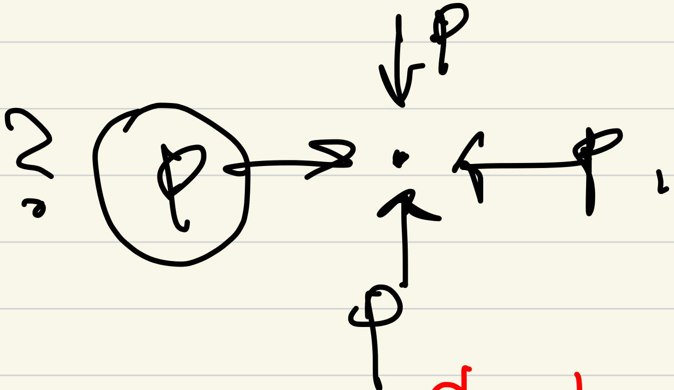
2.1. Pressure and pressure gradient

· at rest → no shear stress, only normal stress.
(fluid pressure)

↓

pressure at a point "in a static fluid" is indep. of orientation.

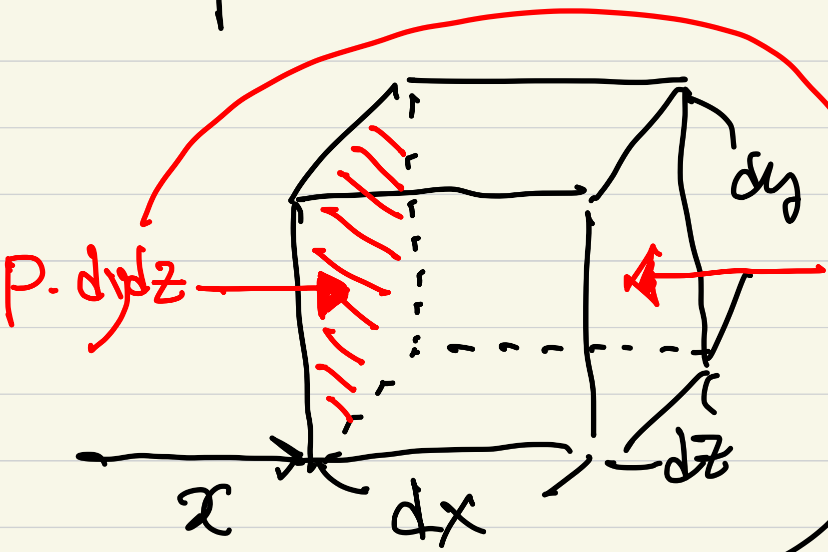


♡ in a moving fluid, → shear & normal stresses.

: pressure is the average of three normal stresses on the element.

$$p = -\frac{1}{3}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})$$ ← Ch.4

* pressure force on a fluid element.



Taylor series expansion.

$(p + \frac{\partial p}{\partial x} dx)\, dydz$.

$$dF_x = \cancel{p \cdot dydz} - (\cancel{p} + \frac{\partial p}{\partial x} dx)\, dydz$$

$$= -\frac{\partial p}{\partial x}\, dxdydz.$$

likewise, $\underset{\text{vector}}{d\underline{F}_{press}} = \underline{\left(-\frac{\partial p}{\partial x}\hat{i} - \frac{\partial p}{\partial y}\hat{j} - \frac{\partial p}{\partial z}\hat{k}\right)} dxdydz.$

$$= -\nabla p\, (\underbrace{dxdydz}_{\text{volume}}).$$

$$\Rightarrow \boxed{d\underline{f}_{press} = -\nabla p.}$$

(net force per unit volume)

2.2 Equilibrium of a fluid element.

- surface force : pressure, shear stress, ...
- body force : gravity, electromagnetic force, ...

$\hookrightarrow d\underline{F}_{grav} = \rho \underline{g} \cdot dx dy dz$,

$d\underline{f}_{grav} = \rho \underline{g}$.

viscous force. (Ch.4) : $\underline{f}_{vis} = \mu \nabla^2 \underline{V}$.

. per unit volume.

$$\boxed{\begin{aligned} \Sigma \underline{f} = \rho \underline{a} &= \underline{f}_{press} + \underline{f}_{grav} + \underline{f}_{vis} \\ &= -\nabla p + \rho \underline{g} + \mu \nabla^2 \underline{V}. \end{aligned}}$$

. special cases

① $\underline{V} = 0$ (or $\underline{a} = 0$; constant velocity).

$\Rightarrow \nabla p = \rho \underline{g}$ (hydrostatic condition)

② rigid-body movement (rotation, translation)

ex) $u_\theta = r\Omega$. $\rightarrow \nabla^2 \underline{V} = 0$.

$\Rightarrow \nabla p = \rho(\underline{g} - \underline{a})$.

③ Irrotational motion. $(\nabla \times \underline{V} = 0) \rightarrow \nabla^2 \underline{V} = 0$.

↳ Bernoulli eq. (will come back to this later).

