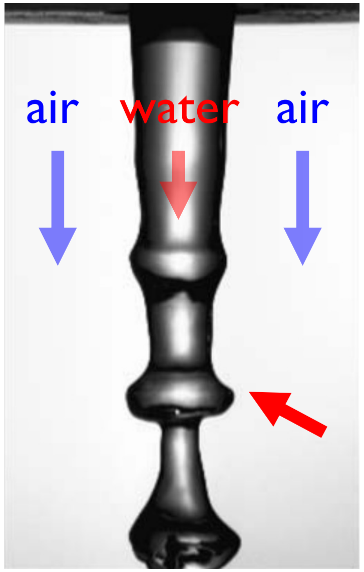
Rayleigh-Plateau Instabilities

(예시)

Keshavarz et al. 2015
Journal of Non-Newtonian Fluid Mechanics

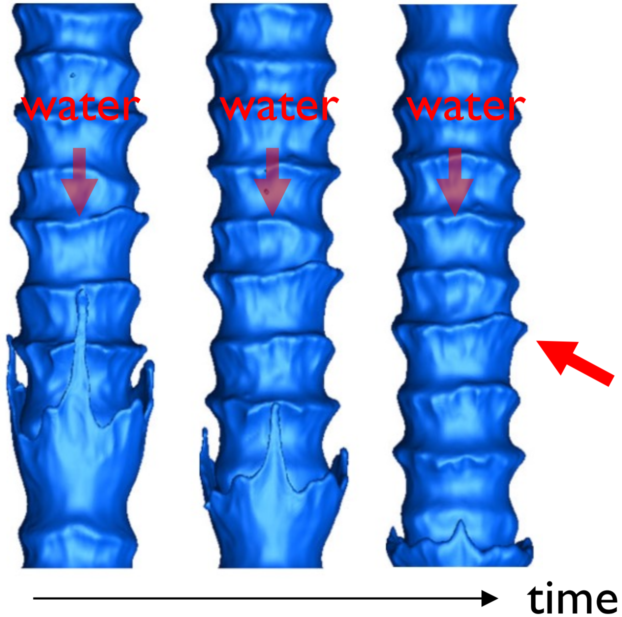
Kelvin-Helmholtz instability

Air-assisted atomization



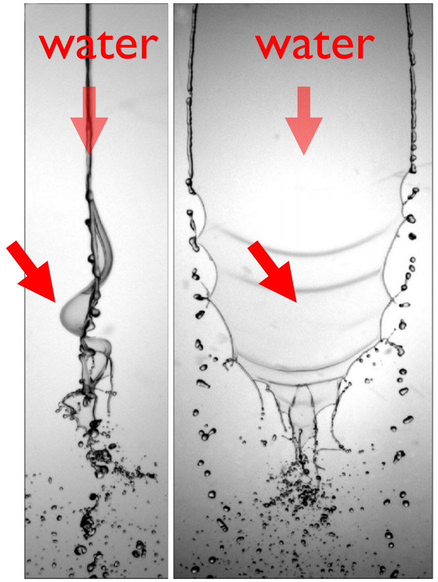
Marmottant & Villermaux 2004,
Journal of Fluid Mechanics

