458.308 Process Control & Design

Lecture 9: Control System Design Based on Frequency Response Analysis

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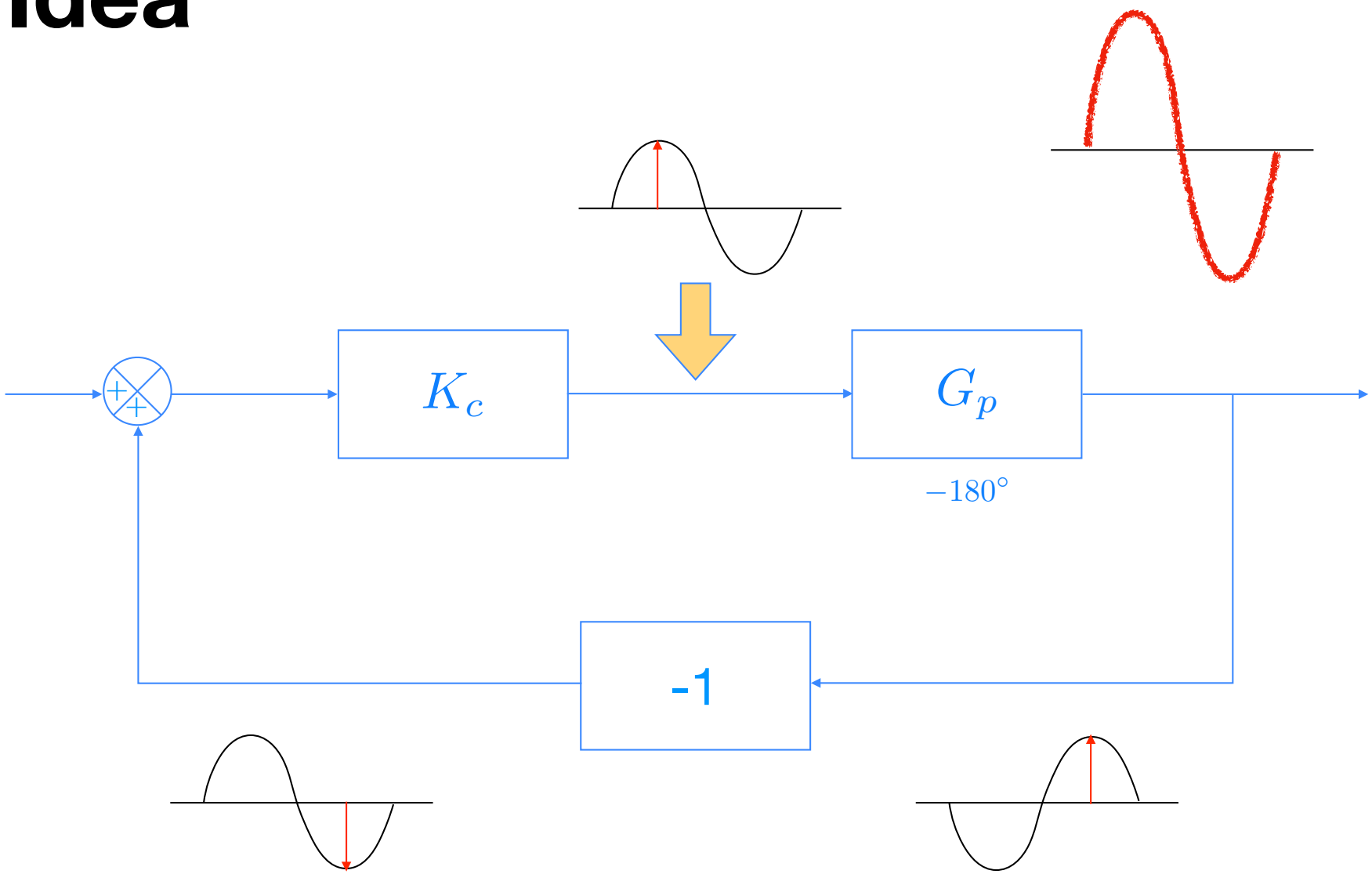
Idea



Signals with the frequency having phase shift of -180° can only be stabilized with AR of $G_{OL} = K_c \cdot G_p(j\omega) < 1$

