1.5. Properties of the velocity field (system vs. control volume)

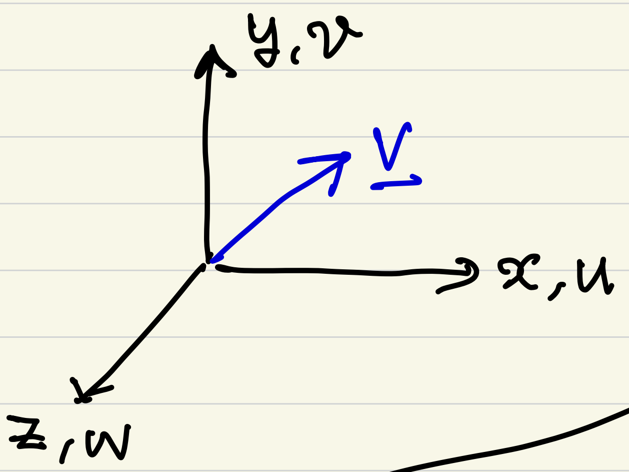
↓

approaches

- Eulerian (C.V.) : $u(x,y,z,t)$
- Lagrangian (system) : $u(t)$

* Velocity field (vectors)

$$\underline{V}(x,y,z,t) = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}.$$



* Acceleration field (of a fluid particle).

$$\underline{a} = \frac{D\underline{V}}{Dt} = \frac{D}{Dt}\underline{V}(x, y, z, t)$$

material (total) derivative

$$= \frac{\partial \underline{V}}{\partial t}\Big|_{\underline{x}} + \frac{\partial \underline{V}}{\partial x}\cdot\underbrace{\frac{dx}{dt}}_{u} + \frac{\partial \underline{V}}{\partial y}\cdot\underbrace{\frac{dy}{dt}}_{v} + \frac{\partial \underline{V}}{\partial z}\cdot\underbrace{\frac{dz}{dt}}_{w}$$

$$= \frac{\partial \underline{V}}{\partial t} + u\frac{\partial \underline{V}}{\partial x} + v\frac{\partial \underline{V}}{\partial y} + w\frac{\partial \underline{V}}{\partial z}.$$

$$= \frac{\partial \underline{V}}{\partial t} + (\underline{V}\cdot\nabla)\underline{V} \quad \left(\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

local acceleration (becomes zero if the

convective acceleration (due to non-uniform velocity distribution)

flow is indep. of time.
steady state)

$$u\frac{\partial u}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}(u^2)$$
(non-linear term)

1.6. Thermodynamic properties of a fluid.

$P,\ \rho,\ T,\ \hat{u},\ h\ (= \hat{u} + P/\rho),\ s,\ C_p,\ C_v \ldots\ \mu,\ k$

$\hat{u}$: internal energy

μ: mtm, k: heat

μ, k: transport properties.

$\rightarrow \rho = \rho(P,T),\ h = h(P,T),\ \mu = \mu(P,T)$ ....

- describes the state of the system.
- normally concerned w/ static system (no flow), but also valid for fluid flows.

(except, non-equilibrium processes like the chemical and nuclear reactions)

· total energy : $e = \hat{u} + \frac{1}{2}V^2 + gz$ ← potential e

internal e (molecular property)

