

↓
zero equation / algebraic model

- Prandtl's mixing length ($\nu_t = \mu_t / \rho = u^* l_m$)

one-equation model

- Spalart-Allmaras, k -model

two-equation model

- k - ϵ , k - ω

⋮
machine learning (optimization)

① Turbulence modulation by particles.

(dispersed)

• 1-way coupling: cont.-phase turb. is unaffected by disp.-phase.

| - very dilute flows ($\phi \equiv V_p / (V_f + V_p) \ll 1$) or

↓ very small density difference. $\rho_p/\rho_f \approx 1.0$.

Volume fraction or mass loading is important.

$$\frac{m_p}{m_f + m_p} \approx \frac{m_p}{m_f} = \frac{\rho_p \cdot V_p}{\rho_f \cdot V_f} \approx \phi \frac{\rho_p}{\rho_f}$$

for heavy particles

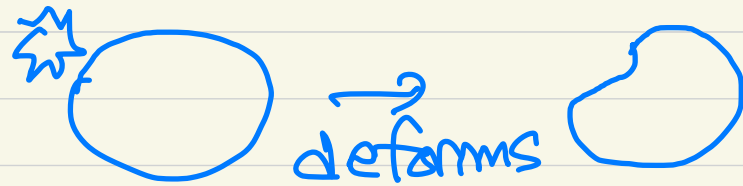
represents geometric effect of particle moving w/ relative velocity to the carrier fluid

⇒ perturbations to the flow.
[water, for example].

associated to gravity or inertia

• 2-way coupling : particles may enhance the turbulence intensity (TI) due to the energy deposited at small scales through work done by

the particles as they move relative to the fluid (typically by gravity) or they may reduce the TI owing to the additional dissipation occurring at the particle/fluid interface, assisted by the no-slip condition (cf. bubbles/drops).



Quantifying the turbulence modulation is tricky as the effect of particles can be distributed non-uniformly across the scales of turbulence.

↳ e.g.) TKE is an integral of the kinetic energy present in the fluctuating velocity field

across the length (or time) scales. The contribution of particles may show up in TKE, but the different effects at different scales may counteract each other and obscure the total TKE.

• Consequence of turbulence modulation by particles.

→ may augment or suppress ⇒ mixing.

heat transfer, chemical reaction, pressure drop,
(drag reduction)

energy conversion, ...

• Mechanisms of turbulence modulation

- Effect on the mean shear. (du/dy).

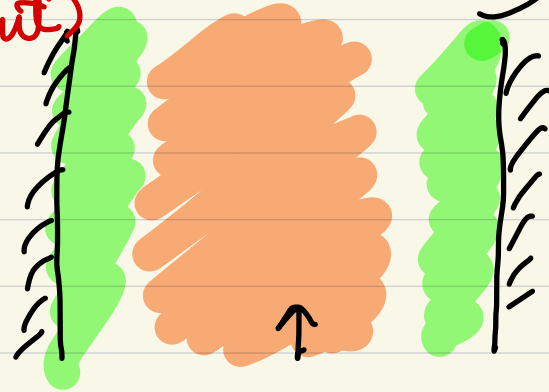
↳ turbulence production

- Unsteady particle wake (typically bubbles)
 - ↳ vortex shedding at higher Re_p .
- Dissipation owing to the forces on the particles.
- others: deformability, wall proximity.

* Turbulence in bubbly flows (in general)

- smaller bubbles tend to suppress turbulence.
- for a given liquid flux, turbulence is enhanced with a larger gas flux.
- energy damping effect on highly deformable bubble may reduce turbulence.
- but the turbulence enhancement is also observed associated w/ shape oscillation.)

- suppression occurs locally differently.
(or enhancement) [e.g. wall proximity]



① Modeling of turbulence modulation.

Inherent production



Turbulence energy
in multiphase flow

Dissipation by particle.



production
by particle



viscous dissipation
(ϵ)



(Kenning & Crowe, 1997)

→ addition of new scales of particles or particle-induced flows.

⇒ interaction w/ existing scales!

- Bubble-induced turbulence [BIT]
 - ↔ shear-induced turbulence (SIT).

- source/origin: collective dynamics of water structure.

- existence of inertial subrange (slope of $-5/3$)
 - ↳ pseudo-turbulence.