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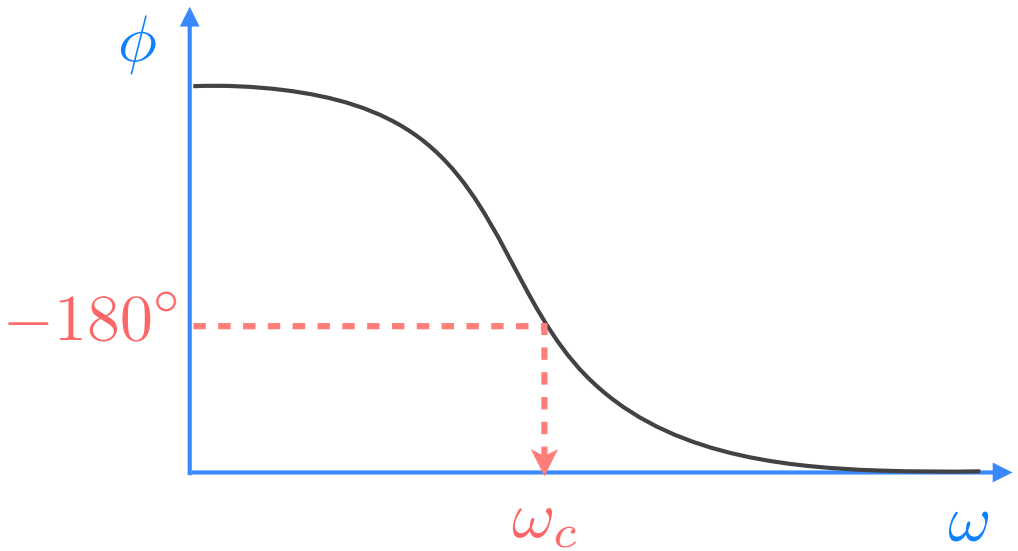
Bode Stability Criterion

1 Find the critical frequency, ω_c, by solving

$$\phi(G_{OL}(j\omega_c)) = -180^\circ \text{ or } -\pi$$

or by reading off ω_c from Bode Diagram of G_{OL}



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Bode Stability Criterion

2 Assumptions: No positive poles in G_{OL}

ϕ crosses -180° only once

Then, the system will be closed-loop stable if and only if

$$\boxed{AR(G_{OL}(j\omega_c)) < 1}$$

3 Advantage of Bode Stability Criterion

- Can quantify stability margin
- Not limited to polynomials (Routh stability)

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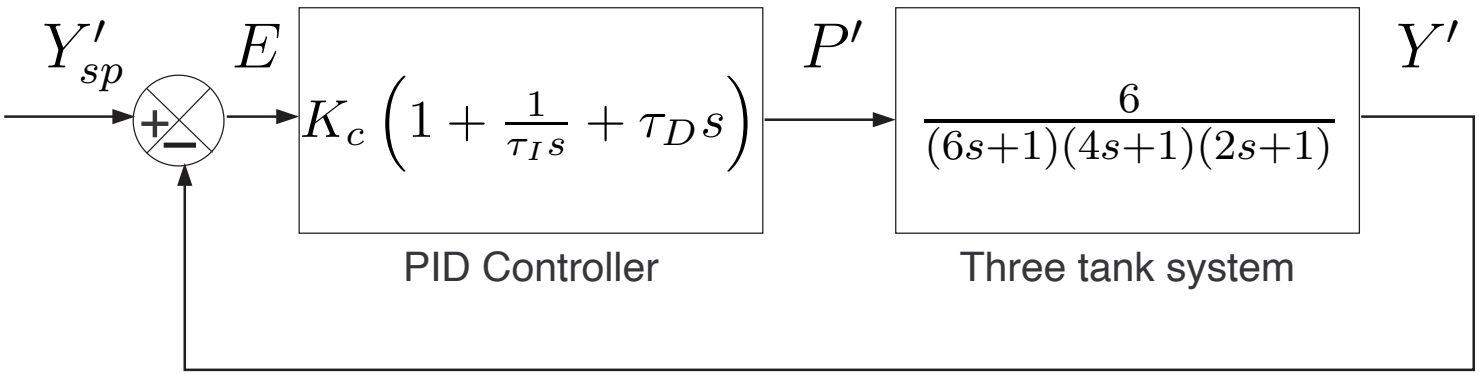
Example



$$G_{OL} = \frac{6}{(6s+1)(4s+1)(2s+1)} \frac{K_c(1+\tau_I s+\tau_I \tau_D s^2)}{\tau_I s}$$

