# Error Budget

"Design a machine of accuracy or uncertainty less than 10um for 300mm travel"

How and where can we start the design in terms of accuracy?

∴We need to get the error budget, error allocation, or error identification, which is to identify the order of importance among the sources of errors to achieve the performance

Error budget is to list of all error sources and their effect on the machine accuracy or total error. It is a powerful tool to identify a order of importance for the sources of error during design optimization. The total error is the kinematic sum of all individual error components, where the individual error components have the *physical causes that can be identified, measured, and controlled, in order to be used for error prediction/compensation/reduction, or for design optimization.

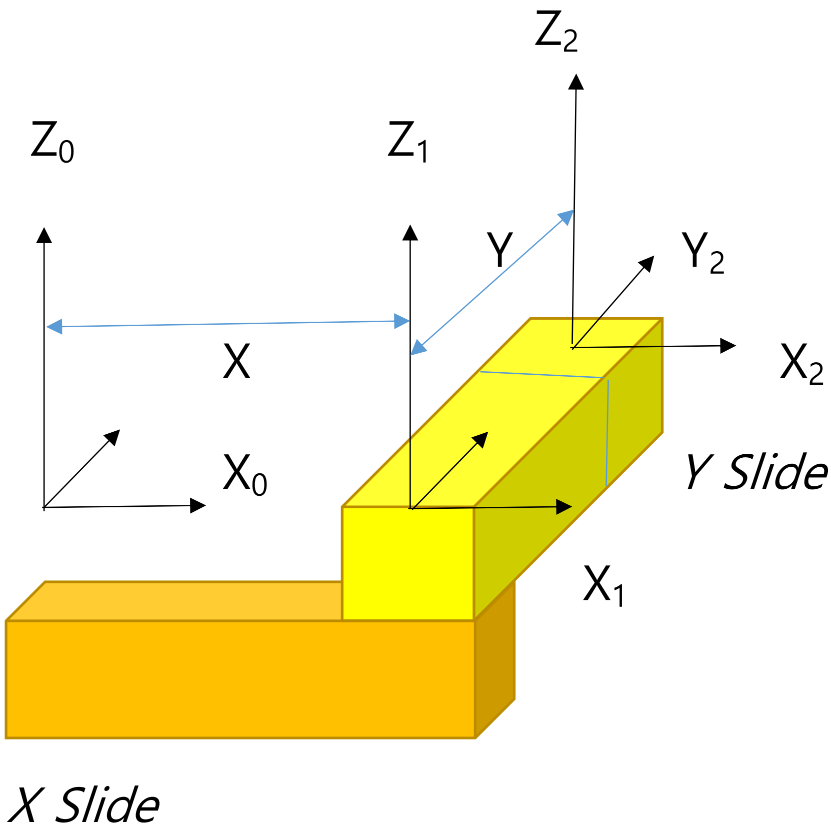
Physical causes: Physical sources of error including all possible errors such as geometric/kinematic errors , coupling mechanism of stiffness, thermal expansion, damping, etc.

<u>Case example: Error budgeting for XY stage</u>

A example task is to indentify the error sources for the XY stage and to perform design optimization.

The <u>error propagation theory</u> provides a very useful tool for the error budget.

A commercial XY stage (Source from Dover Motion)




Kinematic Configuration XY stage;

$[X_0Y_0Z_0]$: Reference Coordinate system

$[X_1Y_1Z_1]$: Coordinate system fixed on X slide

$[X_2Y_2Z_2]$: Coordinate system fixed on Y slide

The influence of geometric error components on the total positioning accuracy of XY stage is to investigate. The functional, or mathematical relationship between them is great help, say, the error propagation of each error components into the total error.