Pressurized atomization



Jarrahbashi et al. 2016,
Journal of Fluid Mechanics

Flapping liquid **sheet**

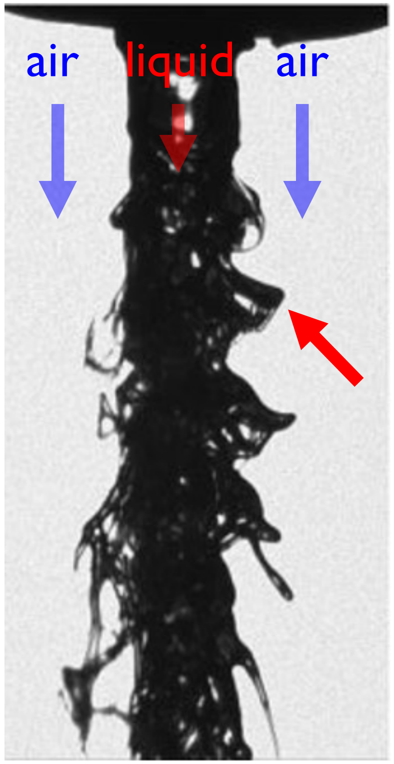


Side view Upper view

Bremond et al. 2007,
Journal of Fluid Mechanics

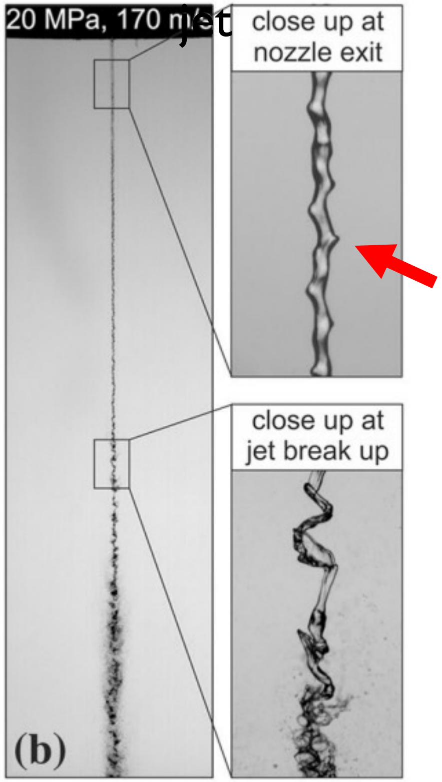
Kelvin-Helmholtz instability

Air-assisted atomization of
polymer solution



Keshavarz et al. 2015,
Journal of Non-Newtonian Fluid Mechanics

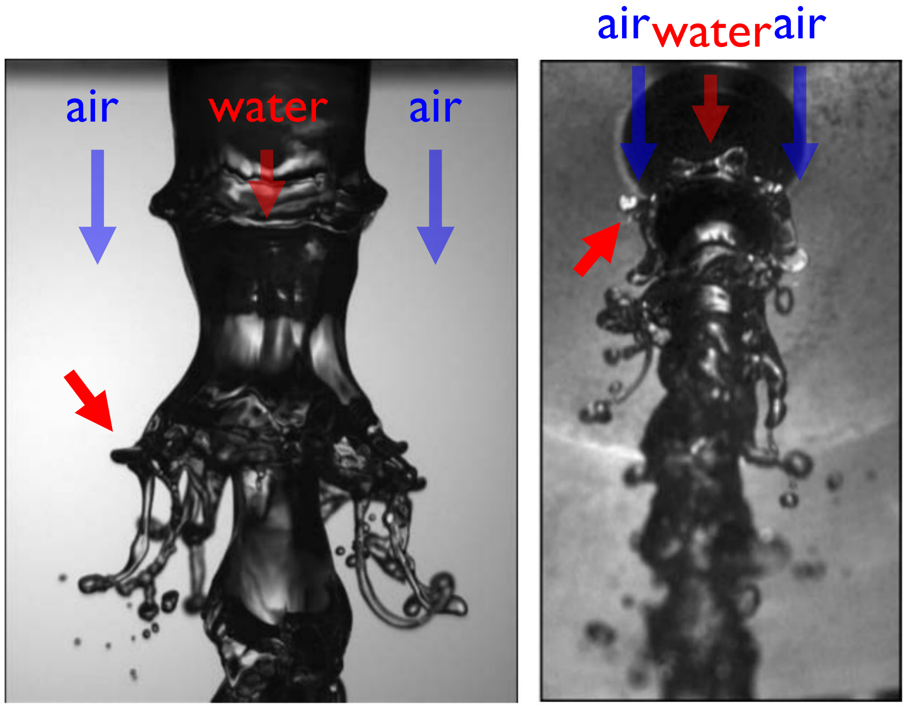
Pressurized atomization of
superheated carbon dioxide jet



