Chapter 7

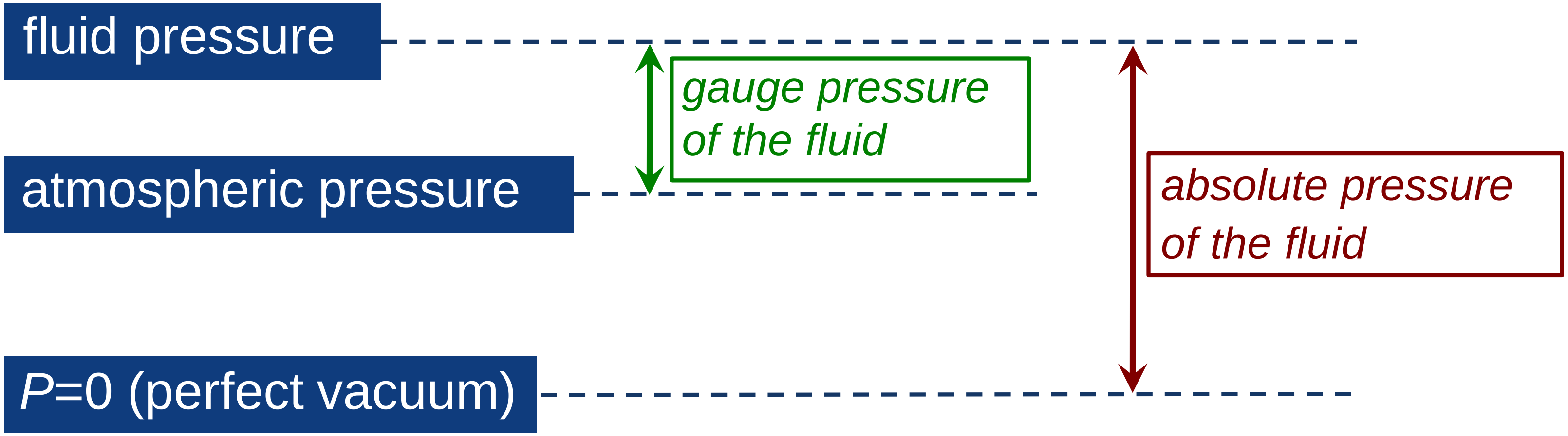
Fluid Flow

What is a fluid?

- Gas
  - loosely associated molecules that are not close together and that travel through space for long distances (many times larger than the molecular diameter) before colliding with each other
- Liquid
  - Molecules that are very close together (on the same order as their molecular diameter) and that are in collision with each other very frequently as they move around each other

The Concept of Pressure

- Absolute pressure
- Gauge pressure
  - Absolute pressure – Atmospheric pressure



Example 7.1

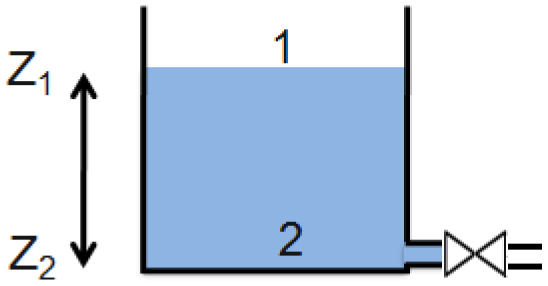
- Absolute pressure

  = Gauge pressure + Atmospheric pressure

  = 34.0 *psig* + 14.2 *psia* (usually 14.7 *psia*)

  = 48.2 *psia*
- *pounds* (*lb$_f$*) *per square inch* (*psi*)
  - *psia*
  - *psig*
- 1 *atm* = 14.7 *psi* = 760 *mmHg* = 101,300 *Pa*

Non-flowing (stagnant) Fluids



$$P_2 - P_1 = \rho\, g\, (z_1 - z_2)$$

ρ is fluid density

z is distance ***UPWARD***

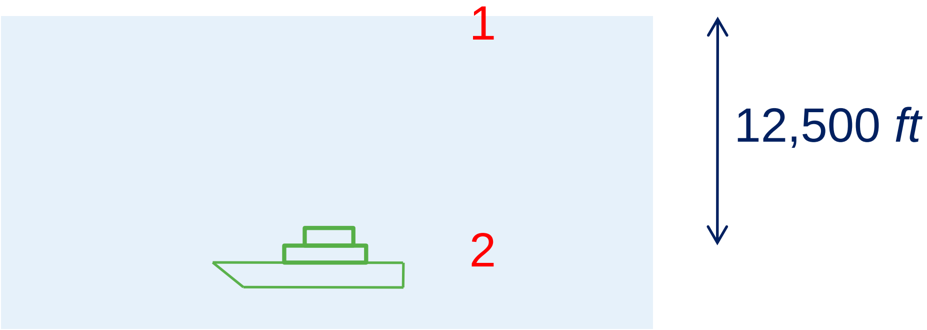
1 and *2* are locations in the liquid

Example

$$P_2 - P_1 = \rho g (z_1 - z_2)$$

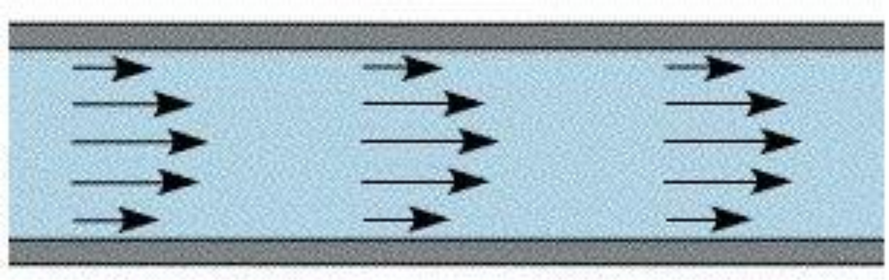
Example: The Titanic sank in 12,500 *ft*. What is the pressure (in *psi*) where she lies?

Note: the density of sea water is ~64.3 lb_m/ft^3

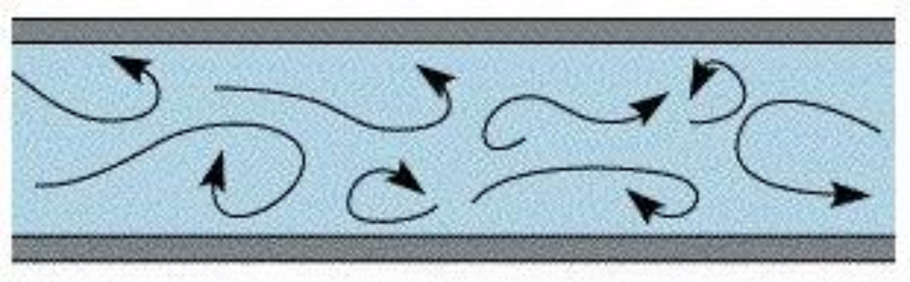


Principles of Fluid Flow

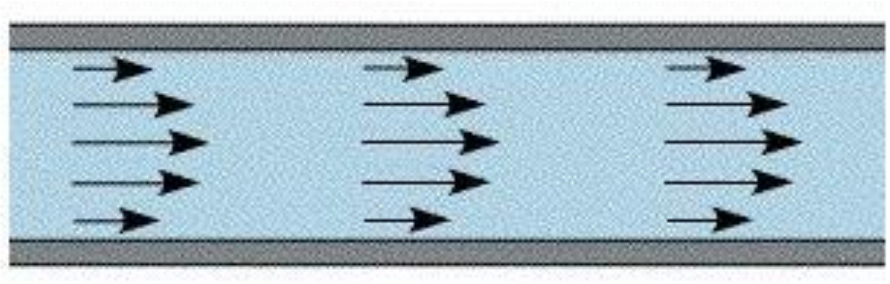
- Laminar flow



- Turbulent flow



Principles of Fluid Flow



- Average velocity (v_{avg})
  - Volumetric flow rate

$$\dot{V} = v_{avg} A_{cs}$$

A_{cs} : cross-sectional area

  - Mass flow rate

$$\rho \dot{V} = \dot{m} = \rho v_{avg} A_{cs}$$

Mechanical Energy Equation

For steady-state incompressible flow
(in the unit of energy per mass of fluid)

$$\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^{2}+gz\right)_{out}-\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^{2}+gz\right)_{in}=w_s-w_f$$

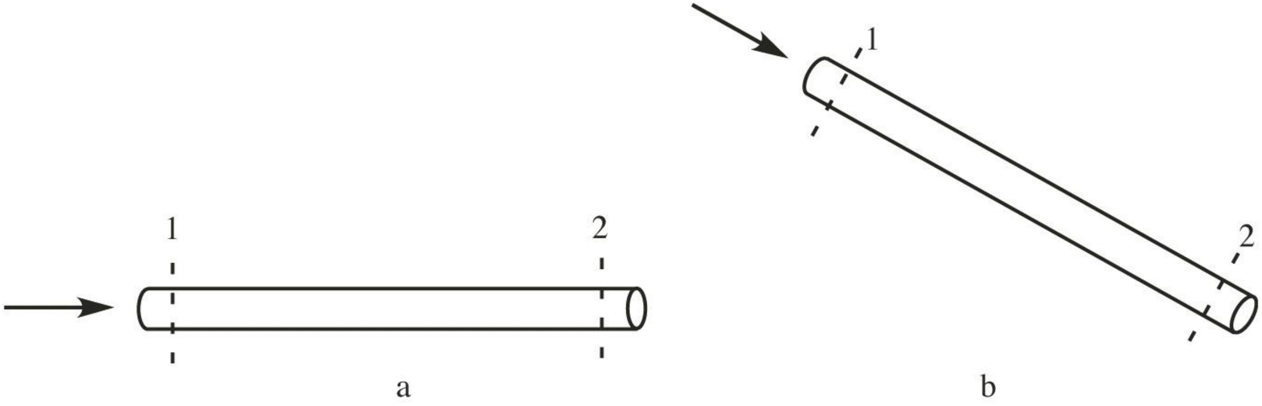


Figure 7.5

Mechanical Energy

- Kinetic energy
  - K.E.: $\frac{1}{2}\, m(v^2)_{avg}$
  - K.E. per mass: $\frac{1}{2}\, (v^2)_{avg} = \frac{1}{2}\, \alpha\, (v_{avg})^2$
  - α : a conversion factor from $(v_{avg})^2$ to $(v^2)_{avg}$
    - Can be assumed to equal 1.0
- Potential energy
  - P.E.: mgz
  - P.E. per mass: gz
- Energy associated with pressure
  - P : force/area, ρ : mass/volume
  - P/ ρ : energy/mass

Work and Friction

- Work (w_s)
  - This kind of work is called “shaft work”.
  - Positive when work is done on the fluid (e.g., by a pump)
  - Negative when the fluid does work on its environment (e.g., in a turbine)

- Friction (w_f)
  - Always positive

Mechanical Energy Equation

$$\frac{P_2-P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = w_s - w_f$$

Increase in fluid mechanical energy
(pressure + kinetic energy + potential energy)

Positive when work is done on the fluid

Always positive

Note 1: each grouping of variables has units of energy per mass of fluid. To cast the equation in terms of “**Power**” (energy/time), multiply all terms by mass flow rate

energy/mass x mass/time = energy/time

Special Case: No Friction or Shaft Work

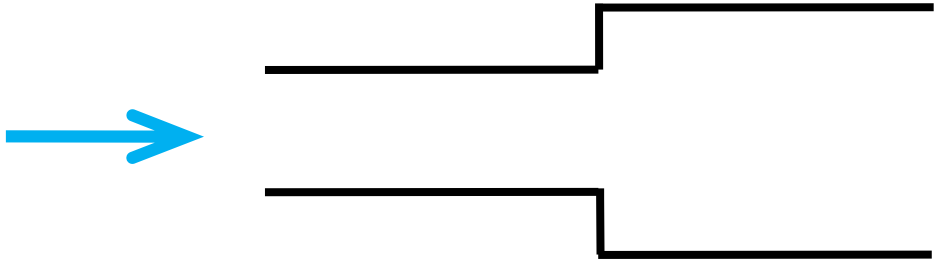
$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

Called the “Bernoulli Equation” after Daniel Bernoulli, a 19th Century fluid mechanics expert

For no work or friction

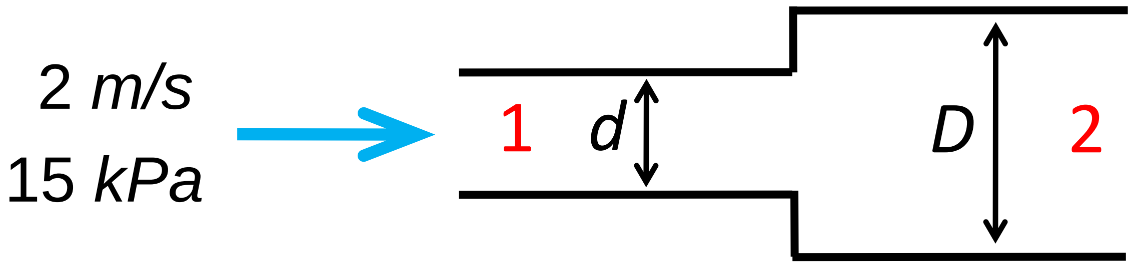
$$\frac{P_2-P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

What happens to the pressure in a horizontal pipe when it expands to a larger diameter?