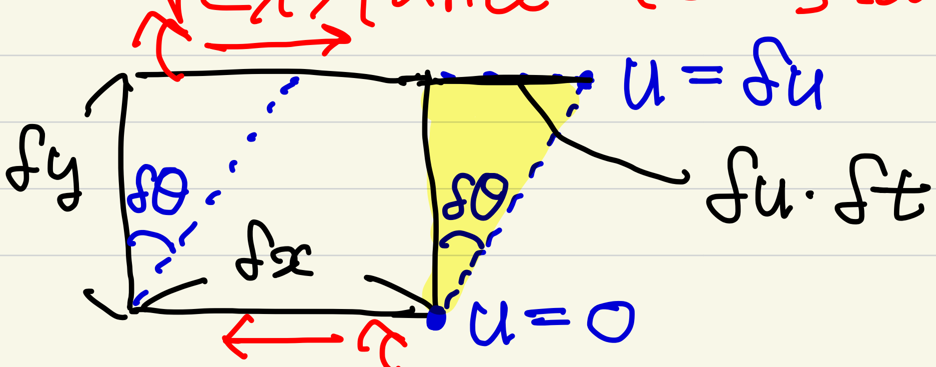
$\hat{u} = \hat{u}(p, T)$

kinetic e

kinematic properties.

1.7 Viscosity and other secondary properties.

↳ quantitative measure of a fluid's resistance to flow.



$\tau \sim \frac{\delta\theta}{\delta t}$ (strain rate)

$$\tan\delta\theta = \frac{\delta u \cdot \delta t}{\delta y}$$

$(\delta\theta \ll 1, \overset{\downarrow}{\delta\theta})$

$$\Rightarrow \lim_{\delta t \to 0} \frac{\delta\theta}{\delta t} = \frac{d\theta}{dt} = \frac{du}{dy}$$

$$\therefore \tau \sim \frac{d\theta}{dt} = \frac{du}{dy}$$ (velocity gradient) or shear. 속도구배.

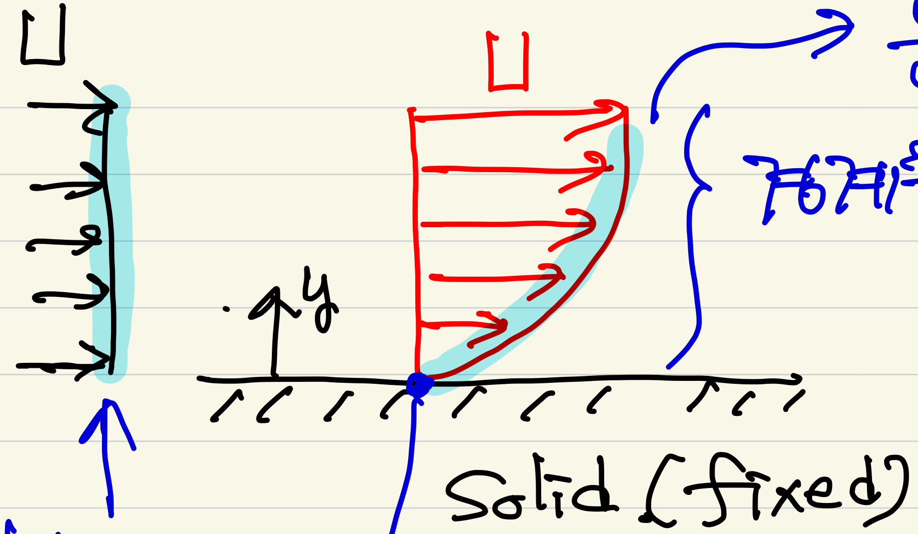
$$\boxed{\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}}$$

μ : proportionality constant.

= viscosity constant. $[ML^{-1}T^{-1}]$

Valid for Newtonian fluid. $\mu = \mu(P, T)$.

cf) for Non-newtonian fluid, $\mu = \mu(P, T, \frac{du}{dy})$.



$\frac{du}{dy} \neq 0$ (near the solid wall)

경계층 (boundary-layer)

마찰저항

$\frac{du}{dy} = 0$

no-slip condition. ($u_{fluid} = u_{wall}$)

* Reynolds Number (Re)

$$Re \equiv \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

V : Characteristic velocity
L : 〃 length

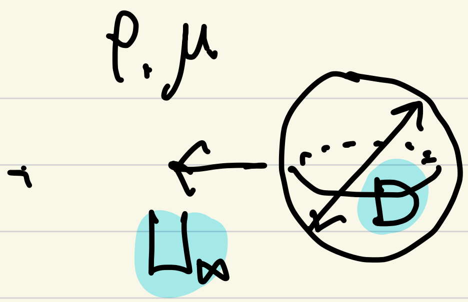
ρ : density, μ : viscosity — fluid property

