Chapter 3

Coupled Flap-Lag-Torsion Dynamics

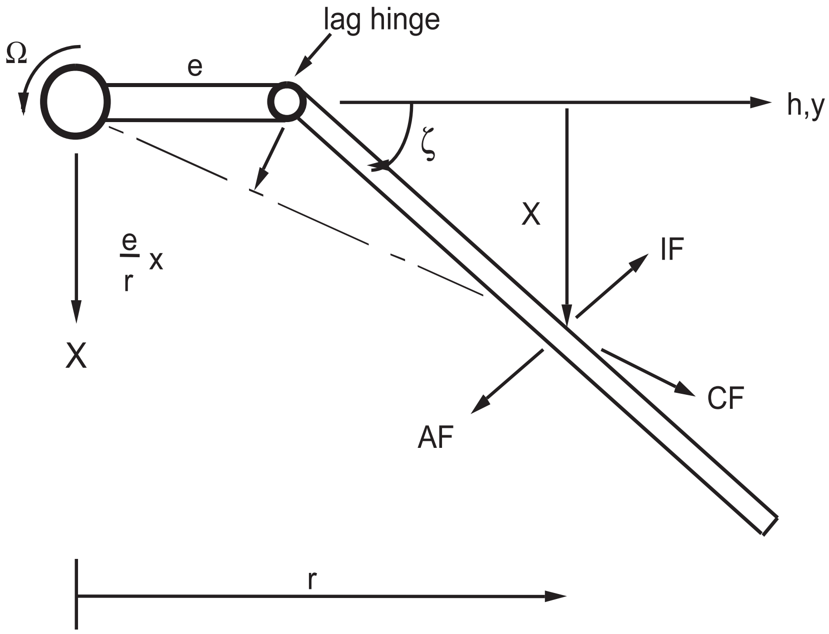
The objective of this chapter is to understand the equations of motion for a blade undergoing flap and lag bending and torsional deflection. In the last chapter, uncoupled flap dynamics was discussed. In this chapter, first the equations of motion for the uncoupled lag and torsion modes are discussed followed by coupled motions. The principle concern here is the structural and inertial terms, then the important coupling terms due to coupled motion due to flap, lag and torsion are identified. The Newtonian approach is used to derive the equations of motion. Also, one can derive these equations using the energy approach, but it is not a physical approach and therefore does not help to understand different forces. The resultant blade forces in the rotating frame and the hub forces in the fixed frame are also derived.

3.1 Lag Dynamics

The dynamics of lag motion is studied below.

3.1.1 Rigid Lag Model with Hinge Offset

The blade is assumed rigid and undergoes a single degree of motion in the plane of rotation. It has a hinge offset at a distance e from the rotation axis. The simple configuration represents an articulated blade with lag hinge. This type of modeling can be an approximate representation for a hingeless blade with a possible leaf spring at the hinge. The lag motion opposes rotation. Let us examine various forces acting on the blade for small angle assumption. For a blade element of length dr



(a) IF: inertia force $m\ddot{x}\,dr = m(r-e)\ddot{\zeta}dr$ arm (r-e) about lag hinge
(b) CF: centrifugal force $m\Omega^2 r\,dr$ arm $\frac{e}{r}(r-e)\zeta$
(c) AF: aerodynamic force F_ζ arm r-e
(d) SF: spring moment about hinge $k_\zeta\zeta$

where m is mass per unit length (lb sec^2/in^2) and F_ζ is external force per unit length (lb/in). Taking moment of all forces about lag hinge

$$\int_e^R m(r-e)^2 dr\ddot{\zeta} + \int_e^R m\Omega^2 r(r-e)\frac{e}{r}dr\,\zeta - \int_e^R F_\zeta(r-e)\,dr$$

$$+k_\zeta\zeta = 0$$

$$\int_e^R m(r-e)^2 dr = \text{mass moment of inertia about lag hinge, } I_\zeta$$

$$I_\zeta(\overset{**}{\zeta} + \nu_\zeta^2\zeta) = \frac{1}{\Omega^2}\int_e^R F_\zeta(r-e)dr \tag{3.1}$$

where ν_ζ is ndimensional lag frequency in terms of rotational speed.

$$\nu_\zeta^2 = \frac{k_\zeta}{I_\zeta\Omega^2} + \frac{e\int_e^R m(r-e)dr}{I_\zeta} \tag{3.2}$$

The second term is due to centrifugal spring and is zero if there is no hinge offset. The first term is due to spring bending at the hinge and this represents the nonrotating natural frequency of the blade made nondimensional with respect to rotational speed

$$\nu_\zeta^2 = \frac{\omega_{\zeta 0}^2}{\Omega^2} + e\frac{S_\zeta}{I_\zeta}$$

where S_ζ is the first moment of mass about lag hinge and I_ζ is the second moment of mass about lag hinge. For a uniform blade

$$S_\zeta = \frac{m}{2}(R-e)^2$$

$$I_\zeta = \frac{m}{3}(R-e)^3$$

The lag frequency becomes

$$\nu_\zeta^2 = \frac{\omega_{\zeta 0}^2}{\Omega^2} + \frac{3}{2}\frac{e}{R-e} \text{ per rev.}$$

For an articulated blade (zero offset)

$$\nu_\zeta = 0$$

This is not a realistic case and there will be no transfer of torque.
For an articulated blade with hinge offset, the lag equation becomes

$$\overset{**}{\zeta} + \nu_\zeta^2 \zeta = \gamma \overline{M}_\zeta \tag{3.3}$$

The lag mode is inherently very low damped and is quite susceptible to various aeroelastic instabilities. In particular, the soft lag rotor can get into mechanical instability called ground resonance. This is the reason that most of the existing rotors have mechanical lag dampers to stabilize the lag motion.

Let us look at uncoupled flap and lag frequencies for hinged blades,

$$\text{Flap: } \nu_\beta^2 = 1 + \frac{3}{2}\frac{e_\beta}{R} + \frac{\omega_{\beta 0}^2}{\Omega^2}$$

$$\text{Lag: } \nu_\zeta^2 = \frac{3}{2}\frac{e_\zeta}{R} + \frac{\omega_{\zeta 0}^2}{\Omega^2}$$

where e_β and e_ζ are respectively the flap and lag hinges from the rotation axis. Generally these are very close, and for making analysis simple these are assumed coincidental.

$$e_\beta = e_\zeta = e$$

For matched stiffness blades, the nonrotating flap and lag frequencies are equal

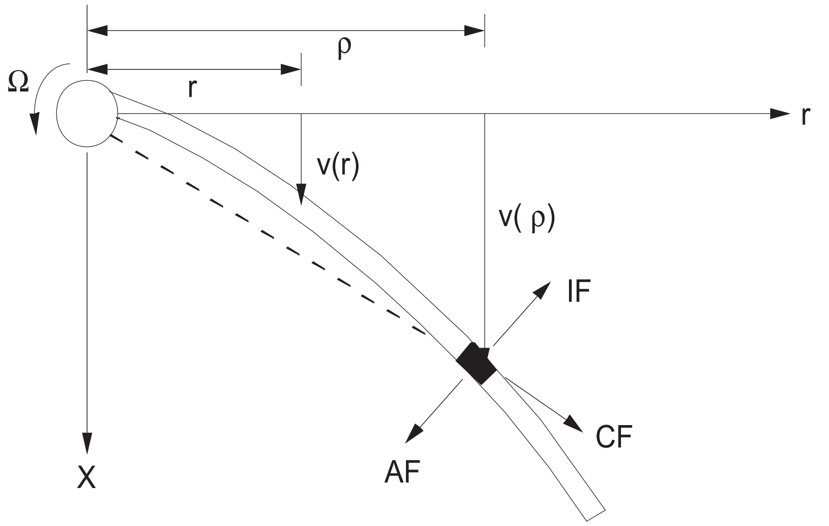
$$\omega_{\beta 0} = \omega_{\zeta 0}$$

This results into an important relationship between flap and lag rotating frequencies.

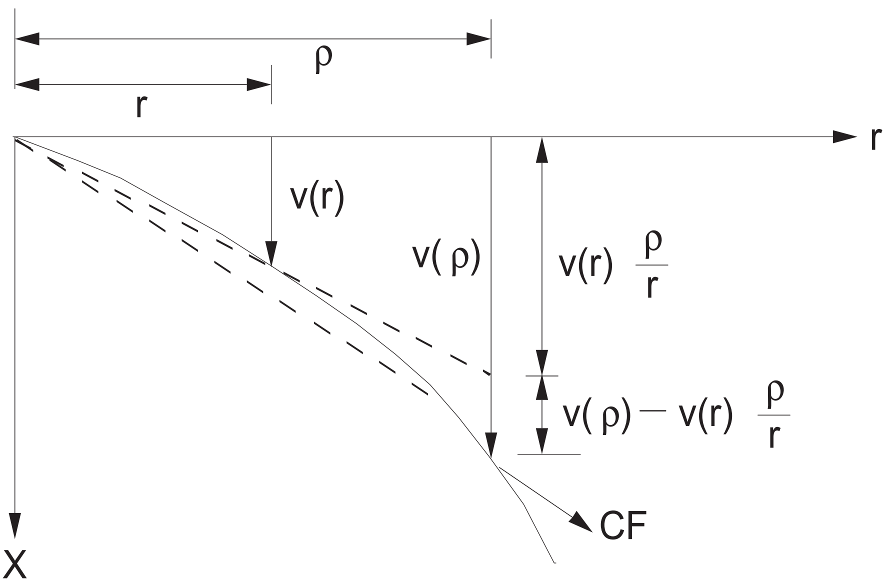
$$\nu_\beta^2 = 1 + \nu_\zeta^2 \tag{3.4}$$

3.1.2 Elastic Lag Model

A better representation for a blade is to assume it as an elastic beam undergoing in-plane bending. At the moment, it is assumed to be pure lag motion and the coupling terms due to other modes of motion will be considered later on. This type of modeling is also applicable to articulated blades if one needs to know more than the fundamental vibration mode. Let us examine the forces acting on an element of length dr a distance ρ



v(ρ) = lag bending deflection at r
v(r) = lag bending deflection at r
z axis is normal to the rotation plane



(a) IF: inertia force $m\ddot{v}(\rho)\, d\rho$ arm $(\rho - r)$ about r
(b) CF: centrifugal force $m\Omega^2 \rho\, d\rho$ arm $\frac{r}{\rho} v(\rho) - v(r)$
(c) AF: aerodynamic force $F_x\ dr$ arm $(\rho - r)$
The lag moment at r is

$$
\begin{aligned}
M_z(r) &= \int_r^R (F_x - m\ddot{v})(\rho - r)\, d\rho \\
&= -\int_r^R m\Omega^2 \rho \{\frac{r}{\rho} v(\rho) - v(r)\} d\rho
\end{aligned}
\qquad (3.5)
$$

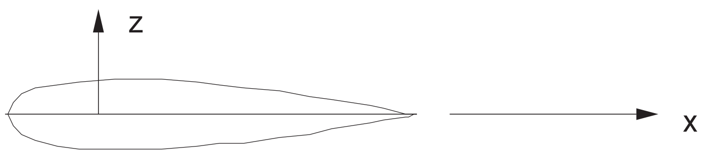
Let us recall the Leibnitz theorem

$$
\begin{aligned}
\phi &= \int_{u_1(r)}^{u_2(r)} f(\rho, r)\, d\rho \\
\frac{\partial \phi}{\partial r} &= \int_{u_i(r)}^{u_2(r)} \frac{\partial f}{\partial r}(\rho, r) d\rho + \frac{\partial u_2(r)}{\partial r} f(u_2(r), r) \\
& - \frac{\partial u_1(r)}{\partial r} f(u_1(r), r)
\end{aligned} \tag{3.6}
$$

The beam bending equation is

$$M_z = EI_z \frac{d^2 v}{dr^2} \tag{3.7}$$

Let us consider an airfoil section



M_z = moment of forces about z axis at station r, in-lb
E = Young's modulus of elasticity, lb/in^2. Typically for aluminum $10.5 \times 10^6 \text{lb/in}^2$
I_z = area moment of inertia about z-axis

$$= \int_{\text{section}} x^2 dA, \ \text{in}^4$$

x and z are section principle axes.
Taking the second derivative of the moment (Eq. (3.7)) and using Leibnitz's theorem one gets

$$\frac{d^2}{dr^2}(EI_z \frac{d^2 v}{dr^2}) - \frac{d}{dr}\left[\int_r^R m\Omega^2 \rho\, d\rho\, \frac{dv}{dr}\right] + m\ddot{v} - m\Omega^2 v = F_x(r, t) \tag{3.8}$$

This equation can be also derived like the flap bending equation. The first term is the flexural term, the second term is due to centrifugal force, the third term is the inertia term and the fourth term is the vertical component of centrifugal force. The boundary conditions here are quite similar to those of flap bending described in Chapter 2.

