Single Particle Motion (Assumption: ① uniform $\vec{E}, \vec{B}$, ② $\vec{F} = m\vec{a} = q(\vec{E} + \vec{v} \times \vec{B})$)

1) $\vec{E} = E_x \hat{x} + E_z \hat{z}$

$$m\dot{v}_x = qE_x \qquad v_x = \frac{q}{m}E_x t + v_{x_0} \qquad x = \frac{1}{2}\frac{q}{m}E_x t^2 + v_{x_0} t + x_0$$

$$m\dot{v}_y = 0 \quad \Rightarrow \quad v_y = v_{y_0} \quad \Rightarrow \quad y = v_{y_0} t + y_0$$

$$m\dot{v}_z = qE_z \qquad v_z = \frac{q}{m}E_z t + v_{z_0} \qquad z = \frac{1}{2}\frac{q}{m}E_z t^2 + v_{z_0} t + z_0$$

2) $\vec{B} = B\hat{z}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{vmatrix} = v_y B\hat{x} - v_x B\hat{y}$$

$$m\dot{v}_x = qBv_y \qquad m\ddot{v}_x = qB\dot{v}_y = -\frac{(qB)^2}{m}v_x \qquad \ddot{v}_x = -\omega_c^2 v_x$$

$$m\dot{v}_y = -qBv_x \quad \Rightarrow \quad m\ddot{v}_y = -qB\dot{v}_x = -\frac{(qB)^2}{m}v_y \quad \Rightarrow \quad \ddot{v}_y = -\omega_c^2 v_y \qquad \left(\omega_c = \frac{|q|B}{m}\right)$$

$$m\dot{v}_z = 0 \qquad m\ddot{v}_z = 0 \qquad \ddot{v}_z = 0$$

$$v_x = A\sin(\omega_c t) + B\cos(\omega_c t) = v_\perp \cos(\omega_c t + \delta)$$

$$m\dot{v}_x = qBv_y \rightarrow v_y = \frac{m}{qB}\left[-v_\perp \omega_c \sin(\omega_c t + \delta)\right]$$

$$= \mp v_\perp \sin(\omega_c t + \delta) \quad \left(\begin{matrix} - : \text{ion} \\ + : \text{electron} \end{matrix}\right)$$

$$\left(\begin{matrix} v_x^2 + v_y^2 = v_\perp^2 \\ v_{x_0} = v_\perp \cos\delta \\ v_{y_0} = \mp v_\perp \sin\delta \\ \tan\delta = \mp \frac{v_{y_0}}{v_{x_0}} \end{matrix}\right)$$

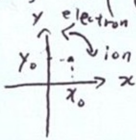
$$x = \frac{v_\perp}{\omega_c}\sin(\omega_c t + \delta) + x_0$$

$$y = \pm \frac{v_\perp}{\omega_c}\cos(\omega_c t + \delta) + y_0$$

$$z = v_{z_0} t + z_0$$

$$(x - x_0)^2 + (y - y_0)^2 = \left(\frac{v_\perp}{\omega_c}\right)^2 = r_L^2 \quad (\text{Larmor radius})$$

$$r_L = \frac{mv_\perp}{|q|B}$$



$\Rightarrow$ diamagnetic

Single Particle Motion

3) $\vec{E} = E_x \hat{x} + E_z \hat{z}$, $\vec{B} = B_z \hat{z}$

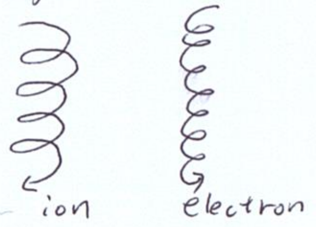
$$\vec{F} = m\vec{a} = q(\vec{E} + \vec{v}\times\vec{B}) = q\{(E_x + v_y B_z)\hat{x} - v_x B_z \hat{y} + E_z \hat{z}\}$$

$$\dot{v}_x = \frac{q}{m}(E_x + v_y B_z) \qquad v_x = v_\perp \cos(w_c t + \delta)$$

$$\dot{v}_y = -\frac{q}{m} B_z v_x \quad \Rightarrow \quad v_y = \mp v_\perp \sin(w_c t + \delta) - \frac{E_x}{B_z}$$

$$\dot{v}_z = \frac{q}{m} E_z \qquad v_z = \frac{q}{m} E_z t + v_{z0}$$

larger   smaller



$$x = \frac{v_\perp}{w_c}\sin(w_c t + \delta) + x_0$$

$$\Rightarrow y = \pm \frac{v_\perp}{w_c}\cos(w_c t + \delta) - \frac{E_x}{B_z} t + y_0 \rightarrow (x - x_0)^2 + \left(y - y_0 + \frac{E_x}{B_z} t\right)^2 = r_L^2$$

$$z = \frac{1}{2}\frac{q}{m} E_z t^2 + v_{z0} t + z_0$$

if) $\int_0^\tau m\frac{d\vec{v}}{dt} = 0$ (~~steady state~~) $= q(\vec{E} + \vec{v}_\perp \times \vec{B}) \rightarrow q(\vec{E} + \vec{v}_\perp \times \vec{B}) \times \vec{B}$

$$\Rightarrow q(\vec{E}\times\vec{B}) + q\{\vec{B}(\vec{B}\cdot\vec{v}_\perp) - \vec{v}_\perp(\vec{B}\cdot\vec{B})\} = q(\vec{E}\times\vec{B}) + q(-B^2 \vec{v}_\perp) = 0$$

$$\therefore v_\perp = \frac{\vec{E}\times\vec{B}}{B^2} = \vec{v}_E \text{ (guiding center)}$$