2.3. Hydrostatic pressure distributions.

$(\underline{V} = 0$ or $\underline{a} = 0$, $\rightarrow \nabla^2 \underline{V} = 0)$.

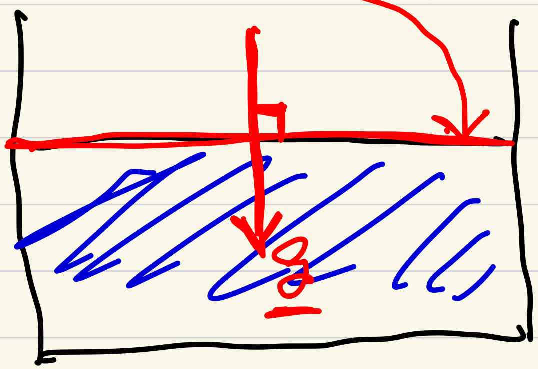
$\rho \underline{a} = -\nabla p + \rho \underline{g} + \mu \nabla^2 \underline{V} \Rightarrow \boxed{\nabla p = \rho \underline{g}}$ hydrostatic distribution.

magnitude and direction of the maximum spatial rate of increase in "p".

$\Rightarrow$ ∇p is perpendicular to the surfaces of constant p.

$\therefore$ A fluid in hydrostatic equilibrium will align its constant-pressure surfaces everywhere normal to the local gravity vector.

(max. pressure increase will be in the direction of gravity, i.e, downward. for a liquid, its free surface is normal to local gravity.



· in a conventional coordinate system.

$$\underline{g} = -g\hat{k}, \quad \nabla p = \rho \underline{g}. \rightarrow \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = -\rho g$$

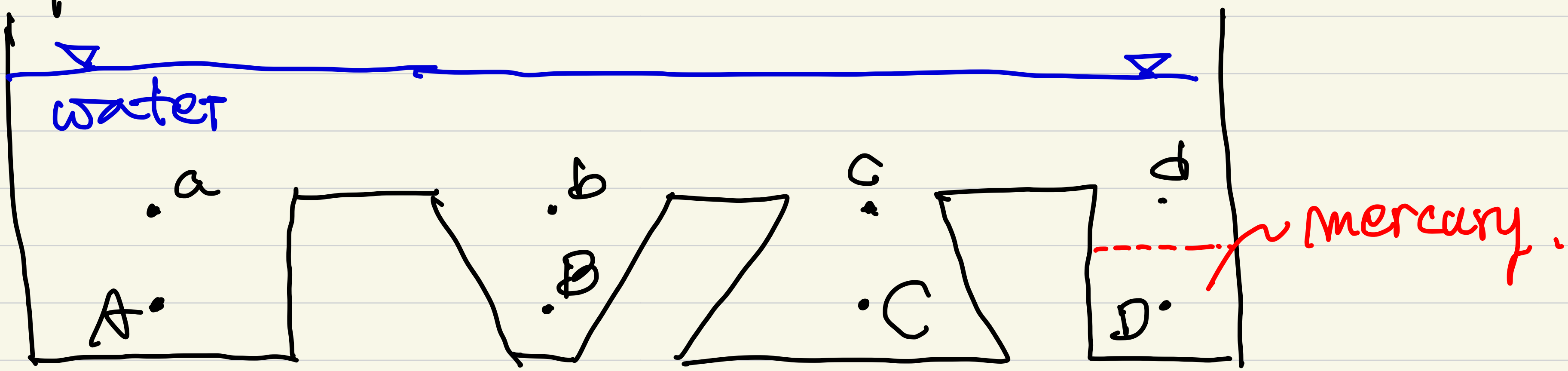
$$\therefore \frac{\partial p}{\partial z} = -\rho g \xrightarrow[\text{integrate.}]{} \boxed{P_2 - P_1 = -\int_1^2 \rho g \, dz.}$$

(Solution for hydrostatic problem.)

"Hydrostatic condition". (Pascal's Law).

Pressure in a continuously distributed uniform static fluid varies only w/ vertical distance, and is indep. of the shape of the container. The pressure is the same at all points on a given horizontal plane in the fluid. The pressure increases w/

depth in the fluid.



$$P_a = P_b = P_c = P_d \,, \quad P_A = P_B = P_C \neq P_D .$$

* Hydrostatic pressure in Liquids.

비압축성.