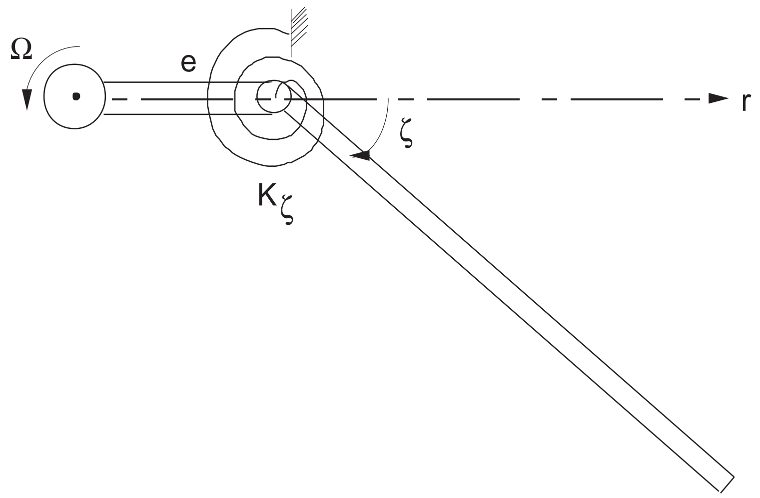
3.1.3 Natural Vibrations of Lag Motion

The natural vibration characteristics of a rotating blade in pure lag bending mode are obtained from the homogeneous solution of lag bending equation (3.8)

$$\frac{d^2}{dr^2}(EI_z \frac{d^2 v}{dr^2}) - \frac{d}{dr}\left[\left(\int_r^R m\Omega^2 \rho\, d\rho\right) \frac{dv}{dr}\right] + m\ddot{v} - m\Omega^2 v = 0 \tag{3.9}$$

The boundary conditions for a hingeless blade are
@ r = 0 v = 0 $\frac{dv}{dr} = 0$
@ r = R $M_z = EI_z \frac{d^2 v}{dr^2} = 0$ (moment)
$S_x = \frac{d}{dr}(EI_z \frac{d^2 v}{dr^2}) = 0$ (shear)

The boundary conditions for a hinged blade with a lag hinge at distance e from rotation axis and a leaf spring k_ζ at the hinge



$@r = e \qquad V = 0$

$$M_z = EI_z \frac{d^2 v}{dr^2} = k_\zeta v \quad \text{(Bending)}$$

$@r = R$

$$M_z = EI_z \frac{d^2 v}{dr^2} = 0 \quad \text{(Bending)}$$

$$S_x = \frac{d}{dr}(EI_z \frac{d^2 v}{dr^2}) = 0 \quad \text{(Shear)}$$

Let us consider a hingeless blade. The closed form (exact) solution is available only for a uniform nonrotating blade. For a rotating blade, one has to apply some approximate method to calculate the natural frequencies and mode shapes. Three approximate methods to solve this problem have been discussed in Chapter 2.

Note that the nonrotating natural vibration characteristics for a uniform beam with different boundary conditions were given in Chapter 2 as beam functions. Recall that the tabulated numerical values by Felgar and Young (1950) were given as follows.

For a cantilever beam, the j^{th} mode shape is expressed as

$$\phi_j(r) = \cosh \lambda_j r - \cos \lambda_j r - \alpha_j (\sinh \lambda_j r - \sin \lambda_j r) \tag{3.10}$$

Mode j	1	2	3	4	j
λ_j	1.8751	4.6941	7.8548	10.995	$(2j-1)\pi/2$
α_j	.7341	1.0185	.9992	1.0	1.0

The nonrotating natural frequency for a uniform beam for j^{th} mode is

$$\omega_{j0} = (\lambda_j)^2 \sqrt{\frac{EI_z}{mR^4}} \tag{3.11}$$

where EI_z = flexural stiffness about chord, lb-in^2

m = mass per unit length, lb-sec^2/in^2

R = blade length, in

Again, these beam modes are orthogonal as discussed in Chapter 2.

3.1.4 Finite Element Formulation

Quite similar to flap bending vibrations, it is quite convenient to calculate lag bending vibration characteristics of a rotating blade using finite element formulation. The procedure is similar to one discussed in art. 2.7. The shape functions for a beam element are the same. The element energy expressions are slightly modified for lag bending of a rotating beam.

$$\text{Kinetic energy} \quad T = \frac{1}{2}\int_0^l m\dot{v}^2\,dx$$

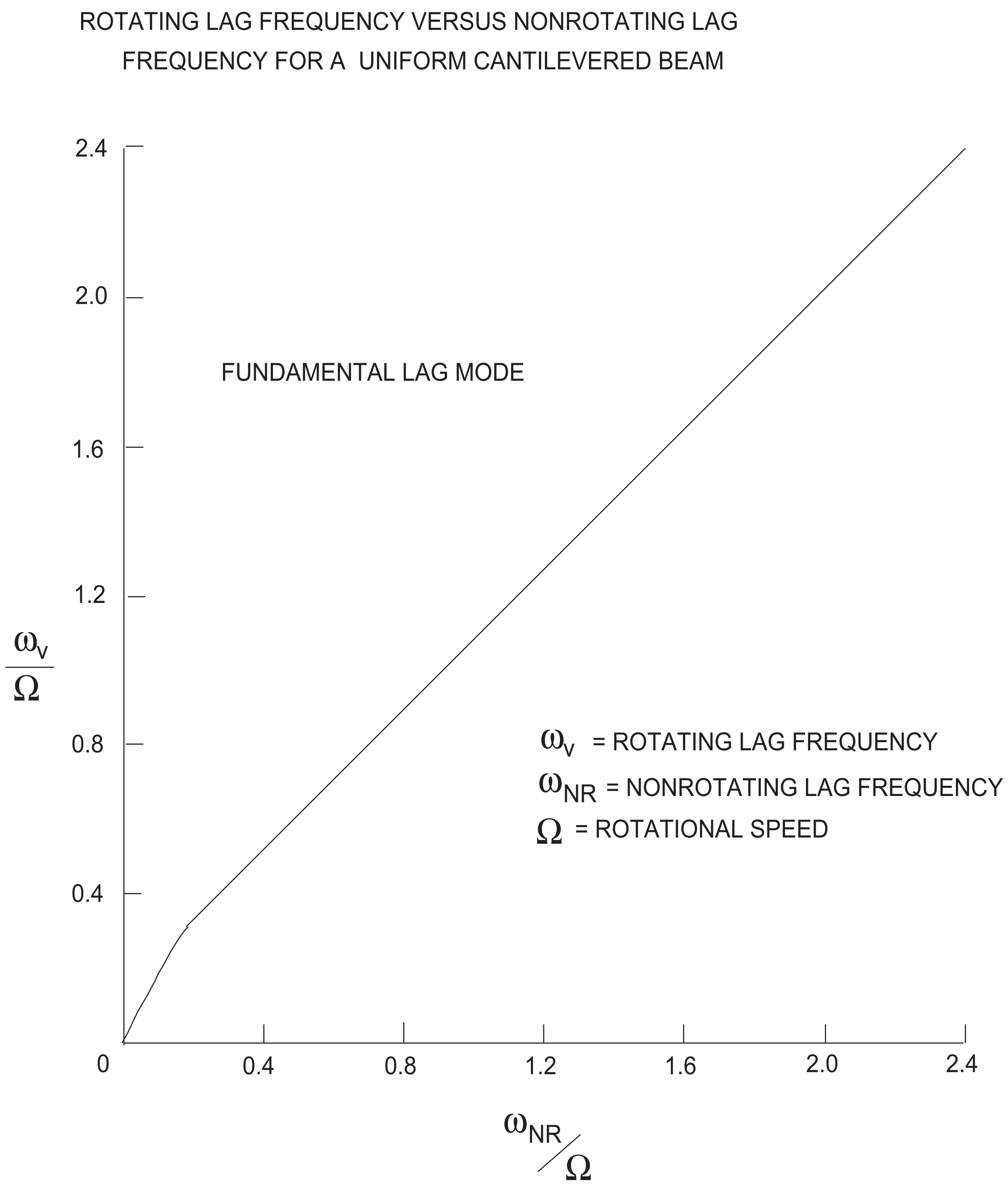
$$\text{Strain energy} \quad v = \frac{1}{2}\int_0^l EI_z(\frac{d^2v}{dx^2}) + \frac{1}{2}\int_0^l T(x)(\frac{dv}{dx})^2 - \frac{1}{2}\int_0^l m\Omega^2 v^2\,dx$$

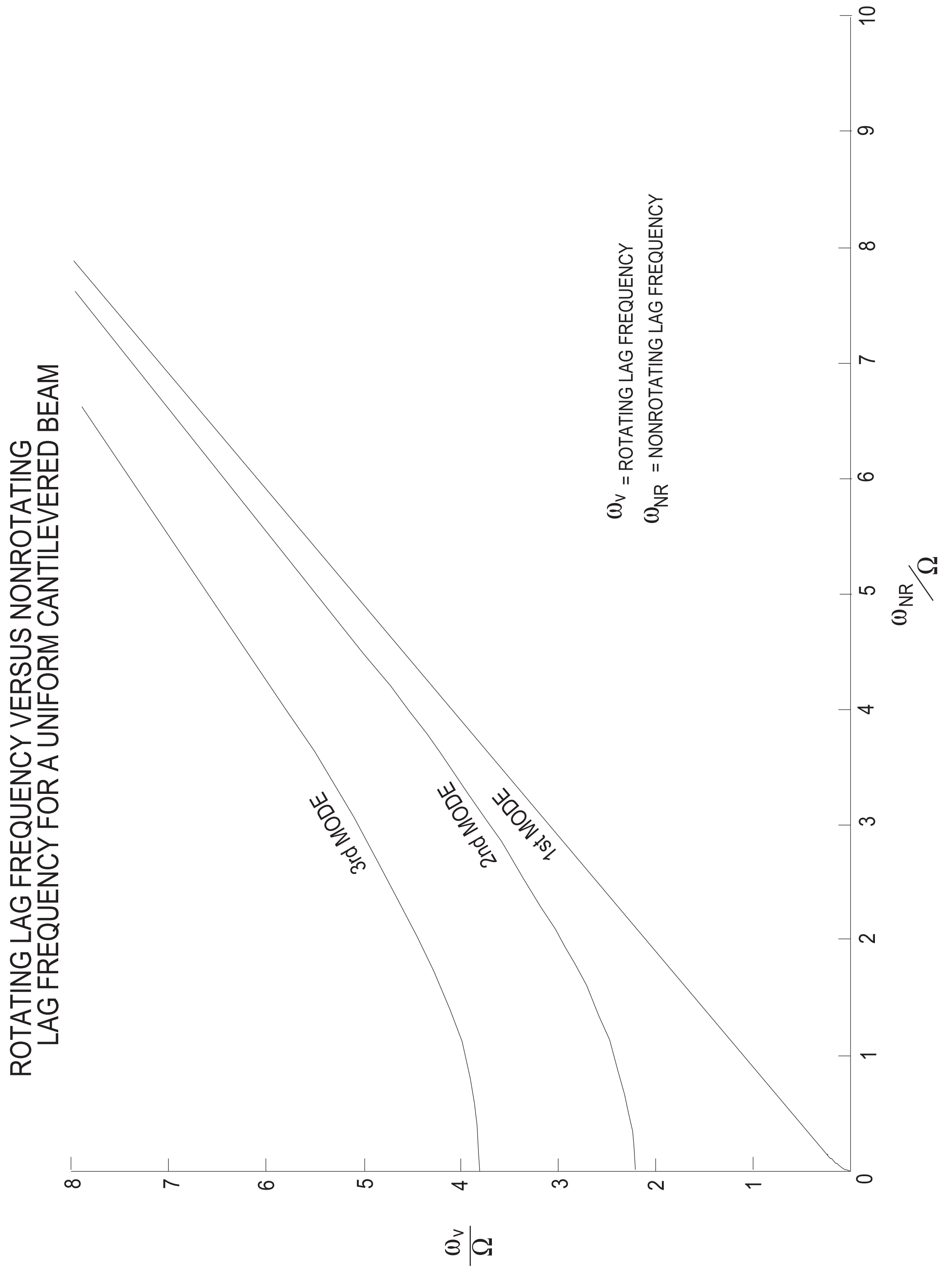
The kinetic energy expression is the same as that of flap bending, whereas for the strain energy expression, the last term is an additional term. The inertial and stiffness matrices are