Translational Errors;

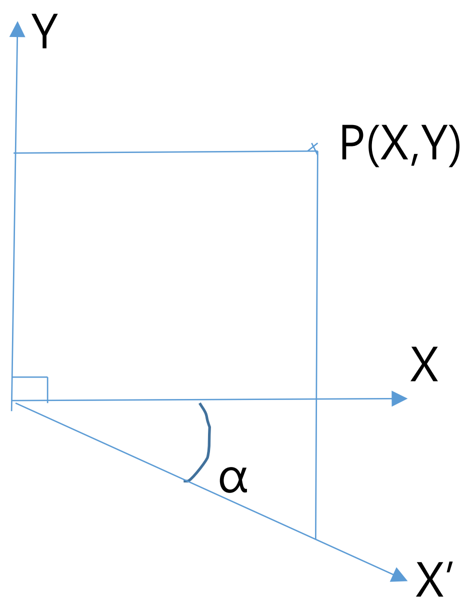
$\delta x(x)$, $\delta y(y)$; positional errors

$\delta y(x)$, $\delta x(y)$; straightness errors

Rotational Error;

$Ez(x)$; Pitch angular error of X axis

Squareness error, $\alpha$, between X and Y axis




Because $\alpha$ is very small,

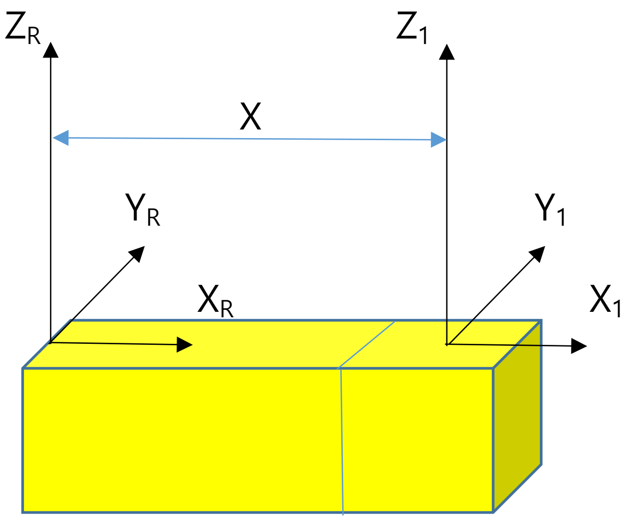
$X_T = X_N \cdot \cos\alpha \fallingdotseq X_N = X$

$Y_T = Y_N - X_N \cdot \sin\alpha \fallingdotseq Y_N - X_N \cdot \alpha = Y - \alpha X$

where

$(X_T, Y_T)$: True coordinates and

$(X_N, Y_N)$: Nominal coordinates=$(X,Y)$

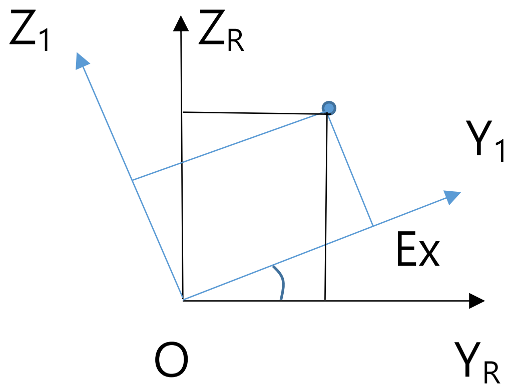



$[O_R X_R Y_R Z_R]$: Reference Coordinate System

$[O_1 X_1 Y_1 Z_1]$: Moving Coordinate System fixed on X-Slide

Two coordinates are initially aligned as the same.

## Roll Motion of Slide, Ex



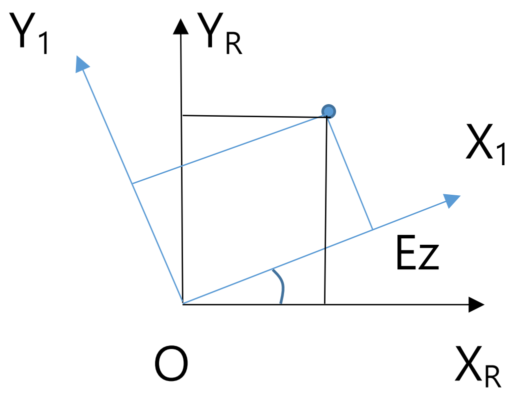

$Y_R = Y_1 \cos Ex - Z_1 \sin Ex \fallingdotseq Y_1 - Z_1 Ex$ ; if $Ex \ll 1$

$Z_R = Y_1 \sin Ex + Z_1 \cos Ex \fallingdotseq Y_1 Ex + Z_1$ ; if $Ex \ll 1$

In 3D Transformation matrix form, $\mathbf{X}_R = \mathbf{T}_R \mathbf{X}_1$; that is

