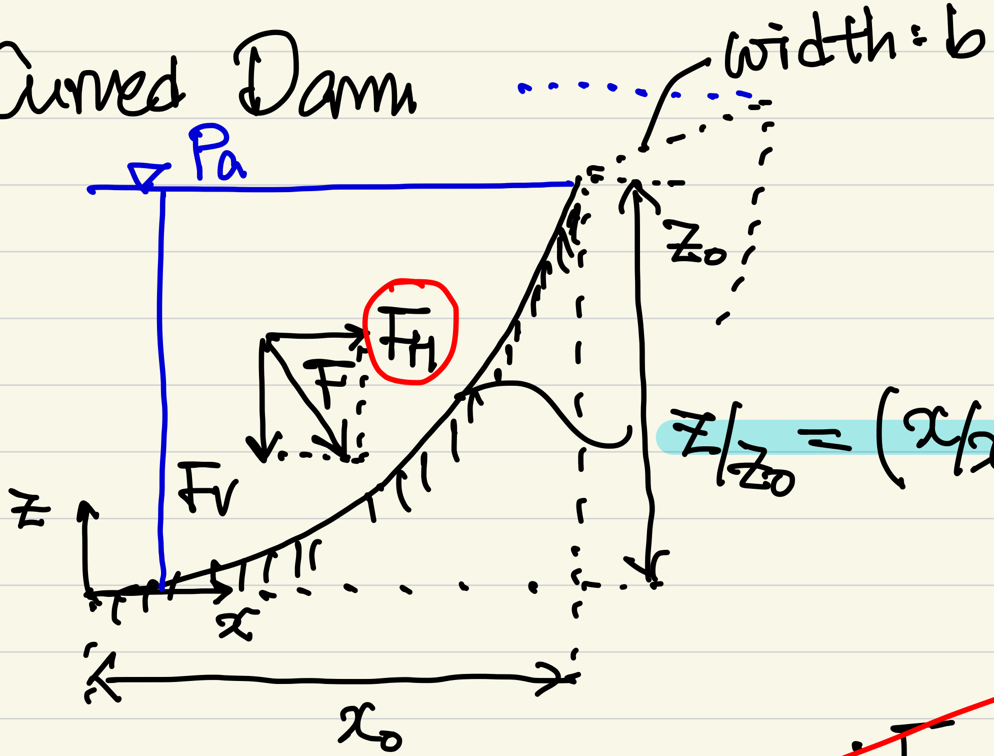
· 중간고사 #1 : 10/11 (화) 11:00~12:30 (1, 2장)
(301동 204호, 305호)

ex) Curved Dam



$$z/z_0 = (x/x_0)^2$$

· $P_a \simeq 0$ ☆

· $F_H = \rho g h_{cg} \cdot A_{proj}$

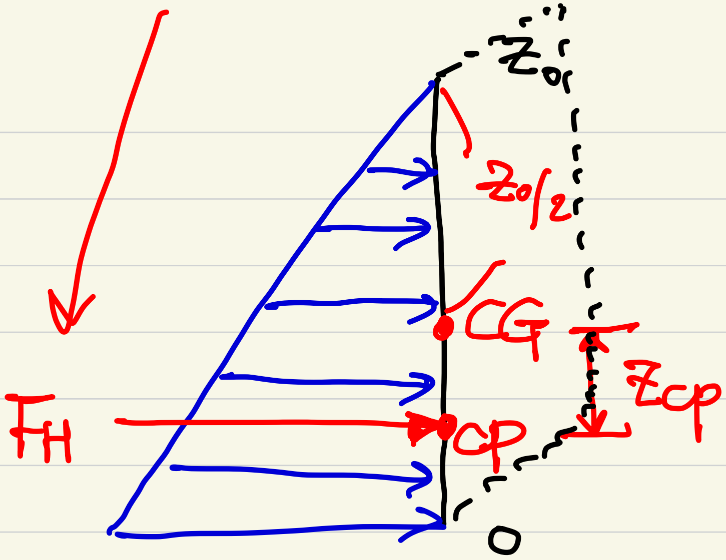
$$= \rho g \cdot \frac{z_0}{2}(z_0 \cdot b)$$