(almost incompressible. ∴ $\rho = \text{const}$)

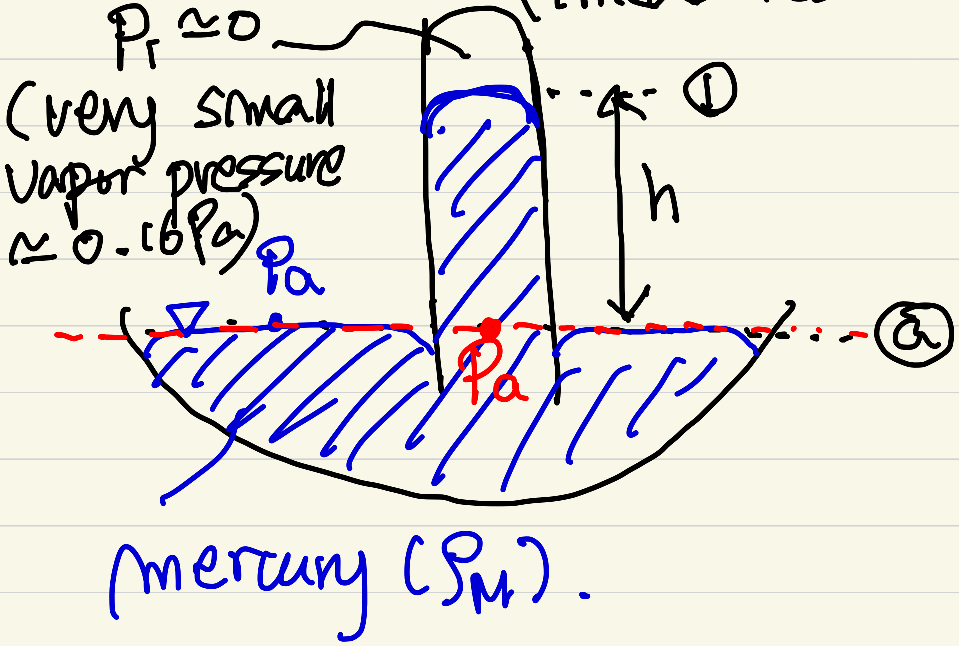
$$P_2 - P_1 = -\int_1^2 \rho g \, dz \; = -\rho g (z_2 - z_1).$$

$$\text{or,} \quad z_2 - z_1 = -\frac{P_2}{\rho g} + \frac{P_1}{\rho g}.$$

pressure head. [L].

* Mercury Barometer

(measures the atmospheric pressure).



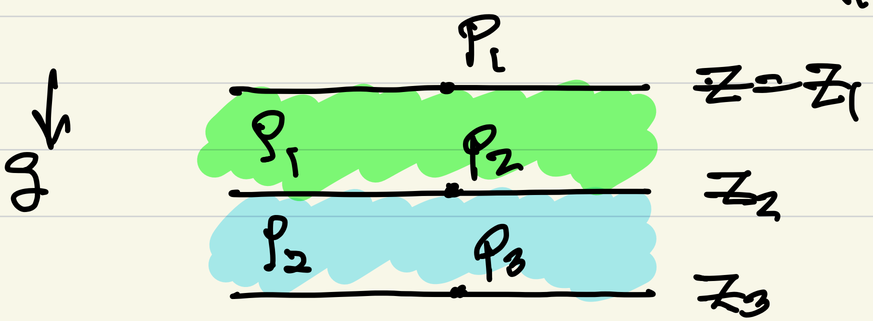
$$P_1 - P_a = -\rho_M g (z_1 - z_a).$$

$$= -\rho_M g h.$$

$$\therefore h = \frac{P_a}{\rho_M g} = 761 \text{ mmHg}.$$

2.4. Application to manometry.

· manometer: a device to measure pressure difference between two points.

