VIII. TURBULENCE IN MULTIPHASE FLOW

⊙ Review of the turbulence in single-phase flow.

"~ an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned."

Hinze (1975) "TURBULENCE".

$$u(t) = \bar{u} + u'$$

Reynolds decomposition.

Characteriss of turbulence.

→ non-linearity of N-S eq.

- disorganized, chaotic, seemingly random
  → statistic analysis (not the deterministic)

- non-repeatable.

- wide range of scales w/ physical significance.

- in general, high Re.

- enhanced diffusion and dissipation.

mass	controllability of the process ↓	mixing ↑
mtm	DR ↑	separation delay
heat	heat loss ↑	heat exchange ↑

- 3D, unsteady, rotational (potential flow can't be turb.)

⊙ Energy Cascade.

"turbulence can be described by large-scale structures that generate turbulent energy which is then cascaded or transferred to small-scale structures where the energy is converted to heat."

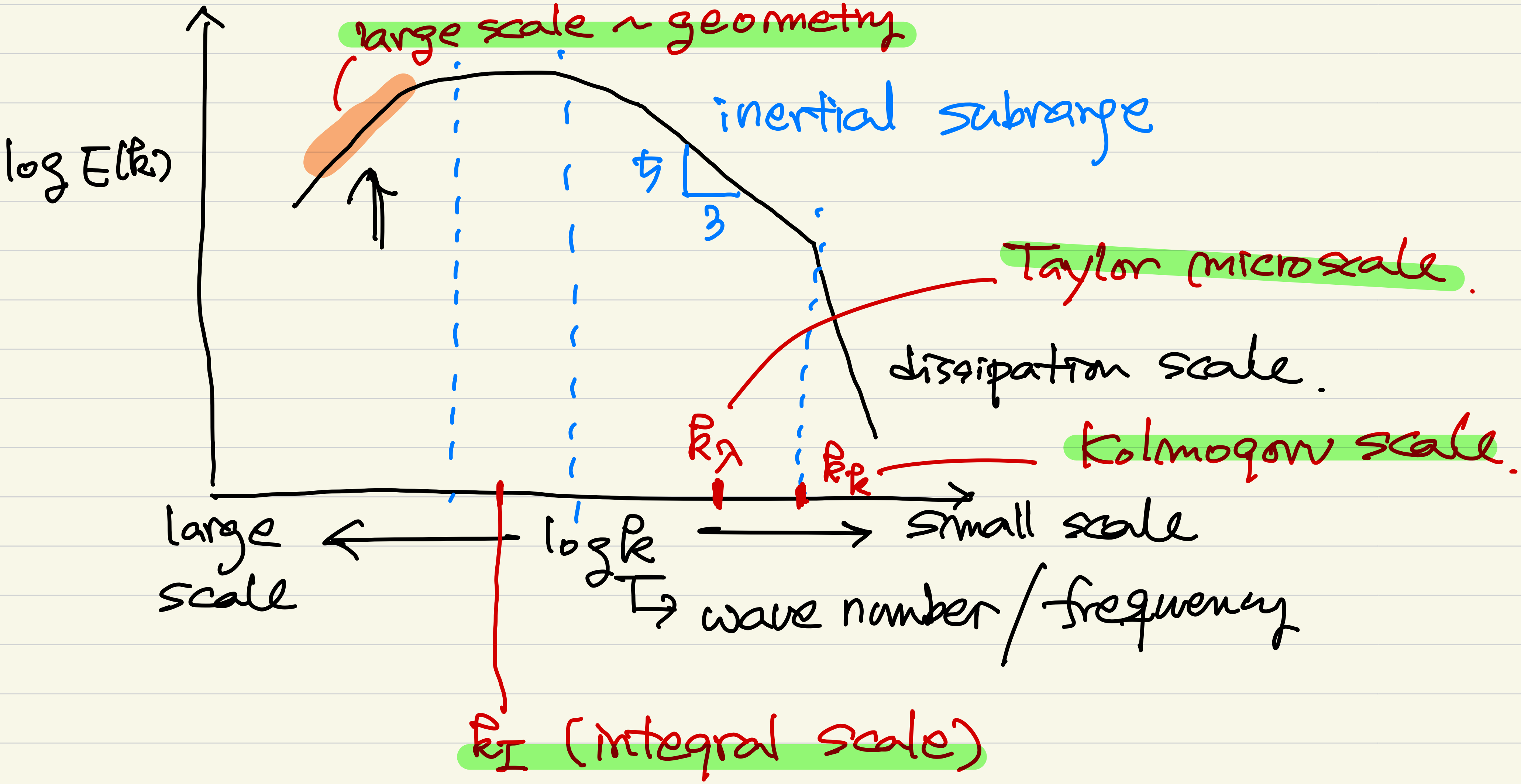
dissipation.

$$\tau = \mu \frac{d\bar{u}}{dy}$$

· "generation" is associated with gradients of mean properties that occur at large scales in the flow.