Which form(s) of energy is (are) decreasing, and which is (are) increasing?

Example: Liquid Flow in an Expanding Pipe



a. What is the average velocity in the larger pipe?

$$v_{avg,2} = v_{avg,1} A_1/A_2 = v_{avg,1} d_1^2/D_2^2 = v_{avg,1}/4 = 0.5\ m/s$$

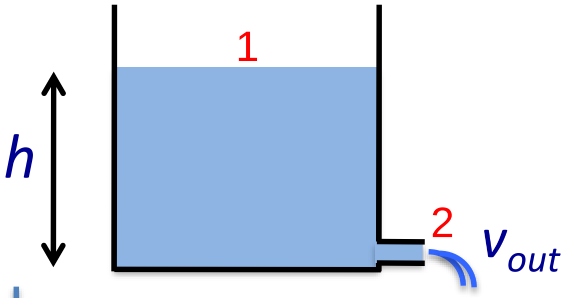
b. What is the pressure in the larger pipe?

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha\,(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

$$P_2 - P_1 = \frac{1}{2}\rho\,\alpha\,(v_1^2 - v_2^2)$$ The Pressure Increases!

Example: An Emptying Tank

Liquid in an open tank flows out through a small outlet near the bottom of the tank. Friction is negligible. What is the outlet velocity as a function of the height of the liquid in the tank?



$$P_1 = P_2 = 0$$

$$v_1 = 0 \qquad v_2 = v_{out}$$

$$z_2 - z_1 = -h$$

$$\alpha \approx 1$$

$$\frac{\overset{0}{\cancel{P_2}} - \overset{0}{\cancel{P_1}}}{\rho} + \frac{1}{2}\overset{1}{\cancel{\alpha}}(v_2^2 - \overset{0}{\cancel{v_1^2}}) + g(\overset{-h}{z_2 - z_1}) = 0$$

$$\frac{1}{2}v_{out}^2 - gh = 0$$

$$\boxed{v_{out} = \sqrt{2gh}}$$

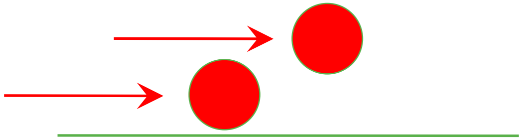
Torricelli's Equation

The Effects of Fluid Friction

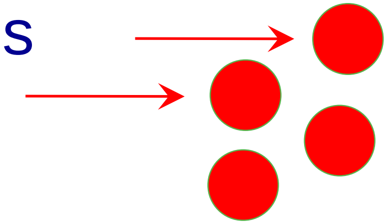
The mechanical energy equation says that friction (w_f) causes mechanical energy to decrease.

Friction is produced in flowing fluid, because fluid molecules...

- Flow past solid boundaries



- Flow past other fluid molecules



Friction in liquid flow through horizontal constant-diameter pipe:

$$v_1 = v_2$$

$$z_2 - z_1 = 0$$

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\,\alpha(\overset{0}{\cancel{v_2^2 - v_1^2}}) + g(\overset{0}{\cancel{z_2 - z_1}}) = -\,w_f$$

$$P_2 = P_1 - \rho w_f$$

Friction in liquid pipe flow *reduces pressure* (not velocity)

Pumps

Example 7.8
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$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha\cancelto{0}{(v_2^{\,2} - v_1^{\,2})} + g\cancelto{0}{(z_2 - z_1)} = w_s - w_f$$

$$P_2 = P_1 + \rho w_{pump} - \rho w_f$$

Pumps

$$Power = \frac{work}{time} = \dot{m}\,\overset{\text{work per mass}}{w_s} = \rho \dot{V}\, w_s$$

$$\text{Pump Efficiency} = \frac{\text{Power delivered to the fluid}}{\text{Power to operate the pump}}$$

Turbines

- The calculated power
  - Power extracted from the fluid using a perfect turbine
  - Actual power delivered by the turbine is smaller than that value. (friction loss, mechanical inefficiencies, etc.)

$$\text{Turbine Efficiency} = \frac{\text{Power delivered by the turbine}}{\text{Power extracted from the fluid}}$$

Chapter 8

Mass Transfer

Mass Transfer

- Molecular Diffusion
  - Concentration difference

- Mass Convection
  - By bulk fluid flow

Molecular Diffusion

- Random movement (in liquids, called Brownian motion)
- Molecules of one species (A) moving through a stationary medium of another species(B)

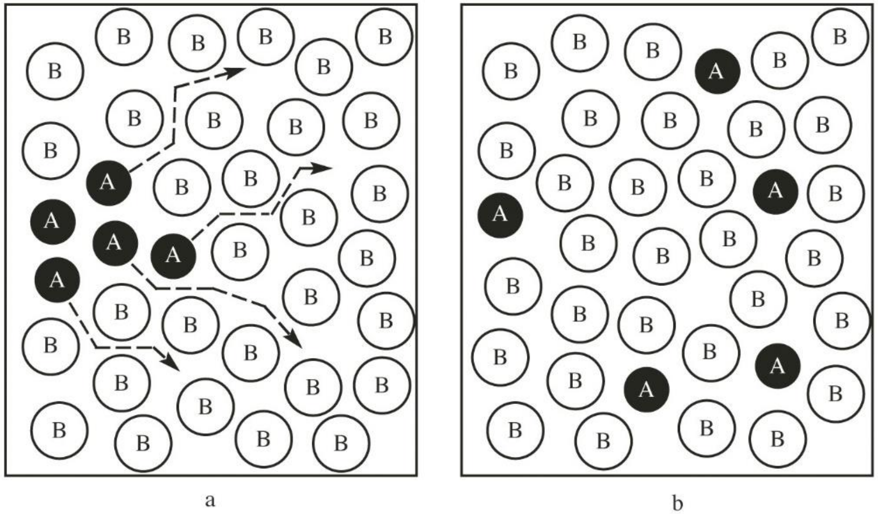

Figure 8.1
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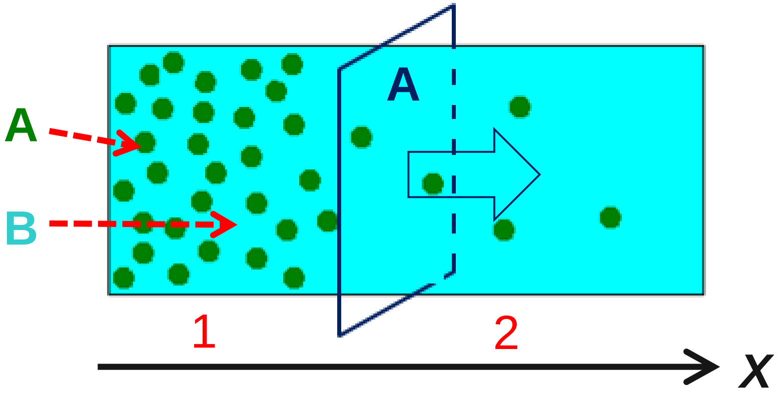
Fick’s Law



(“A” diffusing through “B”)

$$\dot{N}_A = - D_{AB}\, A \, \frac{c_{A,2} - c_{A,1}}{x_2 - x_1}$$

$\dot{N}_A$ = moles of “A” transferred per time from “1” to “2”

D_{AB} = “diffusivity” of “A” diffusing through “B”

A = area through which diffusion occurs (cross-section)

Fick’s Law

$$\dot{N}_A = -D_{AB} A \frac{c_{A,2} - c_{A,1}}{x_2 - x_1}$$

Transfer rate = Driving force / Resistance

Analogy with Ohm’s Law

$$I = \frac{V}{R} \qquad \dot{N}_A = \frac{c_{A,1} - c_{A,2}}{R} = \frac{c_{A,1} - c_{A,2}}{\left(\frac{x_2 - x_1}{D_{AB} A}\right)}$$

What molecular variables affect D_{AB}?

molecular size, shape, charge, temperature

Diffusion in Contact Lens

- “Hard lenses” (polymethylmethacrylate)
  - physically uncomfortable
  - inadequate oxygen diffusion (irritation, inflammation)
- “Soft lenses” (hydrocarbon hydrogels)
  - physically more comfortable
  - inadequate oxygen diffusion
- “Oxygen permeable” (siloxane)
  - physically uncomfortable
  - better oxygen diffusion
- Latest (siloxane hydrogels)
  - physically more comfortable
  - better oxygen diffusion

Mass Convection

- Flow-enhanced transfer of one species moving through another species.

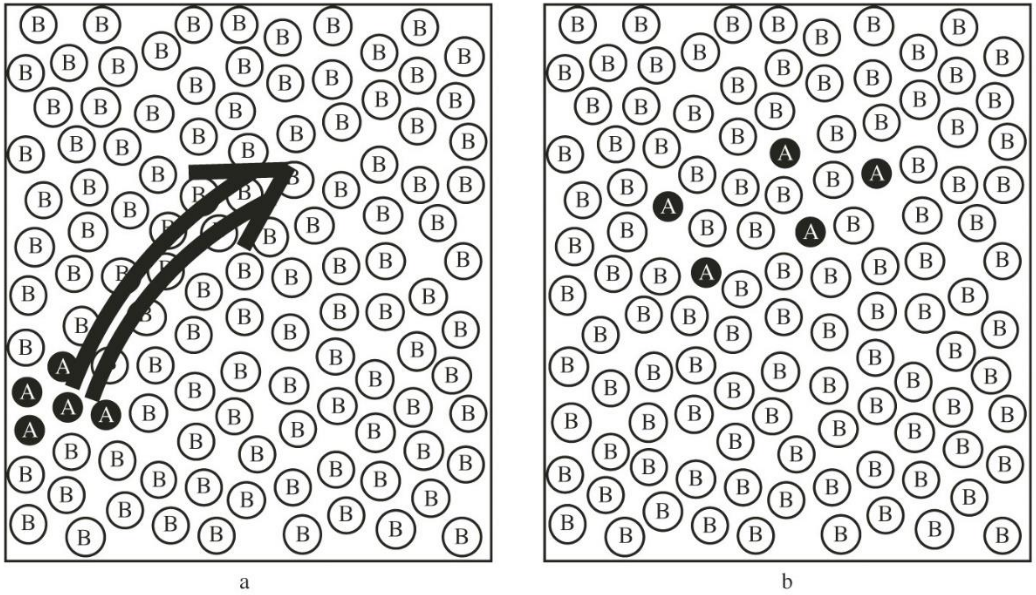


Figure 8.3

Mass Transfer across Phase Boundaries

- Mass convection + Molecular diffusion
  - Mass convection >> Molecular diffusion
- Phase boundaries
  - Liquid/Gas, Solid/Liquid, Liquid/Liquid