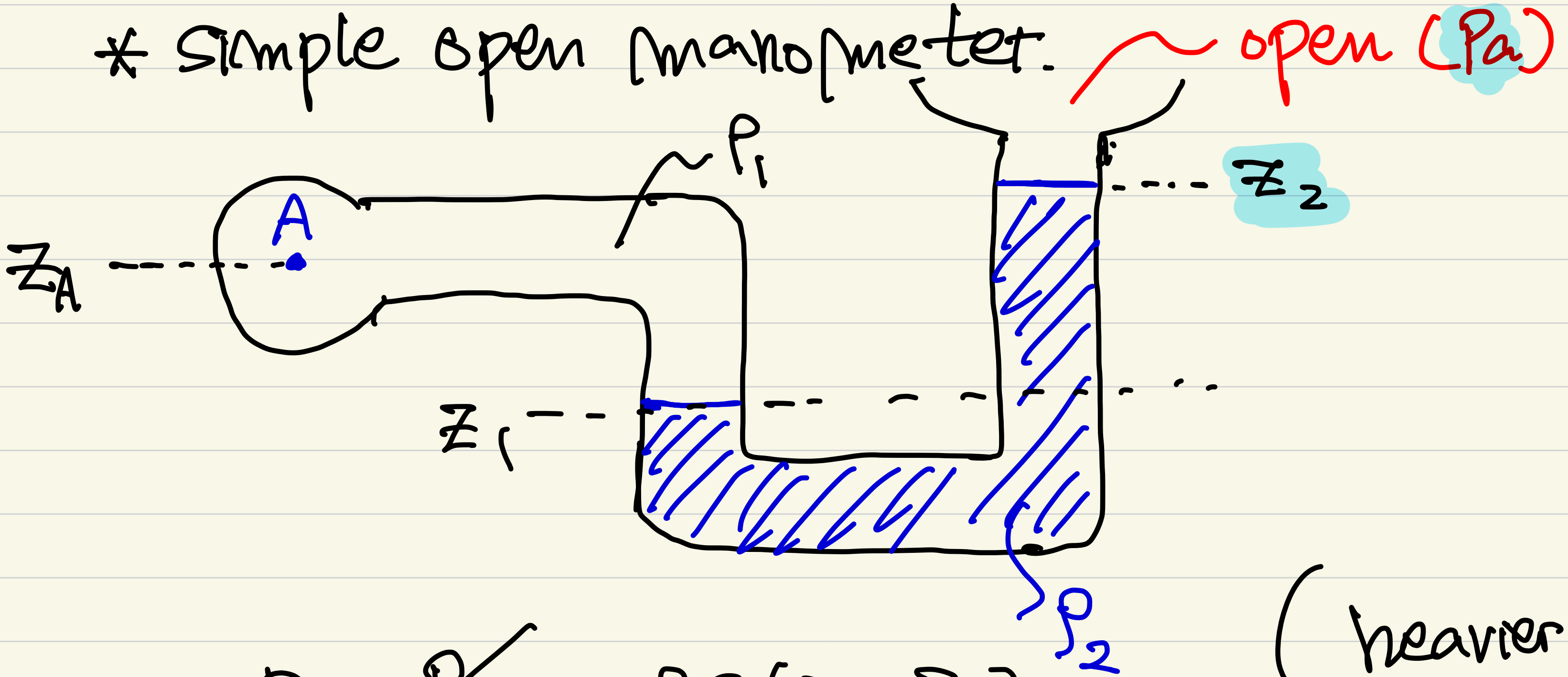


$$P_2 - P_1 = -\rho_1 g (z_2 - z_1)$$

$$P_3 - P_2 = -\rho_2 g (z_3 - z_2).$$

$$\Rightarrow P_3 - P_1 = -\rho_1 g (z_2 - z_1) - \rho_2 g (z_3 - z_2)$$

* simple open manometer.



$$+\left|\begin{array}{l} P_A - \cancel{P_1} = -\rho_1 g (z_A - z_1). \\ \cancel{P_1} - P_a = -\rho_2 g (z_1 - z_2). \end{array}\right.$$

$$\underbrace{P_A - P_a}_{\text{gage pressure.}} = -\rho_1 g (z_A - z_1) - \rho_2 g (z_1 - z_2).$$

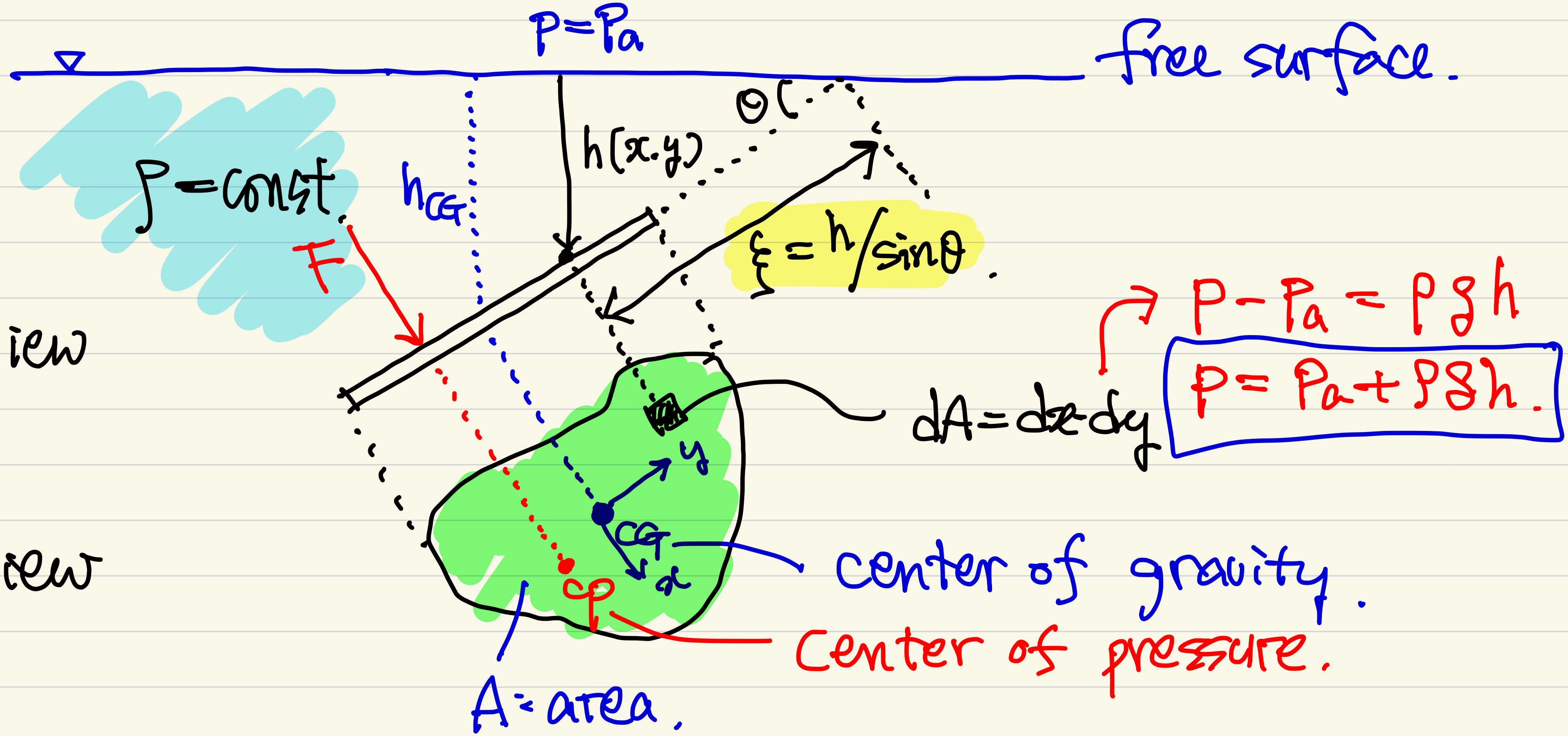
(heavier fluid (ρ_2) is better to make '$z_1 - z_2$' smaller)