$\nu = \mu / \rho$: kinematic viscosity $[L^2 T^{-1}]$

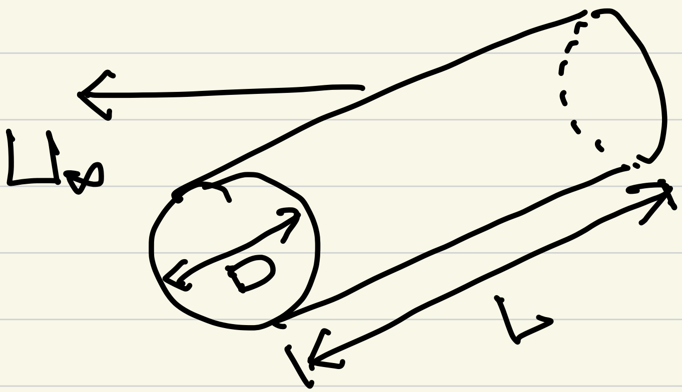
air : 1.5×10^{-5} m^2/s
water : 10^{-6}

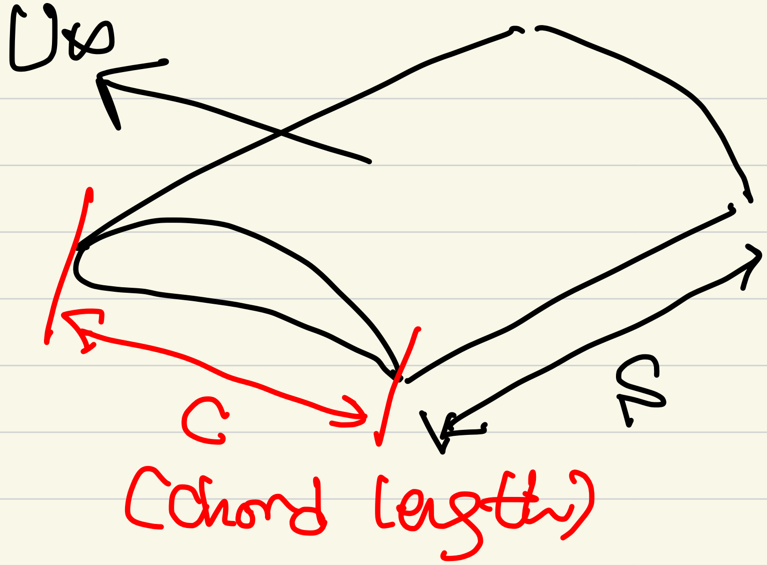
$$Re = \frac{\text{관성력 (inertia)}}{\text{점성력 (viscosity)}}$$

$$\frac{[LT^{-1}] \cdot [L]}{[L^2 T^{-1}]} = [L^0 T^0]$$: non-dimensional variable (무차원수)