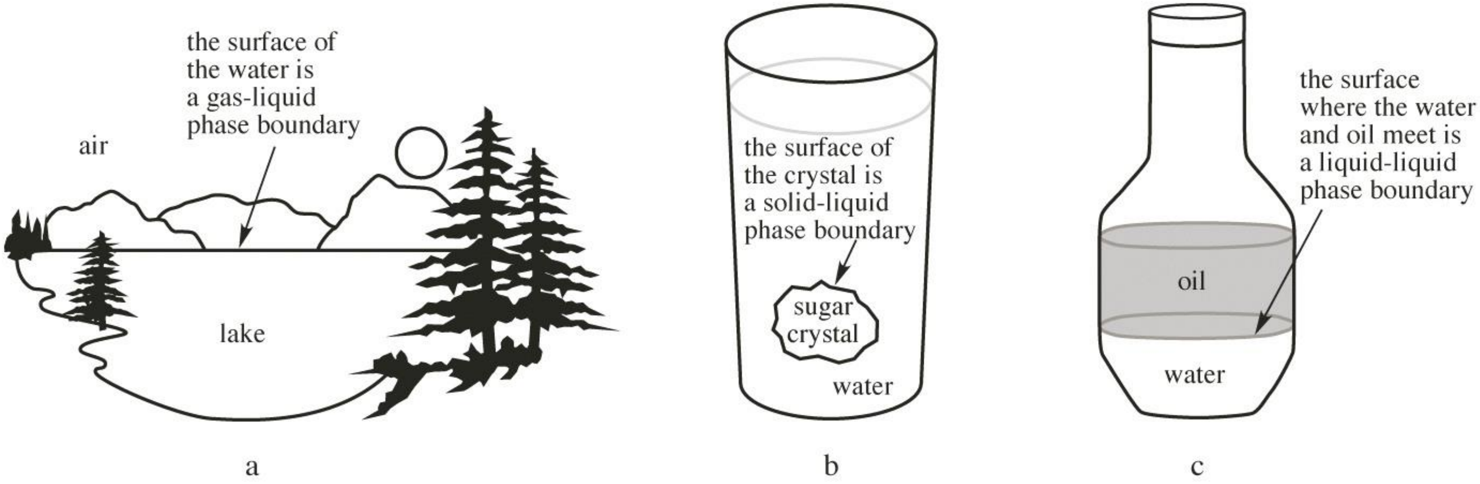


Figure 8.4

Mass Transfer across Phase Boundaries

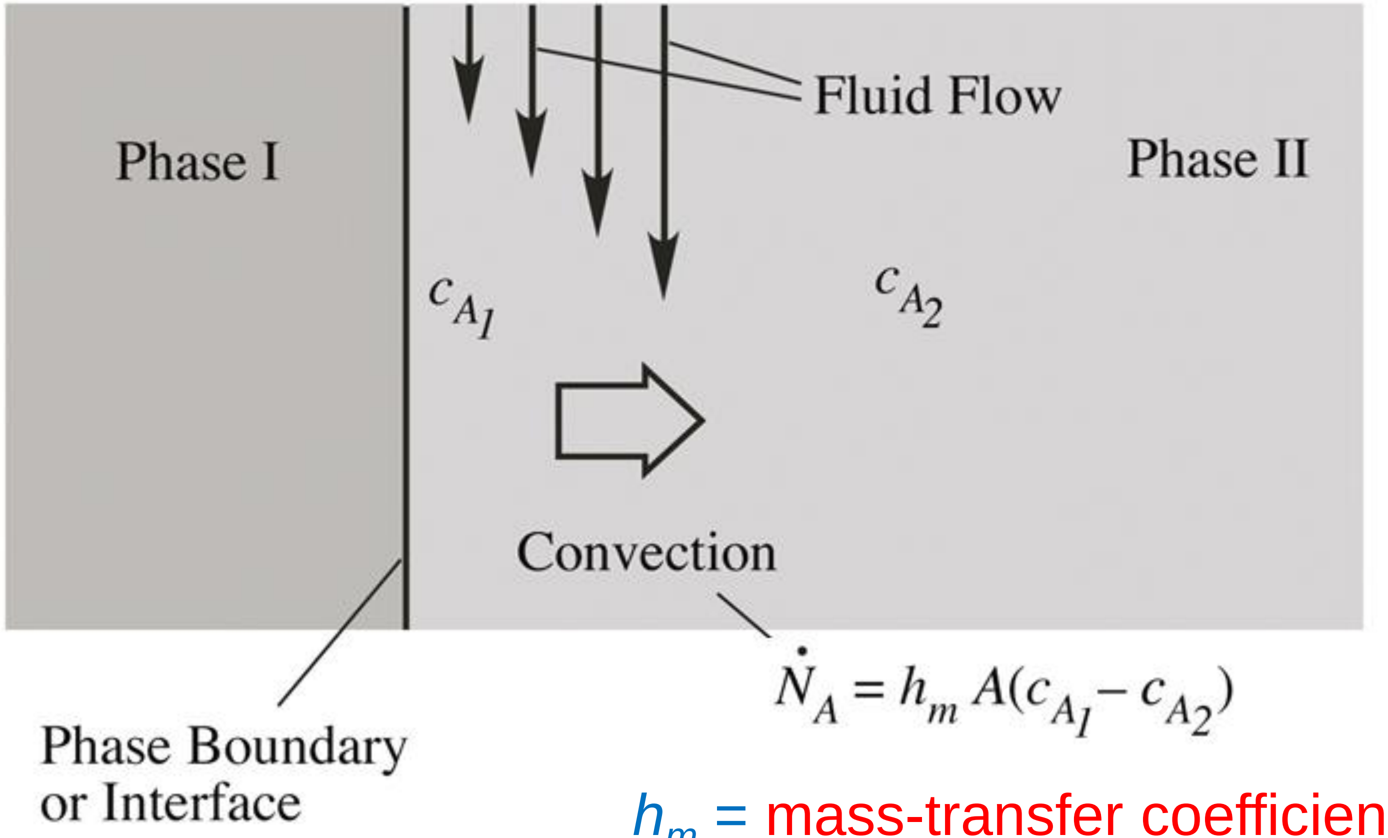


Figure 8.5

h_m = mass-transfer coefficient

c_{A1} = concentration of "A" at the phase boundary in phase II

Mass Transfer across Phase Boundaries

$$\dot{N}_A = h_m A(c_{A,1} - c_{A,2})$$

Analogy with Ohm's Law

$$I = \frac{V}{R} \qquad \dot{N}_A = \frac{c_{A,1} - c_{A,2}}{R} = \frac{c_{A,1} - c_{A,2}}{\left(\frac{1}{h_m A}\right)}$$

What variables affect h_m?

- *flow patterns (depends on geometry, etc.)*
- *molecular size*
- *molecular shape*
- *molecular charge*
- *temperature*

Ex. 8.1. The level of a lake drops throughout the summer due to water evaporation.

(a) How much volume will the lake lose per day to to evaporation?

(b) How long will it take for the water level to drop 1m?

conc. of water at the water surface	$1.0\times10^{-3} kgmol/m^3$
conc. of water in the wind	$0.4\times10^{-3} kgmol/m^3$
area of the lake	$1.7 mi^2$
mass transfer coefficient	$0.012 m/s$
density of the lake water	$1000 kg/m^3$

Multi-Step Mass Transfer

- Membrane Separation

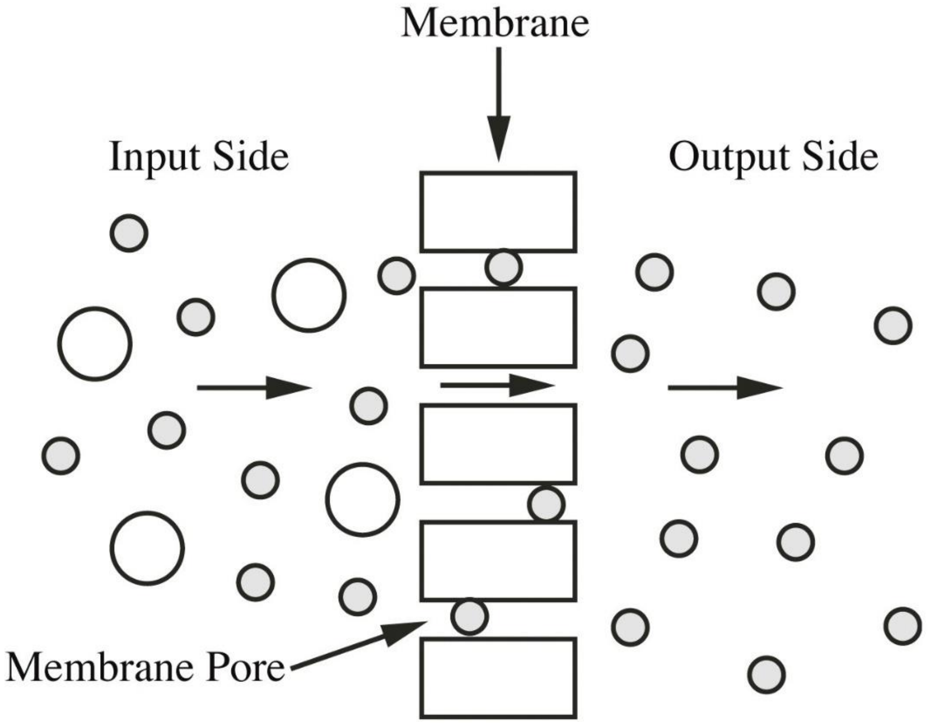


Figure 8.6
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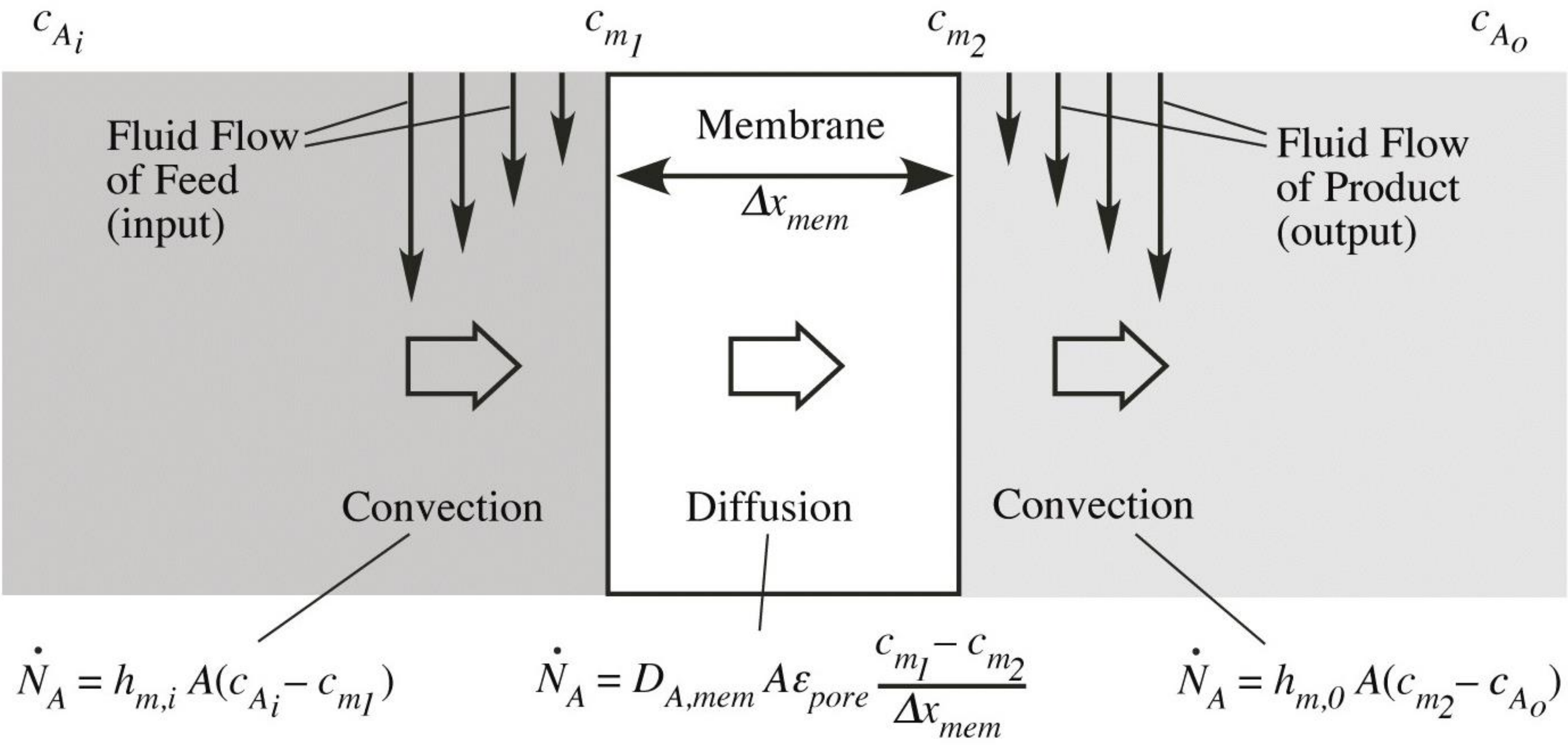
Membrane Separation