2.5. Hydrostatic forces on plane surface.

$$\left(P_2 - P_1 = -\int_1^2 \rho g \, dz \right).$$

side view

plan view



→ Total hydrostatic force on one side, F

$$: F = \int P \cdot dA = \int (P_a + \rho g h) \, dA$$

$$= P_a \cdot A + \rho g \int h \, dA.$$

$$\left(h = \xi \cdot \sin\theta . \quad \boxed{\xi_{CG} = \frac{1}{A} \int \xi \, dA} \text{ by definition} \right)$$

$$F = P_a \cdot A + \rho g \int \xi \cdot \sin\theta \cdot dA$$

$$= P_a \cdot A + \rho g \sin\theta \int \xi \, dA.$$

$$= P_a \cdot A + \rho g \underbrace{\sin\theta \cdot A \cdot \xi_{CG}}_{h_{CG}.} = P_a \cdot A + \rho g A \cdot h_{CG}.$$

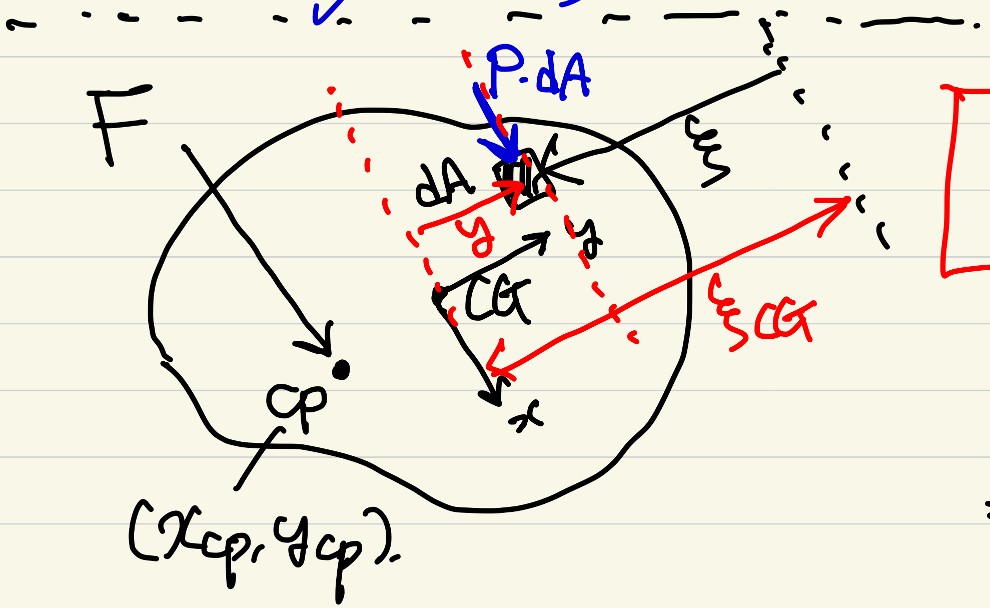
$$= (P_a + \rho g h_{CG}) A. = P_{CG} \cdot A.$$

"The force on one side of any plane submerged in a uniform fluid equals the pressure at the plate centroid (P_{CG}) times the plate area (A), indep. of the plate shape on the angle (θ) at which the plate is slanted.

- Then, where is the force acting?
  - F acts not through the CG.
    (if it is, bending moment is zero!)