· "destruction" is associated with small

Scales and viscous properties of the flow.



- Scales in a turb. flow
  - large scale : represent the flow geometry
    (e.g. pipe diameter, plate length

chord length of the airfoil, ...)

- integral scale : largest size of energetic eddy

turb. motion to generate energy.

$$l \equiv \frac{1}{\|u'\|^2} \int_{-\infty}^{\infty} u'(x,t)\, u'(x+r,t)\, dr$$

norm.

auto-correlation fcn.

- Taylor microscale (intermediate scale)

: corresponding to (or within) the inertial subrange.

$$\lambda^2 \equiv \frac{|u'|^2}{\|S\|^2}$$

$e_{ij} = \frac{\partial u_i}{\partial x_j}$ (deformation rate)

shear rate tensor

$$\underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{R_{ij}} + \boxed{\underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{S_{ij}}}$$

Rotational-rate tensor

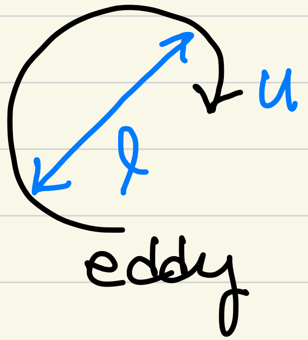
· Kolmogorov's Hypotheses (Kolmogorov, 1941)

- the small-scale turb. quantities are statistically isotropic for high-Re flows (indep. of direction)
- Small scale motions are universal, implying that such scales can be determined by the fluid viscosity (ν) and flow dissipation (ε)

→ smallest scales in a single-phase turb. flow "Kolmogorov scales".

$$\left(\begin{array}{lll} \text{length scale} & : & \eta = (\nu^3/\varepsilon)^{1/4} \\ \text{time} \quad '' & : & \tau = (\nu/\varepsilon)^{1/2} \\ \text{velocity} \quad '' & : & v = (\nu\varepsilon)^{1/4} \end{array}\right)$$

※. dissipation rate (ε) : function of length, velocity scales of large-scale turbulence.



$$\varepsilon \sim \frac{u^2}{\underbrace{l/u}_{\text{time-scale}}} = \frac{\text{kinetic energy of large eddy}}{\text{eddy turnover time scale}}$$

$$= \frac{u^3}{l}.$$

can be estimated from large-scale dynamics (no information of viscosity or small scales required!)

- Modeling the single-phase turbulence.

$$\left\{\begin{aligned} &\frac{\partial u_j}{\partial x_j} = 0 \\ &\frac{\partial u_j}{\partial t} + u_i \frac{\partial u_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \nu \frac{\partial^2 u_j}{\partial x_i^2} \end{aligned}\right.$$

instantaneous

time-average

$$\leftarrow \left\{\begin{aligned} u_j(t) &= \bar{u}_j + u_j' \\ p(t) &= \bar{p} + p' \end{aligned}\right.$$

(Reynolds decomposition)

fluctuating.

↓ time-average.

$$\Rightarrow \left\{\begin{aligned} &\frac{\partial \bar{u}_j}{\partial x_j} = 0 \\ &\frac{\partial \bar{u}_j}{\partial t} + \bar{u}_i \frac{\partial \bar{u}_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_j} + \frac{\partial}{\partial x_i}\left(\nu\frac{\partial \bar{u}_j}{\partial x_i} - \overline{u_i' u_j'}\right) \end{aligned}\right.$$

viscous stress

$$\Downarrow$$

additional transport equations for the turbulence quantities are needed to

$$\tau_{ij} = -\rho\,\overline{u_i' u_j'}$$: turbulent stress (Reynolds stress)

close the system of equation.

→ modeling.

- Practical methods for modeling turbulence in engineering applications involve averaging techniques.

  → Reynolds stress.

- the objective of turbulence modeling is to accurately predict the Reynolds stress for a variety of flows.

- Boussinesq Hypothesis

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} \rightarrow -\rho \overline{u_i u_j} = \mu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$$

eddy viscosity