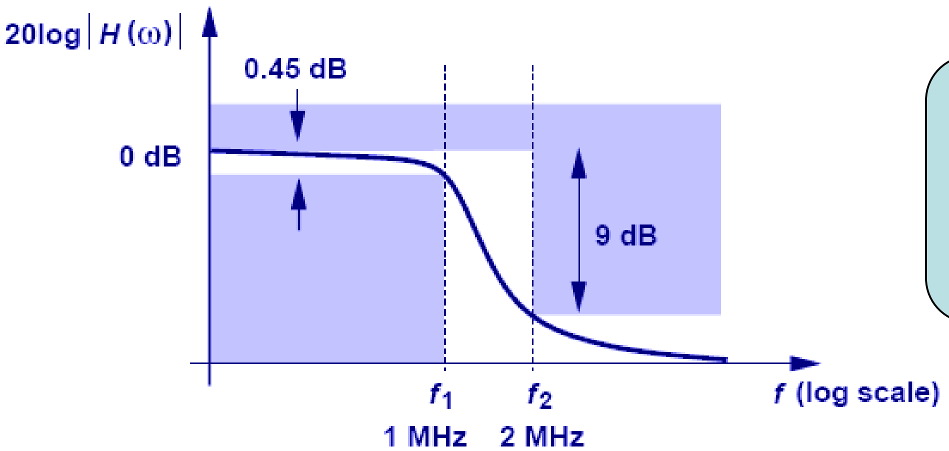
Chapter 15 Analog Filters (supplementary)

- Design Optimization
- Chebyshev polynomial

Design with Margin: Butterworth Filter



Specification: passband flatness of 0.45 dB for f < f_1=1 MHz, stopband attenuation of 9 dB at f_2=2 MHz.

$$|H(f_1 = 1\text{MHz})| = 0.95$$

$$\frac{1}{1+\left(\frac{2\pi f_1}{\omega_0}\right)^{2n}} = 0.95^2$$

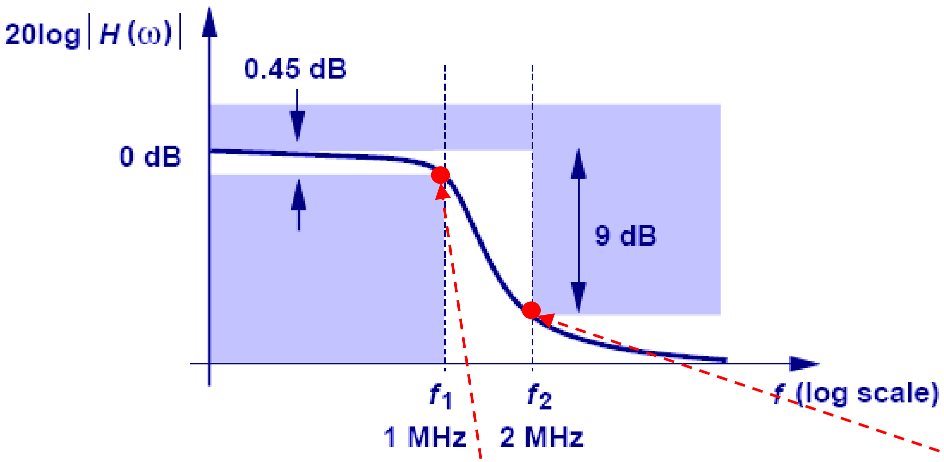
$$|H(f_2 = 2\text{MHz})| = 0.355$$

$$\frac{1}{1+\left(\frac{2\pi f_2}{\omega_0}\right)^{2n}} = 0.355^2$$

$$\left(\frac{f_2}{f_1}\right)^{2n} = 64.2 \Rightarrow f_2 = 2f_1 \Rightarrow n = 3.00225 \to n = 4$$