$$[\widetilde{m}_{ij}]_{\text{lag}} = [\widetilde{m}_{ij}]_{\text{flap}}$$

$$[\widetilde{k}_{ij}]_{\text{lag}} = [\widetilde{m}_{ij}]_{\text{flap}} - m_i\Omega^2 \begin{bmatrix} \frac{13}{35}l & \frac{11}{210}l^2 & \frac{9}{70}l & -\frac{13}{420}l^2 \\ \frac{11}{210}l^2 & \frac{1}{105}l^3 & \frac{13}{420}l^2 & -\frac{1}{140}l^3 \\ \frac{9}{70}l & \frac{13}{420}l^2 & \frac{13}{35}l & -\frac{11}{210}l^2 \\ -\frac{13}{420}l^2 & -\frac{1}{140}l^3 & -\frac{11}{210}l^2 & \frac{1}{105}l^3 \end{bmatrix}$$

In the subsequent figures, the lag vibration results are presented for uniform rotating beams. These results are calculated using finite element analysis. Four to ten finite elements are used; more elements are required for $\omega_{NR}/\Omega < .1$.


ROTATING LAG FREQUENCY VERSUS NONROTATING LAG
FREQUENCY FOR A UNIFORM CANTILEVERED BEAM
FUNDAMENTAL LAG MODE
ω_V = ROTATING LAG FREQUENCY
ω_{NR} = NONROTATING LAG FREQUENCY
Ω = ROTATIONAL SPEED
$\frac{\omega_V}{\Omega}$
ω_{NR}/Ω
0
0.4
0.8
1.2
1.6
2.0
2.4

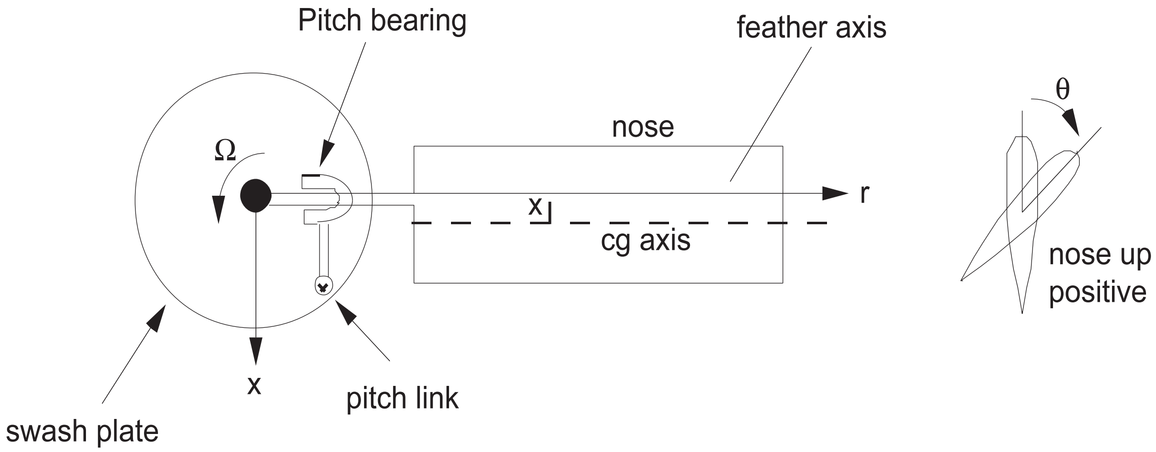


3.2 Torsion Dynamics

As in the case of flap, and lag, the torsion dynamics is studied both using a simple rigid model and a detailed flexible model.

3.2.1 Rigid Torsion Model

This is also called rigid pitch. The blade is assumed rigid and it undergoes a single degree pitch motion about the feathering axis. There is a torsional spring at the root of the blade. This type of modeling is quite satisfactory with helicopter blades because the control system (pitch link) stiffness is less than the blade elastic torsional stiffness. The nose up motion is positive feathering motion. Let us examine the various forces acting on an element dr undergoing torsional motion about the feathering axis,



$\theta =$ pitch angle

$\theta_{\text{con}} =$ control system command pitch

$\theta - \theta_{\text{con}} =$ pitch change due to control flexibility

$x_I =$ chordwise cg offset behind feathering axis

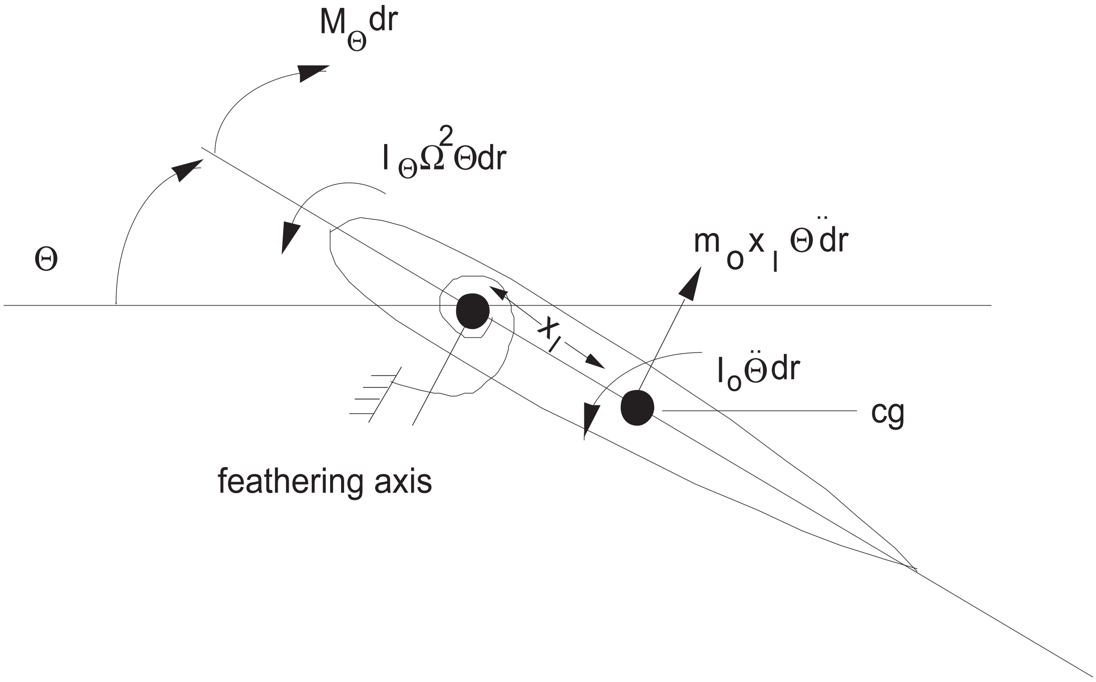
$I_o =$ mass moment of inertia about cg axis per unit length, lb-sec^2

$I_\theta =$ mass moment of inertia about feathering axis per unit length

$= I_o + mx_I^2, \quad lb - sec^2$

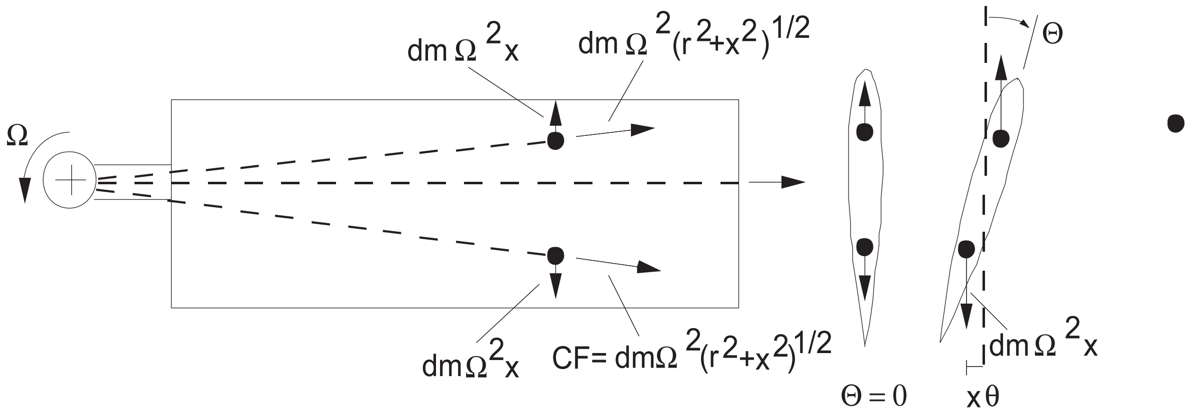
$m =$ mass per unit length lb-sec^2/in^2

$k_\theta =$ control system stiffness in-lb/rad



a) inertia force $mx_I\ddot{\theta}\,dr$ arm x_I about feathering axis
b) inertia torque $I_0\ddot{\theta}\,dr$ about cg axis
c) propellor moment $I_\theta\Omega^2\theta\,dr$ about feathering axis
d) aerodynamic moment $M_\theta\,dr$ about feathering axis
e) spring moment $k_\theta(\theta-\theta_{\text{con}})$

Let us examine carefully the propeller moment caused by the centrifugal action. Let us consider



two mass elements 'dm' on either side of the feathering axis. The elements are being pulled away by the centrifugal force, $dm\,\Omega^2(r^2+x^2)^{1/2}$. The chordwise resolved component

$$dm\,\Omega^2(r^2+x^2)^{1/2}\frac{x}{(r^2+x^2)^{1/2}} = dm\,\Omega^2 x$$

Now let us say that the blade undergoes nose up pitch motion. The chordwise resolved component now produces a nose down couple about the feathering axis

$$\int_{\text{section}} (dm\,\Omega^2 x)(x\theta) = I_\theta\Omega^2\theta$$

This is also called "Tennis racket effect". A tilt of the racket face can be used to produce a pitching motion in the ball. Summing up the torque moments about the feathering axis

$$\int_0^R m\,x_I^2\ddot{\theta}\,dr + \int_0^R I_0\ddot{\theta}\,dr + \int_0^R I_\theta\Omega^2\theta\,dr - \int_0^R M_\theta\,dr$$

$$+k_\theta(\theta - \theta_{\text{con}}) = 0$$

$$\int_0^R I_\theta\,dr = I_f = \text{ total mass moment of inertia about feather axis, lb-sec}^2\text{-in}$$