    ↓

    should balance the bending-moment via stress (pressure) by acting at the high-pressure side.



$$\boxed{F \cdot y_{cp}} = \int y \cdot P \, dA.$$

$$= \int y (P_a + \rho g h) \, dA.$$

$$= \int y (P_a + \rho g \xi \sin\theta) \, dA.$$

$$= P_a \int \cancel{y}\, dA + \rho g \sin\theta \int y\, \xi\, dA.$$

0

$$= \rho g \sin\theta \int y \left(\xi_{CG} - y\right) dA = \rho g \sin\theta \left[\xi_{CG} \int \cancel{y}\, dA - \int y^2 dA\right]$$

0

($I_{xx} = \int y^2 dA$, area moment of inertia)

$\therefore \underline{F} \cdot y_{cp} = -\rho g \sin\theta \cdot I_{xx}$. $\Rightarrow$ $\boxed{y_{cp} = -\rho g \sin\theta \cdot \frac{I_{xx}}{P_{CG} A}}$

$= P_{CG} \cdot A$.

CP is below CG

depends on θ and geo.

as the plate moves deeper, $P_{CG} \uparrow$.

$\rightarrow y_{cp} \rightarrow 0. \rightarrow CP \rightarrow CG.$

· Similarly (moment balance along x-dir).

$$F \cdot x_{cp} = \int x \cdot P\, dA = -\rho g \sin\theta \underline{\int xy\, dA}.$$

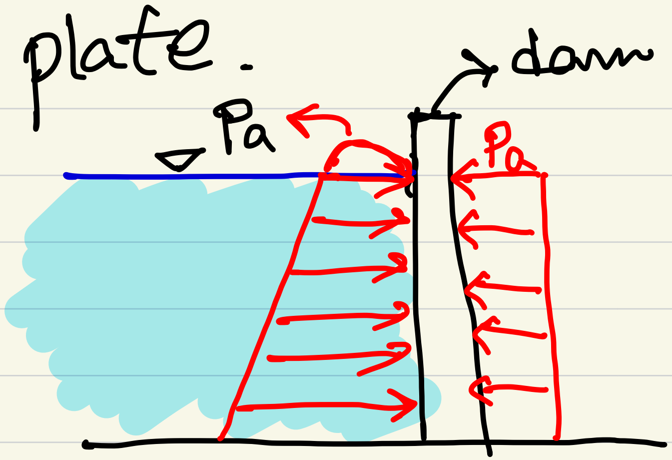
$= I_{xy}$ (product of inertia).

$$\boxed{x_{cp} = -\rho g \sin\theta \frac{I_{xy}}{P_{CG} \cdot A}.}$$

(for symmetric shape, $I_{xy} = 0$, $x_{cp} = 0$).

* <u>Gage pressure</u> formulae.

: P_a (ambient pressure) is neglected in most cases, because it acts on both sides of the



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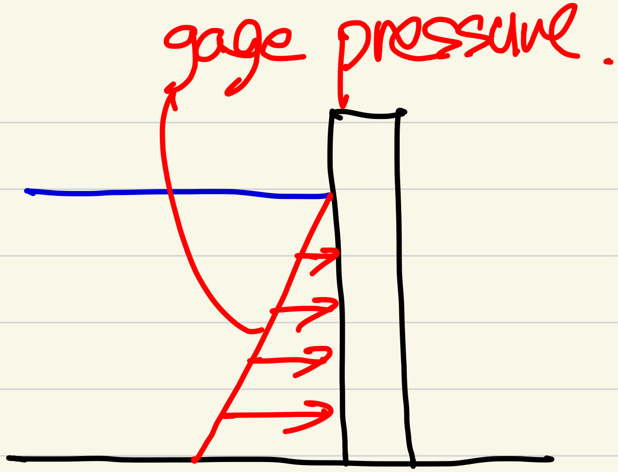


