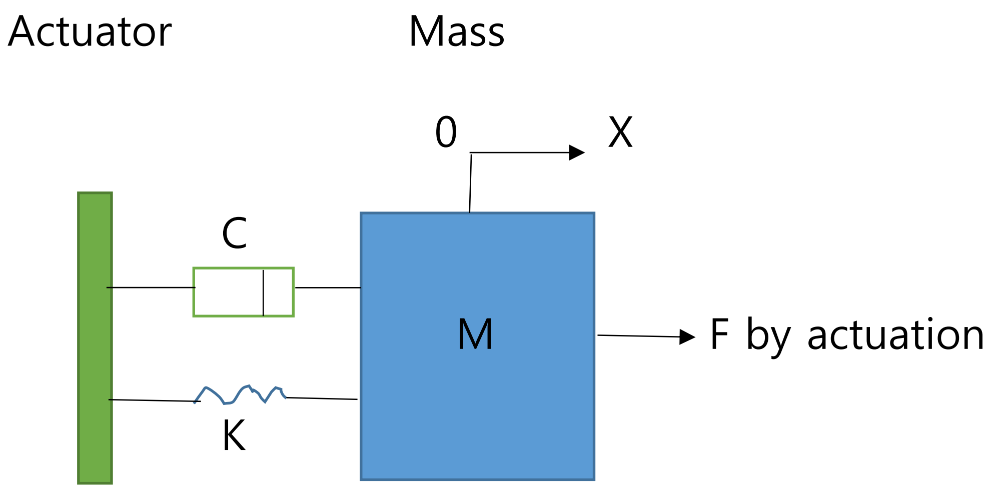
Precision machine Design- Dynamic matching

1. Dynamic matching of components

1) Minimum stiffness of actuator and Maximum servo loop time

The axial stiffness of actuator must be high enough such that the smallest force input to the system should cause a deflection less than the allowable minimum limit. If the deflection is bigger than this limit, the closed loop servo system may not react to the force input properly like rubber string, which is not controllable.



K is the actuator stiffness, and it is in general the higher the better, where M is the mass to be actuated, C is the damper.

Let ΔX_{actual} be the actual displacement under the force increment, ΔF, and $\Delta X_{control}$ is the control target or the allowable minimum limit. In order for the system to be controllable,

$\Delta X_{actual}=\Delta F/K \leq \Delta X_{control}$; thus $\Delta F/\Delta X_{control} \leq K$

$\therefore$K should be high enough such that $K \geq \Delta F/\Delta X_{control}$

Otherwise, the feedback control action will fluctuate continuously between $[0, \Delta X_{actual}]$; which is uncontrollable.

$\Delta F/\Delta X_{control}$ is called as the minimum stiffness of actuator.

When C is assumed as very small, the dynamic response of the actuation is obtained from

$Md^2X/dt^2+CdX/dt+KX=\Delta F$, thus the dynamic response, ΔX becomes

$\Delta X=\Delta F/K+A\sin(\omega_n t+\varphi)$

where ω_n=natural frequency of actuation system=$\sqrt{K/M}$

A=constant

The ΔX signal should be measured from the sensor then fed back to the controller. In digital sampling and control, aliasing is the phenomena such that signals of different frequencies become indistinguishable at the integer number times of frequencies. In order to avoid the aliasing, the LPF(low pass filter) is adopted to attenuate the high frequency signals (in control), or to use the Nyquist criterion (in digital sampling) such that $f_s \geq 2f_0$ where f_s is the sampling frequency and f_0 is the frequency of interest to be detected.

The mechanical time constant, τ_m, of the actuator becomes,

$\tau_m = 2\pi/\omega_n = 2\pi(M/K)^{1/2}$ eq(1)

where ω_n is the natural frequency of actuator.

From the Nyquist Criterion, the servo frequency is at least twice the natural frequency of actuator such that $\omega_{servo} \geq 2\omega_n$. Thus the mechanical time constant, τ_m, should be at least twice of the servo loop time, τ_{servo}. The ratio, $\tau_m/2\tau_{servo}$, approximately gives the number of servo loop or sampling (n). The standard deviation of the average of samplings will be statistically reduced by $\sqrt{n}$ or $(\tau_m/2\tau_{servo})^{1/2}$ when compared to the standard deviation of every single sampling. This is called the behavior of average or averaging effect due to multiple samplings, and it contributes to *make narrower deviation or finer resolution* by the factor of $\sqrt{n}$ or $(\tau_m/2\tau_{servo})^{1/2}$

For a controller of N bits of D/A resolution, the incremental force ΔF can be written as following,

$\Delta F = F_{max}/\ 2^N\ /[\tau_m/2\tau_{servo}]^{1/2}$ eq(2)

where F_{max} is the maximum (or full scale) force that the controller can handle.

Let δ_K is the deflection when ΔF is applied to the system, then

$\delta_K = \Delta F/K$ eq(3)

where K is the stiffness of actuator.