$$= \frac{1}{2}\rho g b z_0^2$$

· $F_V = \rho g \forall$

$$= \rho g \int_0^{x_0} (z_0 - z) b \cdot dx$$

$$= \frac{2}{3}\rho g b x_0 z_0 .$$

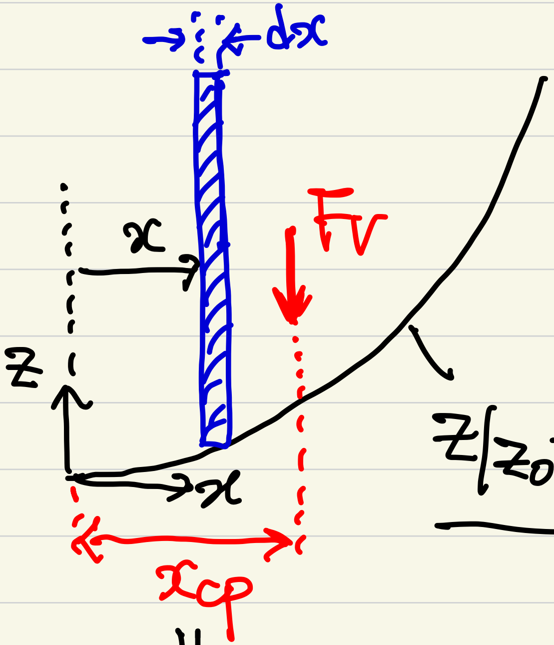


$$\underbrace{F_H}_{\frac{1}{2}\rho g b z_0^2} \cdot z_{cp} = -\rho g \underbrace{\sin\theta}_{=1} \cdot \underbrace{I_{xx}}_{\frac{1}{12} b z_0^3}.$$

$\rightarrow z_{cp} = -\frac{1}{6} z_0$. (or $\frac{1}{3} z_0$ from the bottom)

$\rightarrow$ similarly for F_V.

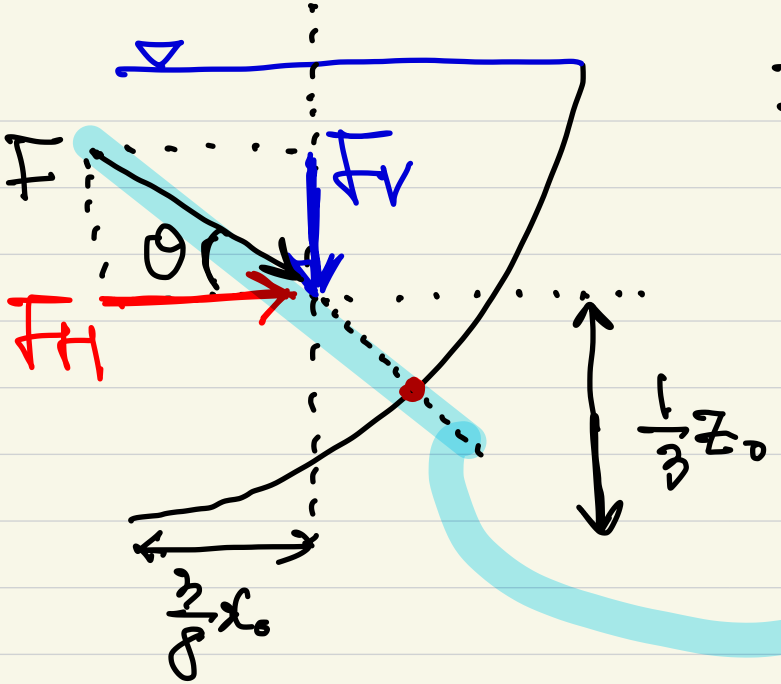
$$F_V \cdot x_{cp} = \int_0^{x_0} \rho g (z_0 - z) b \cdot x \cdot dx$$



$z/z_0 = (x/x_0)^2$

$\therefore x_{cp} = \frac{3}{8} x_0$.

