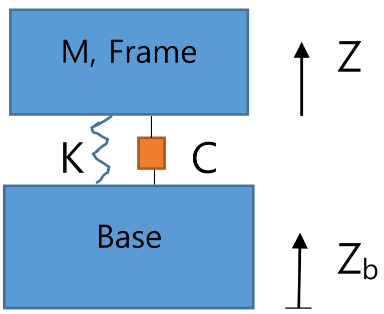
Precision Machine Design-Vibration Isolation

Vibration affects the precision machine seriously.

The capability of being isolated from the disturbing vibration is very important to the precision machine design.

Let a machine body built on machine bed or base.



Ground

Then machine can be modeled as the one DOF spring mass system, where M is the mass of machine frame, K, C are the stiffness and damper of the machine frame built on the machine base that is typically or Granite or Iron.

When the ground is disturbed by external vibration, the ground will experience the movement, Z_b. This ground movement will be transferred to the machine frame, giving the movement of Z, and $\Delta Z = Z - Z_b$ is the strain of interest, or the relative motion between the base and machine frame, that is greatly affecting the machine performance. Thus,

$Md^2Zdt^2=-K(Z-Z_b)-Cd(Z-Z_b)/dt=-K\Delta Z-Cd\Delta Z/dt$

Let $Z=Z\exp(j\omega t)$, and $\Delta Z=\Delta Z\exp(j\omega t)$, and ω=the excited frequency of vibration, the above equation becomes,

$-M\omega^2(Z_b+\Delta Z)=-K\Delta Z-j\omega C\Delta Z$ eq(1)

Thus $\Delta Z/Z_b$, which is the ratio of the strain of interest to the excited vibration, is

$\Delta Z/Z_b=M\omega^2/[K-M\omega^2+j\omega C]$

$= \omega^2/[\omega_n^2-\omega^2+j2\zeta\omega\omega_n]$

where $\omega_n = [K/M]^{1/2}$

=Natural frequency of machine frame stiffness

$2\zeta\omega\omega_n=C/M$ and ζ=damping factor

Thus $|\Delta Z/Z_b|= \omega^2/[(\omega_n^2-\omega^2)^2+(2\zeta\omega\omega_n)^2]^{1/2}$

$=1/[(\alpha^2-1)^2+(2\zeta\alpha)^2]^{1/2}$ eq(2)

where $\alpha=\omega_n/\omega$=ratio of natural freq. to external vibration freq.

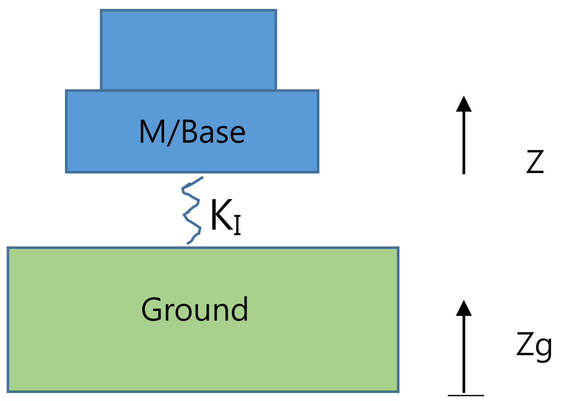
Therefore, from eq(1), in order to ΔZ to be very little or zero, two strategies are possible:

(1) $\alpha=\omega_n/\omega \gg 1$, that is $\omega_n \gg \omega$. Thus very higher natural freq, that is, very higher stiffness of machine frame is required. This is the very reason why the high stiffness structure is desired for the machine frame against the external vibration.

(2) Z_b is zero or very small. The movement of base is to be zero or very small in spite of ground movement by vibration disturbing. This is the vibration isolation of machine base from the ground movement. Detail technology will be explained.

<u>Vibration Isolation</u>:

To add a soft spring of isolation, K_I , between the machine base and ground, then the vibration transmitted to the base from the ground would be greatly reduced



Let Z, Zg be the motion of machine and ground, respectively.

M is the total mass of machine (base+frame).

Motion of equation:

$Md^2Z/dt^2=-K_I(Z-Zg)$

Let $Z=Z\exp(j\omega t)$, where ω is the excited freq. of vibration.

Then the equation becomes,

$(K_I-M\omega^2)Z=K_IZg$

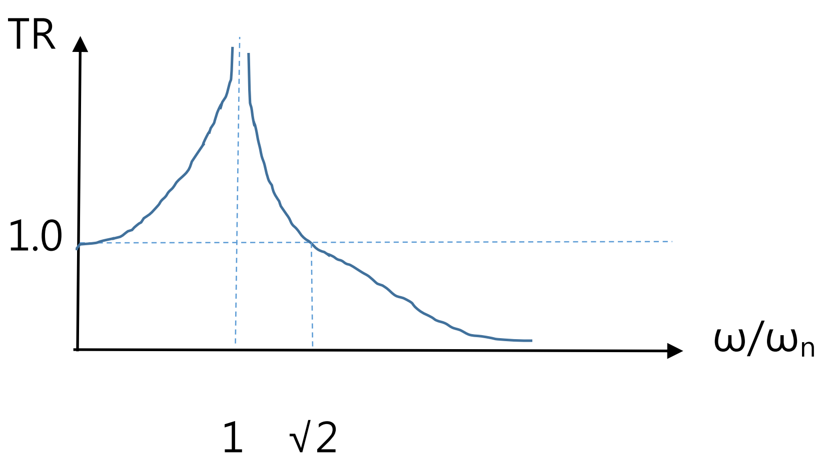
Thus the Z/Zg, ratio of base motion to the ground motion, is

$Z/Zg=K_I/[K_I-M\omega^2]=\omega_n^2/[\omega_n^2-\omega^2]$;

Thus Transmissibility,TR $=|Z/Zg|$

$TR = \omega_n^2/(\omega^2-\omega_n^2)$ if $\omega>\omega_n$ eq(1)

where $\omega_n=[K_I/M]^{1/2}$=Natural freq. of the Isolation.



