similar flow featurs

$$u^+ = \frac{1}{\kappa} \ln y^+ + B$$

No outer layer in internal flows!

→ log-law is valid up to the pipe center.

6.6 Turbulent pipe flow (use log law)

($Re > Re_c \simeq 2000 - 2300$)

pipe radius = R,

$$\frac{u(r)}{u^*} = \frac{1}{\kappa} \ln \frac{(R-r)u^*}{\nu} + B .$$

$$u^* = \sqrt{\frac{\tau_w}{\rho}}$$

bulk velocity, $V = \frac{Q}{A} = \frac{1}{\pi R^2}\int_0^R u \cdot 2\pi r \cdot dr$

$$= \frac{1}{2}u^* \left(\frac{2}{\kappa} \ln \frac{Ru^*}{\nu} + 2B - \frac{3}{\kappa} \right)$$

$$\frac{V}{u^*} = 2.44 \ln \frac{Ru^*}{\nu} + 1.34$$

$$\frac{V}{\sqrt{\frac{\tau_w}{\rho}}} = \left(\frac{8}{f}\right)^{1/2} \qquad \frac{Ru^*}{\nu} = \frac{Vd}{\nu} \cdot \frac{R}{d} \cdot \frac{u^*}{V} = \frac{1}{2} Re_d \left(\frac{f}{8}\right)^{1/2}$$

$$\left(f = \frac{8\tau_w}{\rho V^2} \right)$$

$$\Rightarrow \frac{1}{f^{1/2}} \doteq 1.99 \log_{10}(Re_d \cdot f^{1/2}) - 1.02$$

(Prandtl, 1935 $\frac{1}{f^{1/2}} = 2.0 \log_{10}(Re_d \cdot f^{1/2}) - 0.8$

to fit the data better)

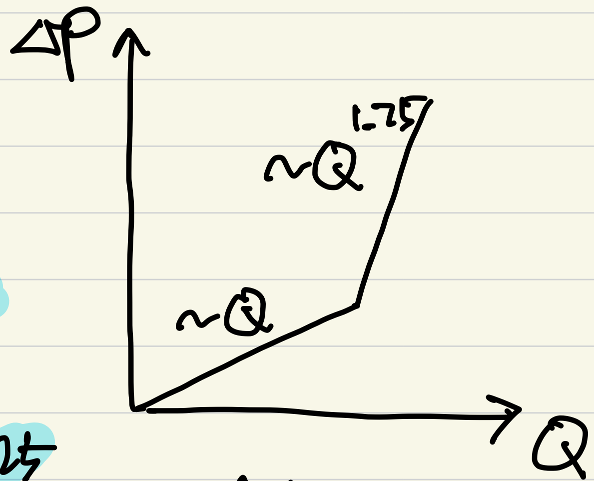
Or, explicitly, $f \doteqdot 0.316 \cdot Re_d^{-1/4}$ (Blasius, 1911)

($Re_d < 10^5$)

$$h_f = \frac{\Delta p}{\rho g} = f \cdot \frac{L}{d} \cdot \frac{V^2}{2g}$$

$$\Rightarrow \Delta p \simeq 0.158\, L \rho^{3/4} \mu^{1/4} \cdot d^{-5/4} \cdot V^{7/4}$$

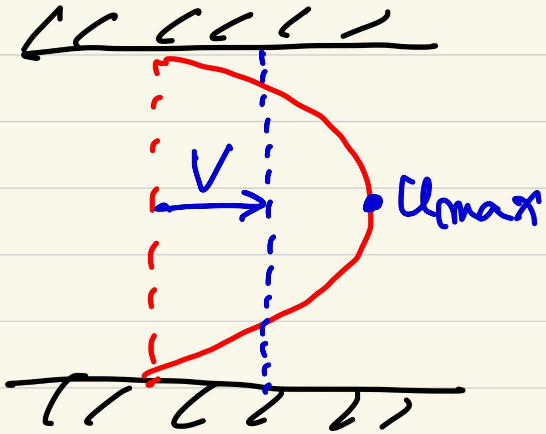
$$= 0.24\, L \rho^{3/4} \mu^{1/4} d^{-4.75}\, Q^{1.75}$$