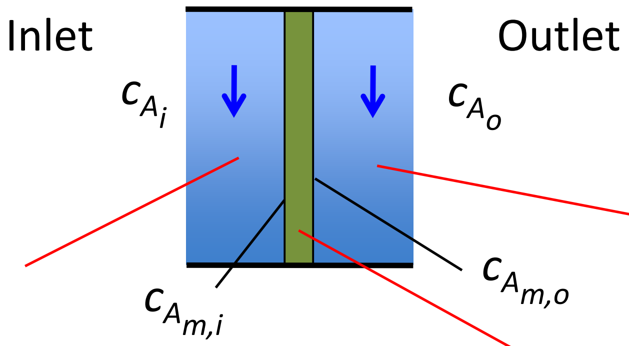
$$\dot{N}_A = h_{m,i}\,A(c_{A_i} - c_{m_1})$$

$$\dot{N}_A = D_{A,mem}\,A\varepsilon_{pore}\,\frac{c_{m_1} - c_{m_2}}{\Delta x_{mem}}$$

$$\dot{N}_A = h_{m,0}\,A(c_{m_2} - c_{A_o})$$

Figure 8.7

Membrane Separation



$$\dot{N}_A = h_{m_i} A(c_{A_i} - c_{A_{m,i}})$$

$$\dot{N}_A = h_{m_o} A(c_{A_{m,o}} - c_{A_o})$$

$$\dot{N}_A = D_{A,m} A \varepsilon_{pore} \frac{c_{A_{m,i}} - c_{A_{m,o}}}{\Delta x_m}$$

Solving for the concentration differences

$$c_{A_i} - c_{A_{m,i}} = \frac{\dot{N}_A}{h_{m_i} A}$$

$$c_{A_{m,i}} - c_{A_{m,o}} = \frac{\dot{N}_A \Delta x_m}{D_{A,m} A \varepsilon_{pore}}$$

$$c_{A_{m,o}} - c_{A_o} = \frac{\dot{N}_A}{h_{m_o} A}$$

Summing these…

$$c_{A_i} - c_{A_o} = \dot{N}_A \left(\frac{1}{h_{m_i} A} + \frac{\Delta x_m}{D_{A,m} A \varepsilon_{pore}} + \frac{1}{h_{m_o} A} \right)$$

Membrane Separation

$$\dot{N}_A = \frac{c_{A_i}-c_{A_o}}{\frac{1}{h_{m_i}A}+\frac{\Delta x_m}{D_{A,m}A\,\varepsilon_{pore}}+\frac{1}{h_{m_o}A}} = \frac{\textit{overall driving force}}{\sum\ \textit{resistances}}$$

Concept: Limiting Resistance

$$\text{Total Resistance} = \frac{1}{h_{m_i}A}+\frac{\Delta x_m}{D_{A,m}A\,\varepsilon_{pore}}+\frac{1}{h_{m_o}A}$$

$$= \begin{array}{l}\text{convection}\\ \text{resistance on}\\ \text{the inlet side}\end{array} + \begin{array}{l}\text{diffusion}\\ \text{resistance in}\\ \text{the membrane}\end{array} + \begin{array}{l}\text{convection}\\ \text{resistance on}\\ \text{the outlet side}\end{array}$$

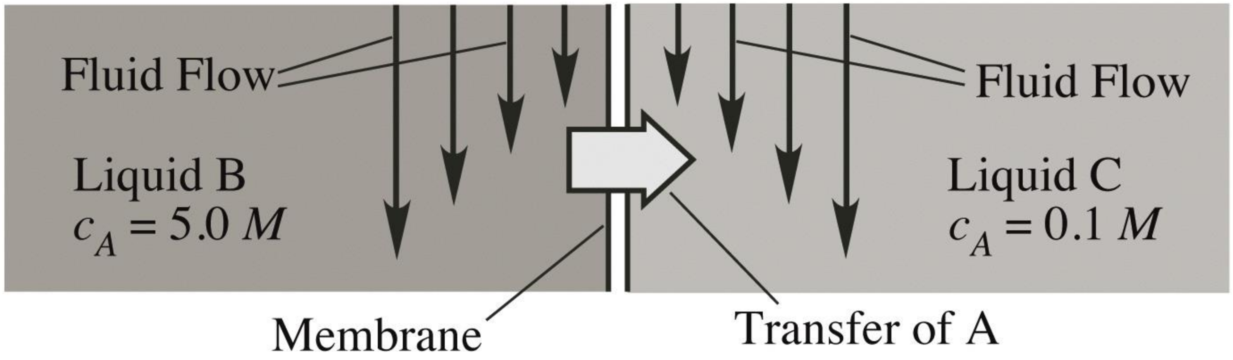
If one resistance >> the others, changing the others will not change the total resistance significantly

Ex. 8.2. Liquid B flows on one side of a membrane, and liquid C flows along the other side. Species A present in both liquids transfers from liquid B into liquid C.

(a) What is the transfer rate of A from B to C?

(b) Calculate the limiting resistance.

conc. of A in liquid B	$5.0M$
conc. of A in liquid C	$0.1M$
thickness of the membrane	$200\mu m$
diffusivity of A in the membrane	$1.0\times10^{-9}m^2/s$
area of membrane	$1m^2$
porosity of membrane	70%
mass transfer coefficient on side B	$7.0\times10^{-4}m/s$
mass transfer coefficient on side C	$3.0\times10^{-4}m/s$



Example 8.2

Ex. 8.3. In patient with severe kidney disease, urea must be removed from the blood with a hemodialyzer. In that device, the blood passes by special membranes through which urea can pass. A salt solution (dialysate) flows on the other side of the membrane to collect the urea and to maintain the desired concentration of vital salts in the blood.

(a) What is the initial removal rate of urea? (Note. This rate will decrease as the urea concentration in the blood decrease.)

(b) One might be tempted to try to increase the removal rate of urea by developing better hemodialyzer membrane. Is such an effort justified?

Blood side	
mass transfer coeff. for the urea	$0.0019 cm/s$
urea conc. within the dialyzer	$0.020 gmol/l$
Dialysate side	
mass transfer coeff. for the urea	$0.0011 cm/s$
urea conc. within the dialyzer	$0.003 gmol/l$
Membrane	
thickness	$0.0016 cm$
diffusivity of urea in the membrane	$1.8 \times 10^{-5} cm^2/s$
total membrane area	$1.2 m^2$
porosity	20%

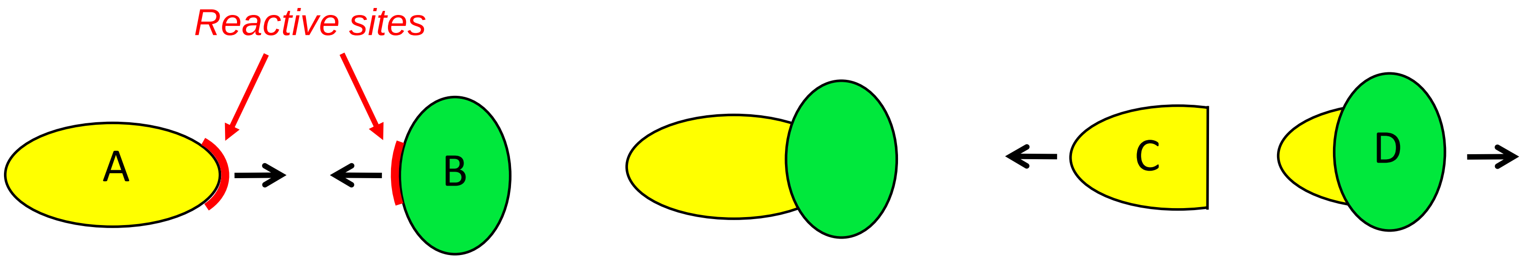
Chapter 9

Reaction Engineering

Reaction Rate

$A + B \rightarrow C + D$

- Frequency of Molecular Collision
  - Depends on concentrations
  - Depends on velocity of moving molecules
    - The velocity depends on the temperature.
- Orientation and Force of the Collision
  - Not all collisions lead to reaction