Engelmeier et al. 2018,
Experiments in Fluids

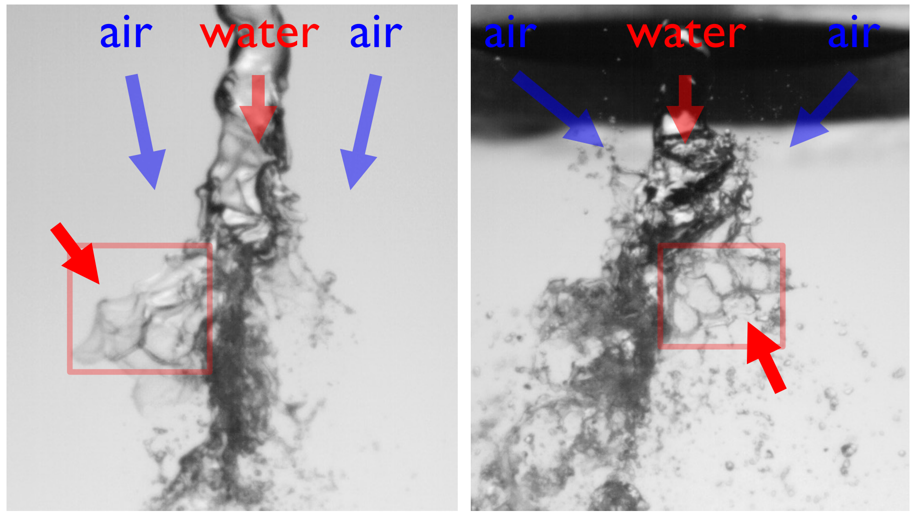
Rayleigh-Taylor instability

Air-assisted atomization



Marmottant & Villermaux 2004,
Journal of Fluid Mechanics

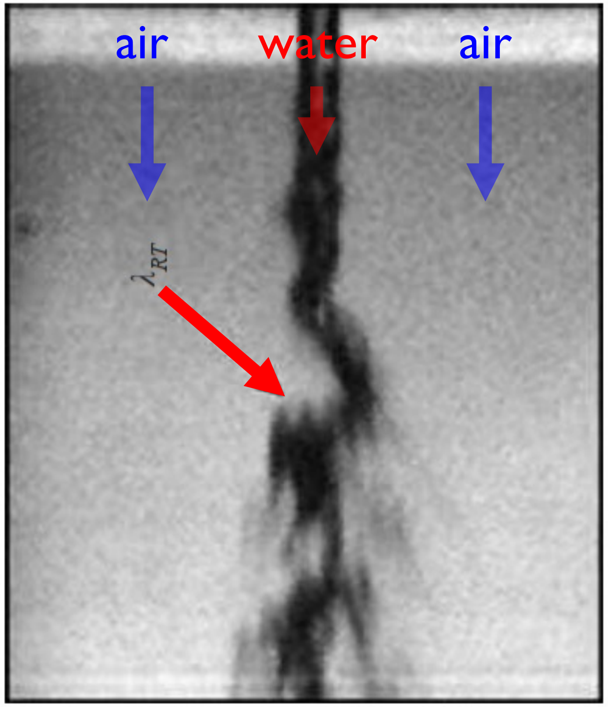
Air-assisted atomization
using the dual-nozzle