From eq(1) to (3),

The stiffness of actuator, K, becomes

$\boldsymbol{K = \Delta F/\delta_K = [F_{max}\tau^{1/2}{}_{servo}\ /[2^N\pi^{1/2}M^{1/4}\delta_K]]^{4/3}}$ eq(4)

: Dynamic matching condition

Eq(4) indicates that the minimum stiffness of actuator can be determined from the target deflection (δ_K) and the servo loop time (τ_{servo}). Eq(4) gives the minimum stiffness of actuator when the target deflection δ_K, and the servo loop time, τ_{servo} are chosen.

In the mean time between the loop time, it can be assumed that the system behaves as the open loop system, thus the system of mass M will travel with constant acceleration, $a=\Delta F/M$, ignoring any damping effect. Thus the distance of travel, δ_M, by the system during the time of τ_{servo} will be

$\delta_M=at^2/2=0.5(\Delta F/M)\tau^2_{servo}$

Thus $\tau_{servo}=[2\delta_M/\Delta F]^{1/2}$ eq(5)

Appying eq(5) to eq(4),

$K=F_{max}\delta_M{}^{1/4}/[2^{N-1/4}\pi^{1/2}\delta_K{}^{5/4}]$ eq(6)

The distance of travel, δ_M , during the time of τ_{servo} can be assumed as the half of the servo error, $\delta_{servo}/2$. Similarly, the minimum target deflection, δ_K can be assumed as the half of the servo error, $\delta_{servo}/2$.

Thus eq(6) becomes,

$\mathbf{K=F_{max}/[2^{N+3/4}\pi^{1/2}\delta_{servo}]}$ eq(7)

: Minimum Stiffness to achieve δ_{servo}

The servo loop time, τ_{servo}, also can be determined from eq(4),(5),(3),(6)

$\boldsymbol{\tau_{servo}}=[2\delta_M M/\Delta F]^{1/2}=[2\delta_M M/(K\delta_K)]^{1/2}$

$=[2\delta_M M 2^{N-1/4}\pi^{1/2}\delta_K{}^{5/4}/ F_{max}\delta_M{}^{1/4}\delta_K]^{1/2}$

$=[2^{N+3/4}\ \pi^{1/2}\ \delta_M{}^{3/4}M\ \delta_K{}^{1/4}/F_{max}]^{1/2}$

$\mathbf{=[2^{N-1/4}\ \pi^{1/2}\ M\delta_{servo}/F_{max}]^{1/2}}$ if $\delta_M=\delta_K=\delta_{servo}/2$ eq(8)

: Maximum Servo loop time to achieve δ_{servo}

Eq(8) gives the maximum servo loop time in order to give the allowable δ_{servo} .

Therefore, in order to decrease the servo error, δ_{servo} , that is to give higher precision,

1) Increase K; stiffness solution (static solution)

2) Decrease τ_{servo} ; control solution (dynamic solution)

Ex) For M=200Kg, F_{max}=1000N, N=12 bit D/A

For δ_{servo}=10nm to be achieved,

① $K \geq \mathbf{F_{max}/[2^{N+3/4}\pi^{1/2}\delta_{servo}]=}$8.192 MN/m (Minimum stiffness), or

② $\tau_{servo} \leq \mathbf{[2^{N-1/4}\pi^{1/2}M\delta_{servo}/F_{max}]^{1/2}}$=0.00349sec=3.49msec (Maximum servo loop time)

For δ_{servo}=1nm to be achieved,

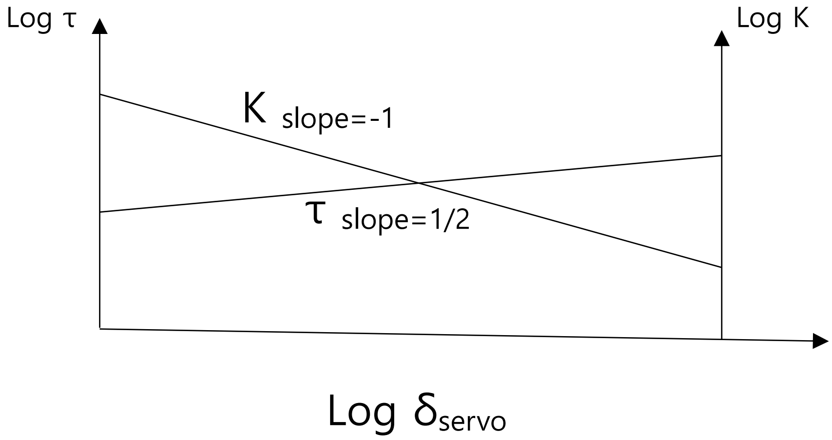
①$K \geq \mathbf{F_{max}/[2^{N+3/4}\pi^{1/2}\delta_{servo}]=}$81.920 MN/m (Minimum stiffness),

or

②$\tau_{servo} \leq \mathbf{[2^{N-1/4}\pi^{1/2}M\delta_{servo}/F_{max}]^{1/2}}$=0.00110sec=1.10msec

(Maximum servo loop time)