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -Ex \\ 0 & Ex & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

Yaw motion of Slide, Ez



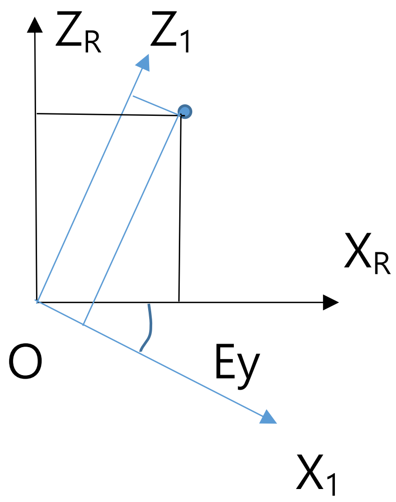

$X_R = X_1\cos Ez - Y_1\sin Ez \fallingdotseq X_1 - Y_1 Ez$ ; if $Ez \ll 1$

$Y_R = X_1\sin Ez + Y_1\cos Ez \fallingdotseq X_1 Ez + Y_1$ ; if $Ez \ll 1$

In 3D Transformation matrix form, $\mathbf{X}_R = \mathbf{T}_Y \mathbf{X}_1$; that is

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & -Ez & 0 \\ Ez & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

Pitch motion of Slide, Ey




$X_R = X_1\cos Ey + Z_1\sin Ey \fallingdotseq X_1 + Z_1 Ey$ ; if $Ey \ll 1$

$Z_R = -X_1\sin Ey + Z_1\cos Ey \fallingdotseq -X_1 Ey + Z_1$ ; if $Ey \ll 1$

And, in 3D Transformation matrix form, $\mathbf{X}_R = \mathbf{T}_p\mathbf{X}_1$; that is

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & 0 & Ey \\ 0 & 1 & 0 \\ -Ey & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

Combining all angular motions by multiplying the 3 matrices to give the transformation matrix for rotational motion of X-slide, **T**x ; where the order of multiplication is arbitrary due to the asymmetric matrices.

$\mathbf{T}_X=\mathbf{T}_R\mathbf{T}_Y\mathbf{T}_P$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -Ex \\ 0 & Ex & 1 \end{bmatrix} \begin{bmatrix} 1 & -Ez & 0 \\ Ez & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & Ey \\ 0 & 1 & 0 \\ -Ey & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -Ez(x) & Ey(x) \\ Ez(x) & 1 & -Ex(x) \\ -Ey(x) & Ex(x) & 1 \end{bmatrix}$$

And, it is noteworthy that all the transformation matrices are asymmetric.

Now for translating motion of X slide is introduced;

Translation in X direction=X+$\delta x(x)$

; nominal position +positional error

Translation in Y direction=$\delta y(x)-\alpha X$

;Y straightness error of X axis + squareness error

Translation in Z direction=$\delta z(x)$

;Z straightness error of X axis

Thus, the translating motion of X-slide is expressed as the column vector, **L**x, that is,

$$\mathbf{L}x=\begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix}$$

Thus a point $P(X_1,Y_1,Z_1)$ on the X slide can be expressed in the reference coordinate system $[X_R,Y_R,Z_R]$;

In the 3D transformation matrix,

$\mathbf{X}_R=\mathbf{T}x\mathbf{X}_1+\mathbf{L}x$ ; eq(1) and that is,

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & -Ez & Ey \\ Ez & 1 & -Ex \\ -Ey & Ex & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix}$$