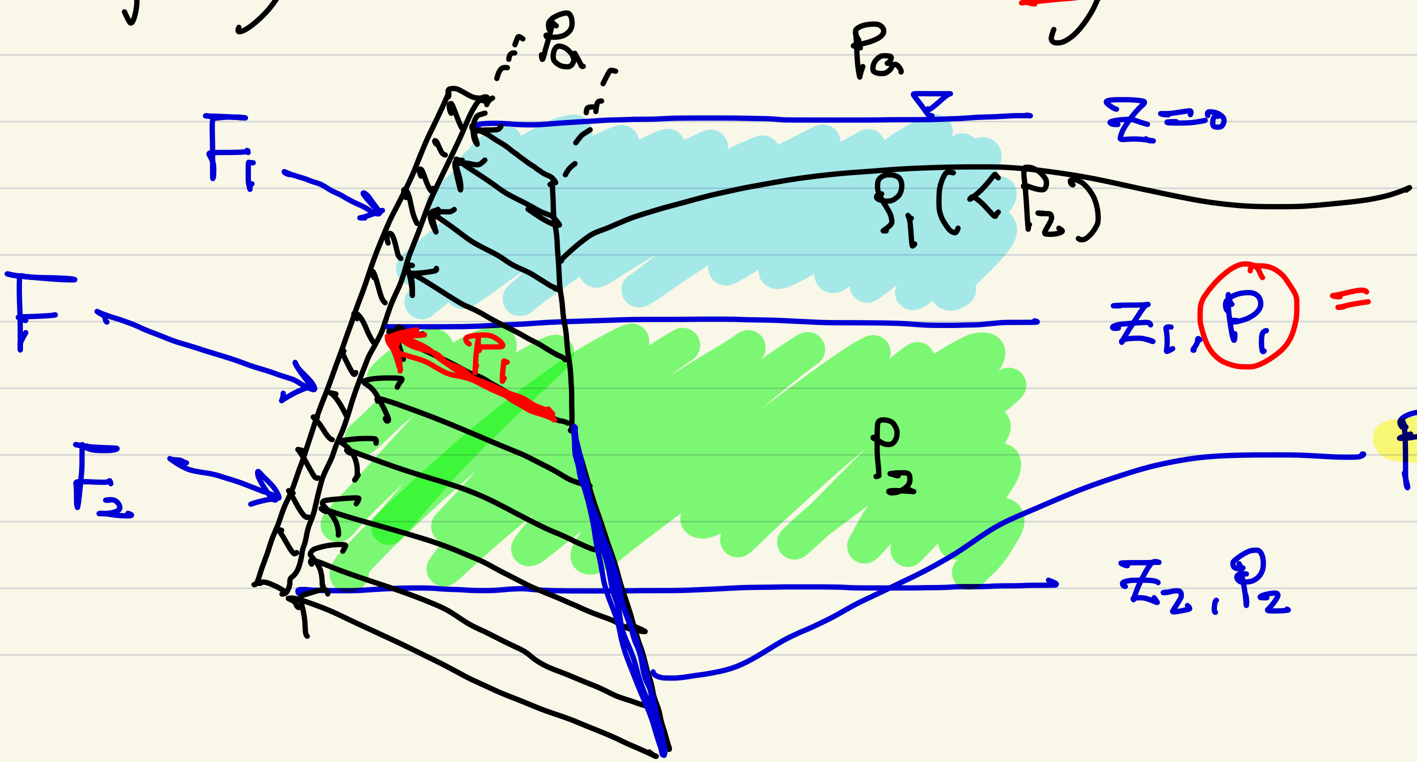
$\frac{3}{8} x_0$.



$$F = \sqrt{F_H^2 + F_V^2}.$$

line of action.

2.7) Hydrostatic forces in layered fluids.



$$p = p_a - \rho_1 g z$$

$$p_1 = p_a - \rho_1 g z_1$$

$$p = p_1 - \rho_2 g(z - z_1)$$

$$\therefore F = \sum_i F_i = \sum_i P_{CG,i} \cdot A_i$$

$$\begin{cases} y_{cp,i} = -\dfrac{\rho_i g \sin\theta_i \cdot I_{xx,i}}{P_{CG,i} \cdot A_i} \\[2ex] x_{cp,i} = -\dfrac{\rho_i g \sin\theta_i \cdot I_{xy,i}}{P_{CG,i} \cdot A_i} \end{cases}$$

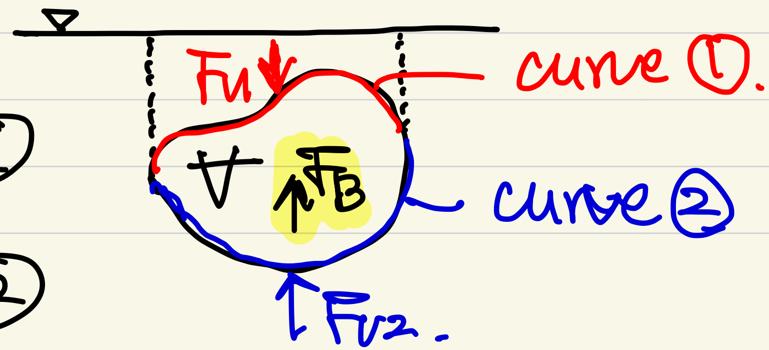
$\rightarrow$ moment summation.

$$\begin{cases} F \cdot y_{cp} = \sum_i F_i \cdot y_{cp,i} \\ F \cdot x_{cp} = \sum_i F_i \cdot x_{cp,i} \end{cases}$$

2.8 Buoyancy and Stability.

(Archimedes' Law).