<u>Graph for K and τ</u>

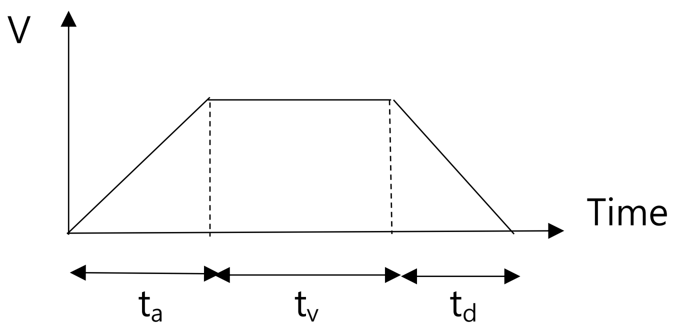


This is the case of the actuator stiffness and servo loop time issue for high precision motion. Additionally, *the natural frequencies of the individual components such as brackets, holders, jigs are carefully avoided from the frequency of excitation or actuation in the system.*

2. Optimum transmission ratio between elements

Velocity profile of actuation:

Among various velocity profiles, the triangular or trapezoidal velocity profiles are the most common as shown in the fig, as they can be conveniently implemented through the controller



Let t_T=total time for move, t_a=time of acceleration=$t_T/3$, t_v=time of constant velocity=$t_T/3$, and t_d=time for deceleration=$t_T/3$

When a is the acceleration/deceleration, the total distance travelled, D, is

$D=0.5a(t_T/3)^2+a(t_T/3)(t_T/3)+0.5a(t_T/3)^2=2at_T^2/9$

Thus for given distance, D, and time for move, t_T

The acceleration and velocity can be determined, which is the first step for the motor and power elements selection.

$a=9D/2t_T^2$ eq(2-1)

$V=a(t_T/3)=3D/2t_T$ eq(2-2)

<u>PR, Power Rate</u> [Watt/s]:

Amount of change in power during a time period of acceleration/deceleration.

Thus for the trapezoidal velocity profile with actuation force,

Power rate during acceleration, PR;

$PR = d(FV)/dt=FdV/dt=Fa=F(F/M)=F^2/M$ eq(2-3)

where F=Actuation force, M=Mass to be actuated.

For rotating actuation;

$PR=d(T\omega)/dt=Td\omega/dt=T\alpha=T(T/J)=T^2/J$ eq(2-4)

where T=Actuation torque[Nm], J=Mass moment of inertia to be rotated[Kgm^2], ω=angular velocity[rad/s],

α=angular acceleration[rad/s^2]

Thus for the velocity profile given,

Power for Load, $P_{Load}=FV$

Power Rate of Load, $PR_{Load}=FV/t_a$

The required power can be assigned larger than the twice of power for load,

$P_{required} \geq 2P_{Load}$ eq(2-5)

The required power rate can be assigned larger than four times of the power rate for load (from G. Newton's paper, Selecting the optimum electric servo motor for incremental positioning applications, 10 symp. Increment. Motion Control Syst. And Dev.BB Kuo(ed.) p5)

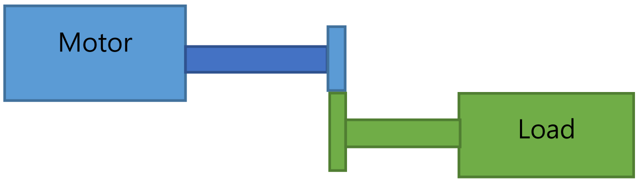
$PR_{required} \geq 4PR_{Load}$ eq(2-6)

Eq(2-5),(2-6) give guidelines for the motor or actuator selection.

<u>Optimal Transmission Ratio</u>

Assume that motor transmits torque to element (or Load);

Transmission Ratio= n : 1



For optimum transmission, the power output from the motor during the acceleration time, t_a, must be equal to the power input to the Load during the same time t_a.

$T_{motor}\omega_{motor}=J_{motor}\alpha_{motor}\omega_{motor}=J_{motor}\alpha^2_{motor}t_a$

$T_{Load}\omega_{Load}=J_{Load}\alpha_{Load}\omega_{Load}=J_{Load}\alpha^2_{Load}t_a$

where α is the angular acceleration, ω is the angular velocity.

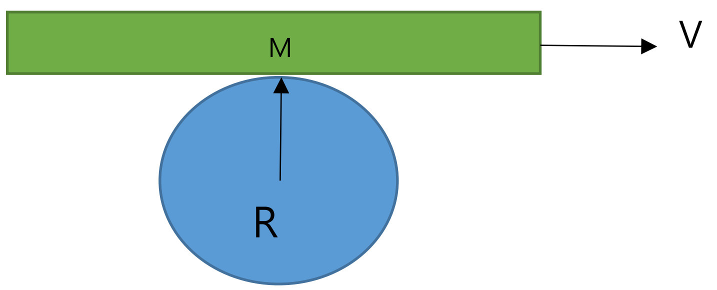
Therefore $J_{motor}\alpha^2_{motor} = J_{Load}\alpha^2_{Load}$ eq(2-7)