$\left(\vec{v}_E = \frac{\vec{E}\times\vec{B}}{B^2} = \frac{q\vec{E}\times\vec{B}}{qB^2} = \frac{\vec{F}_E\times\vec{B}}{qB^2}\right)$ 유추 → ← general force. $\boxed{\vec{v}_f = \frac{\vec{F}\times\vec{B}}{qB^2}}$ $\rightarrow \vec{v}_g = \frac{m\vec{g}\times\vec{B}}{qB^2}$

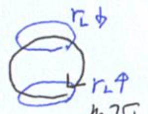
ion   electron

Non-uniform $\vec{B}$

$\boxed{* \, r_L << L_B}$ → r_L가 일정하다 → Talyer 전개 때매

$\vec{B} = B(y)\hat{z}$, $\vec{F} = q(\vec{v}\times\vec{B})$

assumption $\delta = 0$ $B(y) = B_0 + y\frac{\partial B}{\partial y}\Big|_{y=0} + \cdots$ assumption $\frac{\partial B}{\partial y}\Big|_{y=0} = \frac{\partial B}{\partial y}$



$r_L = \frac{m v_\perp}{|B||q|}$

$F_x = 0$ (∇B가 y 방향만 있어서)

$$F_y = -q \underbrace{v_x B}_{\text{외적계산 결과}} = -q(v_\perp \cos w_c t) B = -q(v_\perp \cos w_c t)\left[B_0 + y\frac{\partial B}{\partial y}\right]$$

$$= -q v_\perp \cos w_c t\left[B_0 \pm r_L \cos w_c t \frac{\partial B}{\partial y}\right] = -q v_\perp B_0 \cos w_c t \mp q v_\perp r_L \cos^2 w_c t \frac{\partial B}{\partial y}$$

$$\bar{F}_y = \frac{1}{\tau}\int_0^\tau F_y\, dt = \cancel{\frac{1}{\tau}\int_0^\tau -q v_\perp B_0 \underbrace{\cos w_c t}_{\text{주기 함수}}\, dt} \mp \frac{q v_\perp r_L}{\tau}\int_0^\tau \cos^2 w_c t\, dt \frac{\partial B}{\partial y}$$

$$\boxed{\bar{F}_x = \frac{1}{\tau}\int_0^\tau \mp q v_\perp B \sin w_c t\, dt = 0}$$

(τ : 주기, 한바퀴 돌때)

$$= \mp \frac{q v_\perp r_L}{\tau}\frac{\tau}{2}\frac{\partial B}{\partial y} \qquad \left(\because \cos^2 w_c t = \frac{1}{2}(1 + \cancel{\cos 2 w_c t})\right) \text{ 주기 함수}$$

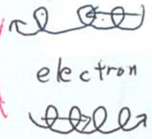
$$= \mp \frac{q v_\perp r_L}{2}\frac{\partial B}{\partial y}$$

$\vec{F} = q(\vec{v}\times\vec{B}) = q(\vec{v}\times\vec{B}_0) + q(\vec{v}\times\vec{B}_1)$ ↑∇B ⊙B

drift velocity

$$\vec{v}_{\nabla B} = \frac{\vec{F}\times\vec{B}}{qB^2} = \frac{\mp q v_\perp r_L \frac{1}{2}\frac{\partial B}{\partial y}\hat{y}\times\vec{B}}{qB^2} = \frac{\mp v_\perp r_L \frac{1}{2}\nabla B\times\vec{B}}{B^2} = \pm\frac{1}{2} v_\perp r_L \frac{\vec{B}\times\nabla B}{B^2}$$

수학적 결과   물리적 결과

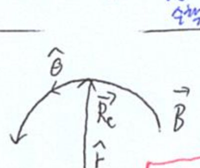


r_L의 변화로 인한 drift

Curvature

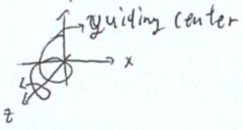
$\vec{B} = B_\theta(r)\hat{\theta}$

$$\vec{F} = \underbrace{\frac{m v_\parallel^2}{R_c}\hat{r}}_{\text{원심력}} = \frac{m v_\parallel^2}{R_c}\frac{\vec{R}_c}{R_c} = \frac{m v_\parallel^2}{R_c^2}\vec{R}_c$$



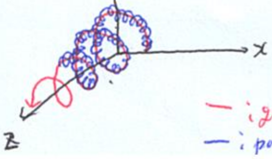
$\vec{v}_\parallel$, $\vec{B}$ → $\hat{\theta}$방향

$$\vec{v}_c = \frac{\vec{F}\times\vec{B}}{qB^2} = \frac{1}{qB^2}\frac{m v_\parallel^2}{R_c^2}\vec{R}_c\times\vec{B}$$



HW

$\vec{B}$ 일때 ∇B 존재하는 이유



— : guiding center
— : particle

$$\boxed{\vec{v}_c = \frac{\vec{F}\times\vec{B}}{qB^2}}$$

$\vec{F}$ = 로렌츠힘 or 원심력 등등