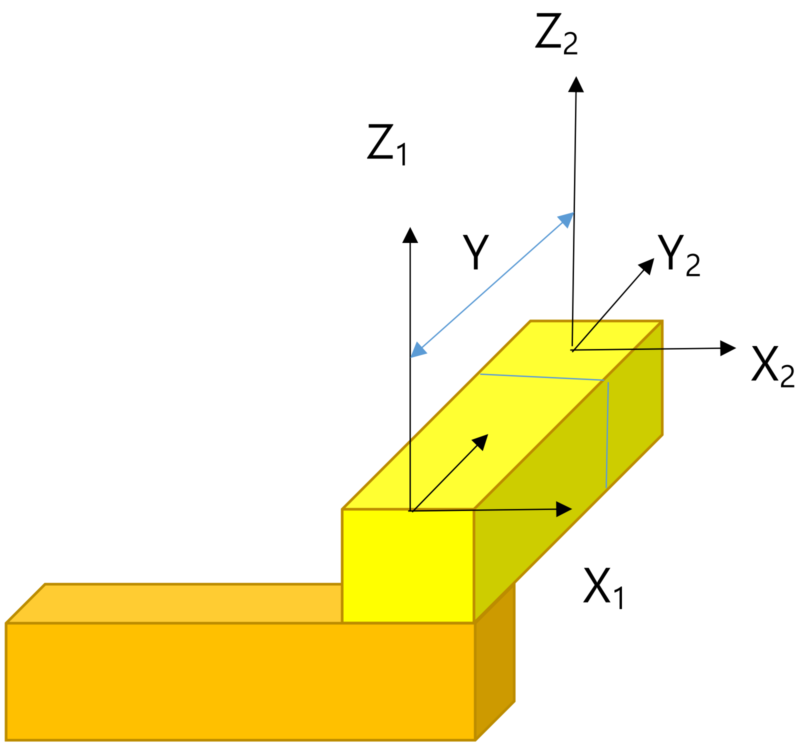
That is,

$$X_R=X+\delta x(x)+X_1-Y_1Ez(x)+Z_1Ey(x)$$

$$Y_R=\delta y(x)-\alpha X+X_1Ez(x)+Y_1-Z_1Ex(x)$$

$$Z_R=\delta z(x)-X_1Ey(x)+Y_1Ex(x)+Z_1$$

Now, introduce the Y-slide motion on the top of the X-slide;




$[O_1X_1Y_1Z_1]$: Moving Coordinates fixed on X slide

[$O_2X_2Y_2Z_2$]: Moving Coordinates fixed on Y slide

The two coordinates are initially aligned as the same.

Angular motion of the Y slide is similarly expressed as the transformation matrix, **T**y, that is

$$\mathbf{T}y= \begin{bmatrix} 1 & -Ez(y) & Ey(y) \\ Ez(y) & 1 & -Ex(y) \\ -Ey(y) & Ex(y) & 1 \end{bmatrix}$$

For translating motion of Y slide,

Translation in X direction=$\delta x(y)$

; X straightness error of Y axis

Translation in Y direction=$Y+\delta y(y)$

; nominal position +positional error

Translation in Z direction=$\delta z(y)$

; Z straightness error of Y axis

Thus the translating motion of Y slide can be expressed as the column vector, **L**y; that is

$$\mathbf{L}y = \begin{bmatrix} \delta x(y) \\ Y+\delta y(y) \\ \delta z(y) \end{bmatrix}$$

Thus a point $P(X_2,Y_2,Z_2)$ on the X slide can be expressed in the $[X_1,Y_1,Z_1]$ coordinate system;

$\mathbf{X}_1=\mathbf{T}y\mathbf{X}_2+\mathbf{L}y$ ; eq(2)

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} 1 & -Ez & Ey \\ Ez & 1 & -Ex \\ -Ey & Ex & 1 \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} + \begin{bmatrix} \delta x(y) \\ Y+\delta y(y) \\ \delta z(y) \end{bmatrix}$$

That is,

$X_1=\delta x(y)+X_2-Y_2Ez(y)+Z_2Ey(y)$

$Y_1=Y+\delta y(x)+X_2Ez(y)+Y_2-Z_2Ex(z)$

$Z_1=\delta z(y)-X_2Ey(y)+Y_2Ex(y)+Z_2$

Applying eq(2) to eq(1), the point $P(X_2,Y_2,Z_2)$ can be expressed in the reference $[X_R Y_R Z_R]$ coordinate system.