When n is the optimum transmission ratio between the motor and the load such that $\alpha_{motor}=n\alpha_{Load}$

Thus $n_{opt}=[J_{Load}/J_{motor}]^{1/2}$ eq(2-8)

For a friction drive;

Friction Drive of Mass, M



Optimum transmission occurs when power is balanced;

During t_a acceleration period,

Power from the motor wheel

$=T\omega=J_{motor}\alpha\omega=J_{motor}\alpha^2 t_a$

Power to friction driven rod

$=FV=M_{Load}a^2t_a=M_{Load}\alpha^2R^2t_a \quad (\because a=\alpha R)$

From the power balance, the optimum radius of wheel, R, can be determined as,

$R=[J_{motor}/M_{Load}]^{1/2}$ eq(2-9)

This is the optimum radius of motor wheel for optimal transmission.

For the lead-screw driven carriage,

M=Mass, V=velocity

Lead

L mm/rev (=Screw pitch)

During t_a acceleration period,

Power from Lead Screw$=T\omega=J_{Screw}\alpha^2t_a$

Power to Carriage$=FV=M_{Carriage}a^2t_a=M_{Carriage}\,(L\alpha/2\pi/1000)^2t_a$

From power balancing,

the optimum screw pitch can be determined as,

$L=2\pi(1000)[J_{Screw}/M_{Carriage}]^{1/2}$ [mm/rev] eq(2-10)

Sometimes it may lead quite small lead, thus requiring very high rotation speed. In this case, the lead can be chosen according to the critical speed criteria such that,

$\omega_n=k^2[EI/(A\rho L^4)]^{1/2}$

$L=2\pi(1000)V_{max}/\omega_{max}$ [mm/rev], where $\omega_{max}<\omega_n$ eq(2-11)

Optimum transmission ratio under large external loads

When large external load is applied to the system due to friction force or cutting force, the optimum transmission ratio can be modified as follows

(source: J.Park and S.Kim,Optimum speed reduction ratio for DC servo drive system, Int.J.Machine Tools and Manufacture, 29(2), 1989)

$n_{opt}=[J_{Load}\sqrt{(1+r)}/J_{motor}]^{1/2}$ eq(2-11)

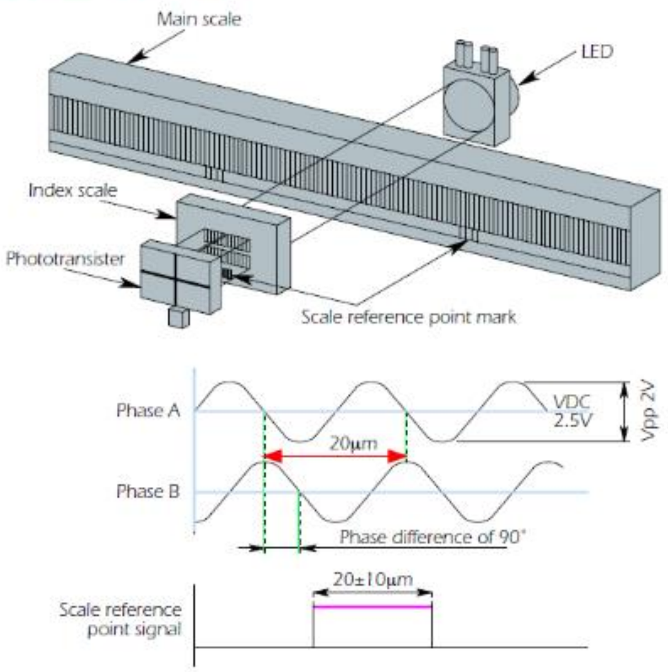
where $r=T^2{}_{Load}t_c{}^2/[c\omega^2{}_{Load}J^2{}_{Load}]$, and

$c=t_c/t_a+t_c/t_d$ where t_c is total time of travel, t_a is the time for acceleration, t_c is the total time for travel.