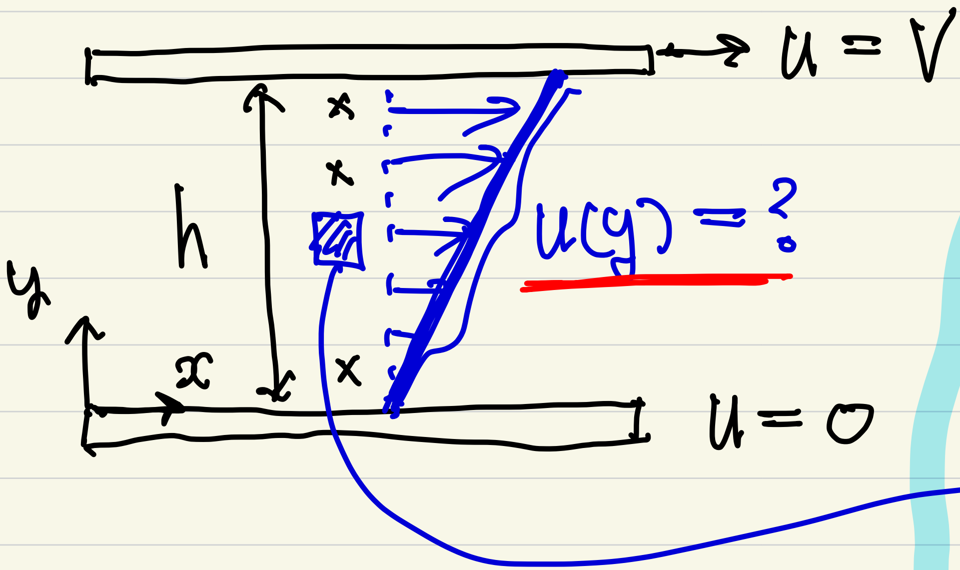


$$Re = \frac{\rho U_\infty D}{\mu}$$

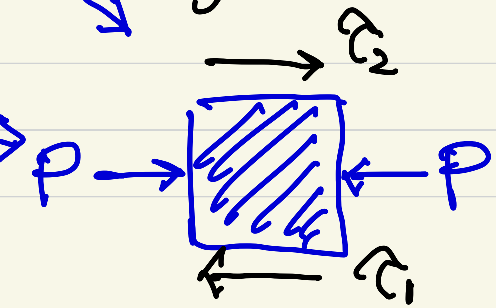




* Flow between plates (Couette flow)



- zero acceleration
- no pressure gradient along the flow direction.



$$\Sigma F_x = (\tau_1 - \tau_2)A = 0 \rightarrow \tau_1 = \tau_2 = \tau \ (= \text{constant})$$

$$\underset{\text{상수}}{\tau} = \underset{\text{상수}}{\mu} \frac{du}{dy} \rightarrow \frac{du}{dy} = \frac{\tau}{\mu} = \text{constant.}$$

$$\therefore u(y) = a + by \quad (\text{linear})$$

@ $y = 0$, $u = 0$

@ $y = h$, $u = V$. $\rightarrow u(y) = V \cdot \frac{y}{h}$.

e.g.) for SAE30 oil at 20°C. $V = 3\,m/s$, $h = 2\,cm$

$$\boxed{\tau = \mu \frac{du}{dy}} = \mu \cdot \frac{V}{h} \doteq 44\,Pa.$$

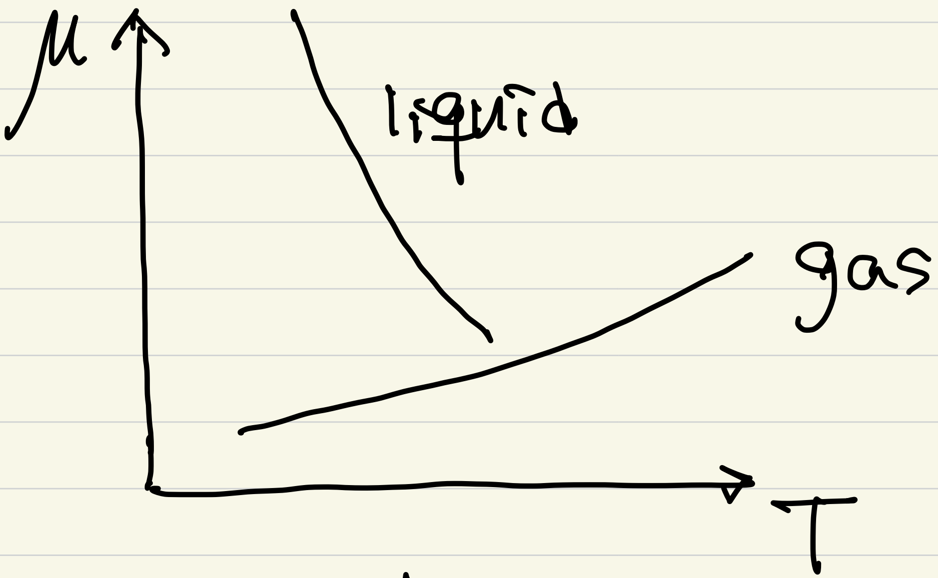
w/ with

w/o without

* Variation of viscosity w/ temperature.