- **The minimum order of the Butterworth filter is four.**

Design with Margin: Butterworth Filter



n=4,

Determine two ω_0s: ω_{0p} and ω_{0s}

Touching passband and stopband

$$|H(f_1 = 1\text{MHz})| = 0.95$$

$$\frac{1}{1+\left(\dfrac{2\pi(1\text{MHz})}{\omega_{0p}}\right)^8} = 0.95^2$$

$$\rightarrow \omega_{0p} = 2\pi(1.32\text{MHz})$$

$$|H(f_2 = 2\text{MHz})| = 0.355$$

$$\frac{1}{1+\left(\dfrac{2\pi(2MHz)}{\omega_{0s}}\right)^8} = 0.355^2$$

$$\rightarrow \omega_{0s} = 2\pi(1.57\text{MHz})$$

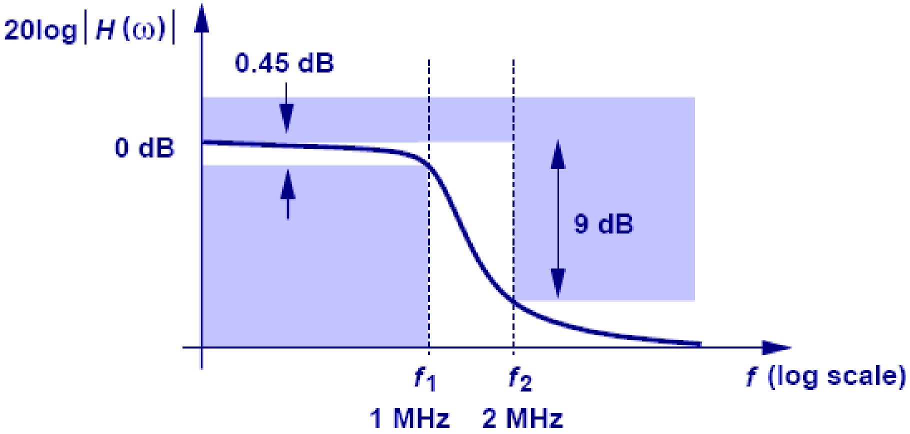
$$\omega_{0,opt} = 2\pi(1.44\text{MHz})$$

<Optimum Design>

Passband Ripple : 0.23dB

Stopband Attenuation : 11.7dB

Design with Margin: Chebyshev Filter



Suppose the filter required in Example 14.24 is realized with third-order Chebyshev response. Determine the attenuation at 2MHz.

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.95 \rightarrow \varepsilon = 0.329$$

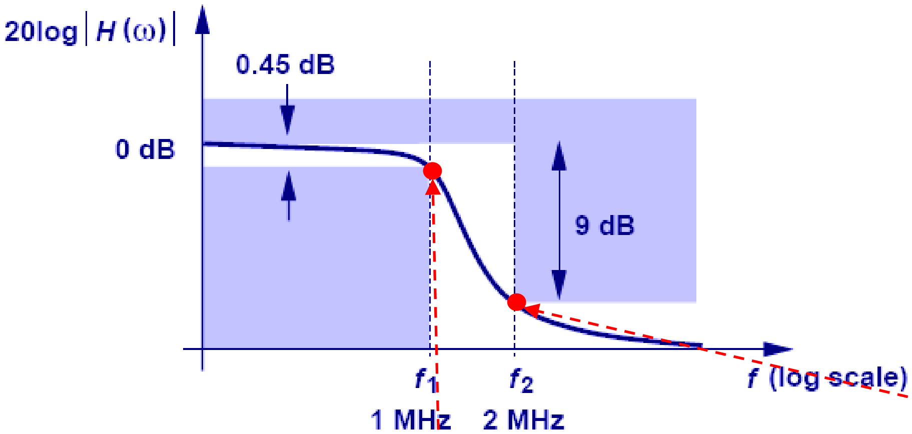
$$\omega_0 = 2\pi\ (1\text{MHz})$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2 C_3^2\left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\left|H(j2\pi(2\text{MHz}))\right| = 0.116 = -18.7\text{dB}$$

- A third-order Chebyshev response provides an attenuation of -18.7 dB a 2MHz.

Design with Margin: Chebyshev Filter



n=3,
Determine ε_p and ε_s
Touching passband and stopband

Passband Ripple : 0.45dB

$$\rightarrow \varepsilon_p = 0.329,\ \omega_0 = 2\pi(1\text{MHz})$$

Stopband Attenuation : 18.7dB

Stopband Attenuation : 9dB

$$\rightarrow \varepsilon_s = 0.101,\ \omega_0 = 2\pi(1\text{MHz})$$

Passband Ripple : 0.04dB

$$\varepsilon_{opt} = 0.182$$

<Optimum Design>

Passband Ripple : 0.14dB

Stopband Attenuation : 13.9dB

Chebyshev polynomial $C_n(x)$

- n=1, x
- n=2, -1 +2x^2
- n=3, -3x +4x^3
- n=4, 1 -8x^2 +8x^4
- n=5, 5x -20x^3 + 16 x^5
- n=6, -1 +18x^2 - 48x^4 + 32x^6
- n=7, -7x +56x^3 -112 x^5 +64x^7
- n=8, 1 -32x^2 +160x^4 -256x^6 +128x^8
- n=9, 9x -120x^3 +432 x^5 -576x^7 +256x^9
- n=10, -1 +50x^2 -400x^4 +1120x^6 -1280x^8 +512x^10