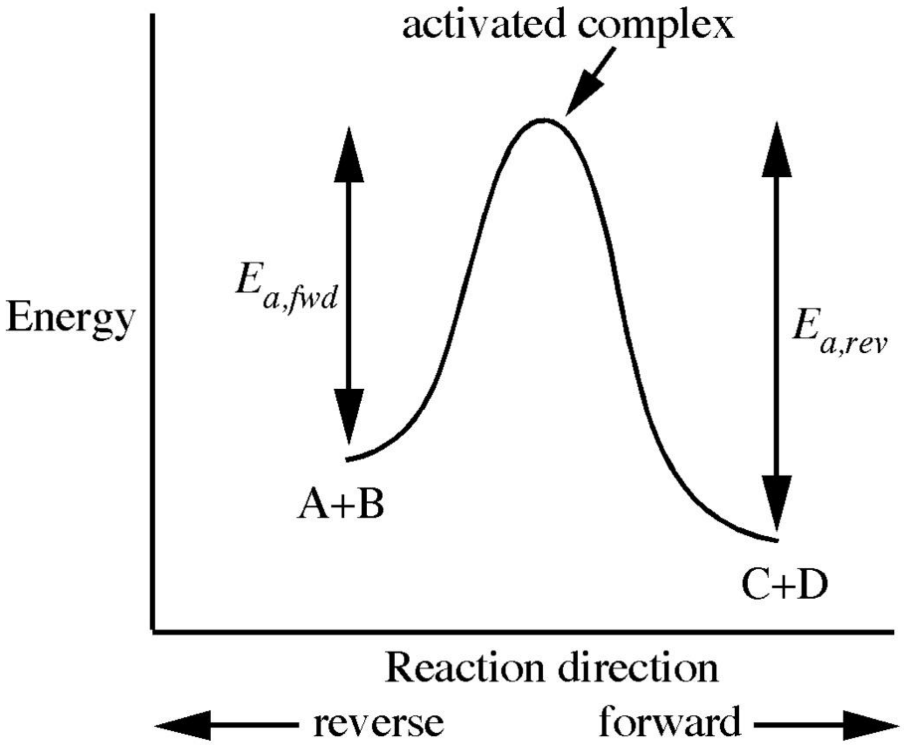
Collision

Activated Complex

Transformation/ Separation

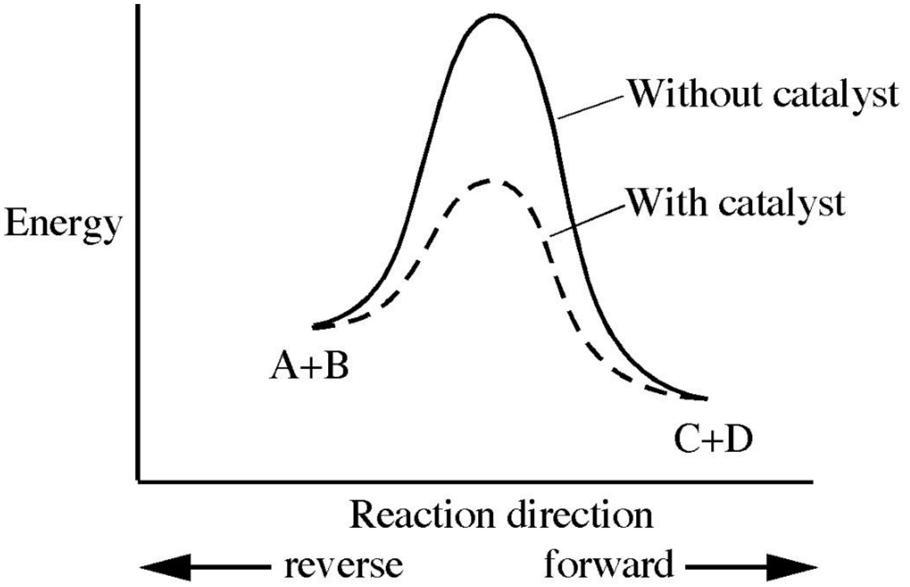
Reaction Rate

- Energy Requirements of the Reaction



Reaction Rate

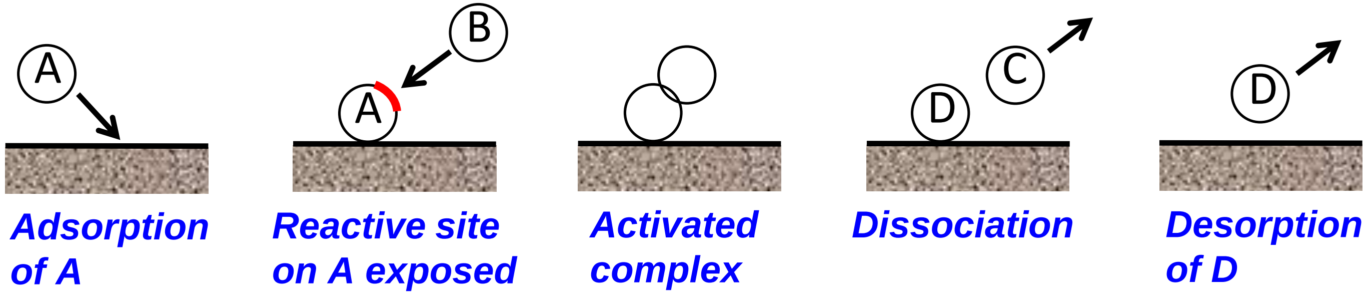
- How could you alter reactor conditions to increase the reaction rate?
  - Increase temperature (collision rate, energy)
  - Increase pressure or concentration (collision rate)
  - Use catalysts (activation energy)



Catalyst

- Catalysts are porous.

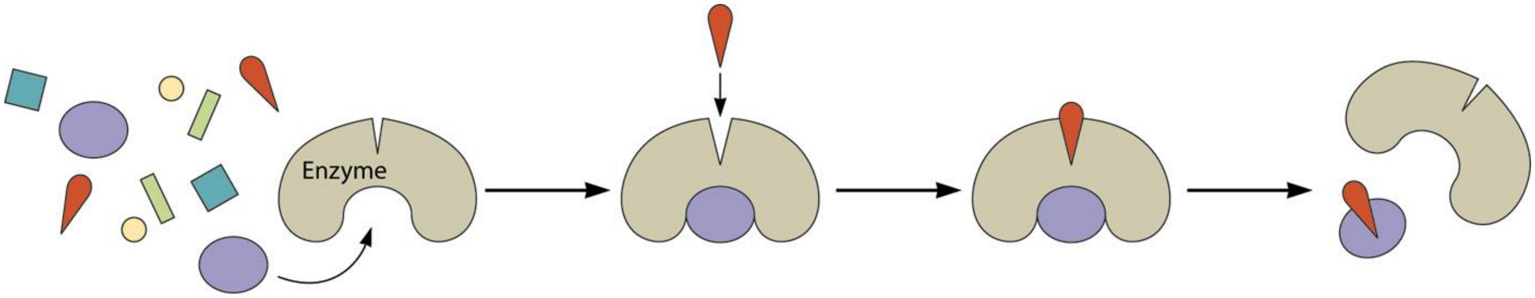
1. Reactants diffuse into the pores.
2. Reaction proceeds.



3. Products diffuse out of the pores.

Biocatalyst (Enzyme)

- Enzyme
  - Protein which accelerates chemical reactions



Reversible Reaction

- Irreversible Reaction
  - when the reverse reaction rate is insignificant compared with the forward reaction rate

$$A + B \rightarrow C + D$$

- Reversible Reaction
  - if the reverse reaction rate is significant

$$A + B \rightleftharpoons C + D$$

Reaction Rate

A + B → C + D

$$\text{Reaction rate} \quad \left(\frac{\text{moles of } A}{\text{time} \quad \text{volume}}\right)$$

$$r_{reaction,A} = k_r \, c_A^{\,n} \, c_B^{\,m}$$

$$r_{consumption,A} = r_{reaction,A} \, V_{reactor}$$

- “Order” of this reaction
  - Overall order: n+m
  - with respect to reactant A: n

Reaction Rate

$$A + B \rightarrow C + D$$

Liquid: $r_{reaction,A} = k_r \, c_A{}^n \, c_B{}^m$

Gas: $r_{reaction,A} = k_r \, p_A{}^n \, p_B{}^m$

(p = partial pressure)

- Reaction Rate Constant

  $$k_r = k_0 \, e^{-E_a / RT}$$

  - k_0 : frequency factor
  - E_a : activation energy

Arrhenius Equation

- Reaction Rate Constant

$$k_r = k_0\, e^{-E_a / RT}$$

  - k_0 : frequency factor (with the same unit as k_r)
  - E_a : activation energy (in units of energy per mole)
  - R : universal gas constant
  - T : absolute temperature

Reaction Rate

- Reversible Reaction

$$\mathrm{A + B} \underset{k_r'}{\overset{k_r}{\rightleftharpoons}} \mathrm{C + D}$$

$$r_{reaction,A} = k_r\, c_A^{\,n}\, c_B^{\,m} - k_r'\, c_C^{\,r}\, c_D^{\,s}$$

Reactor

- Batch Reactor
- Continuous Reactor
  - Plug-Flow Reactor (PFR)
  - Continuously Stirred Tank Reactor (CSTR), Chemostat

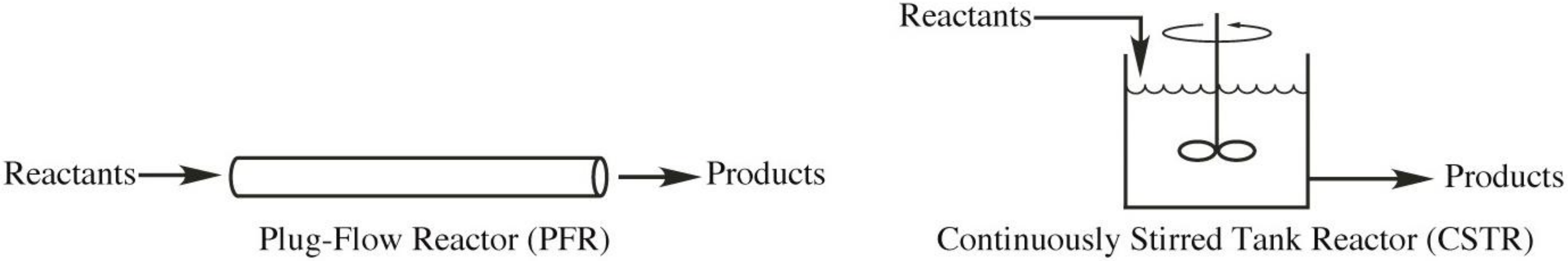
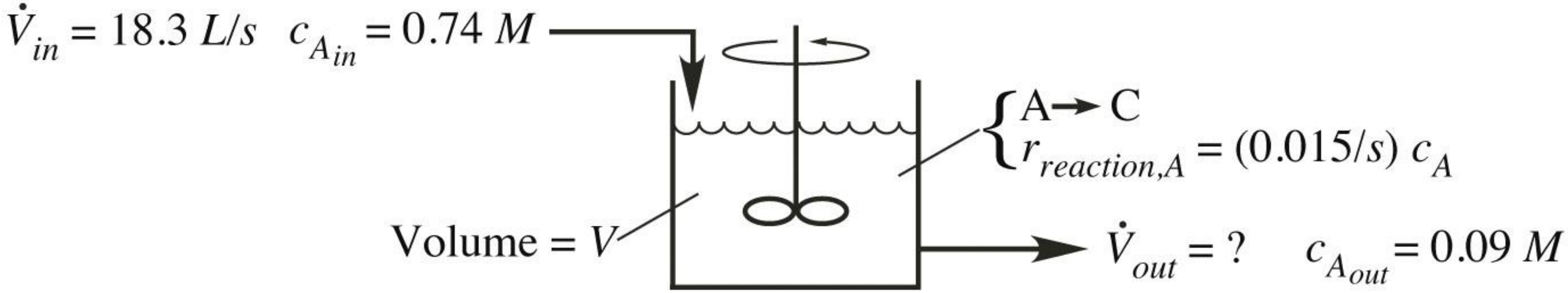


Figure 9.4

Ex. 9.2. Species A in liquid solution (concentration=0.74M) enters a CSTR at 18.3 L/s, where it is consumed by the irreversible reaction

A → C where $r_A = k_r c_A$ ($k_r = 0.015/s$ and c_A is in units of *gmol/L*)

What reactor volume is needed so that the concentration of A leaving the reactor equals 0.09M? The density can be assumed to be constant.