$\mu = \mu(P, T)$. Strong dependency.

P: moderate (weak) dependency.



- gas viscosity.

$$\frac{\mu}{\mu_0} = \begin{cases} \left(T/T_0\right)^{n} & : \text{power law} \Rightarrow \boxed{\text{as } T\uparrow,\ \mu\uparrow} \\ \dfrac{\left(T/T_0\right)^{3/2} \cdot (T_0 + S)}{T + S} & : \text{Sutherland law.} \end{cases}$$

(μ_0 : @ $T_0 = 273K$)

(n, S : constant; $n \approx 0.7$, $S = 110K$)

for air

· liquid viscosity.

$$\ln\frac{\mu}{\mu_0} = a + b\left(\frac{T_0}{T}\right) + c\left(\frac{T_0}{T}\right)^2.$$

$\boxed{\text{as } T\uparrow,\ \mu\downarrow.}$

for water, $T_0 = 273\,K$, $\mu_0 = 0.001792$ kg/(ms)

$a = -1.94$, $b = -4.80$, $C = 6.74$

* thermal conductivity

viscosity → mtm tranfer (transport)

" → heat "

$$\bar{q} = -k\nabla T.$$: Fourier's Law of heat conduction.

rate of heat flow per unit

temperature gradient

area.

- heat moves from hot(T_1) to cold(T_2) spots

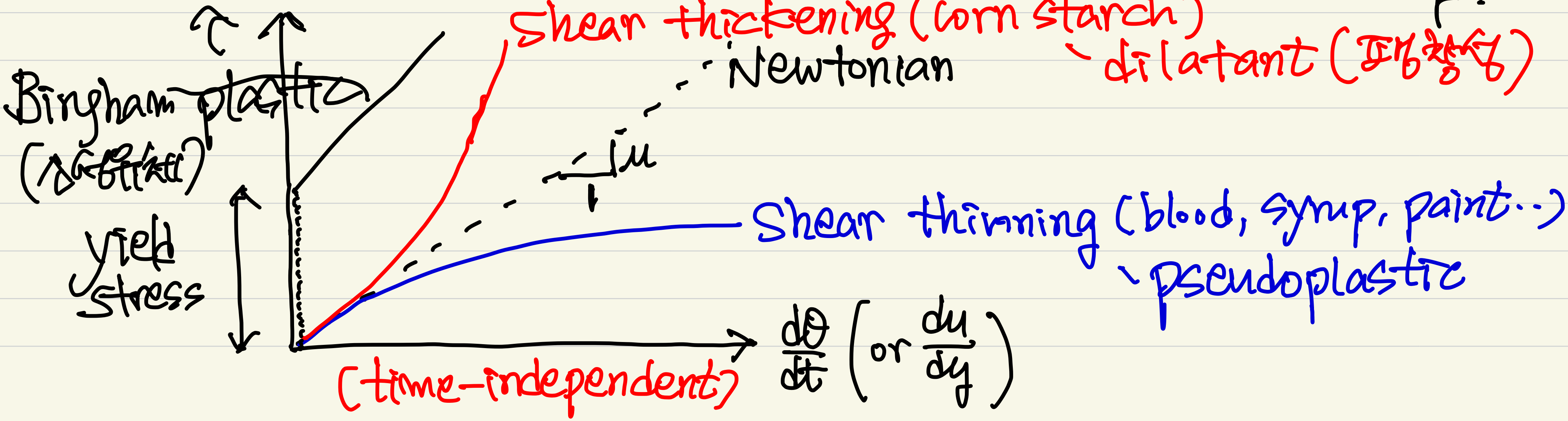
$$T_1 > T_2, \Rightarrow \frac{dT}{dx} < 0$$

