Precision Machine Design :

Error Assessment (Positional Error Calibration)

: Error=True Value- Nominal Value

3-5 Cycles of bi-directional measurement from 0mm to 1000mm at 100mm step, allowing over-travel at the edge of machine travel; to include any reversal error.



# Typical Setup for Error Assessment

(Source: New error correction algorithms minimizing residual positional error for computer integrated calibration/correction system in CNC machine tools, H.J.Pahk, S.W.Lee, Proc. of Institution of Mechanical Engineers, 679-686, Vol213 part C, 1999)

Target	Nom	Laser	Error[m	<u>nm]</u>
0	0	0 (reset)	0	forward
1	100	100.001	0.001	
2	200	200.003	0.003	
3	300	300.004	0.004	
4	400	400.005	0.005	
5	500	500.007	0.007	
6	600	600.008	0.008	
7	700	700.009	0.009	
8	800	800.010	0.010	
9	900	900.012	0.012	
10	1000	1000.014	0.014	overtravel
10	1000	1000.016	0.016	backward
9	900	900.014	0.014	
8	800	800.012	0.012	
7	700	700.010	0.010	
6	600	600.009	0.009	

Target	Nom	Laser	Error[m	<u>m]</u>
5	500	500.008	0.008	
4	400	400.007	0.007	
3	300	300.005	0.005	
2	200	200.004	0.004	
1	100	100.003	0.003	
0	0	0.002	0.002	overtravel
1 <sup>st</sup> cycle				
0	0	0.001	0.001	forward
1	100	100.002	0.002	
2	200	200.004	0.004	
3	300	300.005	0.005	
4	400	400.007	0.007	
5	500	500.009	0.009	
6	600	600.010	0.010	
7	700	700.011	0.011	

Target	Nom	Laser	Error[mm	<u>ו]</u>
8	800	800.012	0.012	
9	900	900.014	0.014	
10	1000	1000.016	0.016	overtravel
10	1000	1000.018	0.018	backward
9	900	900.017	0.017	
8	800	800.015	0.015	
7	700	700.013	0.013	
6	600	600.011	0.011	
5	500	500.009	0.009	
4	400	400.008	0.008	
3	300	300.007	0.007	
2	200	200.005	0.005	
1	100	100.004	0.004	
0	0	0.003	0.003	overtravel
2 <sup>nd</sup> cycle				

Target	Nom	Laser	Error[mm]	
0	0	0.002	0.002	forward
1	100	100.002	0.002	
2	200	200.005	0.005	
3	300	300.006	0.006	
4	400	400.008	0.008	
5	500	500.010	0.010	
6	600	600.011	0.011	
7	700	700.013	0.013	
8	800	800.014	0.014	
9	900	900.016	0.016	
10	1000	1000.018	0.018	overtravel
10	1000	1000.020	0.020	backward
9	900	900.019	0.019	
8	800	800.017	0.017	
7	700	700.015	0.015	

6	600	600.014	0.013	
5	500	500.012	0.011	
4	400	400.010	0.009	
3	300	300.009	0.007	
1	100	100.007	0.005	
0	0	0.005	0.004	
3 <sup>rd</sup> cycle				
•••				

In this way the number of cycle can be increased, and the positional error calibration is performed at every target of machine travel for the bidirectional movement.

# Positional Error Calibration

## Forward measurement

The ith repeat forward positional error at the jth target,  $X_{ji} \uparrow$ 

 $X_{ji}$   $\uparrow$  =the ith measurement error at the jth target

=Laser – Nominal =  $L_{ji} \uparrow -N_j$ 

where  $L_{ji}$  is the ith repeat laser measurement at the jth target, and  $N_j$  is the nominal data.

The forward mean positional error at the jth target is

 $\underline{X}_{j} \uparrow = \Sigma X_{ji} \uparrow / N$ 

where N = number of cycle=1,2..

The forward standard deviation at the jth target is

 $\sigma_{j}\uparrow=\sqrt{\Sigma}(X_{ji}\uparrow-\underline{X}_{j}\uparrow)^{2}/N-1$ 

The *uni-directional repeatability* for forward direction at the jth target,

 $UR_{j} \uparrow = 6 \sigma_{j} \uparrow$ 

And the maximum uni-directional repeatability for the

### forward direction is,

 $UR_{max} \uparrow = Max UR_{j} \uparrow (for j=1,2...)$ 

### Backward measurement

The ith repeat *backward positional error* at the jth target,  $X_{ji} \downarrow$ 

 $X_{ji} \downarrow$  = the ith measurement error at the jth target

$$=Laser-Nominal=L_{ji}\downarrow -N_{j}$$

where  $L_{ji}$  is the ith repeat laser measurement at the jth target, and  $N_j$  is the nominal data.

The backward mean positional error at the jth target is

$$\underline{X}_{j} \downarrow = \Sigma X_{ji} \downarrow / N$$

where N = number of cycle=1,2..