- bubble parameter (b) (Rensen et al, 2005)

$$b \equiv \frac{\alpha V_R^2}{u_o'^2}$$

α : void fraction.

V_R : relative bubble velocity

u_o' : velocity fluctuation

w/o bubbles.

\downarrow
 $b < 1.0$: turbulence-dominated
 ($b = 0$: single-phase flow)
 $b > 1.0$: rising bubbles dominate
 ($b \rightarrow \infty$: bubble swarm
 in a quiescent liq.)

Almeras et al. (2017)

$$: u'_{z,rms} \sim b^{0.4} \quad (b < 0.7)$$

$$\sim b^{1.3} \quad (b > 0.7)$$

- Scaling relation for bubble-induced turbulence
 (two-phase averaged N-S equation for the
 liquid phase w/o surface tension)

$$\nabla \cdot [(1-\alpha) \rho \bar{u} \bar{u}] = - \nabla [(1-\alpha) \bar{p}] + \nabla \cdot [(1-\alpha) \bar{\tau}_m]$$

$$- \nabla \cdot [(1-\alpha) \rho \bar{u}' \bar{u}'] + (1-\alpha) \rho g - \Phi_b$$

↑
interfacial
momentum transfer

if we can ignore the contributions of

(mean shear, pressure gradient and gravity

(→ valid for homogeneous bubble swarm)

$$\nabla \cdot [(1-\alpha) \rho \bar{u}' \bar{u}'] \sim - \Phi_b$$

↑ drag only

very similar to
bubble wake

$$(1-\alpha) \rho U_{z,rms}^{\prime 2} \sim \alpha V_R^2 / d_b$$

$$\frac{U_{z,rms}^{\prime 2}}{U_0^{\prime 2}} \sim \frac{\alpha V_R^2}{(1-\alpha) U_0^{\prime 2}}$$

$$\text{if } \alpha \ll 1, \frac{\alpha V_R^2}{(1-\alpha) U_0^{\prime 2}} \approx \alpha \frac{V_R^2}{U_0^{\prime 2}}$$

total liquid mtrm \sim interfacial mtrm transfer.

- Efforts for algebraic equation model for Reynolds stress. (zero eq. model)

$u(t) = \bar{u} + u'$ $\Rightarrow \tau = \mu \frac{\partial \bar{u}}{\partial y} \rightarrow \tau_{turb} = \mu_t \frac{\partial \bar{u}}{\partial y}$

single-phase flow

in two-phase flow.

$u(t) = \bar{u} + u' + u''$

contribution from bubbles.

(Sato & Sekoguchi, 1975).

modeling of u'' takes the similar approach to u' modeling. (e.g. mixing-length eddy-

viscosity model)

$$-\overline{u''v''} = \epsilon_b \frac{\partial \bar{u}}{\partial r} \sim \frac{\alpha \bar{d}_b \bar{v}_r}{\alpha} \frac{\partial \bar{u}}{\partial r} \quad (\text{Saito \& Sekoguchi, 1975})$$

$\bar{v}_r = \bar{v}_b - \bar{v}_d$: relative velocity.

$$\sim \alpha \bar{d}_b \bar{v}_r \left(\frac{\partial \bar{u}}{\partial r} + \frac{\partial \bar{v}_r}{\partial r} \right) + \bar{d}_b \bar{v}_r \bar{v}_b \frac{\partial \bar{d}}{\partial r}$$

(Hosokawa & Tomiyama, 2013)

$$\sim \bar{d}_b \bar{v}_b \left(1 - 0.4 \bar{d}^{-2/3} \right) \alpha^{1.1} \frac{\partial \bar{u}}{\partial r} + \left(1 - 0.4 \bar{d}^{-2/3} \right) \left(\bar{d}_b \bar{v}_b \bar{d}^{1.1} \frac{\partial \bar{v}_r}{\partial r} + 0.1 \bar{d}_b \bar{v}_b^2 \bar{d}^{-0.1} \frac{\partial \bar{d}}{\partial r} \right)$$

⇓

$$-\overline{u''v''} = f \left(\frac{\partial \bar{u}}{\partial r}, \frac{\partial \bar{v}_r}{\partial r}, \frac{\partial \bar{d}}{\partial r} \right) \quad (\text{Lee \& Park, 2020})$$