<u>Sensors</u>

Linear Optical Scale

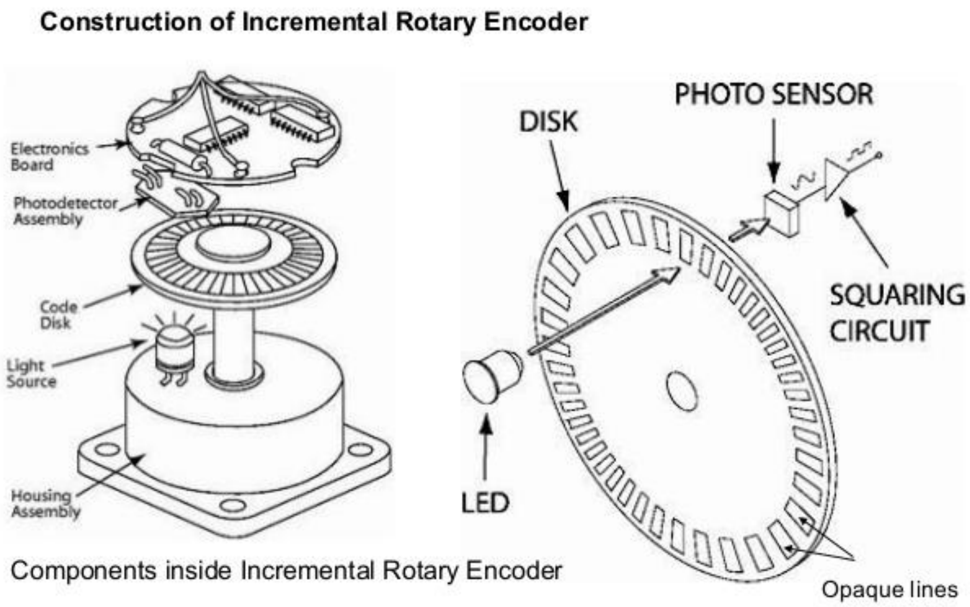
Detecting principle



The detector of each scale unit consists of a light source (LED) and a photoelectric device (phototransistor) which face each other with the main scale and index scale between them. When the main scale moves, relative to the index scale, the quantity of light that is transmitted through the gratings on the index scale varies with the same period as the grating pitch. This variation of light intensity is converted into electrical signals and output as two-phase (A and B) waves with a phase difference of 90°. The display unit divides these signals to determine the direction of the scale movement, and digitally displays the displacement of the scale.

Source: Mitutoyo catalog for optical scale, and interpolation electronics such as the <u>quadrature decoding</u> gives the finer resolution; Pitch/2^N ≒ 0.1 um for Pitch=20um, N=8 (bit)

Rotary Encoder



Source: slide share

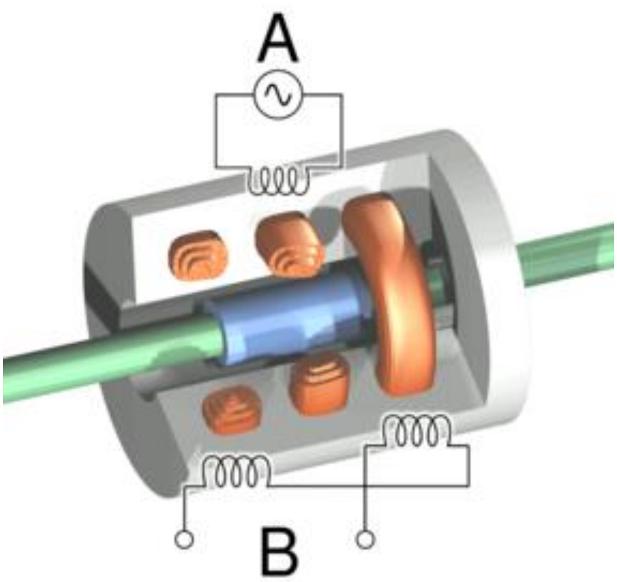
This is rotary version of linear optical scale, and the range is 360° and the resolution is 360°/N, where N is the number of divisions. Electronic interpolation gives very high resolution such as 0.001 deg or less.

Small displacement measurement

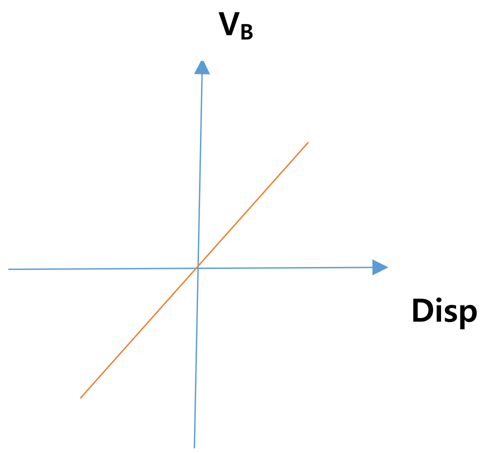
: To measure the small distance up to 10 mm with fine resolution

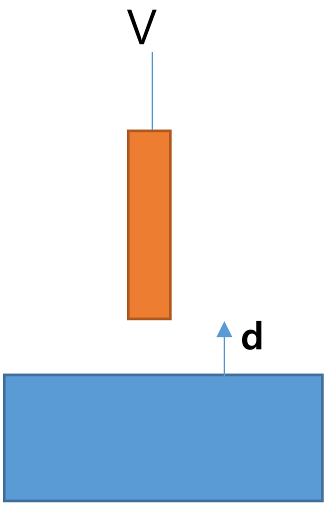
Commercially available sensors for small distance

Type	Range	Resolution
LVDT	~10mm	0.01um
Capacitance Gauge	~1mm	0.001um
Optical sensor	~10mm	0.1um