Example 9.2

Ex. 9.3. In the design of a process, separate liquid streams of pure species A and B will enter a CSTR, where they will be consumed by the irreversible reaction:

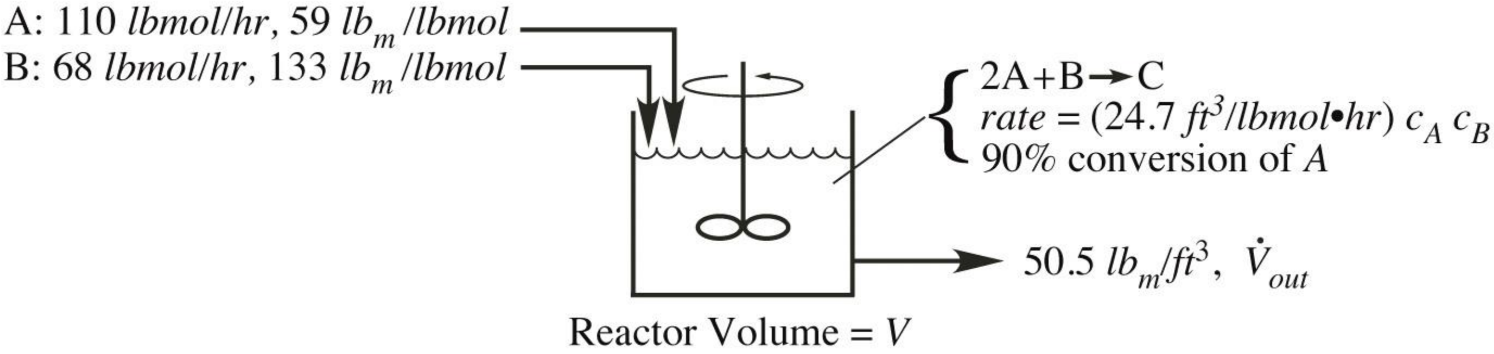
2A+B → C where $r_A = k_r c_A c_B$

$$k_r = 24.7\, ft^3 / lbmol\, hr, \quad c_A, c_B \; in\; lbmol / ft^3$$

$$species A: \dot{n}_A = 110\, lbmol / hr \quad (MW = 59 lb_m / lbmol)$$

$$species B: \dot{n}_B = 68\, lbmol / hr \quad (MW = 133 lb_m / lbmol)$$

In the reactor, 90% of species A is to be reacted, and the output stream will have a density of $50.5 lb_m / ft^3$. What volume must the reactor have?



Example 9.3

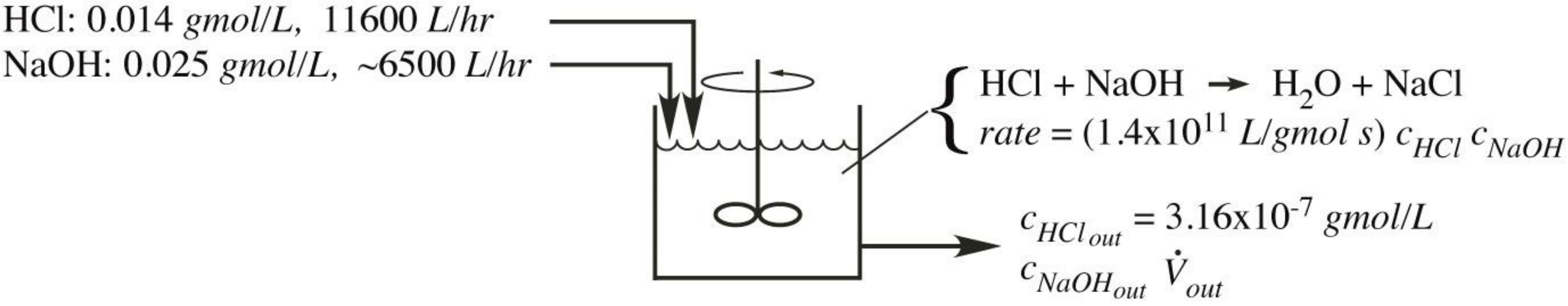


Figure 9.6

State law: the lowest pH allowable for water added to a lake or river is 6.5

$$\rightarrow c_{HCl} = 10^{-6.5} = 3.16 \times 10^{-7}\, M$$

Mole balance on HCl $$c_{HCl,in}\dot{V}_{HCl,in} = c_{HCl,out}\dot{V}_{out} + r_{consumption,HCl}$$

Mole balance on NaOH $$c_{NaOH,in}\dot{V}_{NaOH,in} = c_{NaOH,out}\dot{V}_{out} + r_{consumption,NaOH}$$

Total mass balance $$\dot{V}_{HCl,in} + \dot{V}_{NaOH,in} = \dot{V}_{out}$$

Stoichiometry $$r_{consumption,HCl} = r_{consumption,NaOH}$$

Molar flow rate balance:

The flow rate given in Figure 9.6 is given as an approximate value, because the pH of the final solution is very sensitive to the balance between HCl and NaOH. Thus we will vary the NaOH flow rate carefully as we monitor the pH. For the sake of our present calculation, we need to preset the inlet molar flow rate of NaOH to balance against the molar flow rate of HCl.

$$r_{consumption,HCl} = r_{consumption,NaOH} \qquad c_{NaOH,in}\dot{V}_{NaOH,in} = c_{HCl,in}\dot{V}_{HCl,in}$$

$$\dot{V}_{HCl,in} + \dot{V}_{NaOH,in} = \dot{V}_{out} \qquad \dot{V}_{out} = 11,600L/hr + 6500L/hr = 18,100L/hr$$

$$c_{HCl,in}\dot{V}_{HCl,in} = c_{HCl,out}\dot{V}_{out} + r_{consumption,HCl} \qquad r_{consumption,HCl} = 162gmol/hr$$

$$c_{NaOH,out} = c_{HCl,out}$$

$$r_{consumption,HCl} = k_r c_{HCl,out} c_{NaOH,out} V \qquad V = 3.22L$$

very small reactor -> the reaction proceeds so rapidly that only a very small residence time in the reactor is needed to achieve the desired results.

Chapter 10

Heat Transfer

Energy Balance for a Closed System

- *First Law of Thermodynamics for a closed system* (a fixed volume or space with no streams entering or leaving the system)

$$\Delta E = Q + W \qquad (10.1)$$

E = total energy of a system

Q = heat transferred into the system (e.g. through the boundaries)

W = work done on the system

Energy Balance for an Open System

- Open system
  - a system with streams entering and leaving
- For a steady-state open system

$$\sum_{\substack{output\\streams}} \left\{\dot{m}\,\hat{E}\right\}_{out} - \sum_{\substack{input\\streams}} \left\{\dot{m}\,\hat{E}\right\}_{in} = \dot{Q} + \dot{W} \qquad (10.2)$$

$\dot{m}$ = mass flow rate of a stream (units of mass per time)

$\hat{E}$ = energy per mass of a stream of flowing material

$\dot{Q}$ = rate of transfer of energy across the boundaries of a stream into the system (units of energy per time)

$\dot{W}$ = rate that work is done on a system (units of energy per time)

Energy Balance for an Open System

$$\hat{E}_{total} = \hat{E}_{\text{int}ernal} + \hat{E}_{kinetic} + \hat{E}_{potential} = \hat{U} + \frac{1}{2}\alpha v^2 + gz \tag{10.3}$$

$$\sum_{\substack{output\\streams}} \left\{\dot{m}\,\hat{E}\right\}_{out} - \sum_{\substack{input\\streams}} \left\{\dot{m}\,\hat{E}\right\}_{in} = \dot{Q} + \dot{W} \tag{10.2}$$

$$\sum_{\substack{output\\streams}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{out} - \sum_{\substack{input\\streams}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{in} = \dot{Q} + \dot{W} \tag{10.4}$$

Heat Transfer

- Conduction
  - through a stationary medium
  - by temperature difference
- Convection
  - through a moving medium
- Radiation
  - As electromagnetic waves
  - without a medium (even through a vacuum)
  - e.g. sun radiating its heat to the earth

Conduction

- Fourier's Law of Heat Conduction

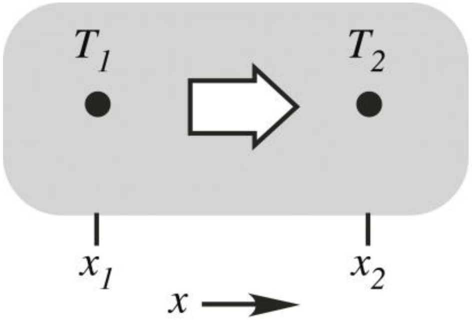


Figure 10.1

$$\dot{Q}_{cond,x} = - k\,A\,\frac{T_2 - T_1}{x_2 - x_1}$$

k = thermal conductivity

A = cross-sectional area through which the heat conducts

- cf. Fick's Law of Diffusion

$$\dot{N}_A = - D_{AB}\,A\,\frac{c_{A,2} - c_{A,1}}{x_2 - x_1}$$

Conductivity (k)

k (W/m℃) @25℃	
air	*.026*
water	*.61*
glass	*1.4*
aluminum	*237*

Which has the lowest k, gas or solid?

What are the best insulations?

Why is a double-pane window
an effective thermal barrier?

Convection

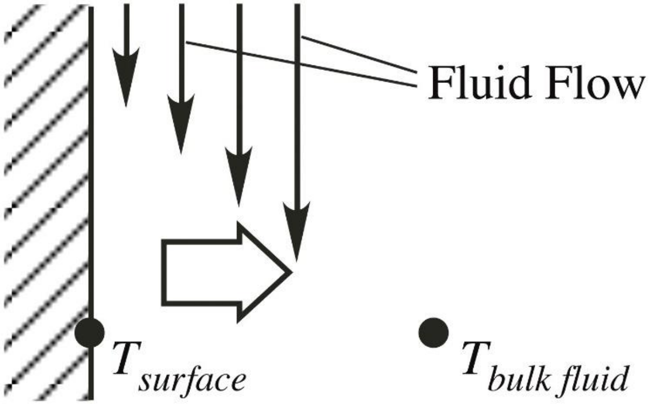


Figure 10.2
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$$\dot{Q}_{conv} = h\,A\,(T_{surface} - T_{bulk\ fluid})$$

h = heat-transfer coefficient (depends on geometry and flow)

A = cross-sectional area

cf. Mass Transfer Rate: $\dot{N}_A = h_m A\,(c_{A,1} - c_{A,2})$

Radiation

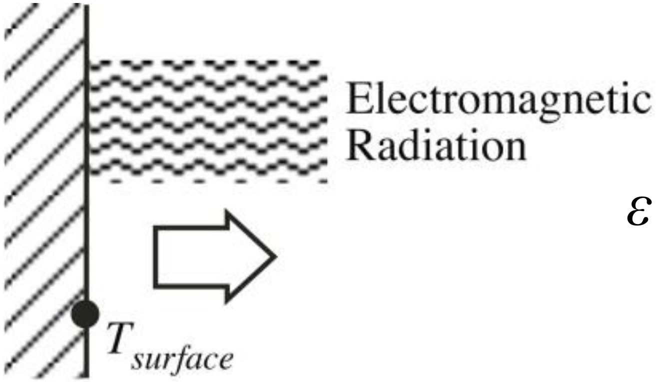

Figure 10.3
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$$\dot{Q}_{rad} = \varepsilon\, \sigma\, A\, (T_{surface})^4$$

ε = emissivity, which indicates how well the surface emits radiation compared with a "perfect" radiator (unit-less)