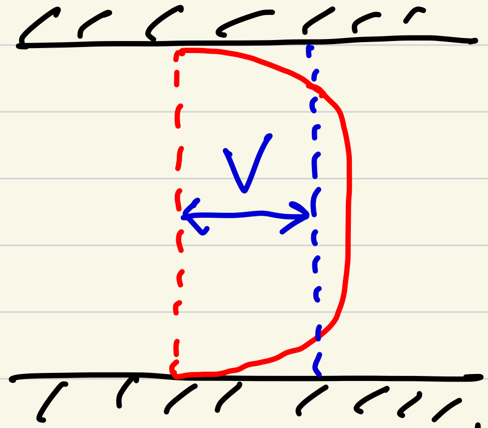
(Hagen, 1839)

· $$\frac{u_{max}}{u^*} = \frac{1}{\kappa} \ln \frac{R u^*}{\nu} + B \quad (@\ r = 0)$$

$$\Rightarrow \frac{V}{u_{max}} \simeq \frac{1}{1 + 1.31\sqrt{f}}$$ ($\simeq 0.8 \sim 0.9$) (cf. 0.5 for lam.)



laminar

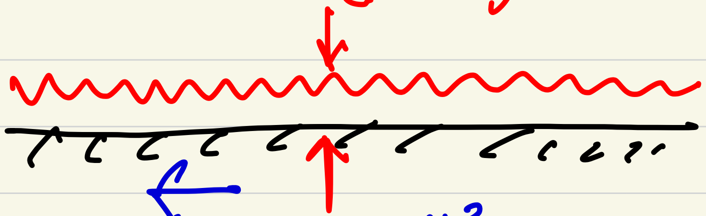


turbulent

* Effect of rough walls.

→

ε (roughness)



$$\Rightarrow \varepsilon^+ \equiv \frac{\varepsilon u^*}{\nu}$$

(non-dimensional roughness)

$$u^* = \sqrt{\frac{\tau_w}{\rho}}$$

$\tau_w = \rho (u^*)^2$

(wall shear stress)

Nikuradse (experiments)

$\varepsilon^+ < 5$: hydraulically smooth

$5 < \varepsilon^+ < 70$: transitional roughness

$\varepsilon^+ > 70$: fully rough.

$\Downarrow$

$$\varepsilon^+ = \frac{\varepsilon u^*}{\nu} = 70.$$

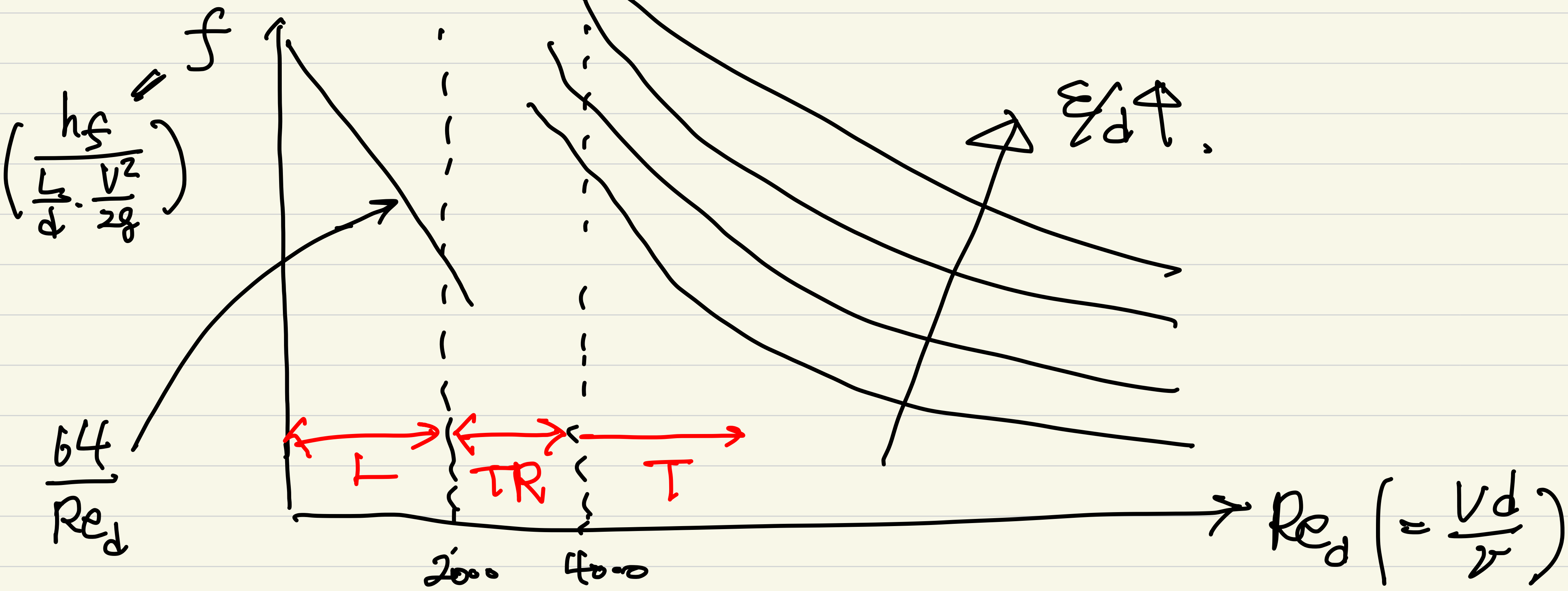
water, $\nu = 10^{-6}$, $V = 10\ m/s$, $d = 10cm$.