This results in

$$I_f(\ddot{\theta} + \Omega^2\theta) + I_f\omega_{\theta 0}^2(\theta - \theta_{\text{con}}) = \int_0^R M_\theta\,dr$$

where $\omega_{\theta 0}$ is the nonrotating natural torsional frequency. Let us nondimensionalize by dividing through $I_b\Omega^2$

$$I_{\underset{f}{*}}(\overset{**}{\theta} + \nu_\theta^2\theta) = \gamma\overline{M}_\theta + I_{\underset{f}{*}}\frac{\omega_{\theta 0}^2}{\Omega^2}\theta_{\text{con}} \tag{3.12}$$

where

$$\overline{M}_\theta = \frac{1}{\rho ac\Omega^2 R^4}\int_0^R M_\theta\,dr$$

and

$$I_{\underset{f}{*}} = I_f/I_b$$

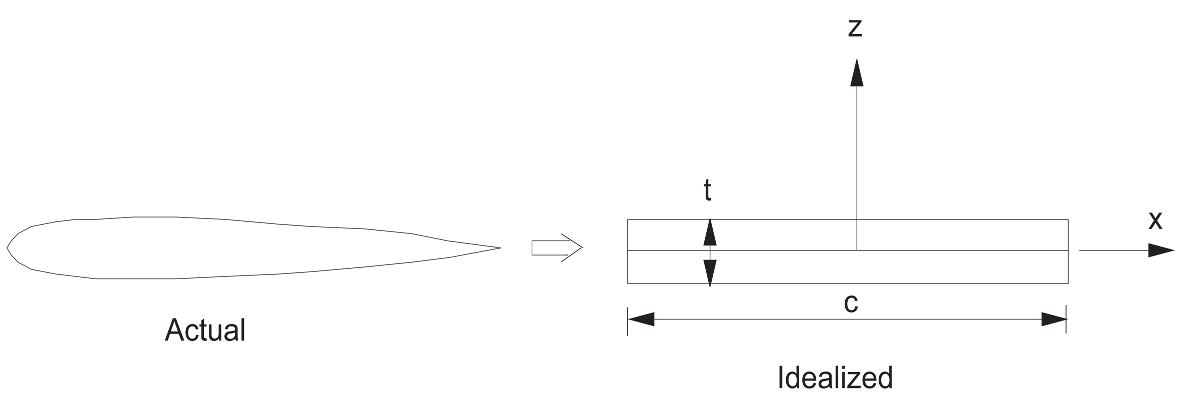
The ν_θ is the rotating natural frequency for torsion mode,

$$\nu_\theta^2 = 1 + \frac{\omega_{\theta 0}^2}{\Omega^2} \tag{3.13}$$

For zero spring, the torsional frequency is equal to the rotational speed and this is caused by propeller moment.

For helicopter blades, typically, the ν_θ varies from 5 to 10. The blades are much stiffer in the torsion mode than the flap and lag modes.

Let us examine an order of magnitude for $I_{\underset{f}{*}}$. Let us consider a solid airfoil, idealized into a rectangular strip.



$$I_0 = \rho_m(I_x + I_z) = \rho_m(\frac{ct^3}{12} + \frac{tc^3}{12})$$

$$\simeq \rho_m \frac{tc^3}{12} \qquad (\rho_m = \text{ mass density})$$

$$I_f = \rho_m \frac{tc^3}{12} R$$

$$I_b = \frac{mR^3}{3}$$

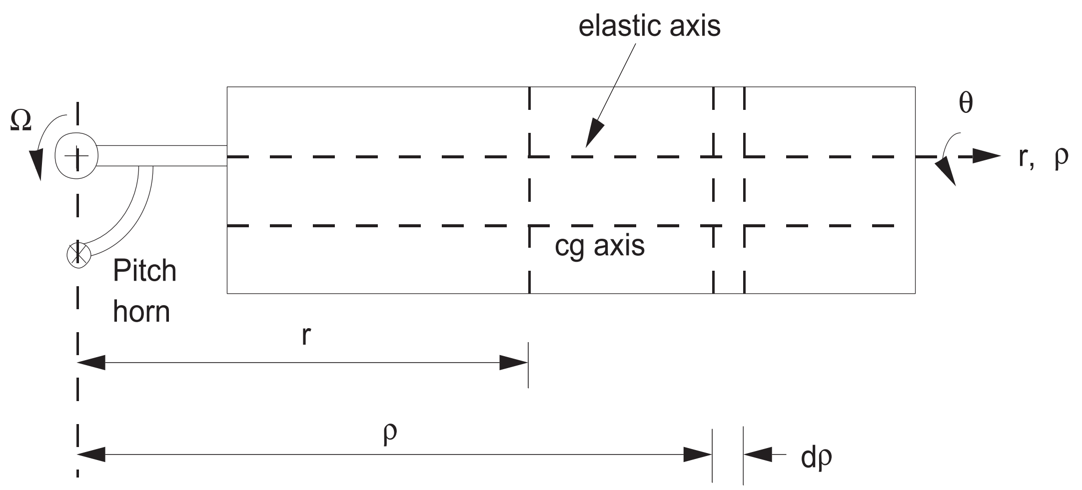
$$\frac{I_f}{I_b} = \rho_m 3 \frac{tc^3 R}{12mR^3} \qquad m = \rho_m ct$$

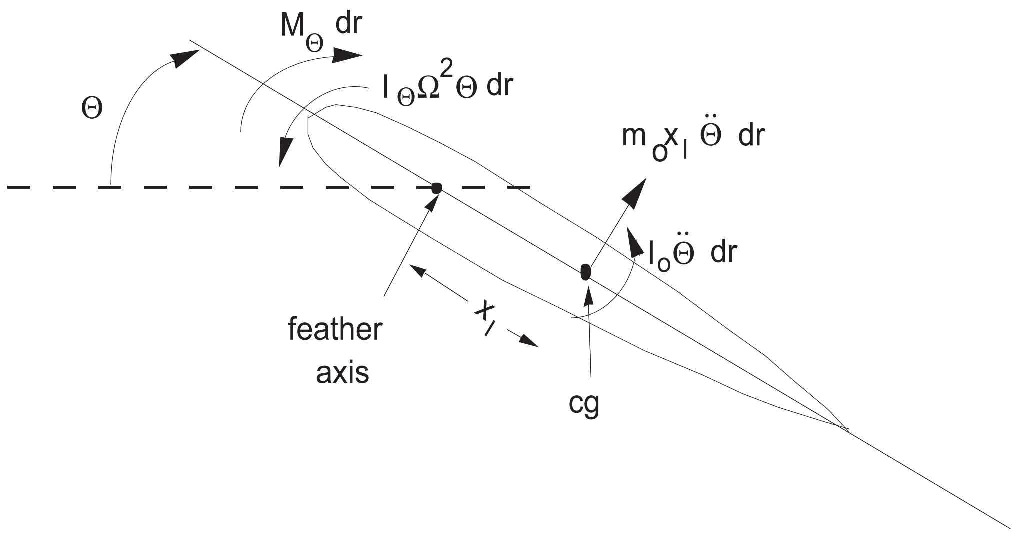
$$= \frac{\rho_m}{\rho_m}\frac{tc^3R}{tcR^3} = \frac{1}{4}(\frac{c}{R})^2$$

For c/R = 20, $I_{\underset{f}{*}}$ = .00063, a very small number.

3.2.2 Elastic Torsion

A better representation for the blade is to consider it as torsionally flexible and it undergoes elastic twist distribution. Again here the analysis is made for the pure torsion mode and coupling terms due to other modes are neglected. The blade twists about the elastic axis and for simplicity of analysis, the elastic axis is assumed to be a straight line coinciding with the feathering axis. Let us say θ is pitch change at a station, which consists of elastic twist ϕ plus rigid pitch due to control flexibility. Let us examine various forces acting at an element at station ρ





a) inertia force $m_0 x_I \ddot{\theta}\, d\rho$ arm x_I about elastic axis
b) inertia torque $I_0\ddot{\theta} d\rho$ about cg axis
c) propeller moment $I_\theta \Omega^2 \theta\, d\rho$ about elastic axis
d) aerodynamic moment $M_\theta\, d\rho$ about elastic axis

Torsional moment at station r

$$M(r) = \int_r^R [M_\theta - m_0 x_I^2 \ddot{\theta} - I_0\ddot{\theta} - I_\theta \Omega^2 \theta] d\rho$$

$$= \int_r^R [M_\theta - I_\theta(\ddot{\theta} + \Omega^2\theta)] d\rho$$

Using the engineering torsion theory for slender bars

$$M(r) = GJ\frac{d\theta}{dr}$$

where θ is the elastic twist distribution. Taking the first derivative of torque, one gets

$$-\frac{d}{dr}(GJ\frac{d\theta}{dr}) + I_\theta(\ddot{\theta} + \Omega^2\theta) = M_\theta \tag{3.14}$$

The effect of control flexibility can be introduced through the boundary condition. Let us say at a distance r_a from the rotation axis, pitch link is located. Also assume that for $r = 0$ to r_a, the blade is rigid torsionally. Boundary conditions are:

$$\begin{array}{lll} @r = r_a & GJ\theta' = -k_\theta(\theta - \theta_{\text{con}}) & \text{(Spring)} \\ @r = R & GJ\theta' = 0 & \text{(Free)} \end{array} \tag{3.15}$$

For cantilevered blades, the boundary conditions are

$$\begin{array}{lll} @r = 0 & \theta = 0 & \text{(fixed)} \\ @r = R & GJ\theta' = 0 & \text{(free)} \end{array} \tag{3.16}$$

Another form of expression for the governing equations to be put in non-dimensional by dividing through Eq. (14) with $I_b\Omega^2$.

$$I_{\overset{*}{\theta}}(\overset{**}{\theta} + \theta) - \frac{d}{d\xi}(\frac{GJ}{I_b\Omega^2}\frac{d\theta}{d\xi}) = M_\theta / I_b\Omega^2 \tag{3.17}$$

where $I_{\overset{*}{\theta}} = I_\theta / I_b$ and $\xi = r/R$.

3.2.3 Natural Vibrations of Torsion Motion

The natural vibration characteristics of a rotating blade in pure torsion mode are obtained from the homogeneous solution of governing equation (14)

$$I_\theta(\ddot{\theta} + \Omega^2\theta) - \frac{d}{dr}(GJ\frac{d\theta}{dr}) = 0 \tag{3.18}$$

The corresponding governing equation for the nonrotating blade is

$$I_\theta\ddot{\theta} - \frac{d}{dr}(GJ\frac{d\theta}{dr}) = 0 \tag{3.19}$$

From the identity of these two equations one obtains

$$\omega_\theta^2 = \omega_{\theta_0}^2 + \Omega^2 \tag{3.20}$$

where ω_θ and ω_{θ_0} are respectively rotating and nonrotating natural frequencies. The mode shapes are the same for the rotating and nonrotating shafts.

3.2.4 Beam Functions for Torsion

The non-rotating natural torsional vibration characteristics are available for uniform bars with different boundary conditions.