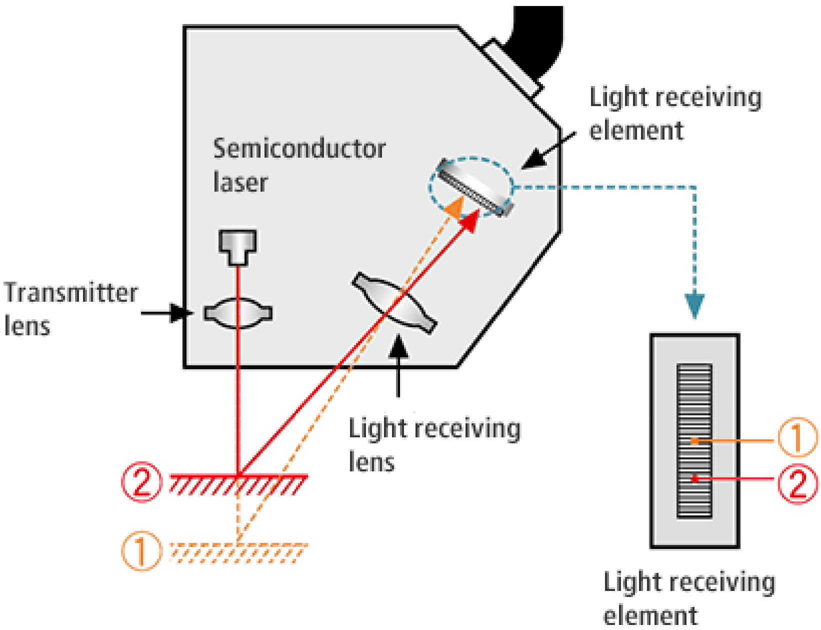


LVDT (Source:Wikipedia)





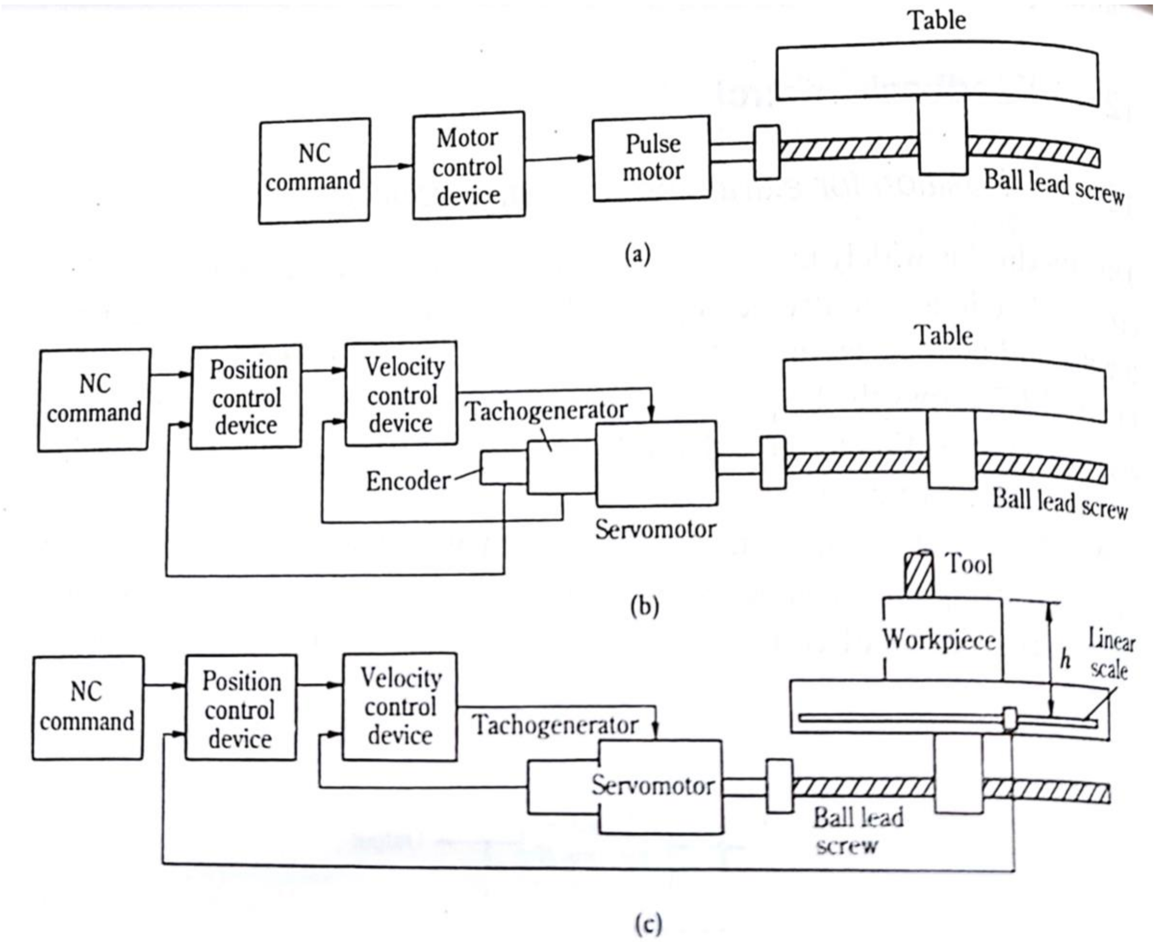
Capacitance Sensor (Source:Wikipedia)



Optical Triangulation Sensor (Source:Keyence.com)

Control Schemes

:There are several schemes to connect the sensor and actuator, that are commonly used in the practical machines or field.