$$\frac{u^*}{V} \simeq \frac{1}{20} \rightarrow u^* = 0.5\ m/s$$

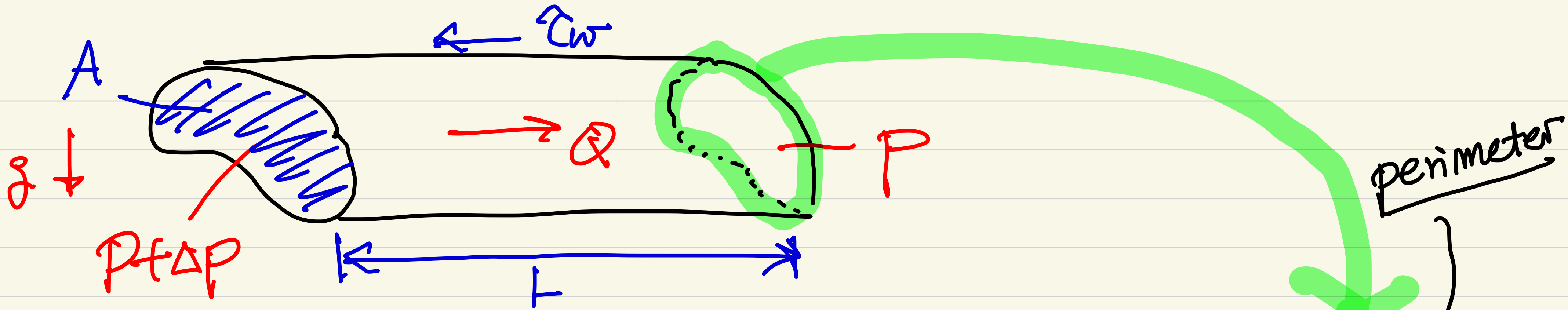
$$\varepsilon = 70 \cdot \nu / u^* = \frac{70 \times 10^{-6}}{0.5} = 0.14\ mm$$

· Moody Chart (1944) for pipe friction.

$$\frac{1}{f^{1/2}} = -2.0 \log\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{Re_d \cdot f^{1/2}}\right)$$



6.8 Flow in non-circular ducts.



Force balance : $\Delta P \cdot A + \rho g A L \cdot \sin\phi - \tau_w \cdot L \cdot P = 0$

$$\Rightarrow h_f = \frac{\Delta P}{\rho g} + \Delta z$$

$$= \frac{\tau_w}{\rho g} \cdot \frac{L}{A/P}$$

cf) circular pipe

$$h_f = \frac{\tau_w}{\rho g} \cdot \frac{L}{d/4}$$

$$\therefore \frac{A}{P} = \frac{D_h}{4} \quad (D_h : \text{hydraulic diameter})$$