For a cantilever bar, the j^{th} mode shape is expressed as

$$\phi_j(r) = \sqrt{2}\sin\lambda_j r \tag{3.21}$$

where

$$\lambda_j = \pi(j - \frac{1}{2})$$

The non-rotating natural torsional frequency for a uniform bar for the j^{th} mode is

$$\omega_{j0} = (\lambda_j)\sqrt{\frac{GJ}{I_\theta R^2}} \quad \text{rad/sec} \tag{3.22}$$

or

$$= (\lambda_j)\sqrt{\frac{GJ}{mk_m^2 R^2}}$$

where

GJ = torsional stiffness, lb-in^2
I_θ = mass moment of inertia per unit length about the elastic axis, lb-sec^2 (= mk_m^2)
R = blade radius, in
m = mass per unit length, lb-sec^2/in^2
k_m = radius of gyration, in

These mode shapes are orthogonal

$$\int_0^R I_\theta\phi_i\phi_j\,dr = 0 \quad \text{for } i \neq j \tag{3.23}$$

The rotating torsional frequency for a particular mode can be obtained from the nonrotating frequency as

$$\omega_j^2 = \omega_{j0}^2 + \Omega^2$$

and the mode shapes are identical.

rotating ϕ_j = nonrotating ϕ_j

Rotating torsion frequency versus nonrotating torsion frequency for uniform cantilevered bar

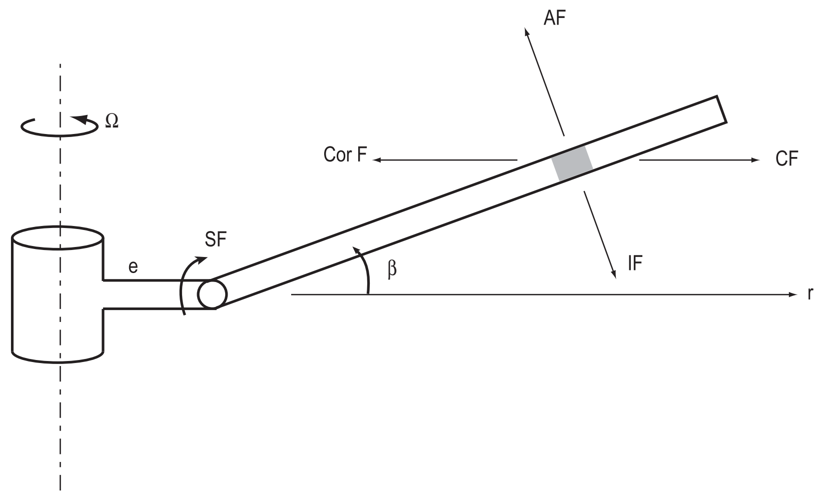
3.3 Coupled Flap-Lag Dynamics

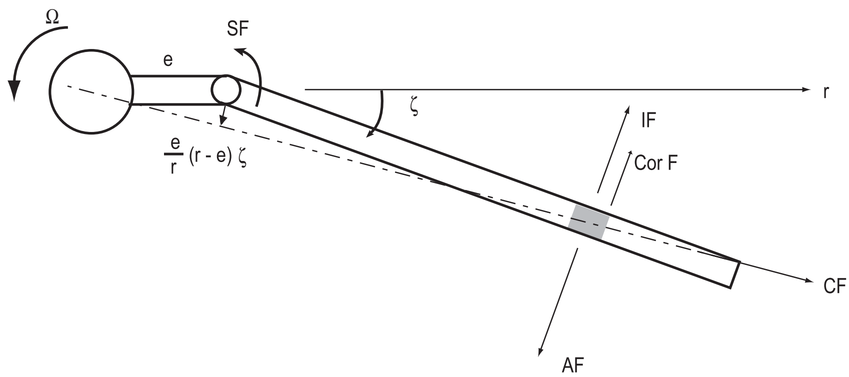
First a rigid model is studied to understand the effect of non-linear flap–lag coupling. Then a detailed flap–lag bending model is considered.

3.3.1 Rigid Model

The blade is assumed rigid and it undergoes two degrees of motion, flap and lag rotations. For simplicity of analysis, it is assumed that the flap and lag hinges are identical. However, a small difference in hinge location can be taken care of by a suitable modification of the rotating flap and lag frequencies. This type of modeling is a good representation for dynamics of an articulated blade with large torsional frequency. It can also be a good approximation for the dynamics of hingeless blades.

Here, the flap and lag motions are coupled due to Coriolis and aerodynamic forces. The flap displacement β is positive up normal to the rotation plane and the lag displacement ζ is positive in the direction opposite to the rotation. Let us examine various forces acting on the element in flap and lag modes for small angles assumption.





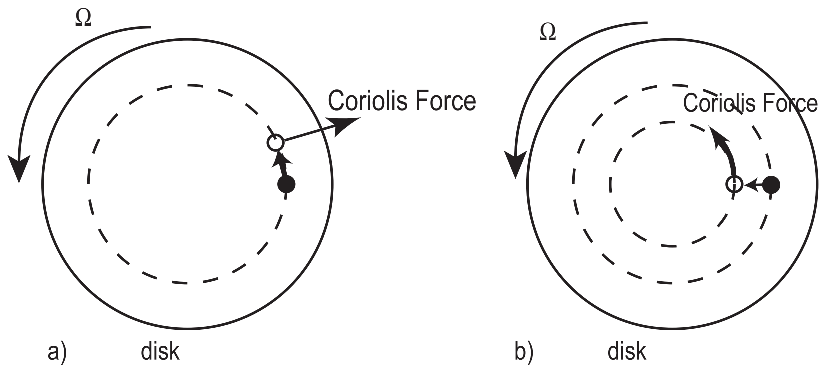
Flap mode forces

(a) IF: inertia force $m(r-e)\ddot{\beta}\,dr$ arm $(r-e)$ about flap hinge
(b) CF: centrifugal force $m\Omega^2 r\,dr$ arm $(r-e)\beta$
(d) Cor F: Coriolis force $2m(r-e)\Omega\dot{\zeta}\,dr$ arm $(r-e)\beta$
(c) AF: aerodynamic force $F_\beta\,dr$ arm $(r-e)$
(e) SF: spring moment due to hinge spring, $k_\beta(\beta-\beta_p)$ where β_p is initial setting.

Lag mode forces

(a) IF: $m(r-e)\ddot{\zeta}$ arm $(r-e)$ about lag hinge
(b) CF: $m\Omega^2 r\,dr$ arm $(r-e)\frac{e}{r}\zeta$
(c) Cor F: $2m(r-e)\beta\Omega\dot{\beta}\,dr$ arm $(r-e)$
(c) AF: $F_\zeta\,dr$ arm $(r-e)$
(d) SF: spring moment $k_\zeta\zeta$

Let us understand first the Coriolis forces. Imagine a person is standing on a circular disk rotating with a speed Ω.



There are two cases. In the first case he attempts to walk on the circumference (radius fixed) in the direction of rotation. He acquires more angular momentum than needed to sustain in equilibrium, so he is pushed towards region needing larger momentum. Thus the person feels a Coriolis force radially outward.

Coriolis force $= 2\Omega m v$

where v is the velocity of the person along the circumferential direction. In the second case, the person moves radially, let us say toward the center. He goes to a region where less angular momentum is required to sustain in equilibrium condition. He gets a push along the direction of rotation. Therefore the person feels a Coriolis force in the circumferential direction. Again, the force expression for this second case is the same as given except that the velocity v is interpreted as the radial velocity.

Taking the moment of forces about the flap hinge

$$\int_e^R \{m(r-e)^2\ddot{\beta} + m\Omega^2(r-e)r\beta - 2m\Omega(r-e)^2\beta\dot{\zeta} - F_\beta(r-e)\}\,dr$$

$$+k_\beta(\beta-\beta_p) = 0$$

or

$$I_\beta(\ddot{\beta} + nu_\beta^2\Omega^2\beta - 2\Omega\dot{\zeta}\beta) = \omega_{\beta 0}^2 I_\beta\beta_p + \int_e^R (r-e)F_\beta\,dr \tag{3.24}$$

where

$$\nu_\beta^2 = \frac{e\int_e^R m(r-e)\,dr}{I_\beta} + \frac{\omega_{\beta 0}^2}{\Omega^2} + 1 \tag{3.25}$$

The ν_β is nondimensional rotating flap frequency (uncoupled) and I_β is flap inertia.

$$\omega_{\beta 0} = \sqrt{\frac{k_\beta}{I_\beta}} \quad \text{rad/sec} \quad \text{nonrotating flap frequency}$$

Taking moments about <u>lag</u> hinge

$$\int_e^R \{m(r-e)^2\ddot{\zeta} + m\Omega^2(r-e)e\zeta + 2m(r-e)^2\Omega\dot{\beta}\beta - F_\zeta(r-e)\}\, dr$$
$$+k_\zeta\zeta = 0$$

or

$$I_\zeta(\ddot{\zeta} + \nu_\zeta^2\Omega^2\zeta + 2\Omega\dot{\beta}\beta) = \int_e^R F_\zeta(r-e) \tag{3.26}$$

where

$$\nu_\zeta^2 = \frac{e\int_e^R m(r-e)\,dr}{I_\zeta} + \frac{w_{\zeta 0}^2}{\Omega^2} \tag{3.27}$$

The ν_ζ is nondimensional rotating lag frequency (uncoupled) and I_ζ is lag inertia.

$$w_{\zeta_0} = \sqrt{\frac{k_\zeta}{I_\zeta}} \text{ rad/sec} \quad \text{nonrotating lag frequency}$$

Dividing the flap and lag equations by $I_b\Omega^2$ and assuming $I_\beta = I_\zeta \simeq I_b$, the nondimensional equations are

$$\begin{array}{ll} \text{Flap:} & \overset{**}{\beta} + \nu_\beta^2\beta - 2\beta\overset{*}{\zeta} = \gamma\overline{M}_\beta + \frac{\omega_{\beta_0}^2}{\Omega^2}\beta_p \\ \text{Lag:} & \overset{**}{\zeta} + \nu_\zeta^2\zeta + 2\beta\overset{*}{\beta} = \gamma\overline{M}_\zeta \end{array} \tag{3.28}$$

where

$$\overline{M}_\beta = \frac{1}{\rho acR^4\Omega^2}\int_e^R F_\beta(r-e)\,dr$$

and

$$\overline{M}_\zeta = \frac{1}{\rho acR^4\Omega^2}\int_e^R F_\zeta(r-e)\,dr$$

The flap-lag equations are coupled inertially through the Coriolis force terms.

Some authors prefer to use lead-lag motion instead of lag motion. The lead-lag displacement has a sign convention opposite to lag displacement. For this convention, there will be a change of sign for Coriolis force terms.

$$\begin{array}{ll} \text{Flap:} & \overset{**}{\beta} + \nu_\beta^2\beta + 2\beta\overset{*}{\zeta} = \gamma\overline{M}_\beta + \frac{\omega_{\beta_0}^2}{\Omega^2}\beta_p \\ \text{Lead-Lag:} & \overset{**}{\zeta} + \nu_\zeta^2\zeta - 2\beta\overset{*}{\beta} = \gamma\overline{M}_\zeta \end{array} \tag{3.29}$$

The flap and lag equations are nonlinear. the equations are linearized by assuming that the dynamic motion is the small perturbation about the steady solution.

$$\beta = \underset{\text{steady}}{\underset{\downarrow}{\beta_0}} + \underset{\text{perturbation}}{\underset{\downarrow}{\beta}}$$

This helps in linearizing the perturbation equations. The β_0 is the steady coning angle.
Perturbation equations

Flap:

Lead-Lag:

$$\begin{aligned} &\overset{**}{\beta} + \nu_\beta^2 \beta - 2\beta_0 \overset{*}{\zeta} = \gamma \overline{M}_\beta \\ &\overset{**}{\zeta} + \nu_\zeta^2 \zeta + 2\beta_0 \overset{*}{\beta} = \gamma \overline{M}_\zeta \end{aligned} \tag{3.30}$$

Since lag moment is much smaller than flap moment, therefore, Coriolis force in lag equation though nonlinear is quite important.

Example: 3.1

The blade and the hub flexibility is represented by two orthogonal spring systems, attached to the hub and the blade inboard and outboard of the pitch bearing respectively. The blade spring system, which rotates during collective pitch changes produces a significant cross coupling of flapping moments with lead-lag deflections and vice versa. The hub spring system does not rotate with the blade pitch and is oriented parallel and perpendicular to the shaft. Obtain the flap-lag equations for the following cases.