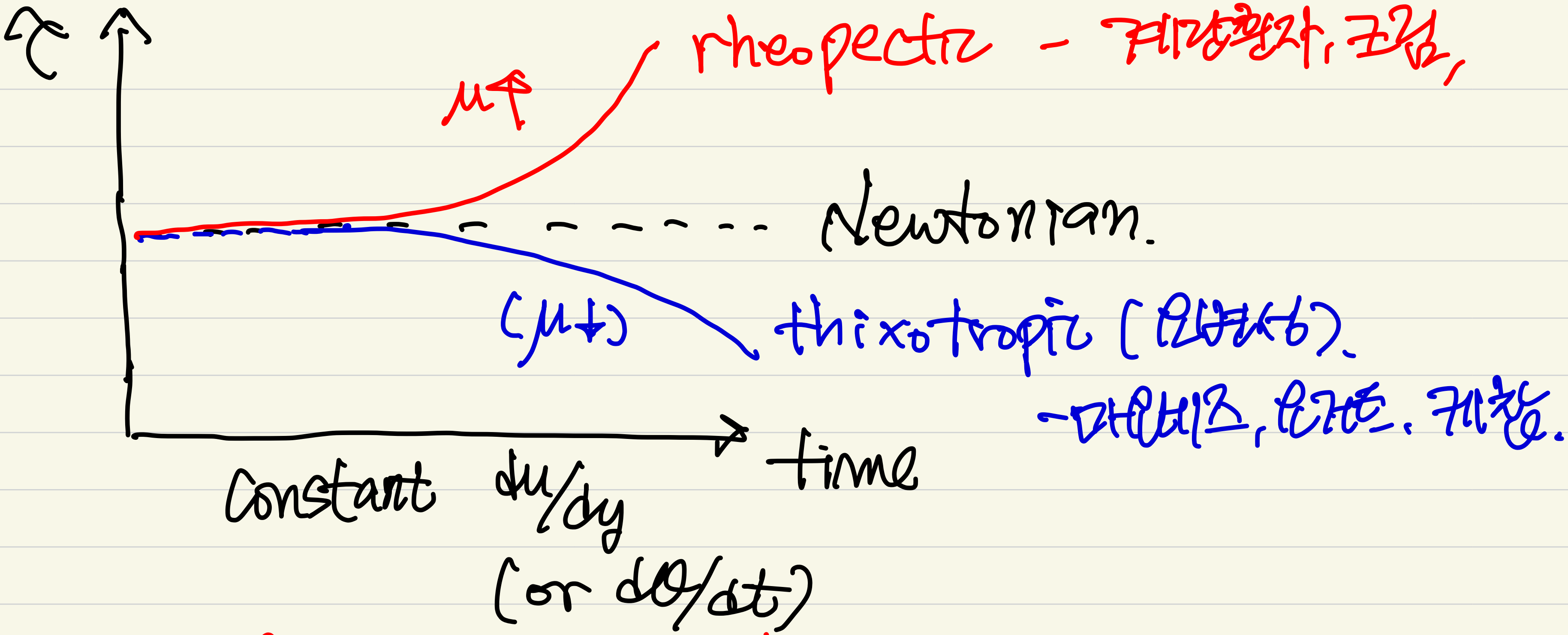
$$T_1 \xrightarrow[x]{} T_2 \Rightarrow \boxed{\bar{q} > 0} \Leftarrow \text{'}-\text{' needed.}$$

- $k = k(P, T)$

* Non-Newtonian fluids (비뉴턴 유체)

: do not follow the linear law of $\tau = \mu \frac{du}{dy}$.





(time-dependent)

$$\rightarrow \mu = \mu\left(P, T, \frac{du}{dy}\right).$$