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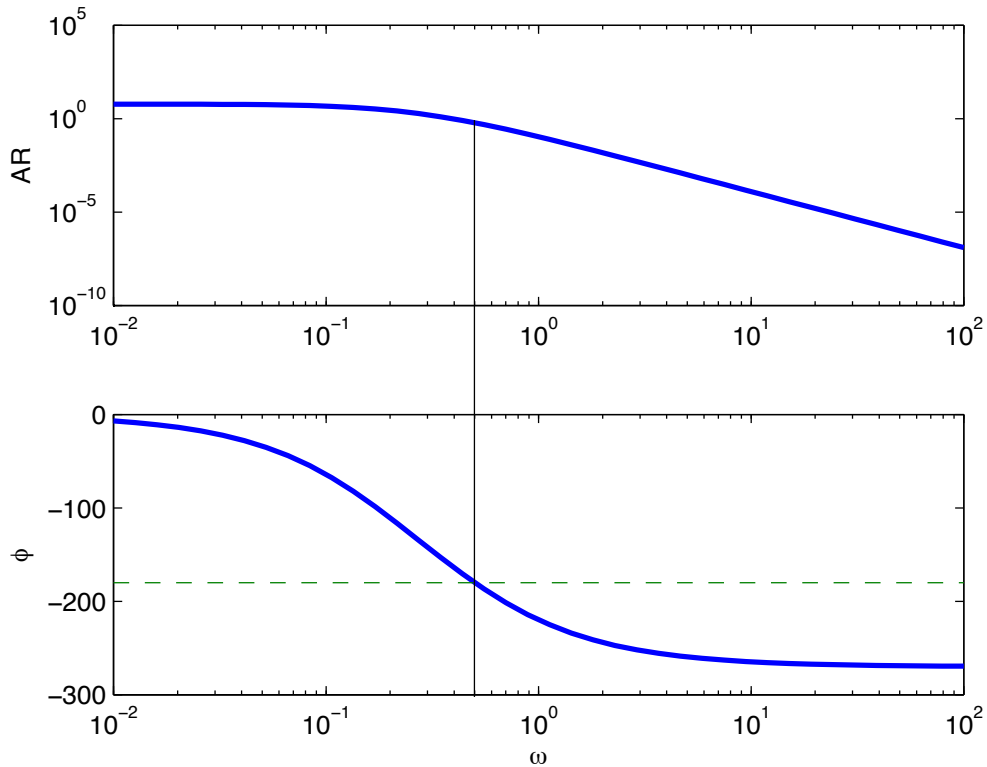
P-Only Control

Bode diagram of G_{OL} with $K_c = 1$



$\omega_c = 0.5,\ AR(G_{OL})$ at $\omega_c = 0.6$ Closed loop is stable with $K_c = 1$