Thus, $\mathbf{X}_R=\mathbf{T}x\{\mathbf{T}y\mathbf{X}_2+\mathbf{L}y\}+\mathbf{L}x$, and if we are ignoring the terms over 2nd order;

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} 1 & -Ez & Ey \\ Ez & 1 & -Ex \\ -Ey & Ex & 1 \end{bmatrix} \begin{bmatrix} \delta x(y)+X_2-Y_2Ez(y)+Z_2Ey(y) \\ Y+\delta y(y)+X_2Ez(y)+Y_2-Z_2Ex(y) \\ \delta z(y)-X_2Ey(y)+Y_2Ex(y)+Z_2 \end{bmatrix} + \begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix}$$

$$X_R=X+\delta x(x)+\delta x(y)+X_2-Y_2Ez(y)+Z_2Ey(y)-(Y+Y_2)Ez(x)+Z_2Ey(x)$$

$$Y_R=Y+\delta y(y)+\delta y(x)-\alpha X+X_2Ez(y)+Y_2-Z_2Ex(y)+X_2Ez(x)-Z_2Ex(x)$$

$$Z_R= \delta z(x)+\delta z(y)-X_2Ey(y)+Y_2Ex(y)+Z_2-X_2Ey(x)+(Y+Y_2)Ex(x)$$

Assuming $\mathbf{X}_2=0$,

and remembering $\Delta X=X_R-X$ and $\Delta Y=Y_R-Y$;

$$\Delta X= \delta x(x)+\delta x(y)-YEz(x) \quad \text{eq(1)'}$$

$$\Delta Y= \delta y(y)+\delta y(x)-\alpha X \quad \text{eq(2)'}$$

These two equation give the total error in X,Y direction respectively, and clearly shows how the geometric error components of each axis are contributing to the total errors.

Initial investigation shows that positional error components, angular error components, straightness error components, and squareness error component are the major error sources. And, they are as following in detail.

Positional error: possible sources are lead screw error due to grade, thermal expansion, backlash error, etc; longer axis gives larger error, typically

Angular error: it is magnified by Y by the Abbe offset, and possible sources are guide straightness, bearing configuration.

Squareness error: it is magnified by X, possible error source is non-square assembly or alignment of Y axis w.r.t. X axis

Straightness error of guide: Non-straightness of guide gives not only the straightness error, but also significant angular error of block moving along the guide, and it can be magnified by the Abbe offset. Typically 1m guide of 1um straightness error gives 8 urad (≒1.6 arcsec) angular error.

Detail error propagation of each error sources as follows;

<u>Total error in X direction, ΔX(x,y)</u>

There are three components, δx(x), δx(y), Ez(x); and they are all from the different error sources and physically identifiable or measurable independently, thus the systematic part and random part can be identified for every geometric error components.

Positional error, δx(x)

= Systematic part ± Random part

=Sδx(x) ± Rδx(x)

Straightness error, δx(y)

= Systematic part ± Random part

=Sδx(y) ± Rδx(y)

Yaw angular error, $Ez(X)$

$= \text{Systematic part} \pm \text{Random part}$

$=SEz(x) \pm REz(x)$

As the geometric error components are physically identifiable and measureable, the systematic part and random part can be evaluated.

The error propagation rules can be applied to the $\Delta X$, total error in X direction as follows, assuming the first order approximation for systematic part and most probable case for the random part.

$\Delta X = \text{Systematic part} \pm \text{Random part}$

$= S\Delta X \pm R\Delta X$ eq(3)

$S\Delta X = \partial F/\partial X \cdot \Delta X + \partial F/\partial Y \cdot \Delta Y + \ldots =$ Sum of systematic part of components from eq(1)',

and $A\Delta X = |\partial F/\partial X \cdot \Delta X| + |\partial F/\partial Y \cdot \Delta Y| + \ldots =$ Absolute sum of the systematic part of components, thus

$S\Delta X = S\delta x(x) + S\delta x(y) - Y \bullet SEz(x)$ eq(4)

$A\Delta X = |S\delta x(x)| + |S\delta x(y)| + |Y \bullet SEz(x)|$ eq(4)'

For the random parts,

RΔX=Square root of the sum of squares, or RMS of random part of components from eq(4),

$= [R^2_{xx}(\partial F/\partial X)^2 + R^2_{yy}(\partial F/\partial Y)^2 + \ldots]^{1/2}$

$= [R\delta x(x)^2 + R\delta x(y)^2 + Y^2 REz(x)^2]^{1/2}$ eq(5)

These equation indicate that the random part as well as the systematic part is also the function of x,y, contributing to the total error in X direction, ΔX(x,y).