① A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces.



- F_{v1} : fluid weight above curve ①
- F_{v2} : " ②

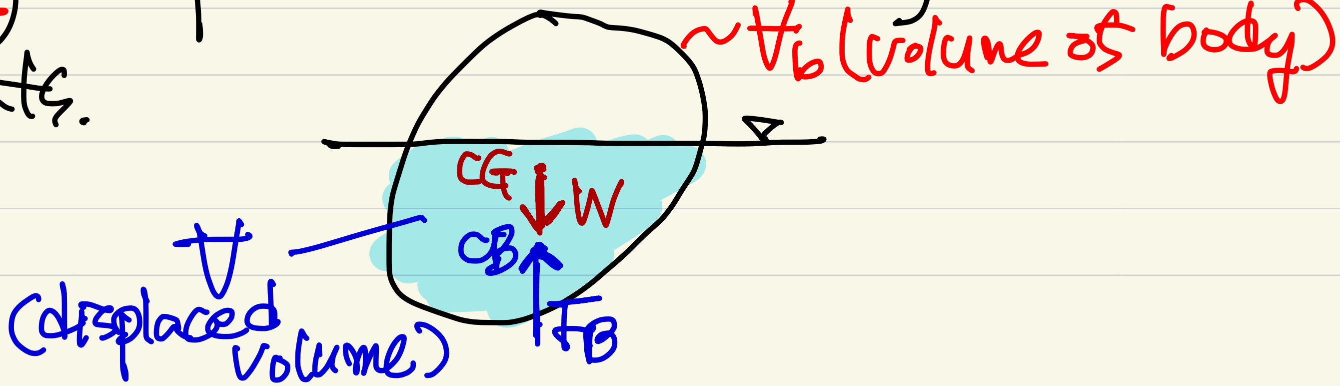
$\Rightarrow F_B = F_{V2} - F_{V1} = \underline{\rho_f} g \forall$ (weight of the body $W = \underline{\rho_b} \cdot g \forall$)

if $\rho_b = \rho_f$: neutrally buoyant.

- line of action of the F_B passes through the center of volume, if ρ is uniform.

  → CB (center of buoyancy) may or may not correspond to actual CM.