Closed loop is stable for $K_c < 1/0.6 (= 1.67)$

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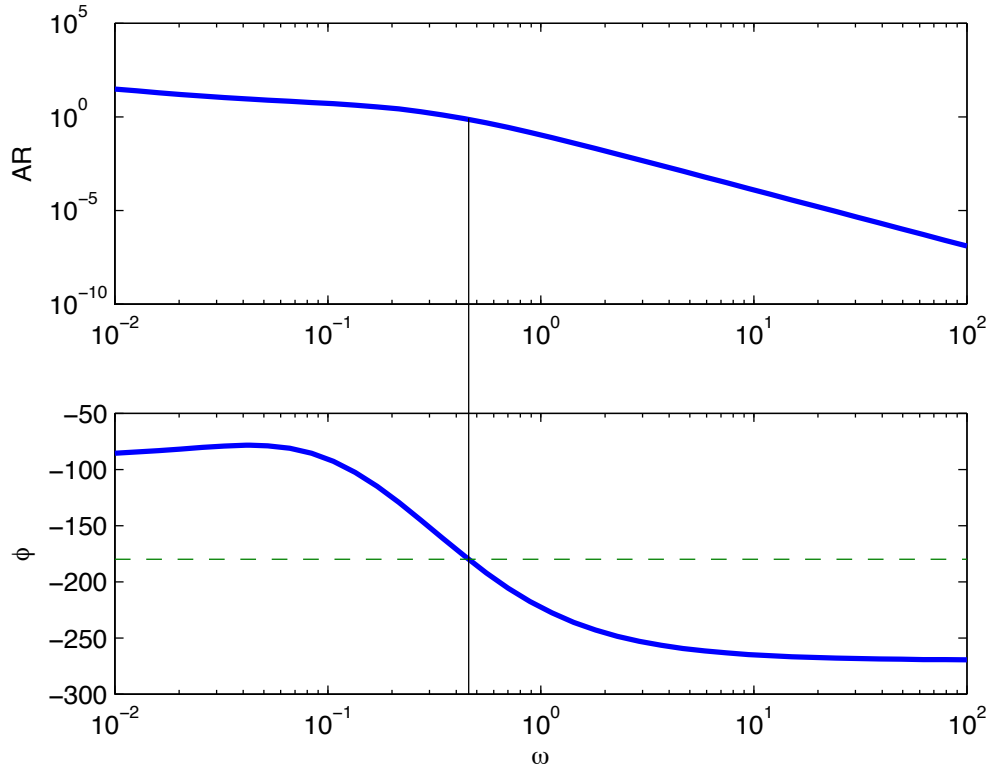
PI Control

Bode diagram of G_{OL} with $K_c = 1,\ 1/\tau_I = 0.05,\ \tau_D = 0$



$\omega_c = 0.457,\ AR(G_{OL})$ at $\omega_c = 0.732$ Closed loop is stable with $K_c = 1$

Closed loop is stable for $K_c < 1/0.732 (= 1.37)$

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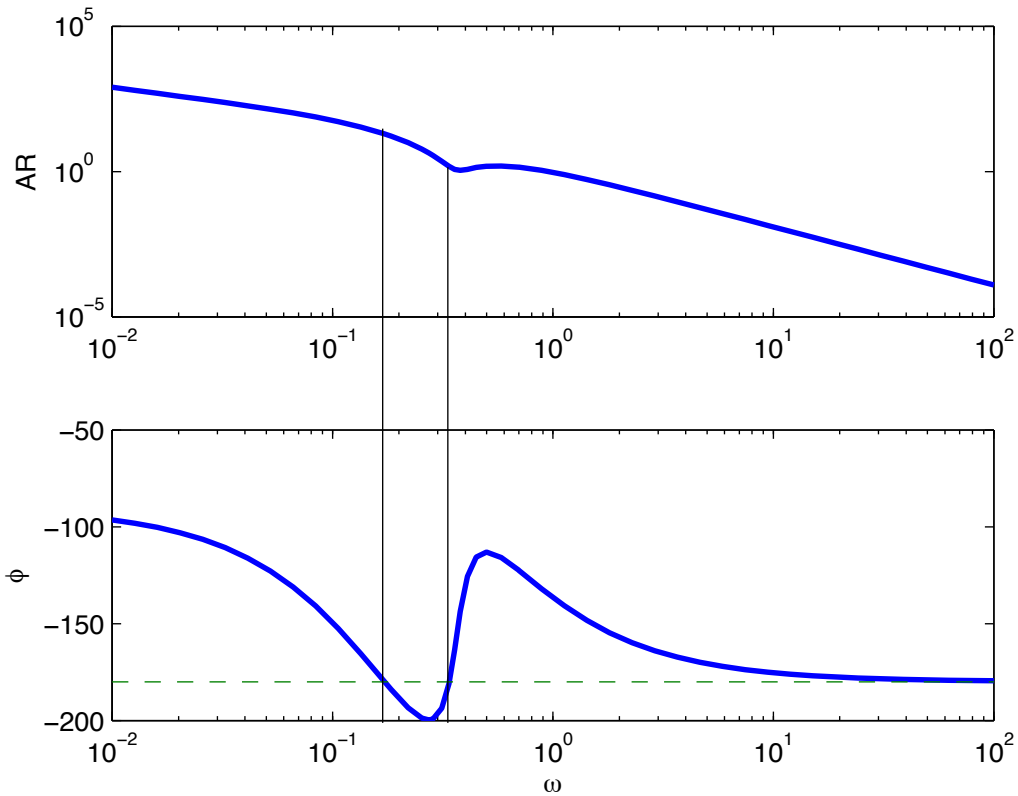
PID Control