Choi & Park 2021,
submitted

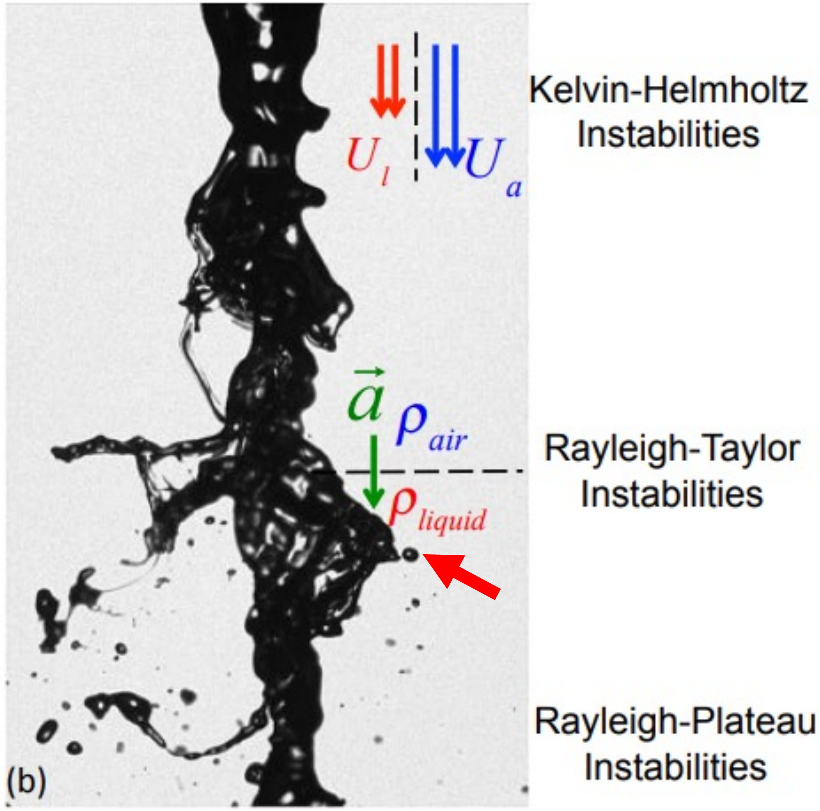
Rayleigh-Taylor instability

Air-assisted atomization



Varga et al. 2003,
Journal of Fluid Mechanics

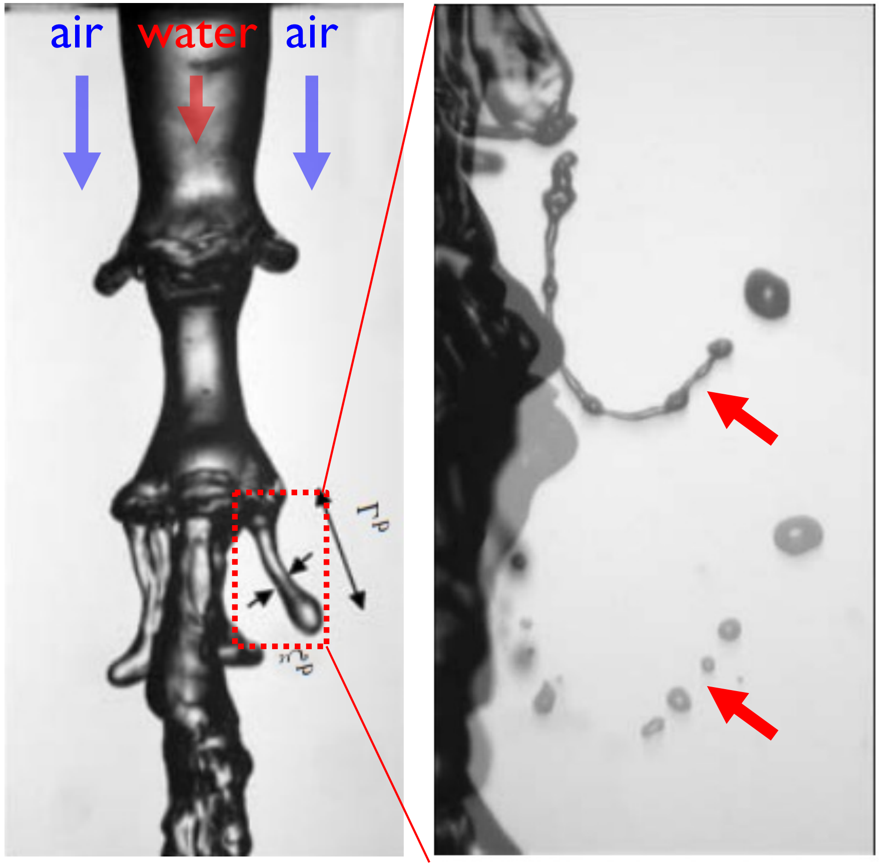
Air-assisted atomization
of polymer solution



Keshavarz et al. 2015,
Journal of Non-Newtonian Fluid Mechanics

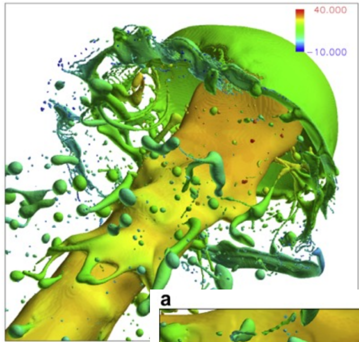
Rayleigh-Plateau instability (High weber number)

Breakup of a ligament at the air-assisted atomization (water & air)