Decibel (dB) is used for the transmissibility, that is defined as,

$dB \equiv 20 Log_{10} TR$ eq(2)

$TR=10^{-2}$ (or 1%), it is -40dB

$TR=10^{-1.5}$ (or 3.16%), it is -30dB

$TR=10^{-1}$ (or 10%), it is -20dB

$TR=10^{-15/20}$ (or 17.8%), it is -15dB

$TR=10^{-1/2}$ (or 31.6%), it is -10dB

$TR=10^{-1/4}$ (or 56.2%), it is -5dB

Observation from TR curve

1) $\omega \le \sqrt{2}\omega_n$ then $TR \ge 1$ ∴ Vibration magnified

2) $\omega = \omega_n$ then $TR \to \infty$ at resonance

∴ To be avoided or damper is needed

3) $\omega > \sqrt{2}\omega_n$ then $TR < 1$ ∴Vibration reduced or isolated

4) $\omega \gg \omega_n$ then $TR \propto 1/\omega^2$ ∴ Efficient Vibration Isolation

This observation leads to followings;

-Vibration Isolation(or VI) system can be worked

-The smaller ω_n, the higher Vibration Isolation

-Very soft spring(of small K_I) is to be used to give the lower ω_n

Practically, ω_n can be lowered down to around 1-2Hz, with appropriate soft springs, whose stiffness becomes, $K_I = M\omega_n^2$

Design Procedure for VI

1. Identification of frequency, ω, to be isolated
2. Choose optimum TR(Transmissibility)
3. Calculate ω_n such that $\omega_n=\omega[TR/(1+TR)]^{1/2}$
4. Determine the isolation spring, such that $K_I=M\omega_n^2$

Ex1) Design example

Vibration measurement shows 10Hz vibration is dominant on the factory floor. We wish to make -10 dB transmissibility.

Mass=10 ton of machine

$TR=-10dB=0.316$, and

$\omega_n=\omega\{TR/(1+TR)\}^{1/2}$

$=(2\pi)(10)(0.490)=30.8[rad/sec] \fallingdotseq 4.9Hz$

Thus $K_I=M\omega_n^2=(10,000)(30.8)^2=9470$ [KN/m]

If we use 4 legs for supporting, then each leg would be allocated to 9470/4=2367[KN/m] of spring, respectively. Because 2500KN/m spring is commercially available, we can choose each leg's spring constant as 2,500N/m, giving K_I as 10,000 KN/m

Thus newly changed natural frequency, ω_n

$\omega_n=[K_I/M]^{1/2}=[10,000,000/10,000]^{1/2}=31.6[rad/sec]=5[Hz]$

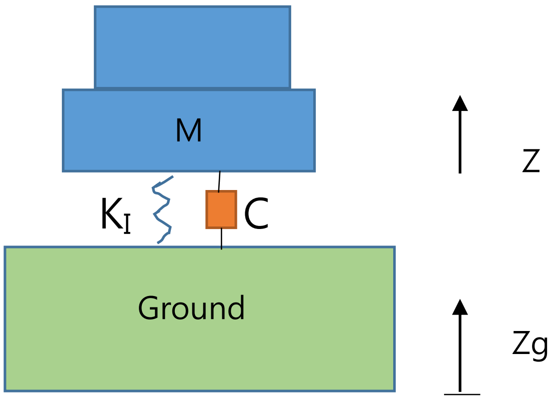
The changed TR would be $5^2/(10^2-5^2)=0.333=-9.54$ dB

(instead of -10dB design target)

<u>The Influence of Damping on VI performance</u>

When there is no damping in the system, the transmitted vibration will never stop, or never decay, and requiring very long time of settling.

A damper, C, can be added to the VI system as in fig.



Motion of equation:

$Md^2Z/dt^2=-K_I(Z-Zg)-C(dZ/dt-dZg/dt)$

Thus

$Md^2Z/dt^2+CdZ/dt+K_IZ=CdZg/dt+K_IZg$

Let $Z=Z\exp(j\omega t)$, $Zg=Zg\exp(j\omega t)$

where ω is the excited freq. of vibration.

Then the equation becomes,

$(K_I-M\omega^2+j\omega C)Z=(K_I+j\omega C)Zg$

Thus the Z/Zg, ratio of base motion to the ground motion, is

$Z/Zg=(K_I+j\omega C)/(K_I-M\omega^2+j\omega C)$

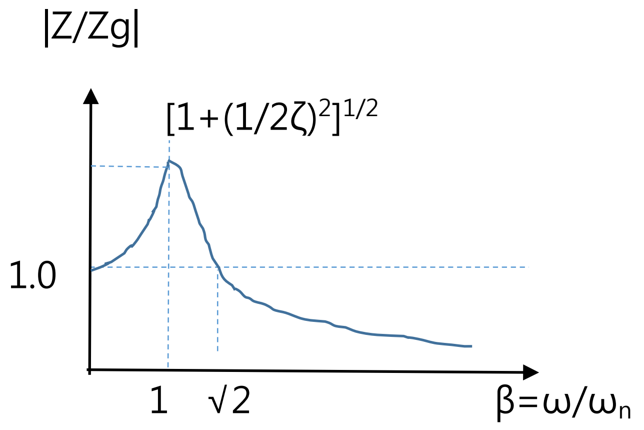
$=(\omega_n^2+j2\zeta\omega\omega_n)/(\omega_n^2-\omega^2+j2\zeta\omega\omega_n)$, where $2\zeta=C/M$