(a)Open Loop (b)Closed loop with Rotary encoder

(c) Closed loop with Linear encoder

(Source from Nakazawa's Principles of Precision Engineering, Oxford University

Press)

(1) Open loop

This method is to drive the actuator without connecting the sensor input; most simple and cheap method.

When NC command or target position is given, motor control device outputs the pulse proportional to the commanded position, then the step motor is actuated by the pulse generated, stepping by each pulses, and the connected lead screw rotates to feed the table. The exact position of feed table is never measured, thus accuracy can be limited, but it has wide application due to the simple configuration and cheap cost.

(2) Closed loop

This method is one of most common methods for motion control, and the configuration typically consists of two parts of control loops: one loop for velocity control with feedback from the tacho-generator, and another loop for position control with feedback from the rotary encoder or linear encoder (linear scale). The position control device generates the signal that corresponds to the gap between the desired position and the actual position, and is in charge for the position to position movement. The velocity control device generates the signal that corresponds to the gap between the desired velocity and actual velocity, and it cares for the velocity control in charge of the path or trajectory.

(3) PID Control

The closed loop control device needs to generate the signal that corresponding to the gap between the desired and actual.

There are several methods to achieve this, and the PID control method is one of the most widely used techniques, as it can be applied to various control applications with a simple design and relatively cheap cost.

When u is the signal and e is the gap to be followed, then u can be calculated as the combination of Proportional part, Integral part, and the Derivative part, and it is called as the PID control; that is,

$u=K_c(e+\int edt/T_i+T_d de/dt)$ in time domain, or

$U(S)=K_c[1+S^{-1}/T_i+ST_d]E(S)$ in S domain

where U(S), E(S) are the Laplace transform of u(t), and e(t), respectively.

Also, K_c is the P gain, $1/T_i$ is the I gain, and T_d is D gain.

The gains are adjusted or tuned from such as the step response, and their characteristics are as follows;

Rise time = the time for rising 90% of target,

Overshoot = the overshoot amount over the target position, Settling-time = the time when the response is within +/-3% tolerance from the target position,

Steady-state error = the permanent error,

=target position - actual position, as t becomes ∞.

As K_c increase,

Rise time↓, Overshoot↑, Settling time=X(not related),

Steady-state error↓

As $1/T_i$ increase,

Rise time↓, Overshoot↑, Settling time↑, Steady-state error=0

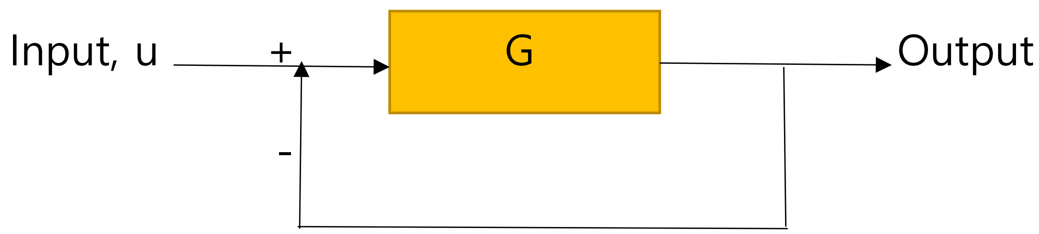
As T_d increase,

Rise time X, Overshoot↓, Settling time↓, Steady-state error=X

Therefore, the gains or parameters can be adjusted or tuned by manual tuning or auto-tuning.

(4) Control scheme for steady-state error reduction

The steady-state error is the permanent error between the target and actual, and it is always desirable to minimize this for precision motion control.



Feedback Control Diagram

For a typical feedback diagram shown, when the target U(S) is commanded, the error E(S) can be derived as

E(S)=U(S)/[1+G(S)], where G(S) is the loop transfer function.

For the step input u(t)=1, or U(S)=1/S,

From the final value-theorem, the steady-state error, e(∞)=lim SE(S), as S->0

=lim 1/[1+G(S)], as S->0

Thus two ways are possible to reduce the steady-state error;

one is to give very high gain for G, then the steady state error decreases inversely. But the maximum gain is limited by the stability of system.