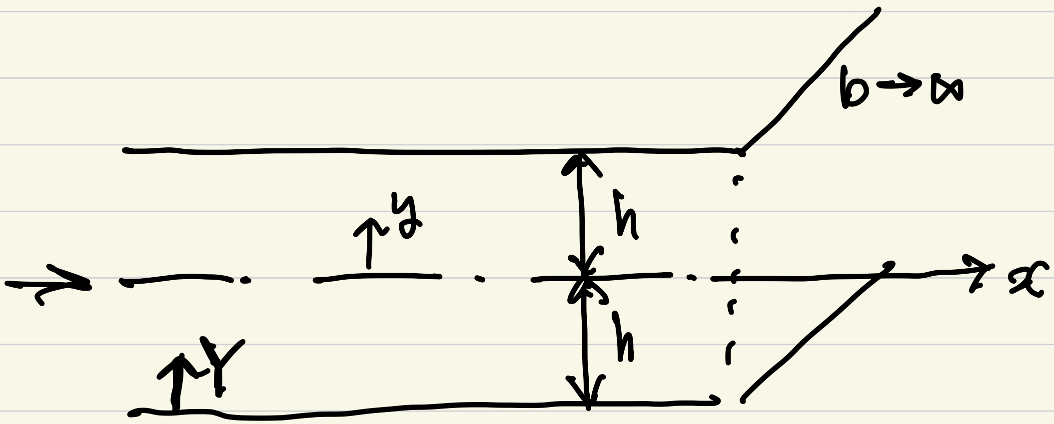
$$\rightarrow \boxed{D_h = 4A/P}$$

for non-circular duct

(compared to the moody chart w/ D_h)

$$f = \begin{cases} 64/Re_{D_h} & (\text{lam}, \pm 40\% \text{ error}) \\ f_{moody}\left(Re_{D_h}, \frac{\varepsilon}{D_h}\right) & (\text{turb}, \pm 15\% \text{ error}) \end{cases}$$

* Flow between parallel plates. (effect of D_h)



- fully-developed. $\left(\frac{\partial}{\partial x} = 0\right)$

→ $u = u(y)$

- for laminar flow

$$\boxed{u = \frac{1}{2\mu}\left[-\frac{d}{dx}(p + \rho g z)\right](h^2 - y^2)}$$

$$V = Q/A = \frac{1}{A}\int u \, dA = \frac{2}{3} u_{max} \quad \left(u_{max} = \frac{h^2}{2\mu} \cdot \frac{\Delta P}{L} \right)$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big)_{y=-h} = \frac{3\mu V}{h}$$

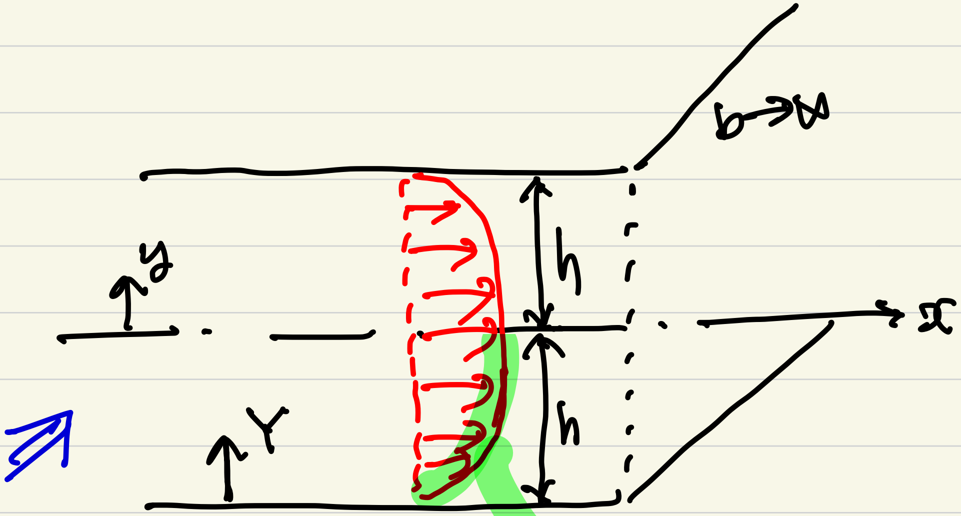
$$f \equiv \frac{8\tau_w}{\rho V^2} = \frac{24}{Re_h} \quad \left(Re_h = \frac{\rho V h}{\mu} \right)$$

if we use $D_h = \dfrac{4 \cdot b \cdot 2h}{(2b+4h)} = 4h$ (as $b \to \infty$)

$$f = \frac{96}{Re_{D_h}} \quad \left(Re_{D_h} = \frac{\rho V D_h}{\mu} \right)$$

$\Downarrow$

very different from $64/Re_{D_h}$.