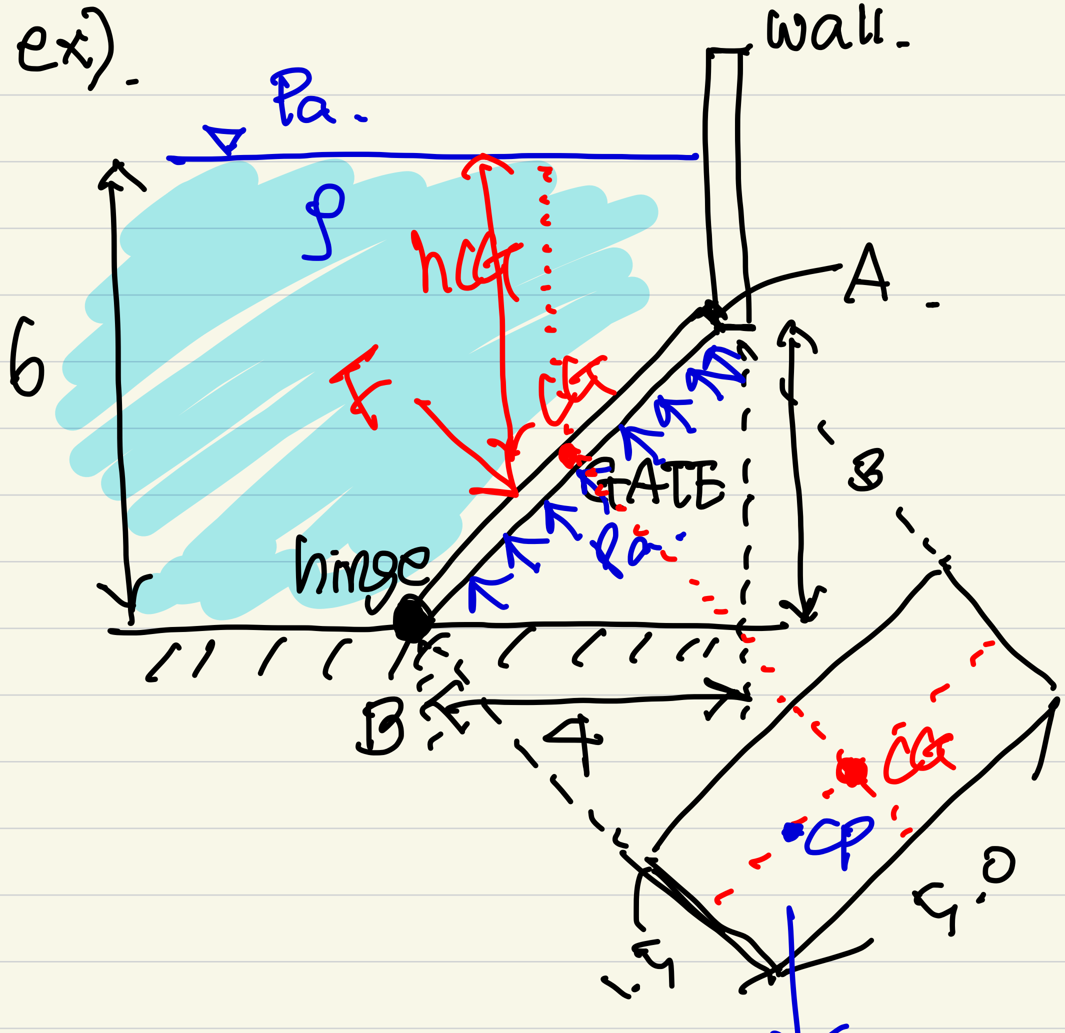
$$\underbrace{P_{CG} - P_a}_{\text{gage pressure}} = \rho g h_{CG} \rightarrow P_{CG} = \rho g h_{CG}$$

$$\Downarrow$$

$$F = \rho g h_{CG} \cdot A$$

$$\left(\begin{aligned} y_{cp} &= -\frac{I_{xx} \cdot \sin\theta}{h_{CG} \cdot A} \\ x_{cp} &= -\frac{I_{xy} \cdot \sin\theta}{h_{CG} \cdot A} \end{aligned} \right)$$

ex).

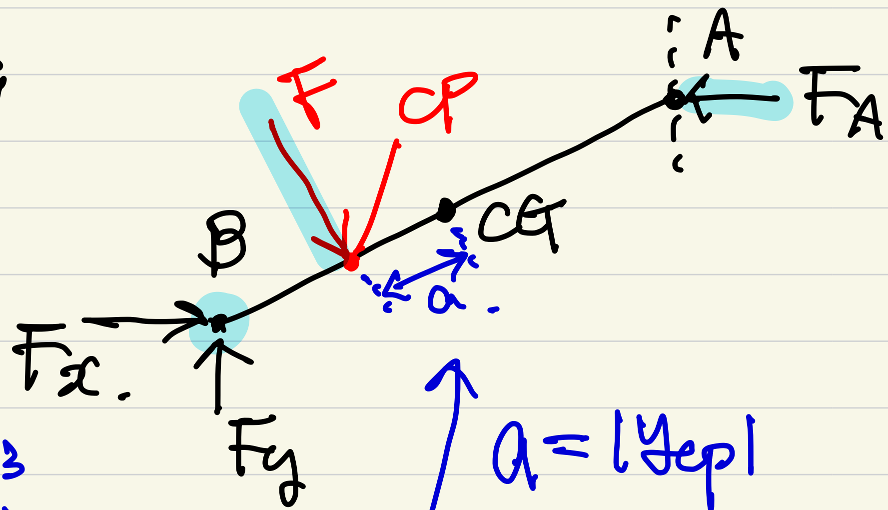


(a) $F_G = ? = P_{CG} \cdot A$.

$= \rho g h_{CG} \cdot A$.

1.5×5.

