

# "Phase Transformation in Materials"

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- **Binary System** mixture/ solution / compound
- Gibbs Free Energy in Binary System

$$G_1 = X_A G_A + X_B G_B \quad J/mol$$
  $G_2 = G_1 + \Delta G_{mix} \quad J/mol$ 

**Ideal solution (\Delta H\_{mix} = 0)**  $\Delta G^{mix} = RT(X_A \ln X_A + X_B \ln X_B)$ 

$$G = X_A G_A + X_B G_B + RT(X_A \ln X_A + X_B \ln X_B)$$

**Regular solution**  $\Delta H_{mix} = P_{AB} \varepsilon$  where  $\varepsilon = \varepsilon_{AB} - \frac{1}{2} (\varepsilon_{AA} + \varepsilon_{BB})$  $G = X_A G_A + X_B G_B + \Omega X_A X_B + RT (X_A \ln X_A + X_B \ln X_B)$ 

- Chemical potential and Activity

$$\mu_{A} = \left(\frac{\partial G'}{\partial n_{A}}\right)_{T, P, n}$$

• 
$$\mu_A = G_A + RT \ln a_A$$

$$\ln\!\left(\frac{a_A}{X_A}\right) = \frac{\Omega}{RT}(1 - X_A)^2$$

 $\frac{a_A}{X_A} = \gamma_A = \text{activity coefficient}$ 

μ는 조성에 의해 결정되기 때문에 dn<sub>A</sub>가 매우 작아서 조성변화 없어야

- **Binary System** mixture/ solution / compound Ideal solution ( $\Delta H_{mix}$ =0) Random distribution

**Regular solution**  $\Delta H_{mix} = P_{AB} \varepsilon$  where  $\varepsilon = \varepsilon_{AB} - \frac{1}{2} (\varepsilon_{AA} + \varepsilon_{BB})$   $\varepsilon \approx 0$ 

$$\Delta H_{mix} > 0 \text{ or } \Delta H_{mix} < 0$$

**Real solution** 



(a)  $\varepsilon < 0$ ,  $\Delta H_{mix} < 0$  (b) Ordered alloys P<sub>AB</sub> ↑ → Internal E ↓

(b)  $\varepsilon > 0$ ,  $\Delta H_{mix} > 0$ Clustering  $P_{AA}$ ,  $P_{BB}$ 

#### **Ordered structure**



(c) when the size difference is large strain effect Interstitial solution

# Q9: Ordered phase II: "Long range order (LRO)"

(1)superlattice, 2)intermediate phase, 3)intermetallic compound)





diate phases: (a) for an intermetallic com-(b) for an intermediate phase with a wide



\* Compound : AB, A<sub>2</sub>B...

 $\rightarrow$  entropy $\downarrow$ 

 $\rightarrow$  covalent, ionic contribution.

 $\rightarrow$  enthalpy more negative  $\downarrow$ 

 $\Delta H_{mix}^{S} << 0$ 

Small composition range → G ↓



Fig. 1.23 Free energy curves for intermed pound with a very narrow stability range, ( stability range.



# Superlattice formation: order-disorder transformation

- $\epsilon < 0$ ,  $\Delta H_{mix} < 0$
- between dissimilar atoms than between similar atoms
- Large electrochemical factor: tendency for the solute atoms to avoid each other and to associate with the solvent atoms
- Size factor just within the favorable limit: lead to atomic rearrangement

so as to relieve the lattice distortion imposed by the solute atoms

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β brass superlattice viewed as two inter-penetrating cubic lattices



 $\varepsilon < 0, \Delta H_{mix} < 0$ 

Fig. 1.21. Part of the Cu-Au phase diagram showing the regions where the Cu<sub>3</sub>Au and CuAu superlattices are stable.



- The entropy of mixing of structure with LRO is extremely small and the degree ۲ of order decrease with increasing temperature until above some critical temperature there is no LRO at all.
- This temperature is a maximum when the composition is the ideal required ٠ for the superlattice.
- The critical temperature for loss of LRO increases with increasing  $\Omega$  or  $\Delta H_{mixi}$ ٠ and in many systems the ordered phase is stable up to the melting point.