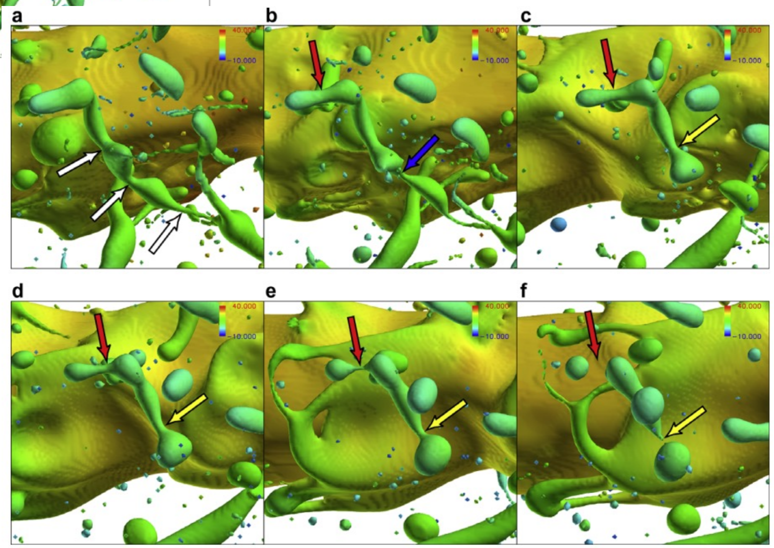
Marmottant & Villermaux 2004, Journal of Fluid Mechanics

Breakup of a ligament at the pressurized atomization (DNS simulation)



(virtual liquid & gas)

Gas density ρ_g (kg/m^3)	Liquid density ρ_l (kg/m^3)	Liquid viscosity μ_l (Pa s)	Gas viscosity μ_g (Pa s)	Surface tension coefficient σ (N/m)
34.5	848	2870e−6	19.7e−6	30.0e−3

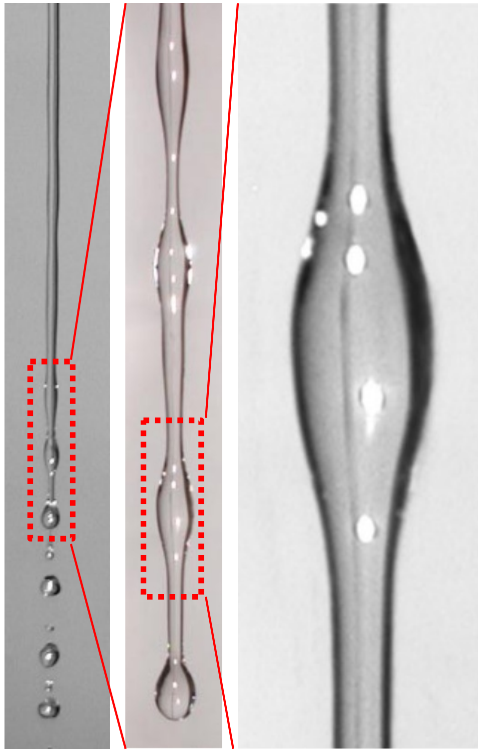


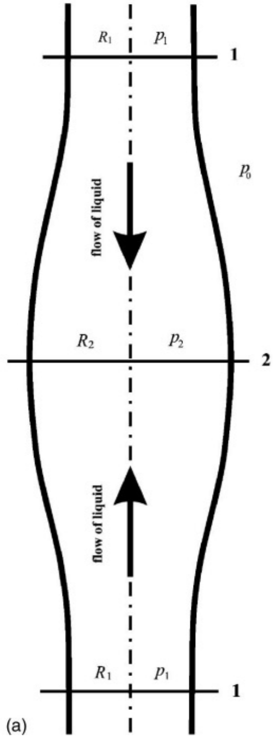
Arrows indicate the breakup point due to the instability

Shinjo & Umemura 2010, International Journal of Multiphase Flow

Rayleigh-Plateau instability (Low weber number)