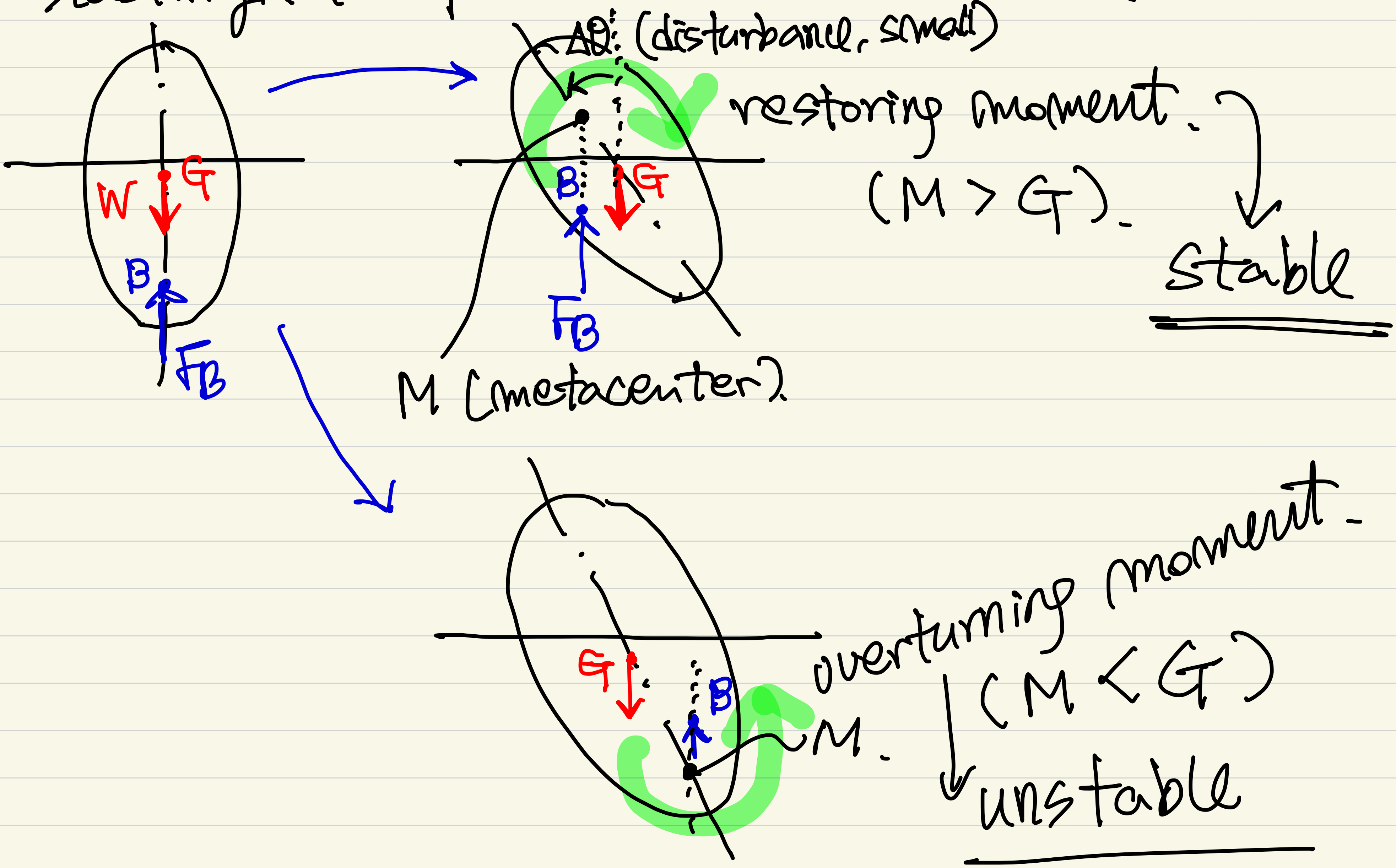
② A floating body displaces its own weight in the fluid it floats.



$$F_B = \rho_f g \forall = W = \rho_b g \forall_b .$$

$(\varepsilon \ll 1)$

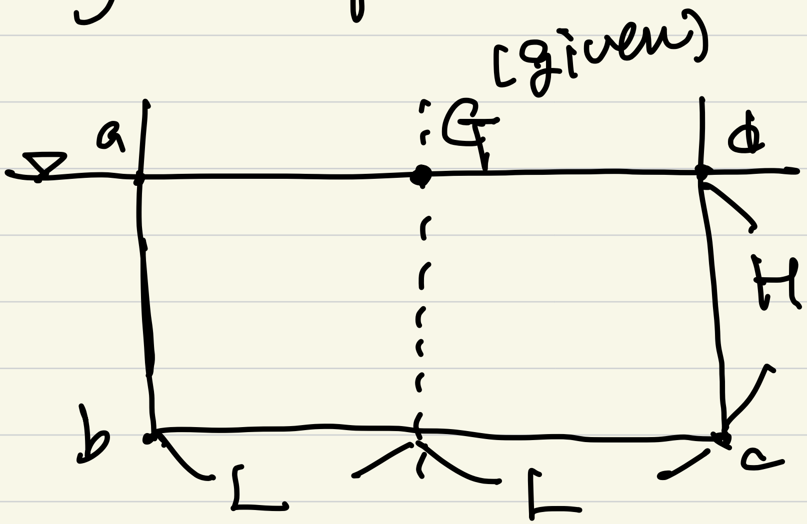
* Stability. (response to a disturbance).



$\overline{MG}$: metacentric height.
(property of the cross-section for the given weight). → indication of the stability
($\overline{MG} > 0$ for the stable condition)

ex). Barge ship.

TA.



L/H for stable condition?