Bode diagram of G_{OL} with $K_c = 1,\ 1/\tau_I = 1.33,\ \tau_D = 10$



Multiple critical frequencies

Bode stability criterion does not work

Nyquist stability criterion

😮Not given in your textbook

$\omega_c = 0.172,\ AR(G_{OL})$ at $\omega_c = 19.8$ $\quad \omega_c = 0.338,\ AR(G_{OL})$ at $\omega_c = 1.48$

Closed loop is in fact stable with $K_c = 1$

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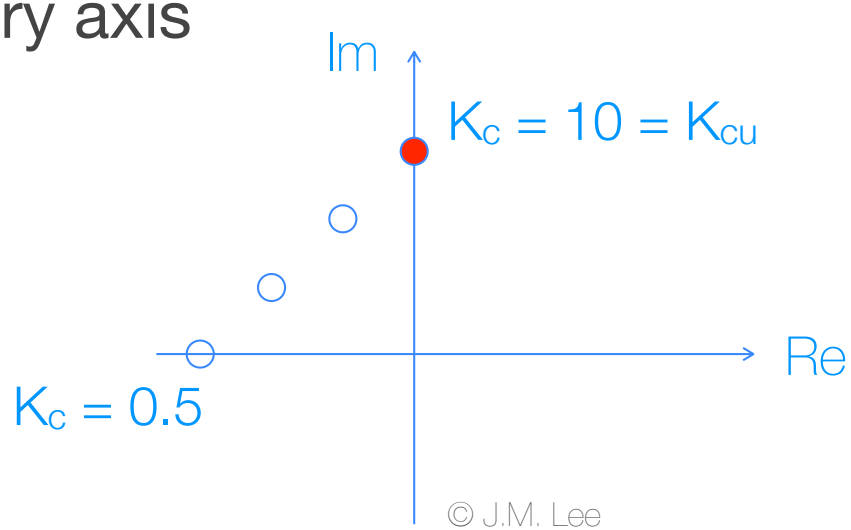
Gain Margin / Phase Margin

Q. How close is a current controller close to instability?

Ultimate controller gain (K_{cu})

K_c that will cause the system on the verge of instability

- The upper bound of Kc from Routh's stability test
- Kc corresponding to $\mathrm{AR}(G_{OL}(j\omega_c)) = 1$ (GM = 1 & PM = 0°)
- Kc corresponding to the pole of closed-loop TF on the imaginary axis



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Gain Margin (GM)

$$\boxed{GM = \frac{1}{AR_{OL}(\omega_c)}}$$

If GM = 1: on the verge of instability

For stability $|AR_{OL}(\omega_c)| < 1 \quad \Rightarrow \quad GM > 1$

If GM = 2, this means Kc can still be doubled while keeping the stability.

The gain margin is the multiplicative factor that the controller gain (or the process gain) can be increased by before the system becomes unstable.

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Phase Margin (PM)

$$PM = 180^\circ + (\phi \text{ of } G_{OL} \text{ at } \omega \text{ where } AR \text{ of } G_{OL} = 1)$$

How much time-delay can be included in the feedback loop before the instability will occur.

The phase angle of the process (or the controller) can be decreased by a factor of P.M. before the closed-loop becomes unstable.