(a) Hub flexible and blade rigid
(b) Blade flexible and hub rigid
(c) Both blade and hub flexible

(a) Hub Flexible and Blade rigid

$$\overset{**}{\beta} + \nu_\beta^2 \beta - 2\beta \overset{*}{\zeta} = \gamma \overline{M}_\beta$$

$$\overset{**}{\zeta} + \nu_\zeta^2 \zeta - 2\beta \overset{*}{\beta} = \gamma \overline{M}_\zeta$$

$$\nu_\beta^2 = 1 + \frac{k_{\beta_H}}{I_\beta \Omega^2}$$

$$\nu_\zeta^2 = \frac{k_{\zeta_H}}{I_\beta \Omega^2}$$

(b) Blade flexible and hub rigid

$$\beta_1 = \beta \cos\theta - \zeta \sin\theta$$

$$\zeta_1 = \beta \sin\theta + \zeta \cos\theta$$

$$M_{\beta_1} = k_{\beta_B}(\beta \cos\theta - \zeta \sin\theta)$$

$$M_{\zeta_1} = k_{\zeta_B}(\beta \sin\theta + \zeta \cos\theta)$$

$$M_\beta = M_{\beta 1} \cos\theta + M_{\zeta 1} \sin\theta$$

$$= k_{\beta_B}(\beta \cos^2\theta - \zeta \sin\theta \cos\theta) + k_{\zeta_B}(\beta \sin^2\theta + \zeta \sin\theta \cos\theta)$$

$$M_\zeta = -M_{\beta_1} \sin\theta + M_{\zeta_1} \cos\theta$$

$$= -k_{\beta_B}(\beta \sin\theta \cos\theta - \zeta \sin^2\theta) + k_{\zeta_\beta}(\beta \sin\theta \cos\theta + \zeta \cos^2\theta)$$

$$M_\beta = \beta(k_{\beta_B} \cos^2\theta + k_{\zeta_B} \sin^2\theta) - \zeta(k_{\beta_B} - k_{\zeta_B}) \sin\theta \cos\theta$$

$$M_\zeta = -\beta(k_{\beta_B} - k_{\zeta_B}) \sin\theta \cos\theta + \zeta(k_{\beta_B} \sin^2\theta + k_{\zeta_B} \cos^2\theta)$$

$$\left\{ \begin{array}{c} M_\beta \\ M_\zeta \end{array} \right\} = \left[\begin{array}{cc} k_{11} & k_{12} \\ k_{21} & k_{22} \end{array} \right] \left\{ \begin{array}{c} \beta \\ \zeta \end{array} \right\}$$

$$k_{11} = k_{\beta_B} \cos^2\theta + k_{\zeta_\beta} \sin^2\theta$$

$$k_{12} = k_{21} = -(k_{\beta_B} - k_{\zeta_\beta}) \sin\theta \cos\theta$$

$$k_{22} = k_{\beta_B} \sin^2\theta + k_{\zeta_\beta} \cos^2\theta$$

(c) Hub flexible blade flexible

$$\left\{ \begin{array}{c} \beta \\ \zeta \end{array} \right\}_{\text{Total}} = \left\{ \begin{array}{c} \beta \\ \zeta \end{array} \right\}_{\text{Blade}} + \left\{ \begin{array}{c} \beta \\ \zeta \end{array} \right\}_{\text{Hub}}$$

$$\left\{ \begin{array}{c} M_\beta \\ M_\zeta \end{array} \right\} = [K_I] \left\{ \begin{array}{c} \beta \\ \zeta \end{array} \right\}_{\text{Blade}}$$

$$\left\{ \begin{array}{c} M_\beta \\ M_\zeta \end{array} \right\} = [K_{II}] \left\{ \begin{array}{c} \beta \\ \zeta \end{array} \right\}_{\text{Hub}}$$

$$[K_{\text{eff}}]^{-1} = [K_I]^{-1} + [K_{II}]^{-1}$$

Assuming a fraction of total stiffness is contributed by blade spring and the rest by hub springs.

$$R = k_\beta / k_{\beta_B} = k_\zeta / k_{\zeta_B}$$

$$\frac{k_\beta}{k_{\beta_H}} = 1 - R = \frac{k_\zeta}{k_{\zeta_H}}$$

$$[K_I] = \left[\begin{array}{cc} \frac{1}{R}(k_\beta \cos^2\theta + k_\zeta \sin^2\theta) & -\frac{1}{R}(k_\beta - k_\zeta)\sin\theta\cos\theta \\ -\frac{1}{R}(k_\beta - k_\zeta)\sin\theta\cos\theta & \frac{1}{R}(k_\beta \sin^2\theta + k_\zeta \cos^2\theta) \end{array} \right]$$

$$[K_{II}] = \left[\begin{array}{cc} \dfrac{k_\beta}{1-R} & 0 \\ 0 & \dfrac{k_\zeta}{1-R} \end{array} \right]$$

$$[K_{\text{eff}} = \frac{1}{\Delta} \left[\begin{array}{cc} k_\beta + R(k_\zeta - k_\beta)\sin^2\theta & \frac{1}{2}R(k_\zeta - k_\beta)\sin 2\theta \\ \frac{1}{2}R(k_\zeta - k_\beta)\sin 2\theta & k_\zeta - R(k_\zeta - k_\beta)\sin^2\theta \end{array} \right]$$

$$\Delta = 1 + R(1-R)\frac{(k_\beta - k_\zeta)^2}{k_\beta k_\zeta} \sin^2\theta$$

3.3.2 Flexible Model

In this section the coupled flap and lag bending dynamics is studied. The blade is assumed as an elastic beam undergoing in-plane and out of plane bending motions. It is also assumed that the forces are applied along the principal axes and there is no structural coupling of bending motions.

Flap bending
(a) IF: $m\ddot{w}(\rho)\, d\rho$ arm $(\rho - r)$ about r
(b) CF: $m\Omega^2\rho\, d\rho$ arm $w(\rho) - w(r)$
(c) Cor F: $2m\Omega\dot{v}(\rho)\, d\rho$ arm $w(\rho) - w(r)$
(d) AF: $F_z\, d\rho$ arm $(\rho - r)$
Taking moment at station r

$$M_x(r) = \int_r^R \{(F_z - m\ddot{w}(\rho))(\rho - r) - (m\Omega^2\rho - 2m\Omega\dot{v})(w(\rho) - w(r))\}\, d\rho \tag{3.31}$$

Lag bending
(a) IF: $m\ddot{v}(\rho)\, d\rho$ arm $(\rho - r)$ about r
(b) CF: $m\Omega^2\rho\, d\rho$ arm $r/\rho\;\; v(\rho) - v(r)$
(c) Cor F$_1$: $2m\Omega\dot{v}$ arm $v(\rho) - v(r)$
(d) Cor F$_2$: $2m\Omega \int_0^\rho (v'\dot{v}' + w'\dot{w}')\, d\rho$ arm $(\rho - r)$
(e) AF: $F_x\, d\rho$ arm $(\rho - r)$

The force Cor F$_2$ is the Coriolis force caused by the radial shortening of the blade. The v and w are inplane and out of plane displacements producing a radial shortening of the blade length

$$-\frac{1}{2}\int_0^\rho (v'^2 + w'^2)\, d\rho$$

This causes radial inward velocity of the element

$$-\int_0^\rho (v'\dot{v}' + w'\dot{w}')d\rho$$

resulting in the coriolis force in the in-plane direction. Taking the lag bending moment at station r

$$\begin{aligned} M_z(r) \;\; = \;\; & \int_r^R \left\{(F_x - m\ddot{v}(\rho))(\rho - r) - m\Omega^2\rho\left(v(\rho)\frac{r}{\rho} - v(r)\right)\right. \\ & \left. +2\Omega\dot{v}m(v(\rho) - v(r)) - 2m\Omega(\int_0^\rho (v'\dot{v}' + w'\dot{w}')d\rho)(\rho - r)\right\} d\rho \end{aligned} \tag{3.32}$$

Differentiate both $M_x(r)$ and $M_z(r)$ twice wrt r from Eqs. (31) and (32).

$$\frac{d^2M_x}{dr^2} = \frac{d^2}{dr^2}\left(EI_x\frac{d^2w}{dr^2}\right)$$

and

$$\frac{d^2M_z}{dr^2} = \frac{d^2}{dr^2}\left(EI_z\frac{d^2v}{dr^2}\right)$$

Using the Leibnitz theorem of integrations, the above equations become:

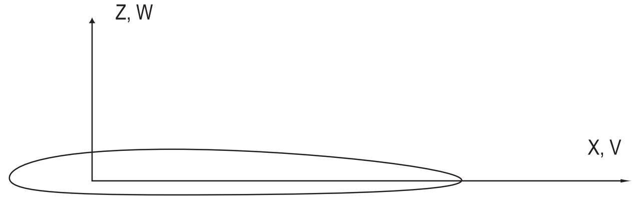
Flap bending Eq.:

$$\begin{aligned} \frac{d^2}{dr^2}\left(EI_x\frac{d^2w}{dr^2}\right) + m\ddot{w} + \frac{d}{dr}(\frac{dw}{dr}\int_r^R 2m\Omega\dot{v}\, d\rho) & \\ -\frac{d}{dr}(\frac{dw}{dr}\int_r^R m\Omega^2\rho\, d\rho) \;\; & = \;\; F_z \end{aligned} \tag{3.33}$$

Lag bending Eq.:

$$\frac{d^2}{dr^2}(EI_z\frac{d^2v}{dr^2}) + m\ddot{v} - m\Omega^2 v - \frac{d}{dr}(\frac{dv}{dr}\int_r^R m\Omega^2\rho\,d\rho)$$
$$+\frac{d}{dr}(2\Omega\frac{dv}{dr}\int_r^R \dot{v}m\,d\rho) + 2m\Omega\int_0^r(\frac{dv}{dr}\frac{d\dot{v}}{dr} + \frac{dw}{dr}\frac{d\dot{w}}{dr})d\rho \quad = \quad F_x \qquad (3.34)$$

Let us consider an airfoil system



I_x = area moment of inertia about x-axis
$= \int_{\text{section}} z^2\,dA,\ \text{in}^4$
I_z = area moment of inertia about z axis
$= \int_{\text{section}} x^2\,dA,\ \text{in}^4$
E = Young's modulus of elasticity, lb/in^2
v = bending deflection along x-axis, in
w = bending deflection along z-axis, in
m = mass per unit length, lb-sec^2/in^2
x,z = section principal axes

The flap and lag bending equations can be nondimensionalized by dividing through with $m_0\Omega^2R$

Flap bending:

$$\frac{d^2}{d\xi^2}(\frac{EI_x}{m_0\Omega^2R^4}\frac{d^2w}{d\xi^2} + \frac{m}{m_0}\overset{**}{w} + \frac{d}{d\xi}(\frac{dw}{d\xi}\int_\xi^1 \frac{m}{m_0}2\overset{*}{v}\ d\xi) - \frac{d}{d\xi}(\frac{dw}{d\xi}\int_\xi^1 \frac{m}{m_0}\xi\,d\xi$$
$$= \frac{F_z}{m_0\Omega^2R} \qquad (3.35)$$

Lag bending:

$$\frac{d^2}{d\xi^2}(\frac{EI_z}{m\Omega^2R^4}\frac{d^2v}{d\xi^2}) + (\overset{**}{v} - v)\frac{m}{m_0} - \frac{d}{d\xi}(\frac{dw}{d\xi}\int_x i^1\frac{m}{m_0}d\xi) + \frac{d}{d\xi}(2\frac{dv}{d\xi}\int_\xi^1 \frac{m}{m_0}\overset{*}{v}\ d\xi)$$
$$+2\frac{m}{m_0}\int_0^\xi(\frac{dv}{d\xi}\frac{d\overset{*}{v}}{d\xi} + \frac{dw}{d\xi}\frac{d\overset{*}{w}}{d\xi})d\xi = \frac{F_x}{m_0\Omega^2R} \qquad (3.36)$$

where

$$v = v/R$$
$$w = w/R$$
$$\xi = r/R$$

Boundary conditions

For hingeless blades

$$@\,r = 0 \qquad v = w = 0$$
$$\frac{dv}{dr} = \frac{dw}{dr} = 0 \qquad \text{(fixed)}$$
$$@\,r = R \qquad EI_z\frac{d^2v}{dr^2} = EI_x\frac{d^2w}{dr^2} = 0$$
$$\frac{d}{dr}(EI_z\frac{d^2v}{dr^2}) = \frac{d}{dr}(EI_x\frac{d^2w}{dr^2}) = 0 \qquad \text{(free)}$$

Using these boundary conditions, the governing coupled equations for flap bending and lag bending can be solved.