# **Order-disorder phase transformation**

- Not classical phase change=<u>~not depend on diffusion process</u>
- change of temperature allowed a continuous re-arrangement of atoms without changing the phase = "2<sup>nd</sup> order transition"
- boundary: ordered lattice & disordered lattice/phase rule could not applied

there are cases in which an ordered phase of one composition exists

in equilibrium with a disordered phase of a different composition.

- Simple composition of the type AB or AB<sub>3</sub> can the transformation

(i.e. at the temperature maximum) be considered diffusionless.



# **Ordered Phase**



# **Intermediate Phase**

 $\varepsilon < 0, \Delta H_{mix} < 0$ 





# **Intermediate Phase**

 $\varepsilon$  < 0,  $\Delta H_{mix}$  < 0 /  $\Delta H_{mix}$  ~ -38 kJ/mol



# **Intermediate Phase**

 $\varepsilon << 0$ ,  $\Delta H_{mix} << 0 / \Delta H_{mix} \sim -142 \text{ kJ/mol}$ 



**1.3 Binary Solutions** 

# **Intermediate Phase**

\_"different crystal structure as either of the pure component"

3 main factors determining <u>the structure of Intermediate phase</u>?

- 1) Relative atomic size
  - Laves phase (size difference: 1.1~1.6 ex: MgCu<sub>2</sub>)
     <u>fill space most efficiently ~ stable</u>
  - Interstitial compound: MX, M<sub>2</sub>X, MX<sub>2</sub>, M<sub>6</sub>X M= Cubic or HCP ex: Zr, Ti, V, Cr, etc, X= H, B, C, and N
- 2) Relative valency electron
  - electron phases ex\_α & β brass
     # of valency electrons per unit cell
     → depending on compositional change

# 3) Electronegativity

- <u>very different electronegativites</u>  $\rightarrow$  ionic bond\_normal valency compounds ex Mg<sub>2</sub>Sn



MgCu<sub>2</sub> (A Laves phase)



Q1: "Clustering"? → Phase separation Metastable vs Stable miscibility gap \* The degree of ordering or clustering will decrease as temp. increases due to the increasing importance of entropy.

High temp.  $\rightarrow$  Entropy effect  $\uparrow \rightarrow$  Solution stability  $\uparrow$ 





# $\epsilon > 0$ , $\Delta H_{mix} > 0 / \Delta H_{mix} \sim +26 \text{ kJ/mol}$



#### Cooling process in the miscibility gap



**Dendrite / interdendrite formation** 

#### **Compositional analysis of as-cast CoCrFeNi/Cu HEA (dendrite)**



- ROI 1, 2 : 1.4 nm x 2 nm x 2 nm
- ROI 3 : 1.2 nm x 2 nm x 23 nm

(1D concentration profile)

1	at%	2	at%
Со	26.19	Со	0.33
Cr	24.15	Cr	0.46
Fe	24.59	Fe	0.39
Ni	19.59	Ni	5.00
Cu	4.74	Cu	93.56
3-1'	at%	3-2'	at%
3-1' Co	at% 25.29	3-2' Co	at% 2.01
3-1' Co Cr	at% 25.29 25.63	3-2' Co Cr	at% 2.01 3.35
3-1' Co Cr Fe	at% 25.29 25.63 23.63	3-2' Co Cr Fe	at% 2.01 3.35 2.56
3-1' Co Cr Fe Ni	at% 25.29 25.63 23.63 20.66	3-2' Co Cr Fe Ni	at% 2.01 3.35 2.56 6.90

- Dendrite region: matrix (4.74 at%Cu) + 2nd phase (93.56 at%Cu)
- No segregation at the interface between Matrix and 2nd phase

## $\varepsilon > 0$ , $\Delta H_{mix} > 0 / \Delta H_{mix} \sim +17 \text{ kJ/mol}$







## $\varepsilon > 0$ , $\Delta H_{mix} > 0 / \Delta H_{mix} \sim +5 \text{ kJ/mol}$







## $\varepsilon >> 0$ , $\Delta H_{mix} >> 0 / \Delta H_{mix} \sim +58 \text{ kJ/mol}$



# Positive heat of mixing relation among constituent elements



Nucleation and growth ↔ Spinodal decomposition without any barrier to the nucleation process