$=(1+j2\zeta\beta)/(1-\beta^2+j2\zeta\beta)$, where $\beta=\omega/\omega_n$

Thus Transmissibility,TR $=|Z/Zg|$

$TR=[1+(2\zeta\beta)^2]^{1/2}/[(\beta^2-1)^2+(2\zeta\beta)^2]^{1/2}$

Graph of TR vs ω/ω_n



The observations are as follows;

(1) For $\omega=\omega_n$; The TR at the resonance is limed, and the maximum value = $[1+(1/2\zeta)^2]^{1/2}$

(2) For $\omega=\sqrt{2}\omega_n$; The TR =1 , same as the case without damper

(3) For $\omega>\sqrt{2}\omega_n$; TR $\propto$ $(1/\beta^2+2\zeta/\beta)$ as ω increase;

thus more slowly decreases than the case without damper. Larger than no damper case by $2\zeta/\beta$

Therefore, the damper generally hurts the VI performance, although it gives limited TR at resonance, giving less settling time. Thus trade-off exists between the VI performance and settling time.

Ex2) A light damper is added to the VI of example1. The logarithmetic decrement, δ, for free vibration is observed as

$\delta=Ln(X_1/X_2)=Ln(1/0.5)=0.3567$

$\zeta=\delta/2\pi=0.110$; Lightly damped system such as air damper

At resonance $TR=[1+(1/2\zeta)^2]^{1/2}=4.654$

At 10Hz excitation freq, then $\omega/\omega_n=2$,

$TR=[1+\{2(0.110)(2)\}^2]^{1/2}/[(4-1)^2+\{2(0.110)(2)\}^2]^{1/2}$

$=0.360$

∴TR@damper=0.360, TR@no-damper=0.333

∴TR increase by about 0.03, when compared to no-damper case

Ex3) For heavy damper case $\zeta=0.5$;

At resonance $TR=[1+(1/2\zeta)^2]^{1/2}=1.414$

At 10Hz excitation freq, then $\omega/\omega_n=2$,

$TR=[1+\{2(0.5)(2)\}^2]^{1/2}/[(4-1)^2+\{2(0.5)(2)\}^2]^{1/2}$

$=\sqrt{5}/\sqrt{13}=0.620$

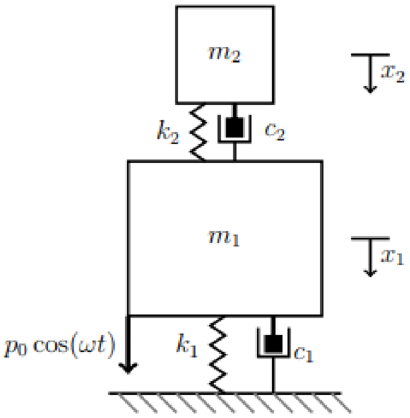
∴TR@damper=0.620, TR@no-damper=0.333

∴TR increase by about 0.3, when compared to no-damper case

∴ The heavier damper, the more seriously hurts to the TR.

➔ Thus light damper is desirable.

Therefore the directly connected heavy damper of heavy is better to be avoided. Instead, indirect damper, such as tuned mass damper, can be desirable for the heavy damping of the VI system. The tuned mass damper is to lower the amplitude of the first oscillator, by transferring it to the second oscillator as in the fig. A lightly damped damper that connected directly, such as air spring, can be used with relatively acceptable increase in TR.

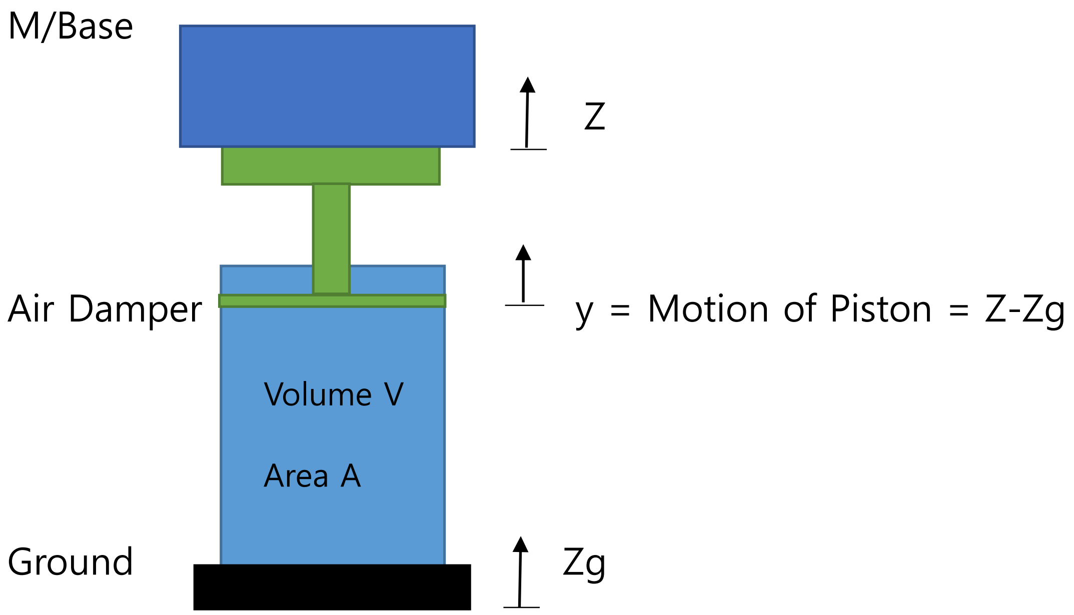


Tuned mass damped system (source:http://www14.informatik.tu-muenchen.de/konferenzen/Jass06/courses/4/Stroscher/Stroscher.ppt.)