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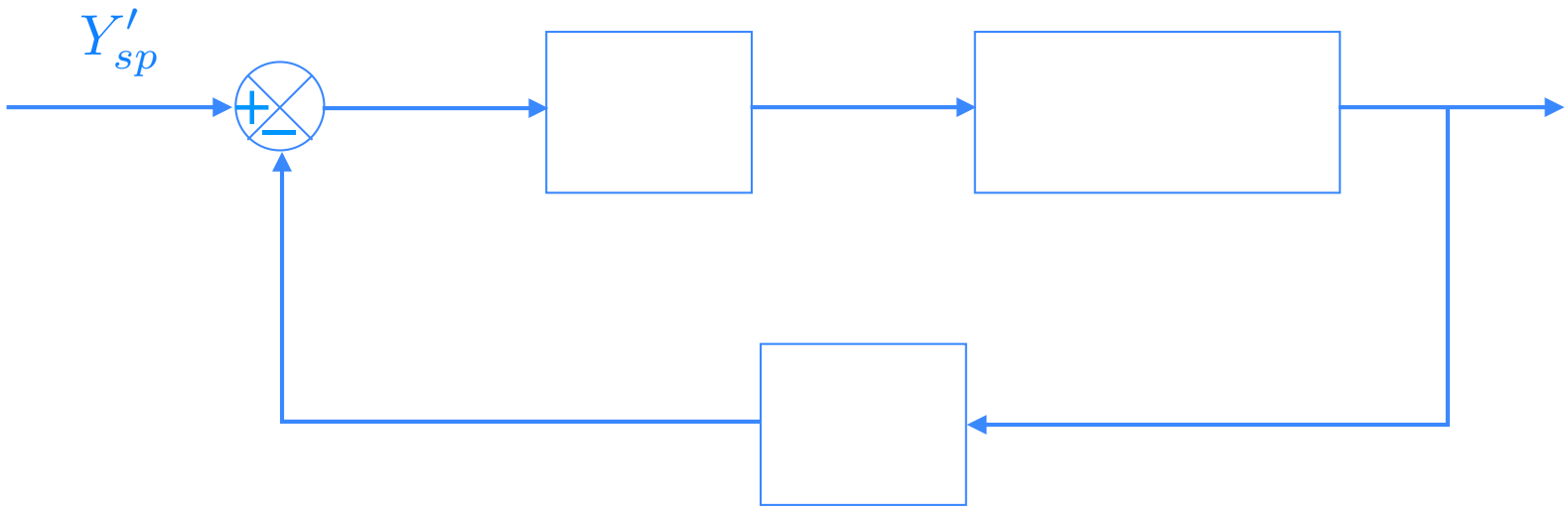
Consider the 2nd order process $\dfrac{1}{(8s+1)^2}$ with measurement delay of 2 (min). Determine the critical frequency and K_c with GM of 1.4 with P controller

1 Draw the closed-loop block diagram



2 G_{OL} = ?

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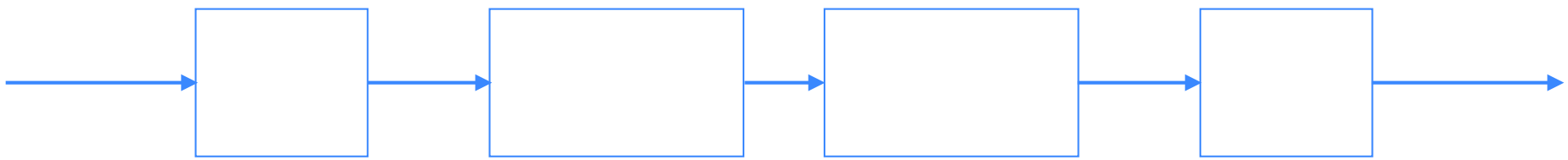
③ Determine the critical frequency.