## \* Ti-Y-Al-Co system





 $(Y_{56}Al_{24}Co_{20})_{25}(Ti_{56}Al_{24}Co_{20})_{75}$ 

 $(Y_{56}Al_{24}Co_{20})_{50}(Ti_{56}Al_{24}Co_{20})_{50}$ 

 $(Y_{56}Al_{24}Co_{20})_{65}(Ti_{56}Al_{24}Co_{20})_{35}$ 

\* La-Zr-Al-Cu-Ni system



#### FIGURE 5.17

Schematic of the miscibility gap and the sequence of phase formation during cooling in the La–Zr–Al–Cu–Ni system. The positions of letters (a) to (d) in the diagram on the left correspond to the schematic microstructures (a) to (d) on the right. (Reprinted from Kündig, A.A. et al., *Acta Mater.*, 52, 2441, 2004. With permission.)



# Phase separation in metallic glasses



- **Binary System** mixture/ solution / compound Ideal solution ( $\Delta H_{mix}$ =0) Random distribution

**Regular solution**  $\Delta H_{mix} = P_{AB} \varepsilon$  where  $\varepsilon = \varepsilon_{AB} - \frac{1}{2} (\varepsilon_{AA} + \varepsilon_{BB})$   $\varepsilon \approx 0$ 

$$\Delta H_{mix} > 0$$
 or  $\Delta H_{mix} < 0$ 

**Real solution** 



(a)  $\varepsilon < 0$ ,  $\Delta H_{mix} < 0$  (b) Ordered alloys P<sub>AB</sub> ↑ → Internal E ↓

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#### **Ordered structure**



(c) when the size difference is large strain effect Interstitial solution



Q2: How can we define equilibrium in heterogeneous systems?

**Equilibrium in Heterogeneous Systems** 

A, B different crystal structure  $\rightarrow$  two free energy curves must be drawn, one for each structure.

We have dealt with the case where the components A and B have the same crystal structure. What would happen when the components A and B have a different crystal structure? → heterogeneous system



1.4



Fig. 1.26 The molar free energy of a two-phase mixture ( $\alpha$ + $\beta$ )

# Lever rule



W6

**Equilibrium in Heterogeneous Systems** 



1.4

### Variation of activity with composition

#### Activity, a : effective concentration for mass action

The most stable state, with the lowest free energy, is usually defined as the state in which the pure component has unit activity of A in pure  $\alpha$ .

when 
$$X_A = 1 \rightarrow a_A^{\alpha} = 1$$

when 
$$X_B = 1 \rightarrow a_B^\beta = 1$$

when  $\alpha$  and  $\beta$  in equil.

$$\mathbf{a}_{\mathsf{A}}^{lpha} = \mathbf{a}_{\mathsf{A}}^{eta}$$
 $\mathbf{a}_{\mathsf{B}}^{lpha} = \mathbf{a}_{\mathsf{B}}^{eta}$ 



Unified activity of two phase

Fig. 1.28 The variation of  $a_A$  and  $a_B$  with composition for a binary system containing two ideal solutions,  $\alpha$  and  $\beta$  35

# **Equilibrium in Heterogeneous Systems**



# Q3: How equilibrium is affected by temperature in complete solid solution?

# 1) Simple Phase Diagrams



#### 1) Simple Phase Diagrams



Fig. 26. Free energy curves for liquid and solid phases in the U-Zr system at 1500°, 1800° and 2000 °K.

# 1) Simple Phase Diagrams

- 1) Variation of temp.: G<sup>L</sup> > G<sup>s</sup>
- 2) Decrease of curvature of G curve
  - (: decrease of -T $\Delta S_{mix}$  effect)

#### Assumption:

- (1) completely miscible in solid and liquid.
- (2) Both are ideal soln.
- (3)  $T_m(A) > T_m(B)$

(4) 
$$T_1 > T_m(A) > T_2 > T_m(B) > T_3$$



# 1) Simple Phase Diagrams

The simplest type of binary phase diagrams is the isomorphous system, in which the two constituents form a continuous solid solution over the entire composition range. An example is the Ni-Cu system.