Air spring

Air shows quite low viscosity characteristics, thus can be used for a good damper with low or moderate damping factor, as shown in the design example above.

Air damper or air cylinder: piston area(A), Volume(V)



Motion of equation

$Md^2Zdt^2=-C(Z-Zg)/dt-(Ps-P)A$ eq(40)

where M=Mass of machine to be supported, C=damping coefficient, Ps=Initial supply pressure of air, P=current air pressure at Y motion of cylinder.

Assuming $C \fallingdotseq 0$ as the air cylinder is lightly damped,

$Md^2Z/dt^2+(Ps-P)A=0$ eq(41)

From the ideal gas equation $P\upsilon^m=const$;

where υ=specific volume [m^3/kg], considering no mass change,

$PV^m=PsVs^m$ eq(42)

where m=1.4 for adiabatic process (usually under high freq.)

m=1 for isothermal process (usually under low freq.)

Remembering the current volume, $V=Vs+yA$

where Vs=Initial volume of cylinder, A=area of cylinder, y=current displacement of cylinder=Z-Zg

Thus $P=Ps(Vs/V)^m=Ps[Vs/(Vs+yA)]^m=Ps[1+yA/Vs]^{-m}$

And $Ps-P=Ps[1-(1+yA/Vs)^{-m}]$ eq(43)

As y is small motion from zero

$f(y)=1-(1+yA/Vs)^{-m} \fallingdotseq f(0)+y\partial f/\partial y|_{y=o}$

$f(0)=0$; $\partial f/\partial y|_{y=o}=(m)(1+yA/Vs)^{-m-1}(A/Vs)|_{y=o}=mA/Vs$

Thus $f(y)=ymA/Vs$

Eq(43) becomes $Ps-P=ymAPs/Vs=(Z-Zg)mAPs/Vs$

Thus eq(41) becomes;

$Md^2Z/dt^2+ZmA^2Ps/Vs=ZgmA^2Ps/Vs$

Thus $Md^2Z/dt^2+KZ=KZg$,

where $K=mA^2Ps/Vs$=Spring constant of air cylinder

Let $Z=Z\exp(j\omega t)$

$(-\omega^2M+K)Z=KZg$

Thus $Z/Zg=K/[K-\omega^2M]=\omega_n^2/(\omega_n^2-\omega^2)$

TR(transmissibility)=$|Z/Zg|= \omega_n^2/(\omega^2-\omega_n^2)$ for $\omega>\omega_n$

where ω_n=Natural frequency of air damper VI

$=[K/M]^{1/2} = [mA^2Ps/Vs/M]^{1/2}$

The advantage of air spring is that **the stiffness can be easily adjusted by changing the air pressure supplied**, while other mechanical springs should be rebuilt for the stiffness adjustment.

There are commercially available pneumatic spring or pneumatic isolators, performing 10dB-15dB reduction in transmissibility, typically.

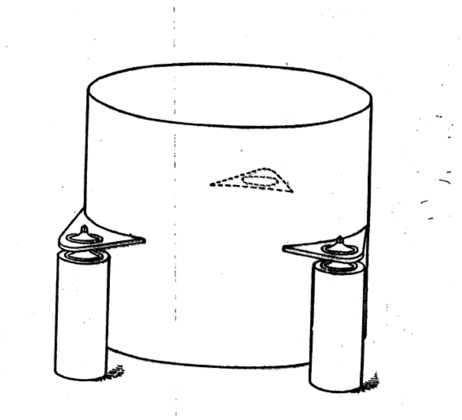
Active Vibration Isolation

The passive type vibration isolation shows quite good TR reduction in many cases, but the residual vibrations can be concerned especially around the natural frequency or some other frequency region. Active vibration isolation can be additionally used in order to remove the residual vibration, the vibration cancelling movement can be performed by the active air supply, electro-magnetic actuator, piezo electric actuator, via the feedback or feedforward control schemes. Active vibration isolation systems are also commercially available, but requiring high cost.

The followings are practical guidelines for VI system design;

(1) Minimize the mass moment of inertial of mass ($J=\int r^2 dm$) to be supported, in order to minimize the inertia of rotation. Because large inertia of rotation will give larger angular momentum, thus it needs more force and energy to be settled into equilibrium. It is a good practice to locate heavier mass is better to be located more centrally.

(2) Locate the centre of supporting plane to be coincided or close to the centre of mass to be supported, in order to reduce the rotatory motion due to the momentum generated by the distance between the two centres. Not only in horizontal direction, but also in vertical direction.