2.9. Pressure distribution in rigid-body motion.

(no relative motion between particles)

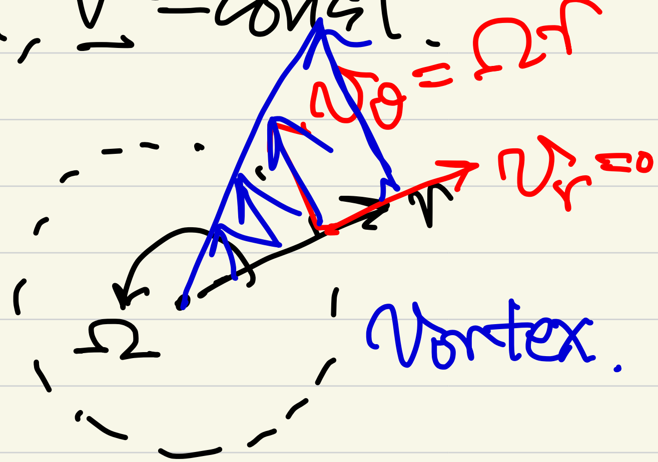
$$\nabla p = \rho(\underline{g} - \underline{a}) + \mu \nabla^2 \underline{V}$$

압력 구배 | 중력 | 가속도 | 0 점성

$\nabla^2 \underline{V} = 0$

translation, $\underline{V}$ = const.

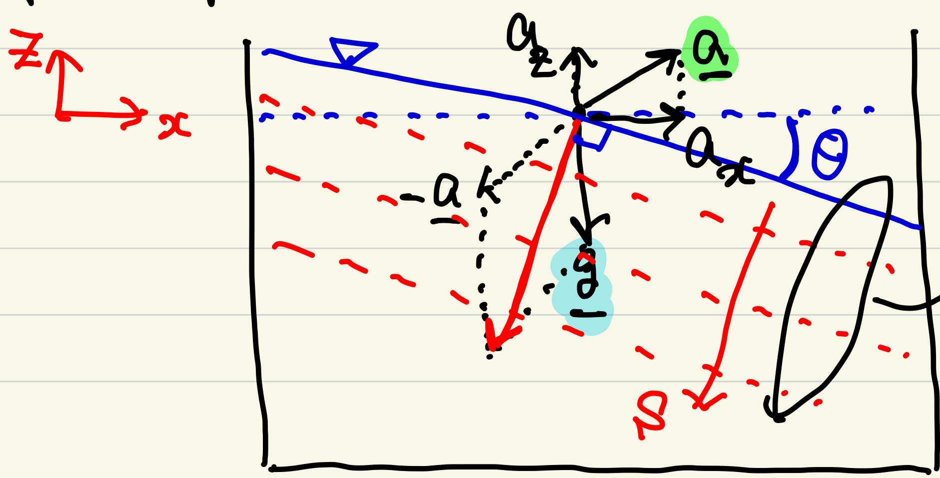
rotation.



$$\therefore \nabla p = \rho(\underline{g} - \underline{a}).$$

(line of constant $p \perp (\underline{g} - \underline{a})$)

* Uniform linear acceleration.