<u>Total error in Y axis, ΔY(x,y)</u>

Similarly, the total error ΔY(x,y) in the Y direction also can be considered as follows;

Geometric error components contributing to Y direction;

Positional error, δy(y)

= Systematic part ± Random part

$= S\delta y(y) \pm R\delta y(y)$

Straightness error, $\delta y(x)$

= Systematic part ± Random part

$= S\delta y(x) \pm R\delta y(x)$

Sqaureness error, $\alpha$

= Systematic part ± Random part

$= S\alpha \pm R\alpha$

Therefore

$\Delta Y$=Systematic part±Random part

$= S\Delta Y \pm R\Delta Y$ eq(6)

$S\Delta Y = \partial F/\partial X \cdot \Delta X + \partial F/\partial Y \cdot \Delta Y + \ldots$ =Sum of systematic part of components from eq(2), and $A\Delta Y = |\partial F/\partial X \cdot \Delta X| + |\partial F/\partial Y \cdot \Delta Y| + \ldots$ = Absolute sum of systematic part of components

$S\Delta Y = S\delta y(y) + S\delta y(x) - X \bullet S\alpha$ eq(7)

$A\Delta Y = |S\delta y(y)| + |S\delta y(x)| + |X \bullet S\alpha|$ eq(7)'

$R\Delta Y$=Square root of the sum of squares, or RMS of random part of components from eq(7)

$= [R^2_{xx}(\partial F/\partial X)^2 + R^2_{yy}(\partial F/\partial Y)^2 + \ldots]^{1/2}$

$= [R\delta y(y)^2 + R\delta y(x)^2 + X^2 R\alpha^2]^{1/2}$ eq(8)

Therefore the full error propagation is considered for the total errors in the X,Y direction, and they are defined as a function of position (X,Y)

Example case of error calculation

An example Case of XY stage at 300mm (or 12 inch) stroke;

Initial error allocation can be assigned as follows;

Positional error; $\delta x(x)=10\pm1$ [um], $\delta y(y)=10\pm1$ [um]

Straightness error; $\delta x(y)=5\pm1$ [um], $\delta y(x)=5\pm1$ [um]

Angular error; $Ez(x)=5\pm1$ [arcsec]

Squareness error; $\alpha=5\pm0.1$ [arcsec]

Therefore,

$A\Delta X=10+5+0.3(4.8)(5)=22.2$ [um]

$R\Delta X=[1^2+1^2+\{(0.3)(4.8)(0.1)\}^2]^{1/2}=1.42$ [um], and

$\therefore \Delta X=22.2\pm1.42$ [um]

$A\Delta Y = 10+5+(0.3)(4.8)(5) = 22.2$ [um]

$R\Delta Y = [1^2+1^2+\{(0.3)(4.8)\ (1)\}^2]^{1/2} = 2.02$ [um]

$\therefore \Delta Y = 22.2 \pm 2.02$ [um]

Therefore the total error, $\Delta A \pm \Delta R$, becomes,

$\Delta A = [A\Delta X^2 + A\Delta Y^2]^{1/2} = [22.2^2 + 22.2^2]^{1/2} = 31.4$

$\Delta R = [R\Delta X^2 + R\Delta Y^2]^{1/2} = [1.42^2 + 2.02^2]^{1/2} = 2.469$

The calculation of error budgeting shows the accuracy of 10um cannot be achieved with the initial error allocation. Thus the total error should be further reduced. When the systematic errors are compensated via such as numerical calculation according to eq(1)' and (2)', the random parts $R\Delta X = \pm 1.42$[um], $R\Delta Y = \pm 2.02$[um], thus $\Delta R = \pm 2.469$[um] are only remaining, thus 10um accuracy can be achieved. But this is a very ideal case, and further reduction on the error allocation has to be made. The squareness error and angular errors are treated more significantly than the rest of errors. The positional errors are also much contributing.

This is a practical procedure for error budgeting, and each causes should be identified, adjusted, and optimized, in such as; precision grade of screw, preloaded screw mechanism, adequate servo-drive, friction, guideway bearing types, preloaded bearing structure; indicator/talyvel/edge/square assisted assembly for guideway with checking of straightness errors, angular errors, etc.

The repeat procedures can be followed until the performance is met with the requirement.

Other error sources such as vibration, machining, or temperature influence can be identified and added to the error budget in terms error accumulation as discussed earlier.

HW3) Perform the error budget analysis for XY stage of 300mmX300mm stroke, where the kinematic chain is

Y->X.