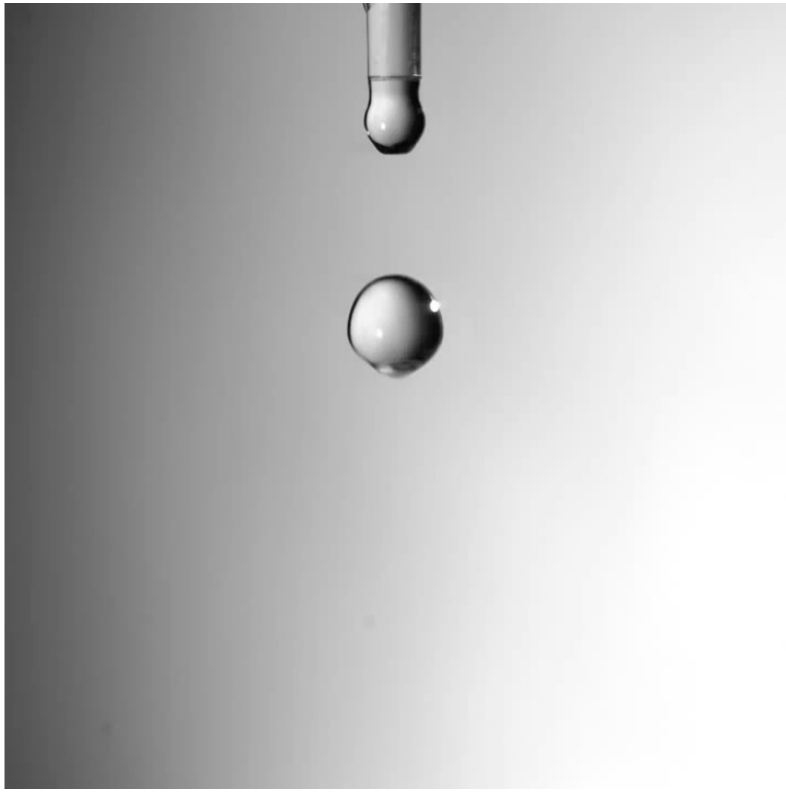
Breakup of the falling water column





Grubelnik & Marhl, 2005,
American Journal of Physics

Breakup of the falling water column



https://www.youtube.com/watch?v=9OOZQxmYnmo

Rayleigh-Plateau instability (Low weber number)

Breakup pattern from the falling tap water

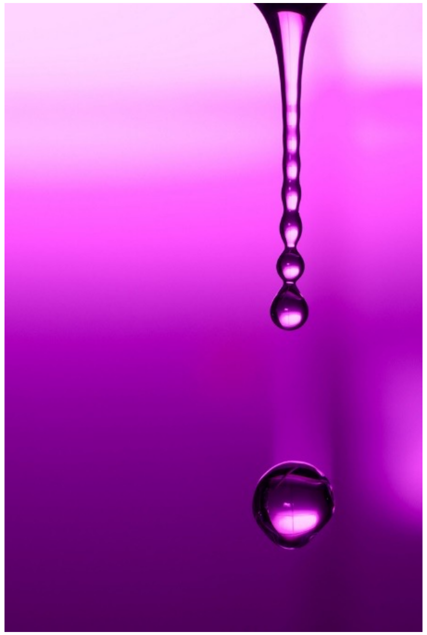
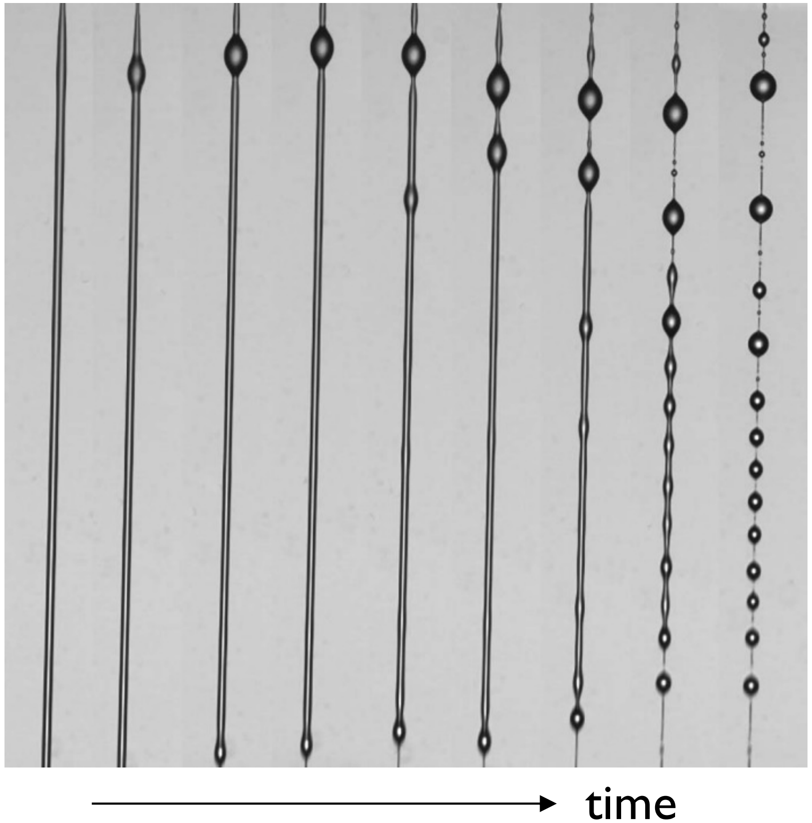