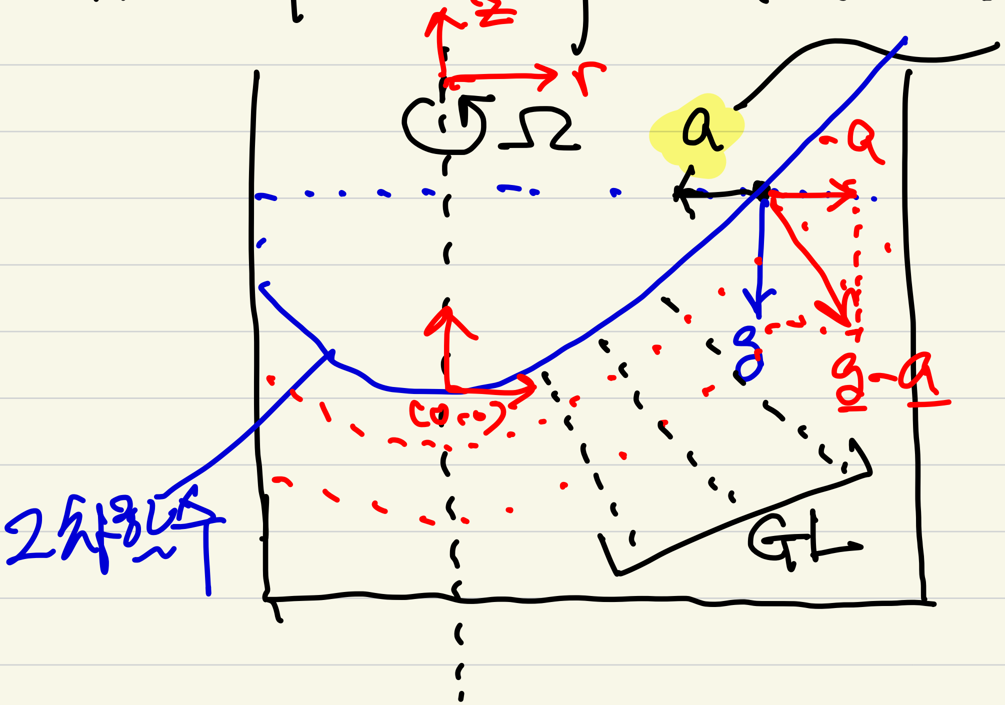


lines of constant pressure.

- $\tan\theta = \dfrac{a_x}{g+a_z} \rightarrow \theta = \tan^{-1}\dfrac{a_x}{g+a_z}$.

- $\Delta p = \rho g z \Rightarrow \dfrac{dp}{ds} = \rho\sqrt{a_x^2+(g+a_z)^2}$.

* rigid-body rotation.



centripetal acceleration.

$$\nabla p = \rho(\underline{g}-\underline{a})$$

$$\underline{a} = \underline{\Omega}\times(\underline{\Omega}\times\underline{r}) = -r\Omega^2\hat{i}_r$$

$$\underline{\Omega} = \Omega\cdot\hat{i}_k$$

$$\nabla p = \boxed{\frac{\partial p}{\partial r}\hat{i}_r} + \boxed{\frac{\partial p}{\partial z}\hat{i}_k}$$

$$= \rho(\underline{g}-\underline{a}) = \boxed{-\rho g\hat{i}_k} + \boxed{\rho r\Omega^2\hat{i}_r}$$

$$\underline{g} = -g\hat{i}_k$$

Euler n eq.

$$\therefore \frac{\partial p}{\partial r} = \rho r \Omega^2, \quad \frac{\partial p}{\partial z} = -\rho g. \Rightarrow p(r,z)$$

$$\rightarrow p(r,z) = \frac{1}{2}\rho r^2 \Omega^2 - \rho g z + C.$$

@ $(r,z) = (0,0)$ (free surface), $p = P_a$.

$$\therefore C = P_a.$$

$$\therefore p = P_a - \rho g z + \frac{1}{2}\rho r^2 \Omega^2.$$

↳ surface of constant pressure (P_1)

$$\therefore \boxed{z = \frac{P_a - P_1}{\rho g} + \frac{\Omega^2 r^2}{2g}}.$$: paraboloids of revolution.

if $P_1 = P_a$. $z = \frac{\Omega^2 r^2}{2g}$. → free surface.

- Lines orthogonal to the constant-pressure surfaces. (i.e. pressure-gradient surfaces).

$$\Rightarrow \left.\frac{dz}{dr}\right|_{GL} = -\frac{1}{(dz/dr)_{p=const}} = -\frac{1}{r\Omega^2/g}.$$

(gradient line)

$$\therefore \left.\frac{dz}{dr}\right|_{GL} = -\frac{g}{r\Omega^2}. \Rightarrow r \simeq C_1 \cdot \exp\left(-\frac{\Omega^2 z}{g}\right).$$

in the absence of friction and Coriolis effects, ∴
the particles will follow these lines!