Another method is to include an Integral gain in the loop transfer function, then G(S) becomes as product of Integral Gain and Loop transfer function, that is, $G(S)=F(S)/S$

Then the steady-state error becomes,

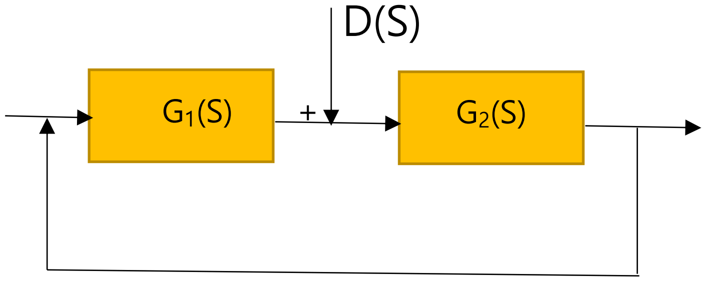
$e(\infty)=\lim 1/[1+G(S)]$, as $S \to 0$

$=\lim 1/[1+F(S)/S]=\lim S/[S+F(S)]$, as $S \to 0$

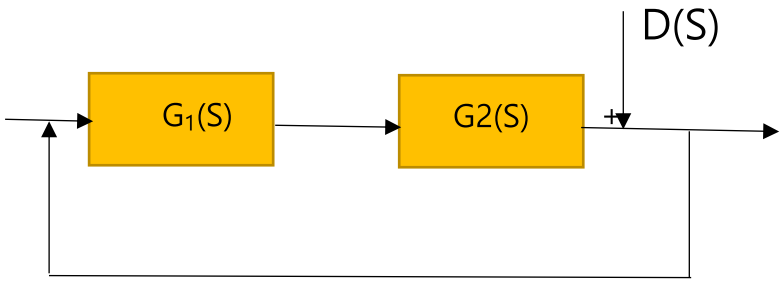
$=0$

Thus the steady-state error becomes zero after the Integral gain is introduced.

(5) Control scheme for noise reduction



Case a: Noise input D before G_2



Case b : Noise input D after G_2

When error or disturbance, D(S), is introduced to the control scheme as shown in fig, the contribution X(S) of the disturbance to the output would be

$X(S)=G_2(S)D(S)/[1+G_1(S)G_2(S)]$ for Case a

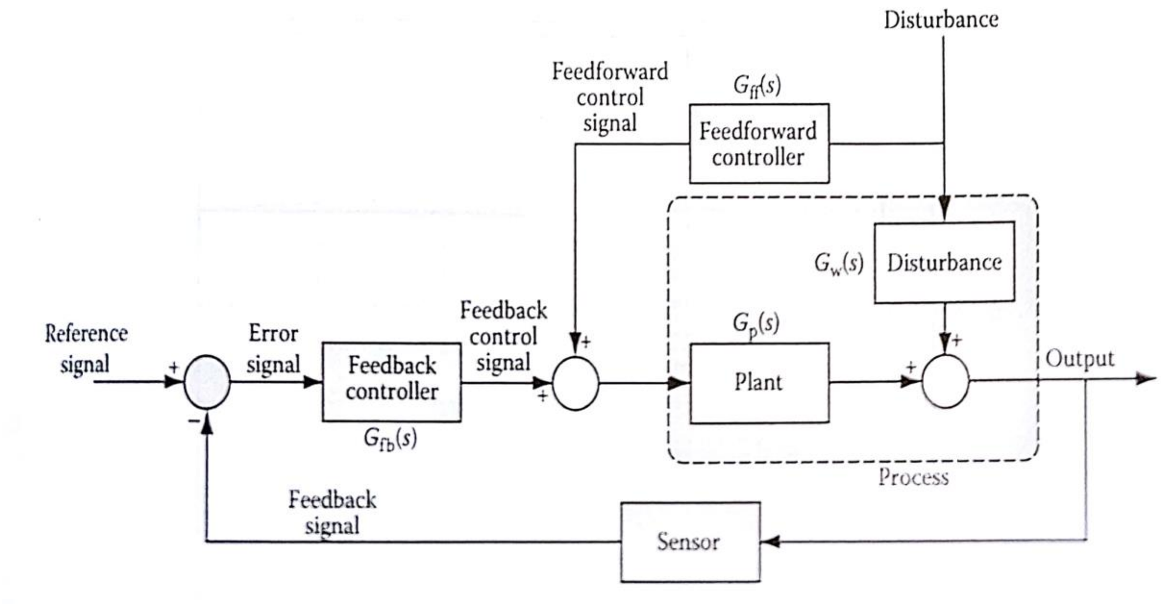
$X(S)=D(S)/[1+G_1(S)G_2(S)]$ for Case b

Thus Case b gives smaller contribution to the output, which is desirable.

Therefore, it is better to increase the gain of transfer function located before the disturbance is input.

(6) Feedforward control scheme for particular application

Feedforward control scheme can be introduced together with the feedback scheme for better performance in the particular application as shown in the fig. Generally, it may give some performance improvement in very limited application, but care should be taken.



Feedforward-feedback combined control scheme

(Source: Mekid's Introduction to Precision Machine Design)

(7) Requirement for sensors

For precision motion of machines, the sensor's resolution is equal to or higher than the control precision. Practically, at least three times higher resolution of sensors are required for practical application.

The response speed of the sensor is also important, and the sensor should be chosen such that 3dB frequency of the sensor may be equal to or higher than the fastest control frequency of the system, because the sensor gives about 70.7% signal for the measured data at the 3dB frequency.