Solidification of alloy  $C_0$  starts on cooing at T<sub>1</sub>. The first solid formed has a composition of  $C_{s1}$  and the liquid  $C_0$ . On further cooling the solid particles grow larger in size and change their composition to  $C_{s2}$  and then  $C_0$ , following the solidus whereas the liquid decrease in volume and changes its composition from  $C_0$  to  $C_{L3}$  following the liquidus. The solidification completes at T<sub>3</sub>.



# Cooling Curves determination of Phase diagrams



# Q4: How equilibrium is affected by temperature in systems with miscibility gap?



How to characterize G<sup>s</sup> mathematically in the region of miscibility gap between e and f?

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# **2)** Systems with miscibility gab $\Delta H_{mix}^{L} = 0 \qquad \Delta H_{mix}^{S} > 0$

- When A and B atoms dislike each other,  $\Delta H_{mix} > 0$
- In this case, the free energy curve at low temperature has a region of negative curvature,  $\frac{d^2G}{dX_2^2} < 0$
- This results in a 'miscibility gap' of  $\alpha$ ' and  $\alpha$ " in the phase diagram



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#### 2) Variant of the simple phase diagram



Fig. 30. Derivation of the phase diagram (Fig. 29b) from free energy curves for the liquid and solid phases.  $T_A > T_B > T_1 > T_2 > T_3$ .

# Q5: How equilibrium is affected by temperature in simple eutectic systems?

**4) Simple Eutectic Systems**  $\Delta H_{mix}^{L} = 0$   $\Delta H_{mix}^{S} >> 0$ 

 ΔH<sub>m</sub>>>0 and the miscibility gap extends to the melting temperature. (when both solids have the same structure.)



Fig. 1.32 The derivation of a eutectic phase diagram where both solid phases have the same crystal structure.

### (when each solid has the different crystal structure.)



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Fig. 1.32 The derivation of a eutectic phase diagram where each solid phases has a different crystal structure.

## Cu-Zn Phase Diagram

#### **Eutectoid and Peritectic Reactions**



Eutectoid: one solid phase transforms into two other solid phases upon cooling

Peritectic: one solid and one liquid phase transform into another solid phase upon cooling

Peritectoid: two other solid phases transform into another solid phase upon cooling



# 2) Variant of the simple phase diagram

 $\Delta H_{mix} < 0$ 





Fig. 32. Phase diagram with a maximum in the liquidus.

### congruent maxima



Fig. 33. Appearance of an ordered  $\alpha'$  phase at low temperatures.

# 5) Phase diagrams containing intermediate phases





5) Phase diagrams containing intermediate phases



# $\theta$ phase in the Cu-Al system is usually denoted as CuAl<sub>2</sub> although the composition X<sub>Cu</sub>=1/3, X<sub>Al</sub>=2/3 is not covered by the $\theta$ field on the phase diagram.



X.L. Liu, I. Ohnuma, R. Kainuma, and K. Ishida, J. Alloys Compds, 264, 201-208 (1998)

# Summary I: Binary phase diagrams

# 1) Simple Phase Diagrams

Both are ideal soln.  $\rightarrow$  1) Variation of temp.:  $G^L > G^s$  2) Decrease of curvature of G curve (:: decrease of -T $\Delta S_{mix}$  effect)

**2)** Systems with miscibility gap  $\Delta H_{mix}^{L} = 0$   $\Delta H_{mix}^{S} > 0$ 1)Variation of temp.: **G**<sup>L</sup> > **G**<sup>s</sup> 2)Decrease of curvature of G curve + Shape change of G curve by H

- **4)** Simple Eutectic Systems  $\Delta H_{mix}^{L} = 0 \quad \Delta H_{mix}^{S} >> 0$  $\rightarrow$  miscibility gap extends to the melting temperature.
- **3) Ordered Alloys**  $\Delta H_{mix}^{L} = 0$   $\Delta H_{mix}^{S} < 0$  $\Delta H_{mix} < 0 \rightarrow A$  atoms and B atoms like each other.  $\rightarrow$  Ordered alloy at low T

 $\Delta H_{mix} << 0 \rightarrow$  The ordered state can extend to the melting temperature.

# 5) Phase diagrams containing intermediate phases Stable composition + Minimum G with stoichiometric composition