Image credit:
N. Morberg)

https://nn.wikipedia.org/wiki/Irritasjon

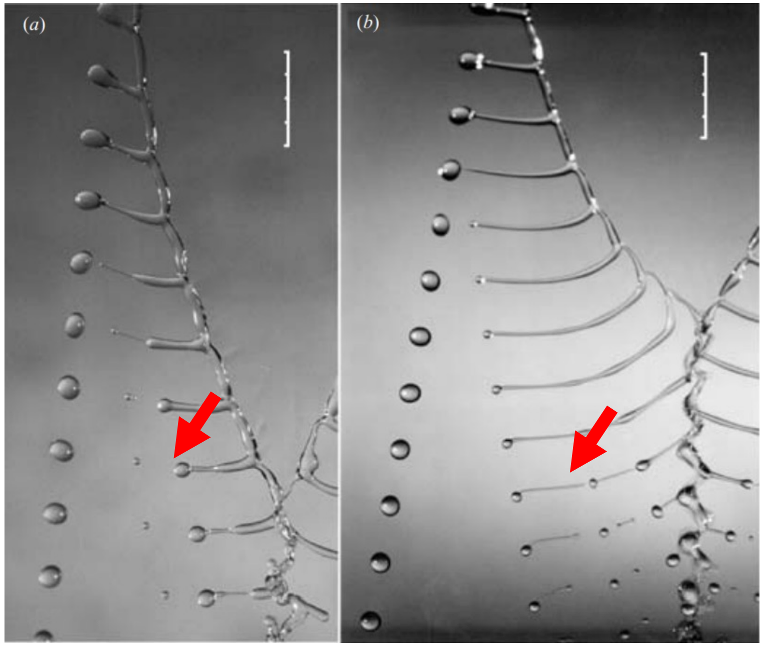
Breakup of polymer solution



Sattler *et al.* 2012,
Physics of Fluids

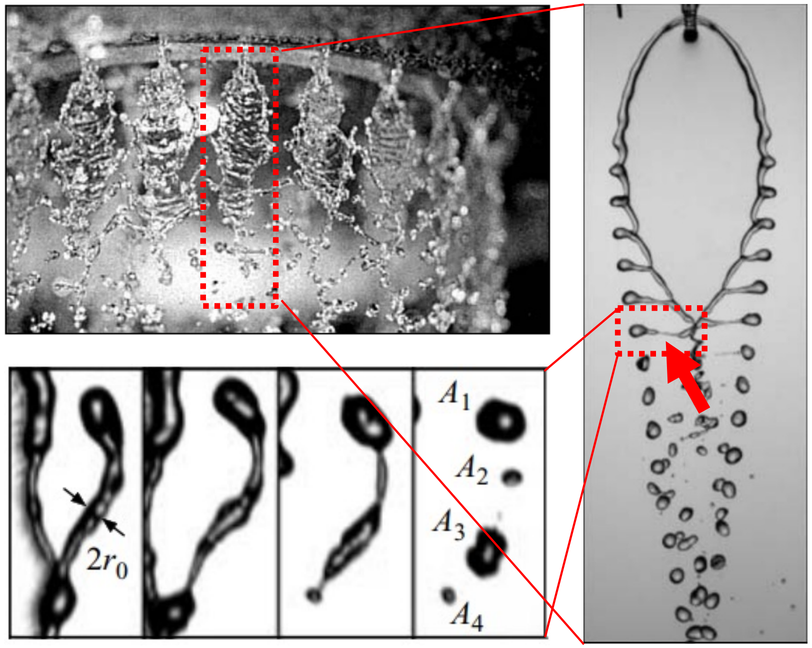
Rayleigh-Plateau instability (liquid sheet)

Collision of laminar jets : fishbone structure



Bush & Hasha 2004, Journal of Fluid Mechanics

Impacting jets in the combustion chamber of the gas generator



Bremond & Villermaux 2006, Journal of Fluid Mechanics