· for turbulent flow

↓

log law

: $\dfrac{u(Y)}{u^*} = \dfrac{1}{\kappa} \ln \dfrac{Y u^*}{\nu} + B \quad (0 < Y < h)$

bulk velocity : $V = \dfrac{Q}{A} = \dfrac{1}{h} \int_0^h u \cdot dY = u^* \left(\dfrac{1}{\kappa} \ln \dfrac{h u^*}{\nu} + B - \dfrac{1}{\kappa} \right)$

B : 5.0

κ : 0.41

$$\frac{V}{u^*} = \boxed{\sqrt{\frac{8}{f}} = \frac{1}{\kappa} \ln \frac{h u^*}{\nu} + B - \frac{1}{\kappa}} \Rightarrow \boxed{\frac{1}{f^{1/2}} = 2.0 \log_{10} \left(Re_{D_h} \cdot f^{1/2} \right) - 1.19.}$$

$\left(u^* \equiv \sqrt{\dfrac{\tau_w}{\rho}} \, , \; f \equiv \dfrac{8 \tau_w}{\rho V^2} \right)$

similar

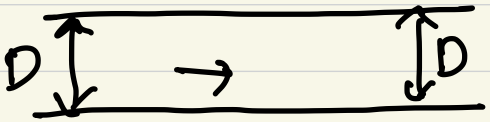
D_h : hydraulic diameter.

$D_h = \dfrac{4A}{P} = 4h.$

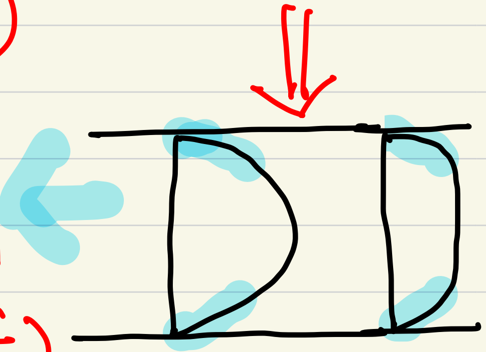
cf) circular pipe flow

$$\frac{1}{f^{1/2}} = 2.0\log\left(0.64\,Re_d\,f^{1/2}\right) - 0.8$$

6.9. Minor or local losses. (Major loss?)

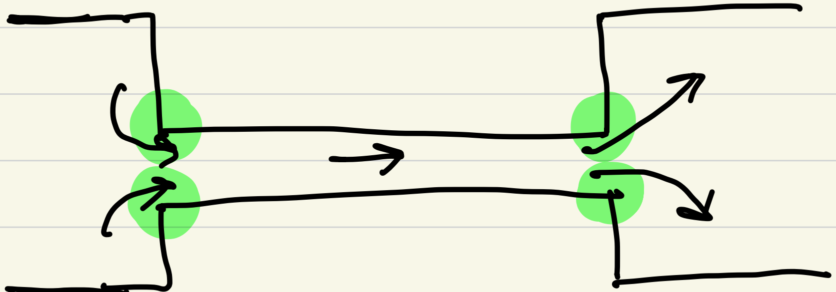


frictional loss

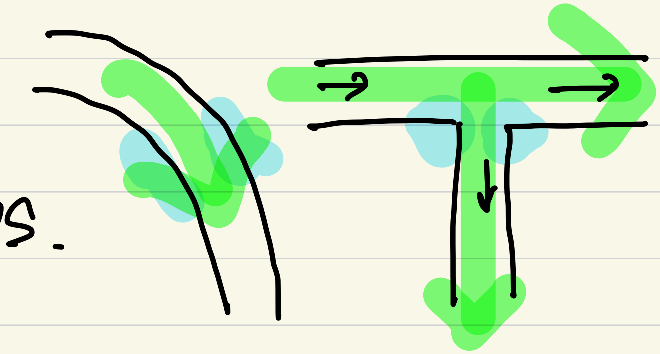


h_f (or f)

- pipe entrance, exit.



- Sudden expansion or contraction



잡낳아? - bends, elblow, tees and other fittings.

$$\Delta p = K \cdot \frac{1}{2}\rho V^2$$

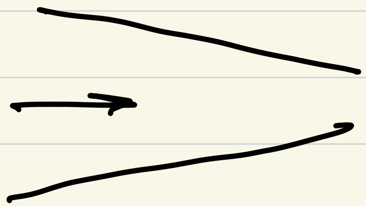
loss coefficient

$$\Rightarrow h_{total} = \underset{(h_f)}{h_{major}} + h_{minor}$$

$$= f \cdot \frac{L}{d} \cdot \frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