(b) Forces at A and B ?



$$\begin{cases} I_{xx} = \frac{1}{12} b L^3 \\ I_{xy} = 0. \end{cases}$$

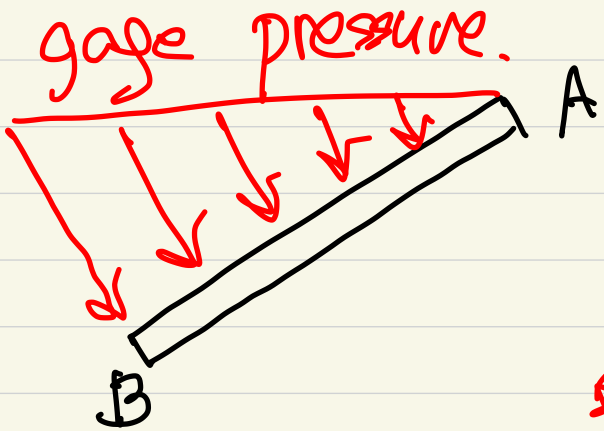
$$a = |y_{cp}| = \left| \frac{I_{xx} \cdot \sin\theta}{h_{CG} \cdot A} \right|$$

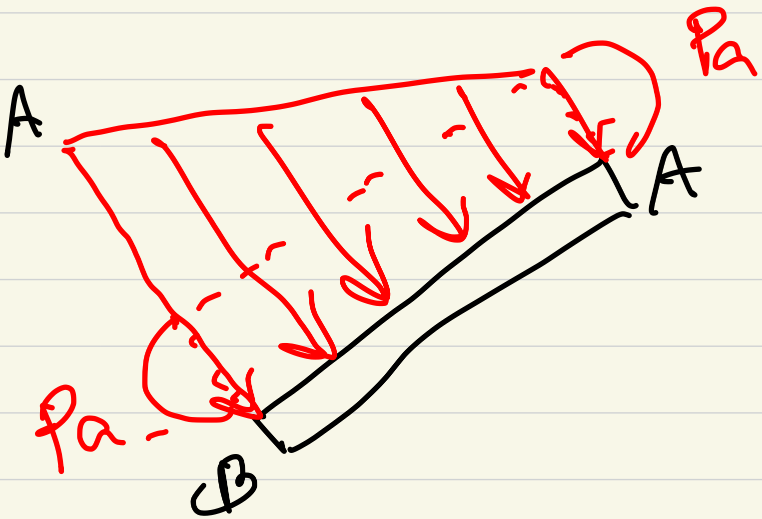
$$\sum F_{\rightarrow} = F_x - F_A + F\sin\theta = 0$$

$$\sum F_{\uparrow} = F_y - F\cos\theta = 0$$

$$\circlearrowleft + \sum M_B = F\cdot(2.5-a) - F_A\cdot 3 = 0$$

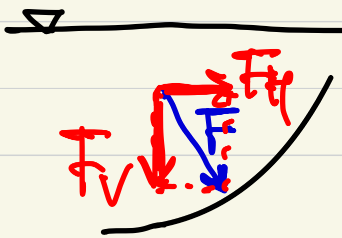
$\rightarrow F_x, F_y, F_A.$

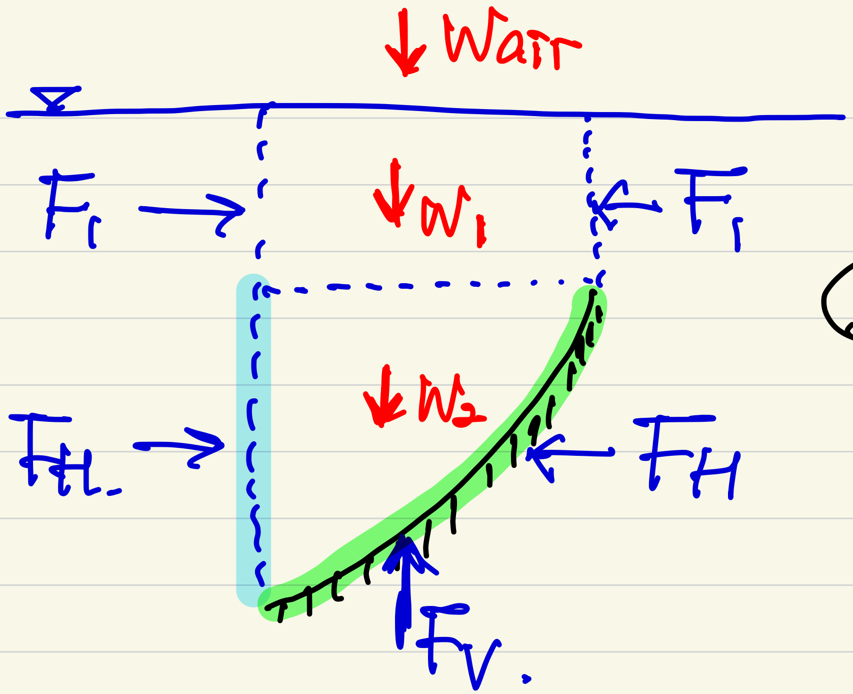




26. Hydrostatic forces on curved surfaces.

↳ try to separate F_H and F_V.





① F_H equals the force on the plane area formed by the projection of curved surface onto a vertical plane.

② $F_V = W_{air} + W_1 + W_2$.