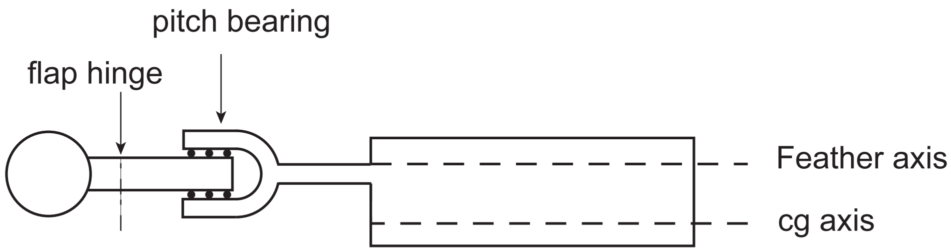
If there is a pitch change there is another coupling term called structural coupling which needs to be included. This effect will be included later on for coupled flap bending, lag bending and torsion equations.

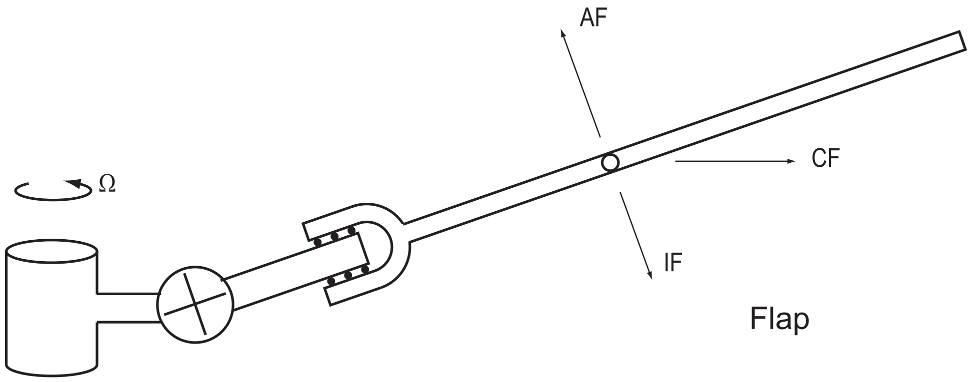
3.4 Coupled Pitch-Flap Dynamics

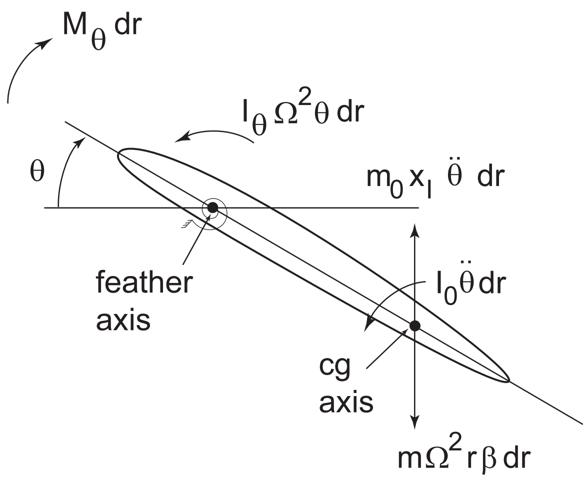
The rigid and flexible blade models are discussed below

3.4.1 Rigid Model

The blade is assumed rigid and it undergoes two degrees of motion, flap and feather rotations. This model represents an articulated blade with flexible control system. It is also assumed that the pitch bearing is outboard for the flap hinge. Also it is assumed that the flap motion does not introduce pitch change, which means pitch-flap coupling 'δ_3' is neglected.







Flap mode forces

(a) IF: $m(r-e)\ddot{\beta}\,dr$ arm $(r-e)$ about flap hinge

$-mx_I\ddot{\theta}\,dr$ arm $(r-e)$

(b) CF: $m\Omega^2 r\,dr$ arm $((r-e)\beta - x_I\theta)$

(c) AF: $F_\beta\,dr$ arm $(r-e)$

(d) SM: $k_\beta(\beta-\beta_p)$

Pitch mode forces

(a) IF: $mx_1\ddot{\theta}\,dr$ arm x_I about feathering axis

$I_0\ddot{\theta}\,dr$ torque

$-m(r-e)\ddot{\beta}\,dr$ arm x_I

(b) CF: $I_\theta\Omega^2\theta\,dr$ torque

$-m\Omega^2 r\beta\,dr$ arm x_I

(c) AF: $M_\theta\,dr$ torque

(d) SM: $k_\theta(\theta-\theta_{\text{con}})$ torque

Taking moment of forces about flap hinge

$$\int_e^R \{m(r-e)^2\ddot{\beta} - mx_I(r-e)\theta + m\Omega^2 r(r-e)\beta - m\Omega^2 x_I\theta$$

$$-F_\beta(r-e)\}dr + k_\beta(\beta-\beta_p) = 0$$

Moments about pitch bearing

$$\int_{e_\theta}^R \{I_0\ddot{\theta} + mx_1^2\ddot{\theta} - m(r-e)x_I\ddot{\beta} + I_\theta\Omega^2\theta - M_\theta + m\Omega^2 r\beta x_I\}dr + k_\theta(\theta-\theta_{\text{con}})$$

$$= 0$$

Writing these equations in nondimensional form

$$\overset{**}{\beta} + \nu_\beta^2\beta - I_{\underset{x}{*}}(\overset{**}{\theta} + \theta) = \gamma\overline{M}_\beta + \frac{\omega_{\beta 0}^2}{\Omega^2}\beta_p \tag{3.37}$$

$$I_{\underset{f}{*}}(\overset{**}{\beta} + \nu_\beta^2\theta) - I_{\underset{x}{*}}(\overset{**}{\beta} + \beta) = \gamma\overline{M}_\theta + I_{\underset{f}{*}}\frac{\omega_{\theta 0}^2}{\Omega^2}\theta_{\text{con}} \tag{3.38}$$

where ν_β and ν_θ are respectively rotating flap and feather natural frequencies and

$$I_{\underset{x}{*}} = \frac{I_x}{I_b} = \int_{e_\theta}^R x_I rm\,dr/I_b$$

$$\overline{M}_\beta = \frac{1}{\rho ac\Omega^2 R^4}\int_e^R F_\beta(r-e)\,dr$$

$$\overline{M}_\theta = \frac{1}{\rho ac\Omega^2 R^4}\int_{e_\theta}^R M_\theta\,dr$$

and e_θ is the pitch bearing offset from the rotation axis. The pitch-flap equations are coupled through inertial and centrifugal force terms. If cg and feather axis are coincident, $I_{\underset{x}{*}}$ becomes zero and these coupling terms are eliminated.

For uniform blades

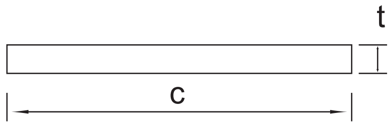
$$I_{\underset{x}{*}} = x_I\frac{mR^2}{2}/\frac{mR^3}{3} = \frac{3}{2}\frac{x_I}{R}$$

Typically $x_I = .1c$ and $\frac{c}{R} = 20$

$$I_{\underset{x}{*}} = \frac{3}{400} = .0075(\sim \epsilon^2)$$

$$I_{\underset{f}{*}} = \frac{I_f}{I_b}$$

Let us consider a rectangular strip



$$I_f = \rho \frac{c^3 t}{12} R$$

$$= \frac{mc^2}{12} R$$

$$I_{\underset{f}{*}} = \frac{1}{4}(\frac{c}{R})^2 = \frac{1}{1600} \sim (\epsilon^2 \text{ to } \epsilon^3)$$

Let us examine now flap and pitch equations. In the flap equations, the $I_{\underset{x}{*}}$ is of the order of ϵ^2 where as other terms are of order unity. The coupling has a negligible influence in the flap equation. In the pitch equation, the $I_{\underset{x}{*}}$ is of the order of ϵ^2 and the $I_{\underset{f}{*}}$ is of the same order. Therefore the coupling term plays an important role in the feather equation. One can write it as

$$\overset{**}{\beta} + \nu_\beta^2 \beta = \gamma \overline{M}_\beta + \frac{\omega_{\beta 0}^2}{\Omega^2} \beta_p \tag{3.39}$$

$$I_{\underset{f}{*}}(\overset{**}{\theta} + \nu_\theta^2 \theta) - (\overset{**}{\beta} + \beta) = \gamma \overline{M}_\theta + I_{\underset{f}{*}} \frac{\omega_{\theta 0}^2}{\Omega^2} \theta_{\text{con}}$$

Example: 3.2

Write the equation of motion and the boundary conditions for flap bending and elastic twist of a rotating blade with a flap hinge and pitch bearing located at a distance e from the rotation axis.

Flap Bending equation

$$(EIw'')'' - [(\int_r^R m\Omega^2 \rho \, d\rho) w']' + m\ddot{w} - mx_I \ddot{\theta}$$
$$+ [(r\theta)' \int_r^R m\Omega^2 x_I \, d\rho]' = f_z(r,t)$$

Boundary conditions

@ $r = e$ $\quad w = 0, \quad EIw'' = k_\beta w' \quad (k_\beta =$ bending spring at hinge$)$

@ $r = R$ $\quad EIw'' = 0, \quad (EIw'')'' = 0$

Torsion Equation

$$-(GJ\theta')' + I_\theta \ddot{\theta} + I_\theta \Omega^2 \theta - mx_I \ddot{w}$$
$$+ r[w' \int_r^R m\Omega^2 x_I \, d\rho]' \quad = \quad M_\theta(r,t)$$

Boundary conditions

@ $r = e$ $\quad GJ\theta' = -k_\theta(\theta - \theta_{\text{con}}$

$(k_\theta =$ control spring stiffness$)$

@ $r = R$ $\quad GJ\theta' = 0$

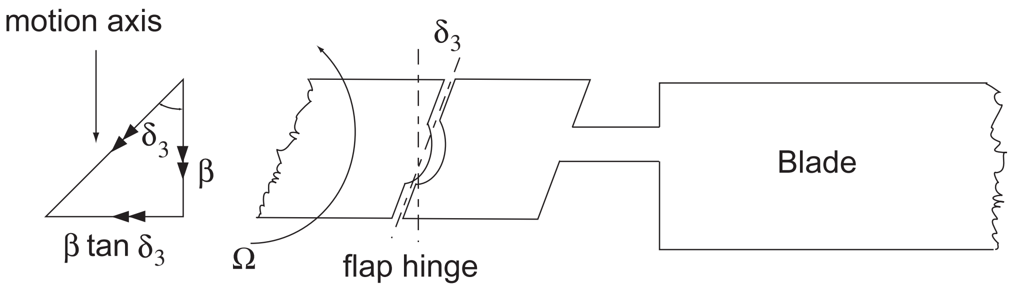
3.4.2 Kinematic Pitch-Flap Coupling: δ_3 Effect

Pitch-flap coupling is a kinematic feedback of the flapping displacement to the blade pitch motion.

$$\Delta\theta = -k_{p\beta}\beta \tag{3.40}$$

where $k_{p\beta}$ is the pitch-flap coupling and is positive when the flap up motion results in a nose down pitch motion. This will act as an aerodynamic spring for the flap mode because the lower pitch means less lift. The pitch-flap coupling plays an important role in flight stability and handling qualities of the vehicle as well as aeroelastic stability of the blade. There are many ways to achieve pitch-flap coupling.