Hint: $\phi = -\tan^{-1}(\tau\omega)$ for $\dfrac{1}{\tau s + 1}$

G_{OL} is equivalent to



P.A.: 0 -2ω

$$\therefore \quad \phi_{OL} = 0 - \tan^{-1}(8\omega_c) - \tan^{-1}(8\omega_c) - 2\omega_c = -\pi$$

$$-2\tan^{-1}(8\omega_c) - 2\omega_c = -\pi \qquad \omega_c = 0.35\left[\frac{\text{rad}}{\text{min}}\right]$$

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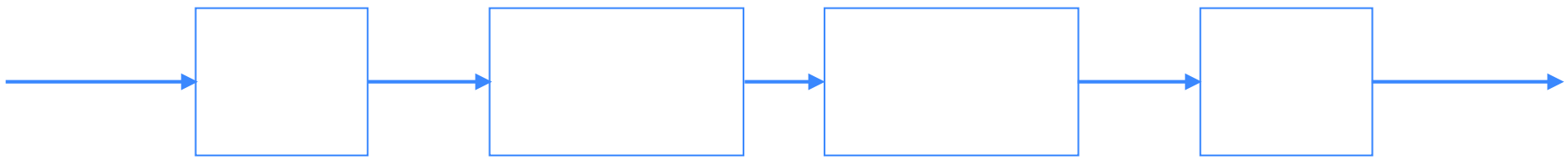
③ Determine the critical frequency.

Hint: $\phi = -\tan^{-1}(\tau\omega)$ for $\dfrac{1}{\tau s + 1}$

G_{OL} is equivalent to



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Kc with GM = 1.4?

$$AR_{OL}(\omega_c) = K_c \cdot \frac{1}{\sqrt{(8\omega_c)^2+1}} \cdot \frac{1}{\sqrt{(8\omega_c)^2+1}} \cdot \left|e^{-2\omega_c j}\right|$$

$$= \frac{K_c}{(8\omega_c)^2+1} = \frac{K_c}{(8\times 0.35)^2+1} = \frac{K_c}{8.84}$$

$$GM = 1.4 = \frac{1}{\frac{K_c}{8.84}} \qquad K_c = \frac{8.84}{1.4} = 6.31$$

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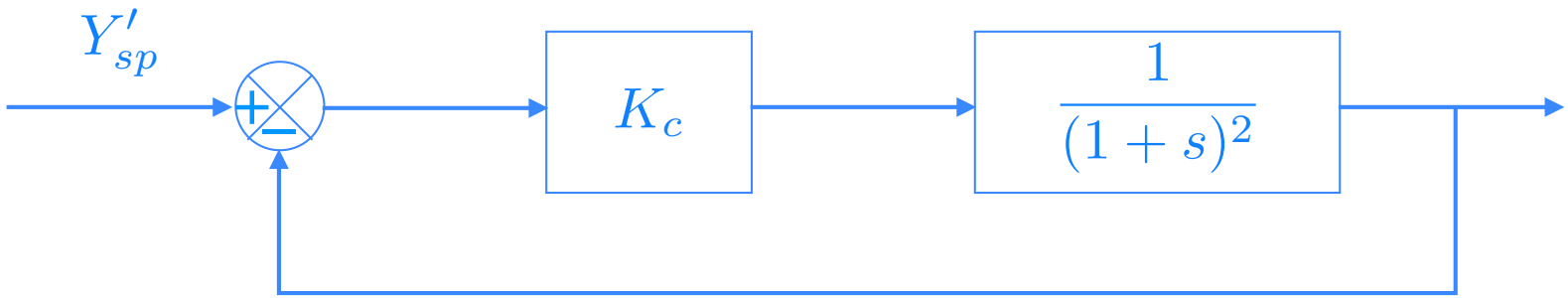
For the 2nd order process $\dfrac{1}{(1+s)^2}$, determine Kc with 30° PM when used with P-only controller



$$PM = 30^\circ = 180^\circ + \phi_{OL}$$

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Relationship with Continuous Cycling and Z-N Tuning

- With P only control and Kc = 1, G_{OL} = G. Plot Bode diagram of G only.
- Let ω_c be the frequency where phase angle of G becomes -180°. Ultimate Gain: K_{cu} = 1/(A.R. of G at $\omega = \omega_c$).
- P controller with such a K_c makes the A.R. of G_{OL}(= K_cG) to be 1 at $\omega = \omega_c$.
- Gain margin with Kc = K_{cu} is 1 (no margin).
- Ultimate period: $P_u = 2\pi/\omega_c$
- Z-N tuning rule uses the information on K_{cu} and P_u (ωc and A.R. of G at $\omega = \omega c$) to define P/PI/PID parameters with sufficient gain and phase margins.

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