Centre of mass located on the supporting plane

(source: Debra's Tutorial on Vibration Isolation)

(3) Locate the supporting points at or close to radius of gyration(r_G)of mass to be supported, in order to minimize the moment induced by the dynamic inertia force. This is because the concentrated mass is assumed to be located at radius of gyration from the axis of rotation during the rotary motion. Points of support which are located with offsets from the r_G location can cause the coupled moment, which can

generate bending moment to the machine structure, which is not desirable to the structure of precision

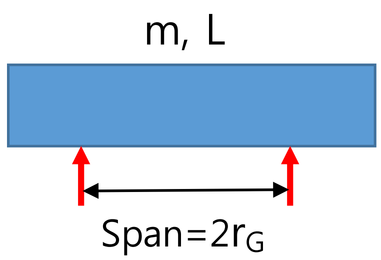
The following fig shows one dimensional case, but it can be expanded to multi-dimensional cases.

Ex) For a rod of length L, mass m;

I=Mass moment of inertia about the center=$\int r^2 dm$

$=mL^2/12=mr_G^2$;

thus $r_G=L/2\sqrt{3}$, and span of support=$2r_G=L/\sqrt{3}=0.577L$



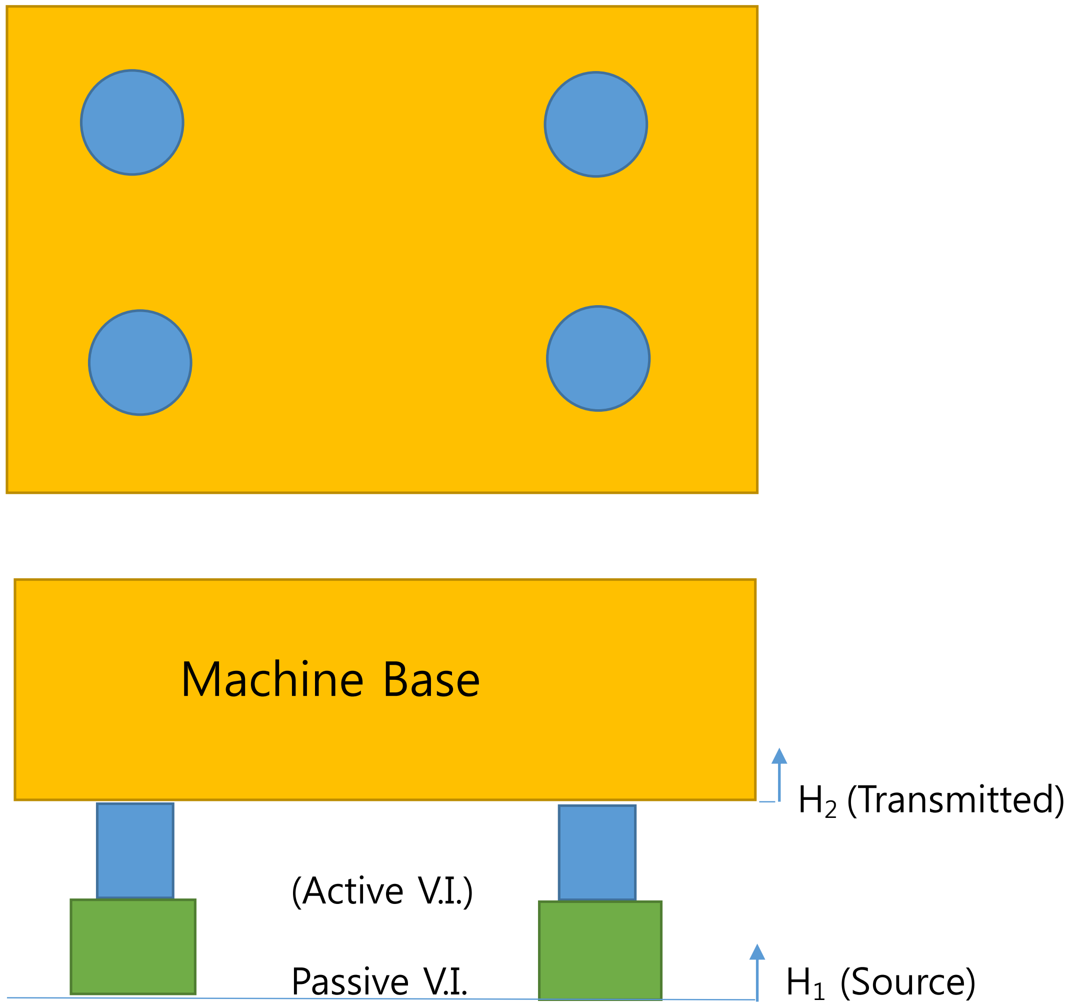
[*It is interesting to recognize that Airy points($L/\sqrt{3}=0.577L$), Golden ratio of Greek rectangle ($B=0.618A$), supporting or suspension points($L/\sqrt{3}=0.577L$) are all very close each other. It is noteworthy that they are all around **0.6**]

(4) Avoid heavy dampers that directly connected to the isolation system; instead use light damper, internal dampers such as tuned mass dampers, where applicable. This is because the direct damper can damage the VI performance in view of transmissibility.

Two types of practical vibration isolation

Passive Vibration Isolation: It is to isolate passively; it is to reduce the transmissibility via using passive elements such as pneumatic cylinders, rubber, dampers, etc. The performance of -20dB is a typical target of high performance commercial pneumatic isolators, although the practical performance is -10 to -15dB

Active Vibration Isolation: It is to isolate actively; it is to superpose the complimentary vibratory motion to the residual vibration, via adapting actuators such as electromagnetic actuators, piezo actuators, etc. This is to isolate vibration at low frequency range around less than 20Hz range, typically, giving -20dB as atypical target.