(a) Skewed Flapping Hinge



If flap hinge is not normal to blade axis the flap motion will be accompanied with the pitch change of the blade. Let us say there is flap motion β then there will be reduction in pitch by

$$\Delta\theta = -\tan\delta_3\beta$$

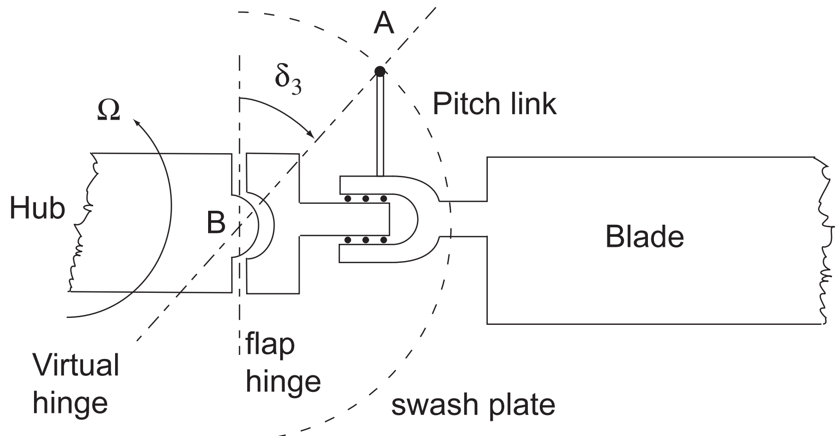
therefore

$$k_{p\beta} = \tan\delta_3$$

where δ_3 is skewing of the flap hinge in radians. The negative pitch-flap coupling can be achieved by skewing the flap hinge in the opposite direction.

(b) Location of pitch link

The second way to introduce pitch-flap coupling is through the pitch control system.



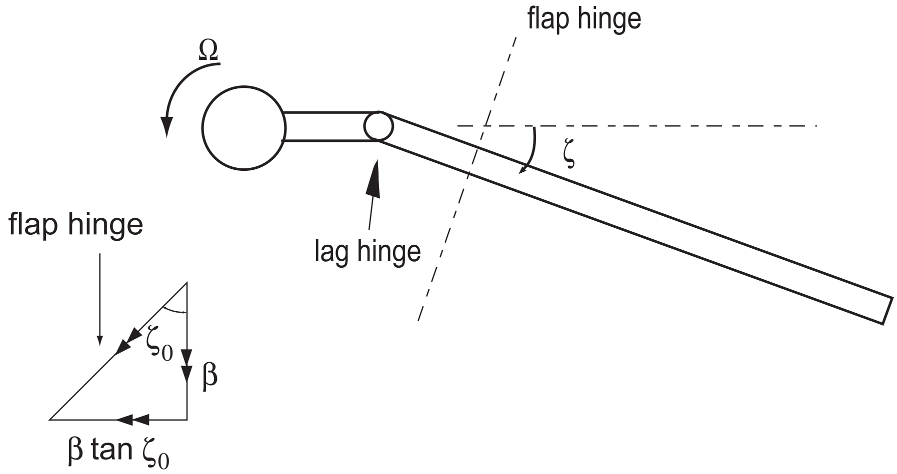
The pitch setting of the blade is obtained by vertical motion of the pitch link which is connected at one end to the moving part of the swashplate and at the end to the blade through the pitch horn. If the posittion of pitch link (A) is not in line with the flap hinge, it will form a virtual hinge. Now if the blade flaps, it will result into a change in the pitch producing pitch-flap coupling. This is possible only if the pitch bearing is outboard of the flap hinge.

$$k_{p\beta} = \tan\delta_3 \tag{3.41}$$

The negative pitch-flap coupling can be obtained by locating the pitch link on the other side of the blade.

(c) Position of Lag Hinge

If flap hinge is outboard of lag hinge, this will result in the δ_3 effect due to steady lag deflection.



The flap motion results in a change of pitch, since the efective flap hinge gets skewed.

$$\delta_3 = \zeta_0$$

This type of pitch-flap coupling is possible even for hingeless blades.

(d) Using Feedback System

Any amount of pitch-flap coupling can be introduced using a feedback system.

$$\begin{array}{ccccc} \Delta\theta & = & -k_{p\beta} & & \beta \\ \swarrow & & \downarrow & & \searrow \\ \text{feedback} & & \text{gain} & & \text{signal from the} \\ \text{on pitch} & & & & \text{pick up on the} \\ \text{link} & & & & \text{blade} \end{array}$$

3.4.3 δ_3 Effect in Hover

Let us examine the rigid flap equation in hovering flight. For a constant pitch blade with uniform inflow and without pitch-flap coupling, the equation of motion is

$$\overset{**}{\beta} + \frac{\gamma}{8}\overset{*}{\beta} + \nu_\beta^2\beta = \frac{\gamma}{8}\theta - \frac{\gamma\lambda}{6}$$

With pitch-flap coupling, the blade pitch gets modified

$$\theta = \theta + \Delta\theta$$

where

$$\Delta\theta = -k_{p\beta}\beta$$

So, the new flapping equation becomes

$$\overset{**}{\beta} + \frac{\gamma}{8}\overset{*}{\beta} + (\nu_\beta^2 + \frac{\gamma}{8}k_{p\beta})\beta = \frac{\gamma}{8}\theta - \frac{\gamma\lambda}{6}$$

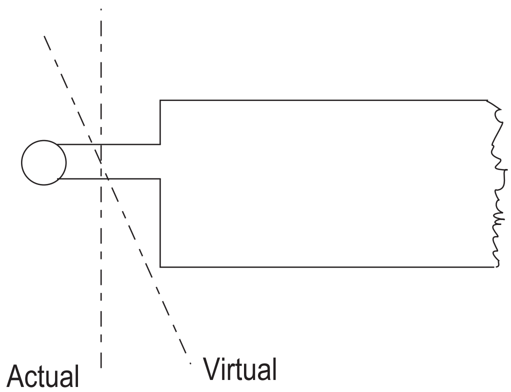
The new rotating flap frequency, ν_{β_e}

$$\nu_{\beta_e}^2 = \nu_\beta^2 + \frac{\gamma}{8}k_{p\beta} \tag{3.42}$$

The pitch-flap coupling has a direct effect on rotating flap frequency. Through a negative pitch-flap coupling, it is possible to reduce the rotating flap frequency below one.

<u>Example: 3.3</u>

Through the skewing of the flap hinge axis, the rotating flap frequency of a blade is reduced by 25%. Calculate the skew angle for a rotor with a Lock number of 8, and the hinge offset is given as 6% of the blade length.



Flap frequency with pitch-flap coupling

$$\nu_{\beta_e}^2 = \nu_\beta^2 + \frac{\gamma}{8}k_{p_\beta}$$

$$\nu_\beta^2 = 1 + \frac{3}{2} \times .06 = 1.09$$
$$\nu_{\beta_e} = .75\nu_\beta$$

$$\begin{aligned} k_{p_\beta} &= -1.09(1 - \frac{9}{16}) = -.4769 \\ &= \tan\delta_3 \end{aligned}$$

$$\delta_3 = 25.5°$$

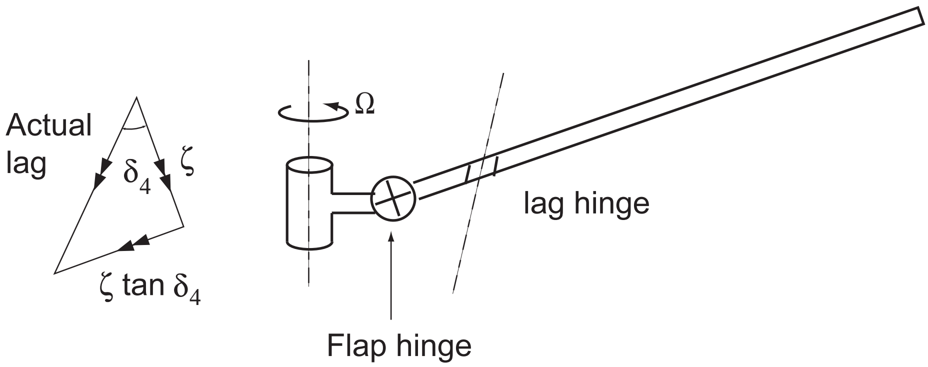
3.4.4 Kinematic Pitch-Lag Coupling: δ_4 Effect

Pitch-lag coupling is a kinematic feedback of lag displacement to the blade pitch motion

$$\delta\theta = -k_{p\zeta}\zeta \tag{3.43}$$

where $k_{p\zeta}$ is the pitch-lag coupling and is positive when lag back produces a nose down pitch motion. The pitch-lag coupling has considerable influence on aeroelastic stability of the blades. Again there are many ways to achieve pitch-lag coupling.

(a) Skewed Lag Hinge



If lag hinge is not normal to the blade axis, the lag motion will be accompanied with the pitch change of the blade. If ζ is the lag displacement then, the reduction of blade pitch will be

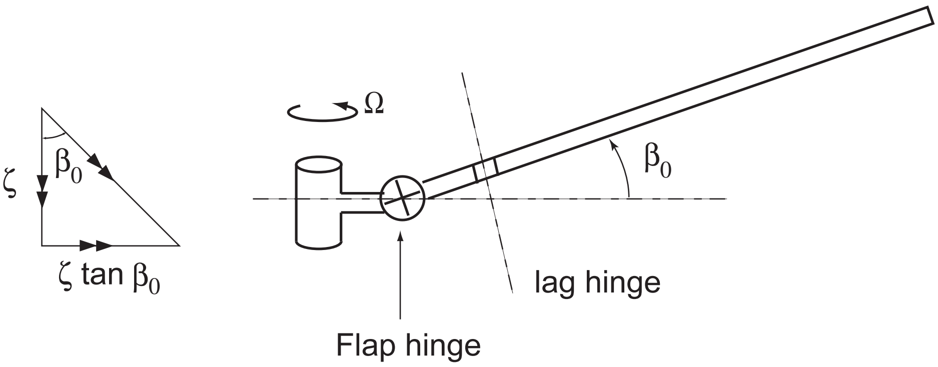
$$\Delta\theta = -\tan\alpha_4\zeta$$

Therefore

$$k_{p\zeta} = \tan\alpha_4 \tag{3.44}$$

where δ_4 is skewing of lag hinge in radians. The negative pitch-lag coupling can be achieved by skewing the lag hinge in the opposite direction.

(b) Position of Flap Hinge



If lag hinge is outboard of flap hinge this will result into α_4 effect due to steady flap deflection.

$$\alpha_4 = -\beta_0$$

(c) Pitch Control Linkage

Through a creative design of pitch control linkages at the root end of blades, desired pitch-lag coupling can be introduced.

(d) Using Feedback System

Any amount of pitch-lag coupling can be introduced using a feedback system

$$\underset{\substack{\swarrow \\ \text{feedback on} \\ \text{pitch link}}}{\Delta\theta} = \underset{\substack{\downarrow \\ \text{gain}}}{-k_{p\zeta}} \quad \underset{\substack{\searrow \\ \text{signal from the} \\ \text{pickup on the blade}}}{\zeta}$$