σ = Stefan-Boltzmann constant

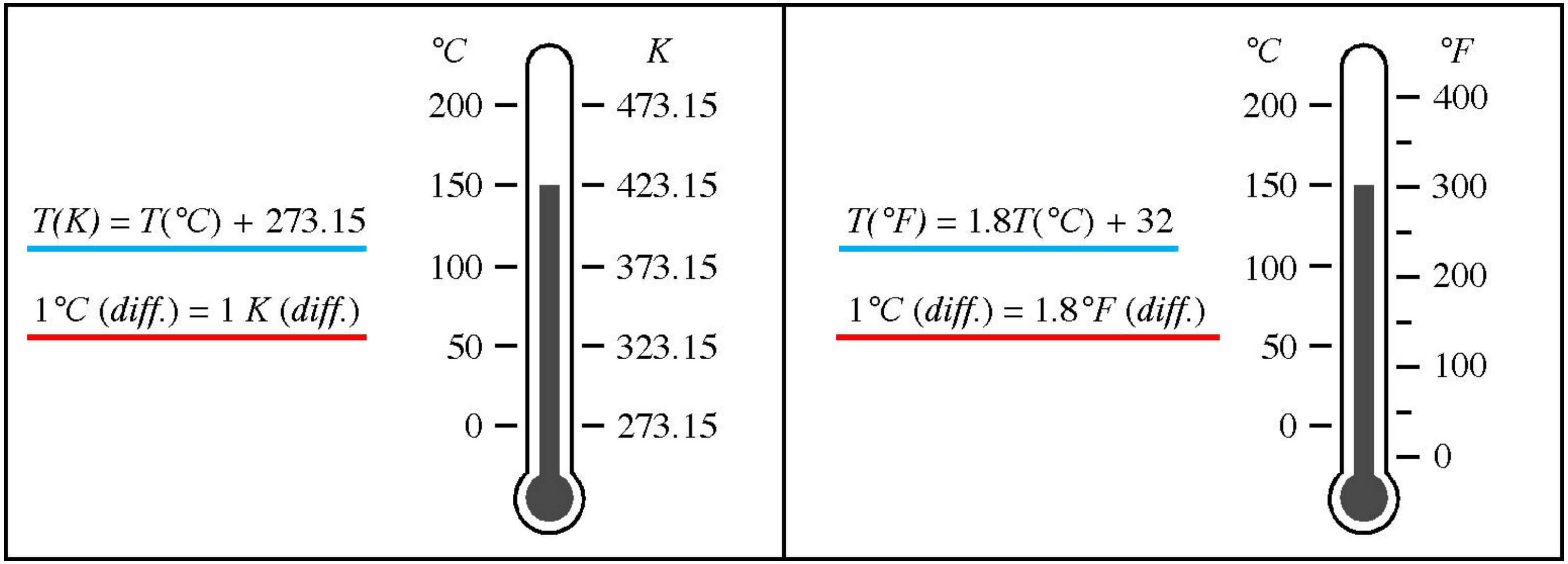
$(5.67 \times 10^{-8}\ W / m^2\, K^4)$

A = area of the radiating surface

T = absolute surface temperature (K)

$$\dot{Q}_{rad,net} = \varepsilon_1\, \sigma\, A_1\, (T_{surface,1})^4 - \varepsilon_2\, \sigma\, A_2\, (T_{surface,2})^4$$

Temperature & Temp. Difference



$T(K) = T(°C) + 273.15$

$1°C\ (diff.) = 1\ K\ (diff.)$

$T(°F) = 1.8T(°C) + 32$

$1°C\ (diff.) = 1.8°F\ (diff.)$

Ex. 10.1. A typical value for the thermal conductivity of steel is 53 *W/m K*. What is the corresponding value in unit of *Btu/hr ft °F*?

ΔT

Rate of Work

- Rate of shaft work ($\dot{W}_s$)
  - Energy per time
  - Positive when work is done on the system (such as in a pump, “push” the fluid)
  - Negative when work is done by the fluid (such as in turbine)
- Rate of flow work ($\dot{W}_{PV}$)
  - Work resulting from the displacement of fluid during flow
  - Similar to the pressure-volume work associated with the compression or expansion of a closed system

Energy Balance

$$\dot{W}_{PV} = \sum_{\substack{\text{input}\\\text{streams}}} (P\dot{V})_{in} - \sum_{\substack{\text{output}\\\text{streams}}} (P\dot{V})_{out} = \sum_{\substack{\text{input}\\\text{streams}}} (\dot{m}P\hat{V})_{in} - \sum_{\substack{\text{output}\\\text{streams}}} (\dot{m}P\hat{V})_{out} \tag{10.8}$$

$$\dot{W} = \dot{W}_s + \dot{W}_{PV} \tag{10.9}$$

$$\sum_{\substack{\text{output}\\\text{streams}}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{out} - \sum_{\substack{\text{input}\\\text{streams}}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{in} = \dot{Q} + \dot{W} \tag{10.4}$$

$$\sum_{\substack{\text{outpu}\\\text{streams}}} \left\{\dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]\right\}_{out} - \sum_{\substack{\text{input}\\\text{streams}}} \left\{\dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]\right\}_{in} = \dot{Q} + \dot{W}_s + \dot{W}_{PV} \tag{10.10}$$

Energy Balance

$$\sum_{\substack{\text{outpu}\\\text{streams}}}\left\{\dot{m}\left[\hat{U}+P\hat{V}+\frac{1}{2}\alpha v^2+gz\right]\right\}_{out}-\sum_{\substack{\text{input}\\\text{streams}}}\left\{\dot{m}\left[\hat{U}+P\hat{V}+\frac{1}{2}\alpha v^2+gz\right]\right\}_{in}$$

$$=\dot{Q}+\dot{W}_s \qquad (10.11)$$

Compare Eq. 10.11 with mechanical energy balance.

$$\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^2+gz\right)_{out}-\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^2+gz\right)_{in}=w_s-w_f$$

(7.8a)

Energy Balance

$$\hat{H} = \hat{U} + P\hat{V} \qquad (10.12)$$

Most common form of
the steady-state open-system energy balance.

$$\boxed{\sum_{\substack{output\\streams}} \left\{\dot{m}\left[\hat{H} + \frac{1}{2}\alpha v^2 + gz\right]\right\}_{out} - \sum_{\substack{input\\streams}} \left\{\dot{m}\left[\hat{H} + \frac{1}{2}\alpha v^2 + gz\right]\right\}_{in} = \dot{Q} + \dot{W}_s}$$

(10.13)

Energy Balance

Steady-state energy balance
with negligible change in kinetic and potential energies
and with no shaft work

$$\sum_{\substack{\text{output}\\ \text{streams}}} \left\{ \dot{m} \left[\hat{H} + \cancelto{0}{\frac{1}{2}\alpha v^2 + gz} \right] \right\}_{out} - \sum_{\substack{\text{input}\\ \text{streams}}} \left\{ \dot{m} \left[\hat{H} + \cancelto{0}{\frac{1}{2}\alpha v^2 + gz} \right] \right\}_{in} = \dot{Q} + \cancelto{0}{\dot{W}_s}$$

(10.13)

$$\sum_{\substack{\text{output}\\ \text{streams}}} \left(\dot{m}\ \hat{H} \right)_{out} - \sum_{\substack{\text{input}\\ \text{streams}}} \left(\dot{m}\ \hat{H} \right)_{in} = \dot{Q}$$

(10.14)

Energy Balance

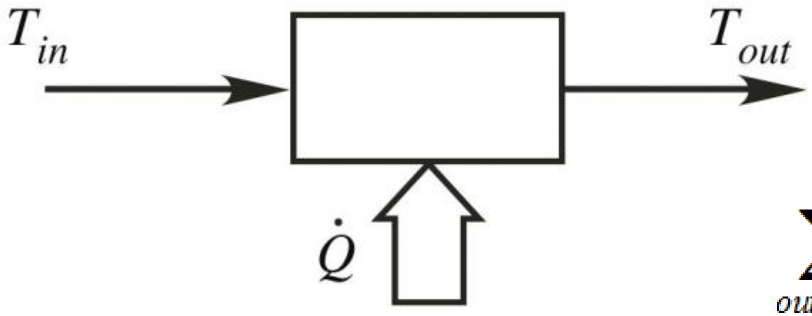


Figure 10.5
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$$\underset{\substack{\text{output}\\\text{streams}}}{\sum}\left(\dot{m}\,\hat{H}\right)_{out} - \underset{\substack{\text{input}\\\text{streams}}}{\sum}\left(\dot{m}\,\hat{H}\right)_{in} = \dot{Q} \qquad (10.14)$$

(i) Sensible Heating/Cooling:

$T_{out} > T_{in}$ (No phase change)

$$\sum_{out} \dot{m}\bar{C}_p(T-T_{ref}) - \sum_{in} \dot{m}\bar{C}_p(T-T_{ref}) = \dot{Q} \qquad (10.16)$$

(ii) Phase Change:

$T_{out} = T_{in}$ (Phase change)

$$\dot{m}_{phase\ change}\,\Delta\hat{H}_{phase\ change} = \dot{Q} \qquad (10.18)$$

(iii) Chemical Reaction:

$T_{out} = T_{in}$

$$r_{consumption,A}\,\Delta\tilde{H}_{reaction,A} = \dot{Q} \qquad (10.20)$$

Heat-Exchangers

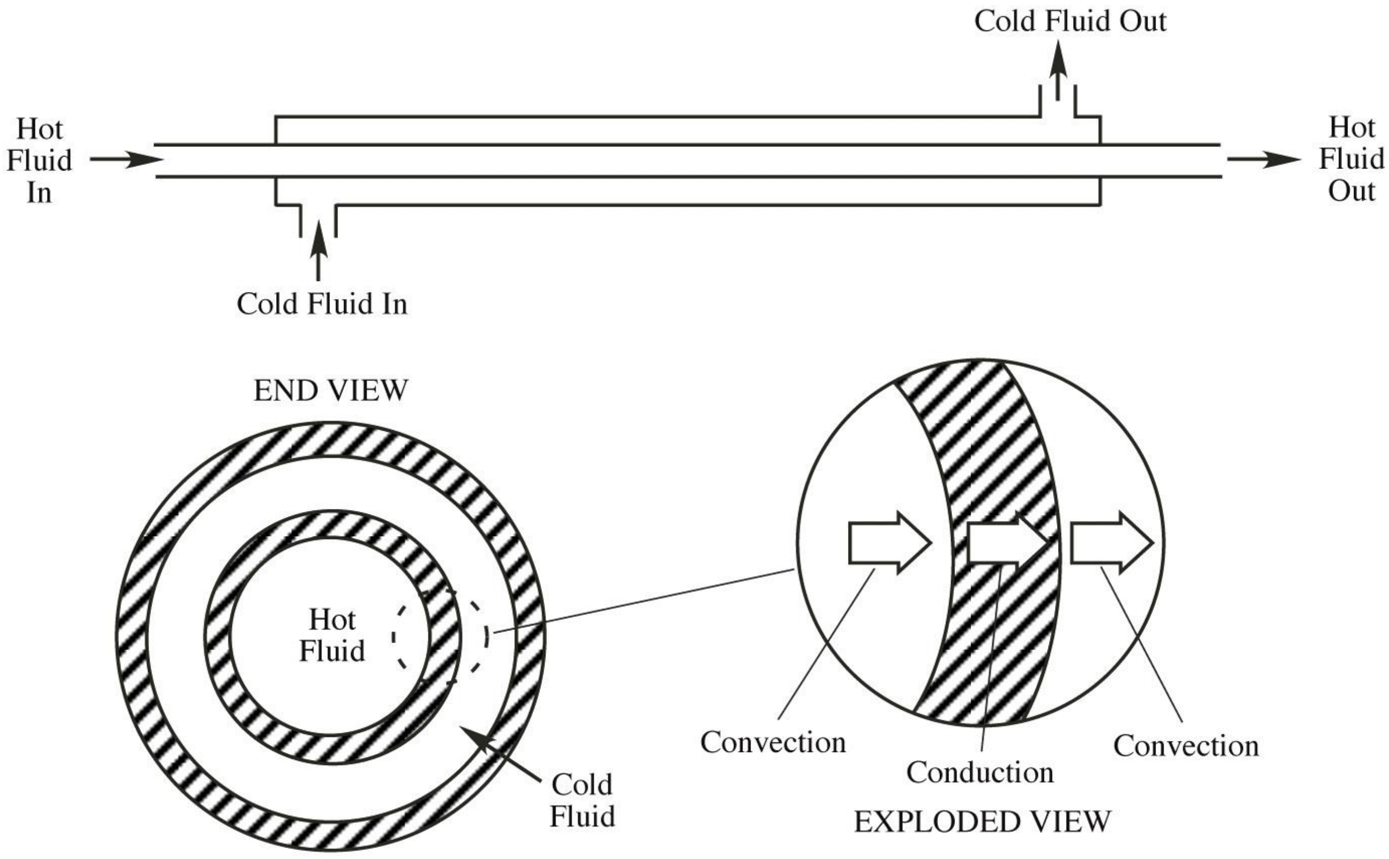

Figure 10.10

Concentric-Cylinder Heat Exchanger

Heat-Exchangers

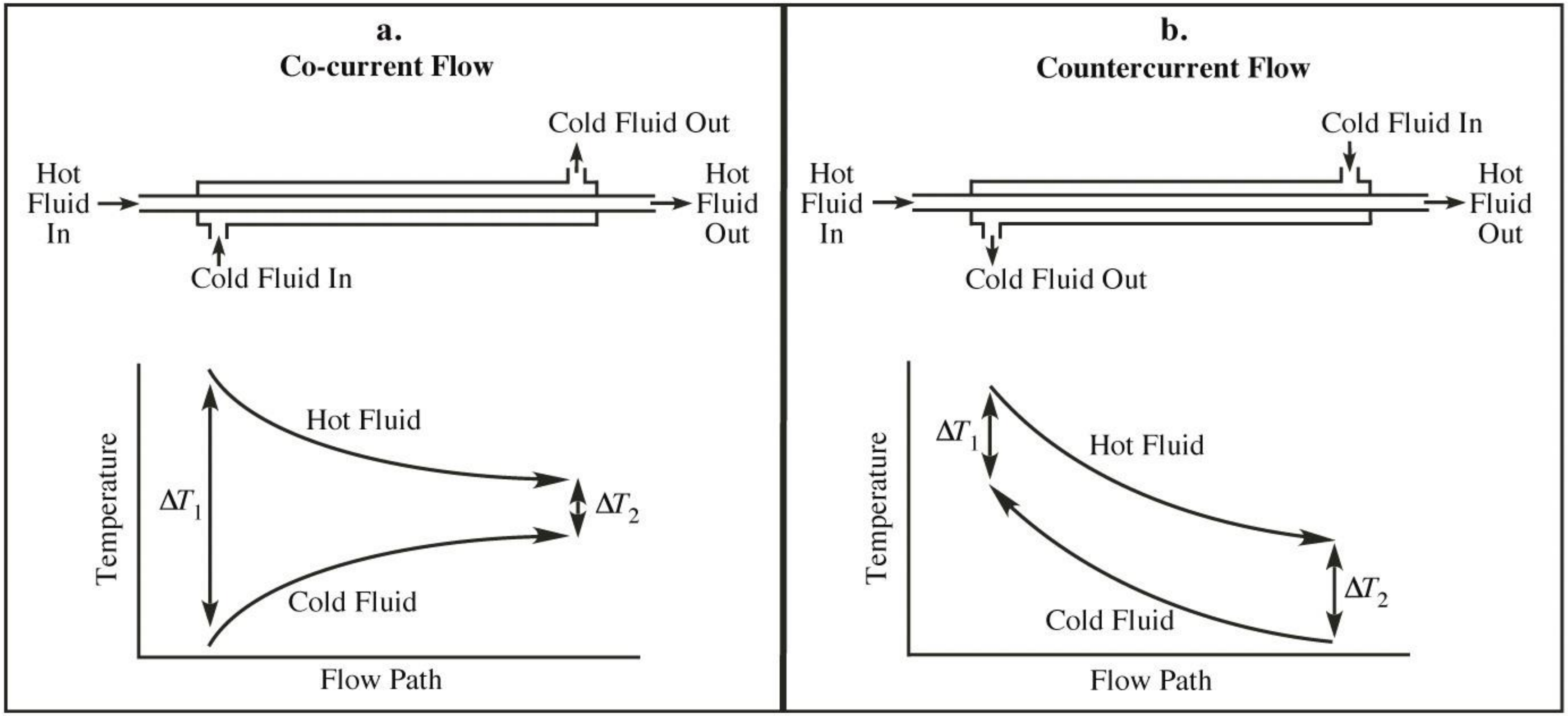

Figure 10.11

Heat-Exchangers

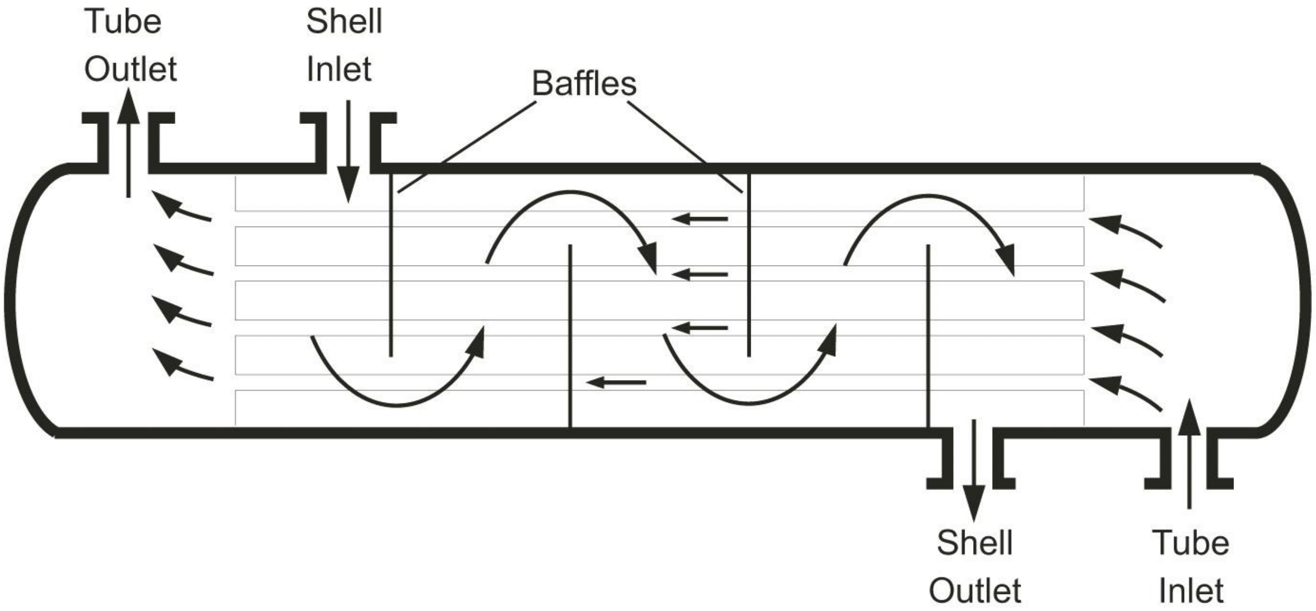

Figure 10.12
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Shell-and-Tube Heat Exchanger

Heat-Exchangers

(i) Sensible Heating/Cooling:

$$\left[\dot{m}C_p(T_{out}-T_{in})\right]_{hot} = -\dot{Q}_{duty} \qquad (10.24a)$$

$$\left[\dot{m}C_p(T_{out}-T_{in})\right]_{cold} = \dot{Q}_{duty} \qquad (10.24b)$$

(ii) Phase Change:

$$\left[\dot{m}\Delta\hat{H}_{phase\,change}\right]_{hot} = -\dot{Q}_{duty} \qquad (10.24c)$$

$$\left[\dot{m}\Delta\hat{H}_{phase\,change}\right]_{cold} = \dot{Q}_{duty} \qquad (10.24d)$$

(iii) Chemical Reaction:

$$\left[r_{consumption,A}\Delta\tilde{H}_{reaction,A}\right]_{hot} = -\dot{Q}_{duty} \qquad (10.24e)$$

$$\left[r_{consumption,A}\Delta\tilde{H}_{reaction,A}\right]_{cold} = \dot{Q}_{duty} \qquad (10.24f)$$

Heat-Exchangers

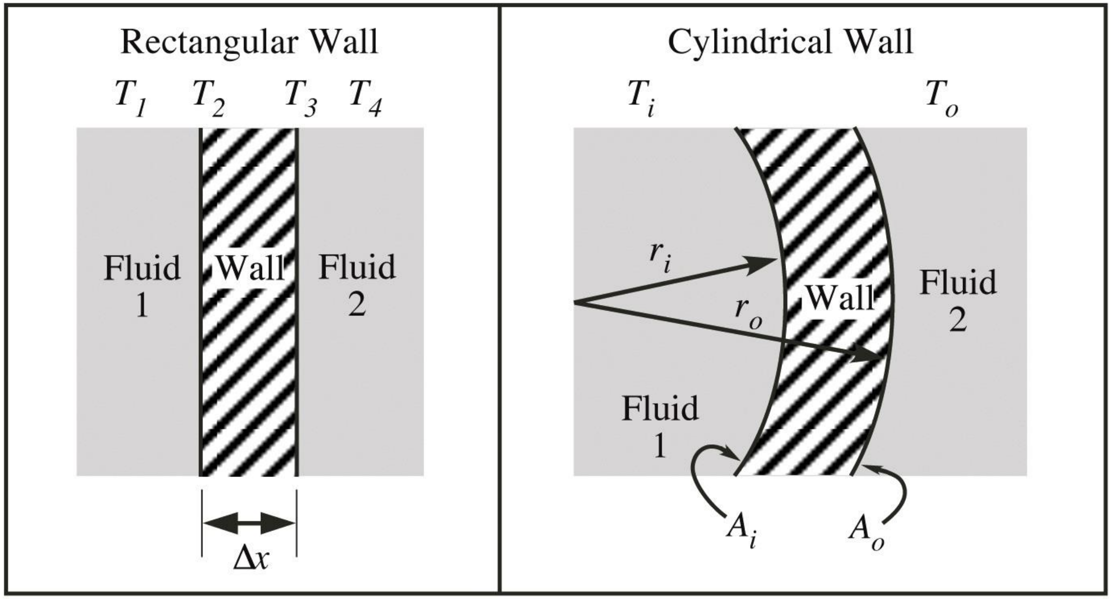

Figure 10.15
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$$\dot{Q} = h_1 A(T_1 - T_2) = kA\frac{T_2 - T_3}{\Delta x} = h_2 A(T_3 - T_4) \qquad (10.25)$$

Homework

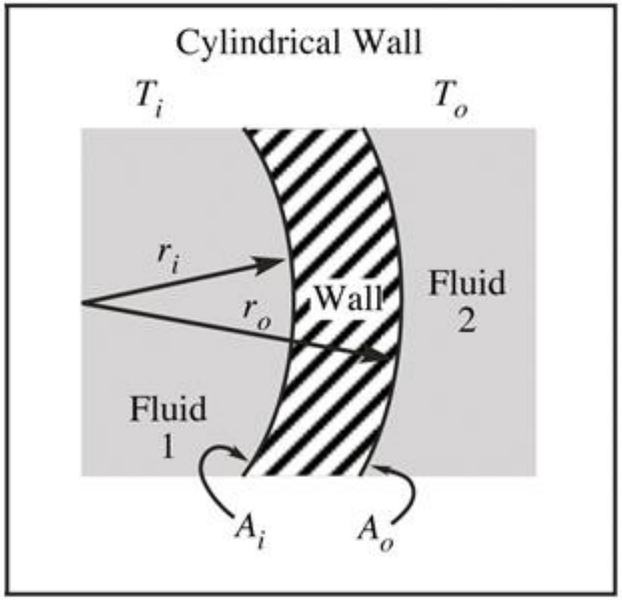
- Derive following equations.

$$\dot{Q} = \frac{T_1 - T_4}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}} \qquad (10.26)$$

$$\dot{Q} = \frac{T_i - T_o}{\frac{1}{h_i A_i} + \frac{\ln(r_o / r_i)}{2\pi k L} + \frac{1}{h_o A_o}} \qquad (10.27)$$

Heat-Exchangers

- T_i and T_o change along the length of the device.
- The prediction of the values of h_i and h_o is complex.



$$\dot{Q}_{duty} = U_o A \Delta T_{ave} \qquad (10.28)$$

U_o : overall heat transfer coefficient

$$\Delta T_{\log mean} = \frac{\Delta T_1 - \Delta T_2}{\ln \dfrac{\Delta T_1}{\Delta T_2}} \qquad (10.29)$$

log mean temperature difference