Floor (Vibrating)

Displacement: $u=A\sin(\omega t+\varphi)$

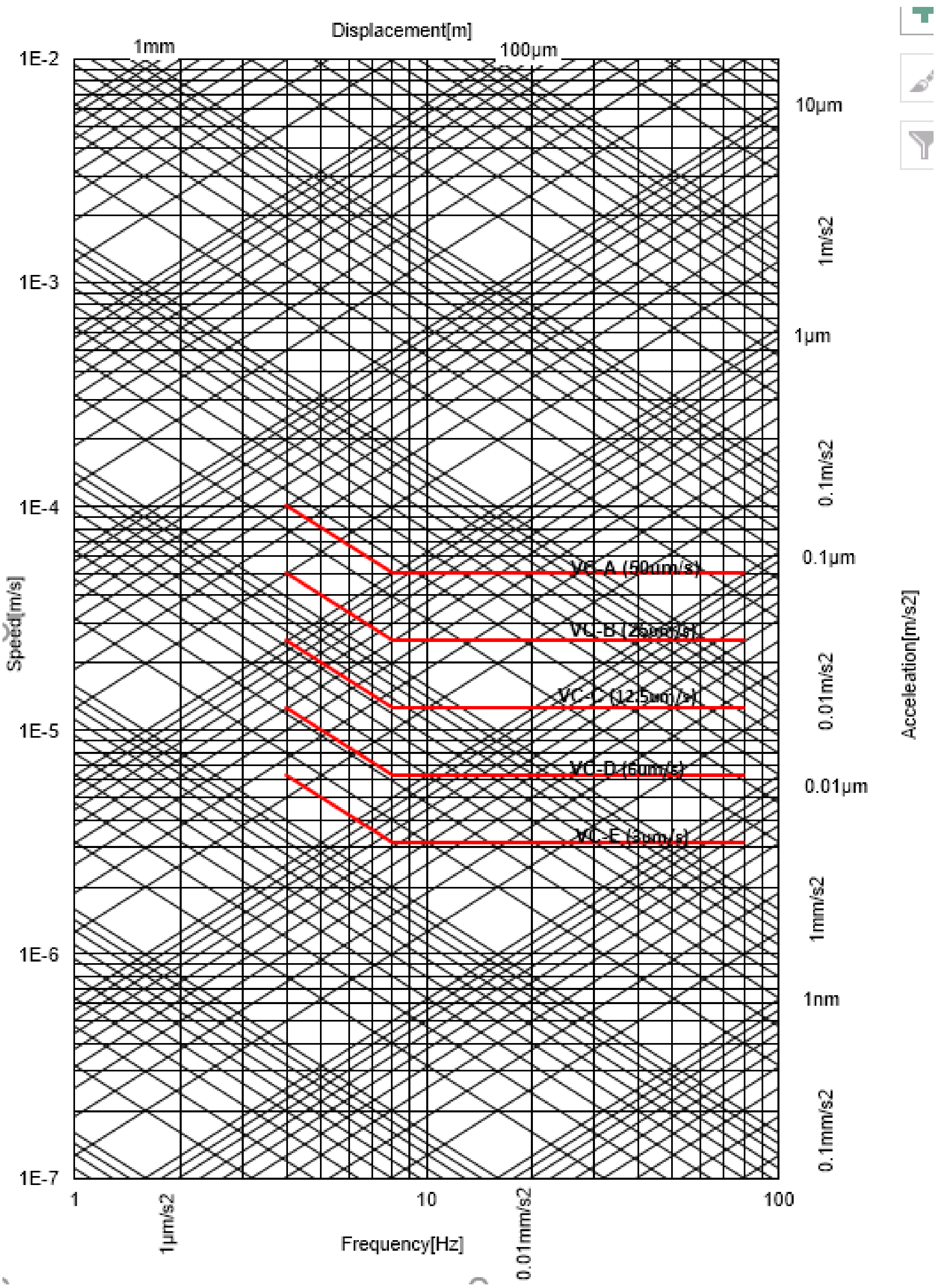
Velocity: $du/dt=\omega A\cos(\omega t+\varphi)$