The backward standard deviation at the jth target is

 $\sigma_{j}\downarrow=\sqrt{\Sigma}(X_{ji}\downarrow-\underline{X}_{j}\downarrow)^{2}/N-1$ 

The *uni-directional repeatability for backward direction* at the jth target,

 $UR_{j}\downarrow = \ 6 \ \sigma_{j}\downarrow$ 

And the *maximum uni-directional repeatability* for the forward direction is,

 $UR_{max} \downarrow = Max UR_{j} \downarrow \text{ (for } j=1,2...)$ 

#### Mean positional error

The *mean positional error* or system positional error at the jth target

 $X_j$ =(Forward mean positional error + Backward mean positional error)/2

 $= (\underline{X}_{j} \uparrow + \underline{X}_{j} \downarrow)/2$ 

The *reversal error* (hysteresis error, or backlash error) at the jth target is,

R<sub>j</sub>=Backward mean positional error- Forward mean positional error

$$= \underline{X}_{j} \downarrow - \underline{X}_{j} \uparrow$$

The maximum reversal error, R<sub>max</sub>, is

 $R_{max}$ = Max |  $R_j$  | (j=1,2...)

The maximum uni-directional repeatability is,  $UR_{max} = Max (UR_j \uparrow, UR_j \downarrow) = Max(6\sigma_j \uparrow, 6\sigma_j \downarrow), \text{ for } j=1,2...$ 

The *bi-directional repeatability* at the jth target is  $BR_j = |R_j| + 3(\sigma_j \uparrow + \sigma_j \downarrow)$ 

The maximum bi-directional repeatability is  $BR_{max}=Max BR_{j}=Max (|R_{j}| + 3\sigma_{j} \uparrow + 3\sigma_{j} \downarrow), \text{ for } j=1,2...$ 

### Positional error calibration



Error Calibration along a machine axis

(Note: s =  $\sigma$ , sample standard deviation, and  $2\sigma$  for 95% probability,  $3\sigma$  for 99% probability)

### **Evaluation of Accuracy**

### 1. Bandwidth Method(ANSI)

:The greatest difference in the positional error representation,

Linear Displacement Accuracy, or Accuracy =Maximum error – Minimum error =Max  $(X_j \uparrow \pm 3\sigma_j \uparrow, X_j \downarrow \pm 3\sigma_j \downarrow)$  (j=1,2...) - Min  $(X_j \uparrow \pm 3\sigma_j \uparrow, X_j \downarrow \pm 3\sigma_j \downarrow)$  (j=1,2...) (\*Discuss pros. and cons.)



The arrow indicates the Linear Displacement Accuracy, or Accuracy.

2. Tolerance Template Method (DIN, Europe)

:To describe the machine accuracy in relation to the length of travel.

The accuracy or uncertainty can be expressed as the function of the distance travelled, as indicated as red line.

 $\mathsf{U} = \mathsf{A} + \mathsf{K}\mathsf{L} \leq \mathsf{B}$ 

where A= offset in [um]

= maximum uni/bi-directional repeatability

=Max ( $6\sigma_i \uparrow$ ,  $6\sigma_i \downarrow$ ) for uni-directional

=Max (  $|R_j| + 3\sigma_j \uparrow + 3\sigma_j \downarrow$ ) for bi-directional

B=Maximum error in [um]

K=slope to just enclose the error data over the machine travel, [um/m]

L=length of machine travel, [mm]



This is one of the most widely accepted method, and many industry currently use it as the accuracy or uncertainty evaluation.

Ex)  $U=2.0+L/100 \le 10$  [um]

HW1 For the given positional error data,

- (1) Calculate the positional errors, and
- (2) Evaluate the positional accuracy



Source:wikipedia

Face diagonals and space diagonals in the 2D/3D working space

The tolerance template can be extended to the 2D;

 $U_2 = A_2 + K_2 L \leq B_2$ 

Where  $U_2$ =accuracy (or uncertainty) in 2D work space

A<sub>2</sub>=maximum repeatability in the length measurement error in 2D work space, [um]

K<sub>2</sub>=slope to just enclose the length measurement error data over the 2D work space, [um/m]

B<sub>2</sub>=maximum length measurement error in the 2D work space, [um]

L=length of machine travel along the 2D work space, such as face diagonals, [mm]

Also in the 3D working space;

 $U_3 = A_3 + K_3 L \leq B_3$ 

where U<sub>3</sub>=accuracy (or uncertainty) in 3D space, [um]

A<sub>3</sub>=maximum repeatability in the length measurement error in 3D work space, [um]

 $K_3$ =slope to just enclose the length measurement error data over the 3D work space, [um/m]

 $B_3$ =maximum length measurement error in the 3D work space, [um]

L=length of machine travel along the 3D work space, such as space diagonals, [mm]