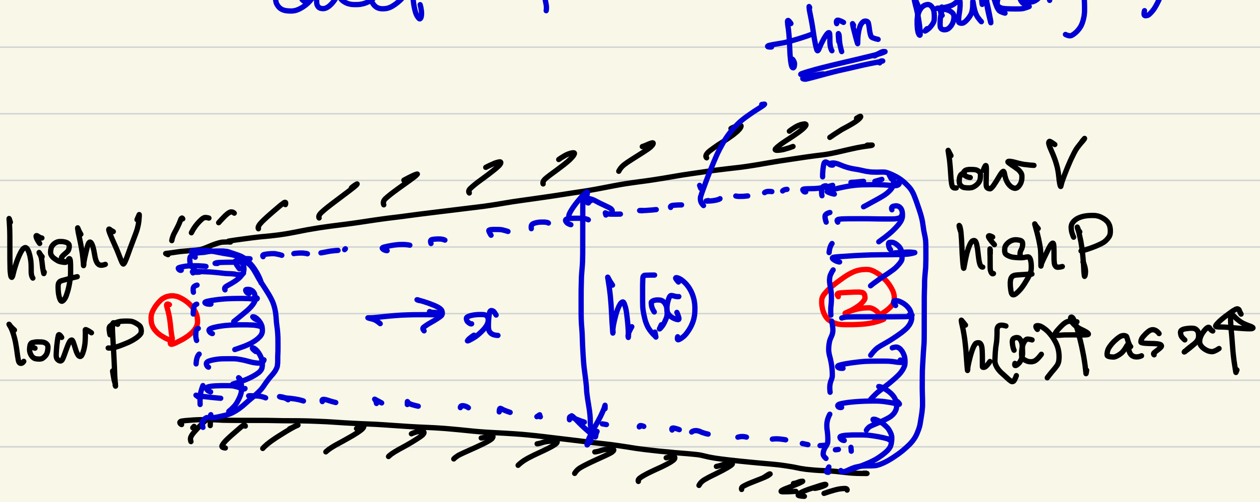
6.11 Diffuser performance

($\leftrightarrow$ Nozzle)

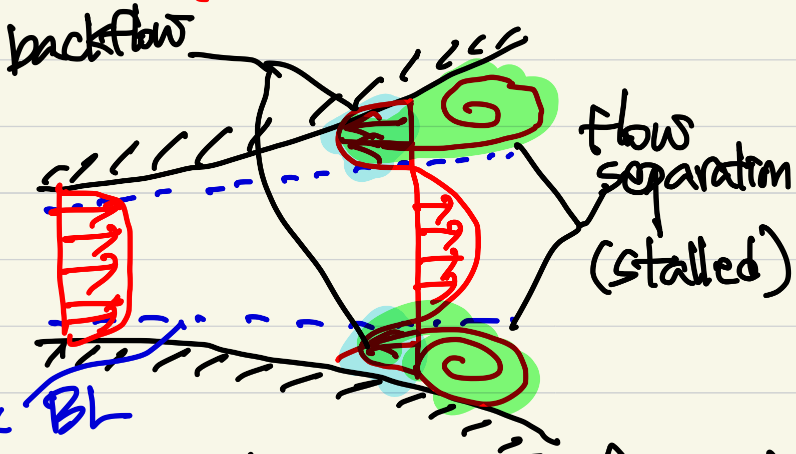


to increase pressure and reduce kinetic energy of duct flows

$V \downarrow$ → increase area



ideal (good performance)



actual (poor performance)

$$p + \frac{1}{2}\rho V^2 = \text{constant} = P_0 \ (\underline{\text{ideal}})$$

(w/o loss)

$$P_{01} = P_1 + \frac{1}{2}\rho V_1^2 = P_{02} = P_2 + \frac{1}{2}\rho V_2^2$$

$$C_p \equiv (P_2 - P_1)/(P_{01} - P_1)$$

(pressure <u>recovery</u> coefficient)

$$= 1 - \left(\frac{V_2}{V_1}\right)^2 = 1 - \left(\frac{A_1}{A_2}\right)^2$$

$$(\because Q_1 = V_1 A_1 = Q_2 = V_2 A_2)$$

$\Rightarrow$ conventional : $A_1 : A_2 = 1 : 5$

$\rightarrow C_p = 0.96.$

real application $C_p = 0.86 \sim \underline{0.24}$

(stall)

6.12. Fluid meters (Velocity, Volume flow)

intrusive $\leftarrow$
① pitot tube
② hot-wire anemometer [열선유속계]

non-intrusive $\leftarrow$
③ LDV (Laser-Doppler velocimetry)
④ PIV (Particle image velocimetry)