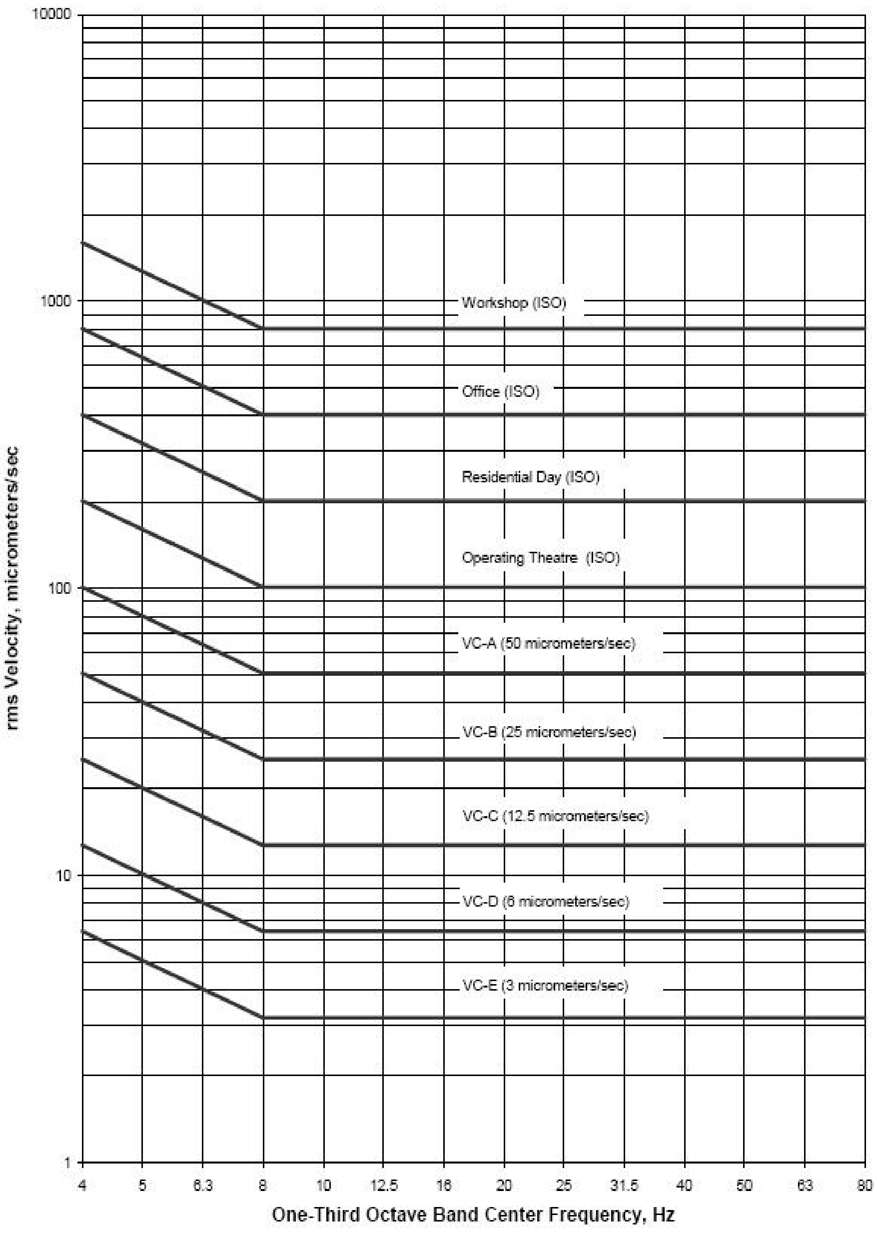
Acceleration: $d^2u/dt^2=-\omega^2A\sin(\omega t+\varphi)=-\omega^2u$

(Velocity Unit: $1\ \text{gal}=1\ \text{cm}/\text{s}^2$

$1\text{G}= 9.8\text{m}/\text{s}^2=980\ \text{cm}/\text{s}^2 \fallingdotseq 1000\ \text{gal}$)