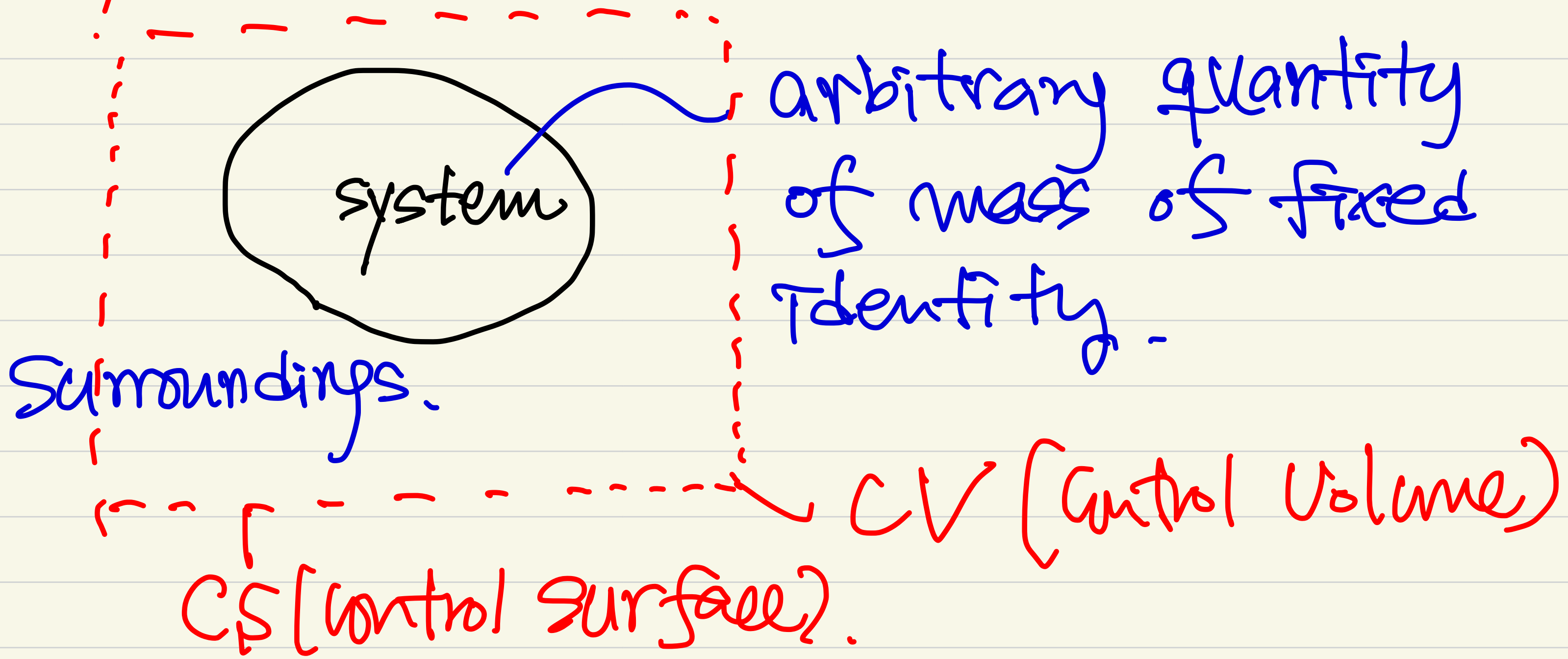
Depending on their density, small particles/bubbles will fall/rise along these exponential lines.

ㄴ중간#1. 10/11. 11:00 - 12:30. 305, 204.

CH. 3 Integral relation for a Control Volume.

3.1. Basic Physical Laws of Fluid Mechanics.

* System vs. Control Volume.



⇒ Law of mechanics : Interaction between system and surroundings.

1) Conservation of mass : $m_{sys} = \text{constant}$, $\frac{dm}{dt} = 0$.

2) " momentum : $\underline{F} = m\underline{a} = m\frac{d\underline{v}}{dt}$

$$= \frac{d}{dt}(m\underline{V})$$

3) " angular mtm

: $\underline{M} = \frac{d}{dt}(\underline{H})$. $\underline{H} = \sum(\underline{r} \times \underline{V})\delta m$.

(total moment)

4) " energy : $dE = \delta Q - \delta W$.

$$\left(\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}\right).$$

5) 2^{ND} law of thermodynamics : $ds \geq \frac{dQ}{T}$.

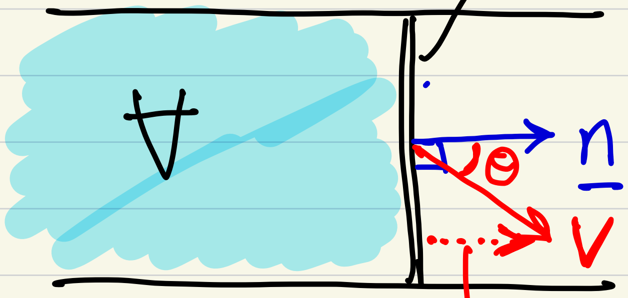
6) state equations, $p = p(\rho, T)$, $e = e(\rho, T)$ .....

↳ Control Volume form.

* Volume and mass flow rate.

· piston.

area : A



(normal vector).

$$d\forall = (V\cdot dt)\cdot A.$$

$$\therefore \frac{d\forall}{dt} = VA.$$

$V\cdot\cos\theta. \rightarrow d\forall = (V\cdot\cos\theta\cdot)\,dt\cdot A.$

$$= (\underline{V}\cdot\underline{n})\,dt\,A.$$