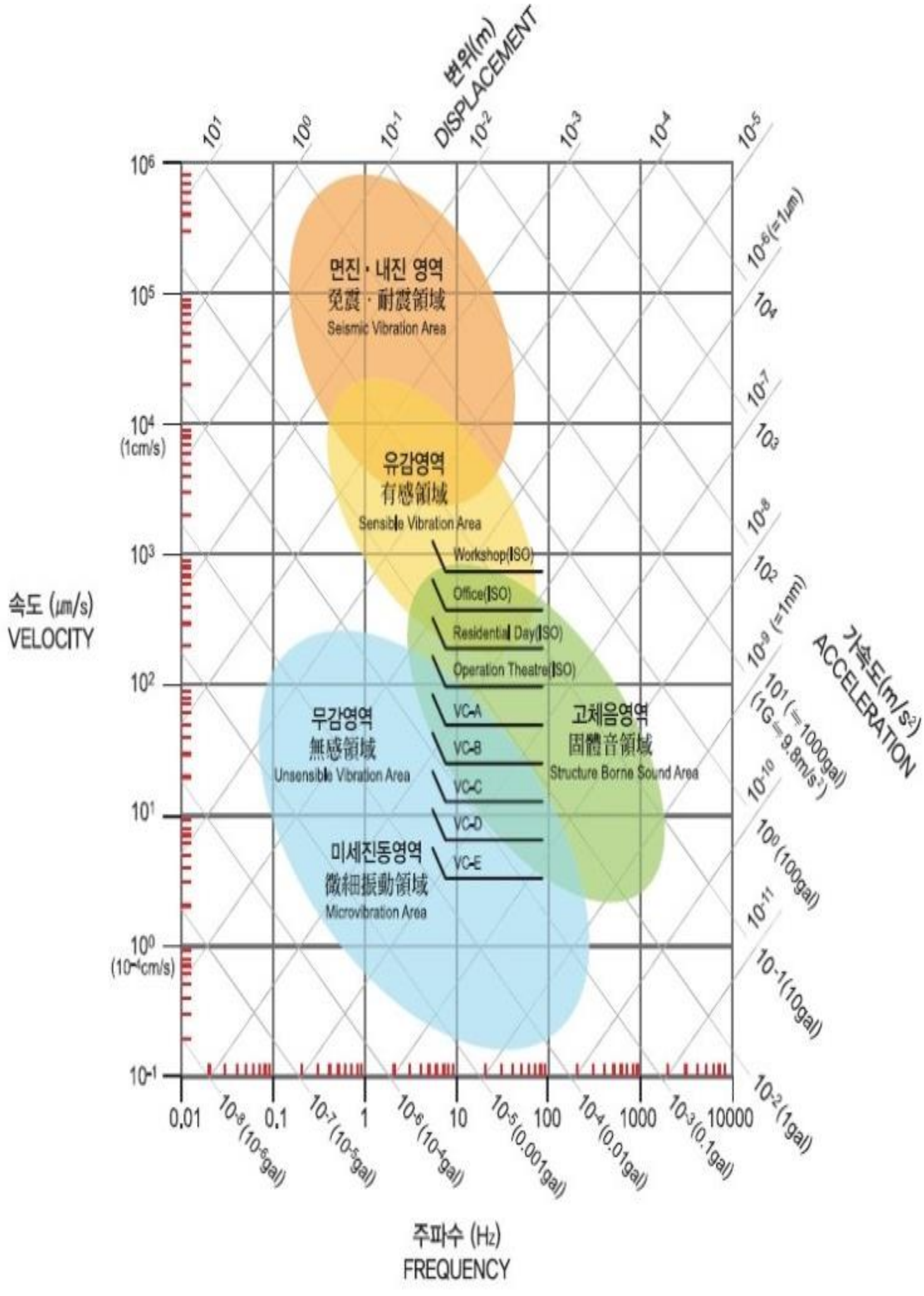
Vibration Criteria Chart, or VC-Chart





10000
1000
100
10
1
rms Velocity, micrometers/sec
Workshop (ISO)
Office (ISO)
Residential Day (ISO)
Operating Theatre (ISO)
VC-A (50 micrometers/sec)
VC-B (25 micrometers/sec)
VC-C (12.5 micrometers/sec)
VC-D (6 micrometers/sec)
VC-E (3 micrometers/sec)
4
5
6.3
8
10
12.5
16
20
25
31.5
40
50
63
80
One-Third Octave Band Center Frequency, Hz

Criterion Curve	Max Level micrmeters/sec ,rms (dB)	Detail Size, microns	Description of Use
Workshop (ISO)	800 (90)	N/A	Distinctly felt vibration. Appropriate to workshops and non-sensitive areas.
Office (ISO)	400 (84)	N/A	Felt vibration. Appropriate to offices and non-sensitive areas
Residential Day (ISO)	200 (74)	75	Barely felt vibration. Appropriate to sleep areas in most instances. Probably adequate for computer equipment, probe test equipment and lower-power (to 20X) microscopes.
Op. Theatre (ISO)	100 (72)	25	Vibration not felt. Suitable for sensitive sleep areas. Suitable in most instances for microscopes to 100X and for other equipment of low sensitivity.
VC-A	50 (66)	**8**	Adequate in most instances for optical microscopes to 400X, microbalances, optical balances, proximity and projection aligners, etc.
VC-B	25 (60)	**3**	An appropriate standard for optical microscopes to 1000X, inspection and lithography equipment (including steppers) to 3 micron line-widths.
VC-C	12.5 (54)	**1**	A good standard for most lithography and inspection equipment to 1 micron detail size.
VC-D	6 (48)	**0.3**	Suitable in most instances for the most demanding equipment including electron microscopes (TEMs and SEMs) and E-Beam systems, operation to the limits of their capacity.
VC-E	3 (42)	**0.1**	A difficult criterion to achieve in most instances. Assumed to be adequate for the most demanding of sensitive systems including long path, laser-based, small target systems and other systems.



변위(m)
DISPLACEMENT
먼진·내진 영역
免震·耐震領域
Seismic Vibration Area
유감영역
有感領域
Sensible Vibration Area
Workshop(ISO)
Office(ISO)
Residential Day(ISO)
Operation Theatre(ISO)
VC-A
VC-B
VC-C
VC-D
VC-E
무감영역
無感領域
Unsensible Vibration Area
미세진동영역
微細振動領域
Microvibration Area
고체음영역
固體音領域
Structure Borne Sound Area
속도 (㎛/s)
VELOCITY
가속도(m/s²)
ACCELERATION
주파수 (Hz)
FREQUENCY

일반 진동 규제치 • General Vibration Criteria, BBN-Criterion

Description / Class	Facility Equipment or Use	Vibration Criteria	
		4~8 [Hz] RMS Acceleration	8~80[Hz] RMS Velocity
일반적인 진동환경	일반사업장	4gal (변위16μm)	800μm/s
	사무실	2gal (변위 8μm)	400μm/s
	거주지 및 Computer System	1gal (변위 4μm)	200μm/s
	100× 현미경, 로봇수술실, Operation Room, 일반연구실 기타	0.5gal (변위 2μm)	100μm/s
정밀진동 Class : A	400×현미경, 측정실 Optical or Other Balance Optical Comparators, 전자장비, 생산설비 등 ※검사, Probe Test, 생산 지원설비 및 장치	0.25gal (변위 1μm)	50μm/s
정밀진동 Class : B	400× 이상 현미경, 精密안과, 신경계 수술실, 防振설비를 갖춘 광학장비, 반도체 생산설비 등 ※Aligner, Steppers등 3μm이상 선폭 노광장치	0.13gal (변위 0.5μm)	25μm/s
정밀진동 Class : C	30000× 전자현미경, Magnetic Resonance Imager, 반도체 생산설비 ※ Aligner, Steppers등 1μm 선폭 노광장치 → 1M DRAM정도	0.06gal (변위 0.25μm)	12μm/s
정밀진동 Class : D	30000×이상 전자현미경, Mass Spectrometer, 세포이식 장치, 반도체 생산설비 ※ Aligner, Steppers등 0.5μm선폭 노광장치 → 4M DRAM 정도	0.03gal (변위 0.12μm)	6μm/s
정밀진동 Class : E	Unisolated Laser and Optical Research System, 반도체 생산설비 ※ Aligner, Steppers등 0.25μm 선폭 노광장치 → 64M DRAM 정도	0.015gal (변위 0.06μm)	3μm/s

※ 일반적인 정밀장비의 정밀도, 분해능에 따른 바닥의 허용진동기준임.

Source:http://blog.naver.com/lee_jinhwan/50174644378