Precision Design

Kinematic Design

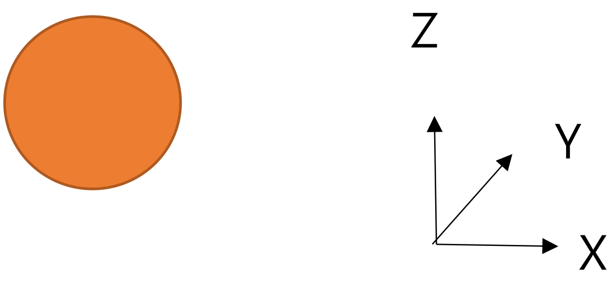
Machine: System supposed to make motion that is moving along the controlled, guided, constrained trajectory. Thus an important design principle is to follow the kinematics, i.e. motion of science, consisting of DOFs and Constraints.

DOFs: Number of Independent Motions allowed to a body, or number of independent coordinates explaining for a body in motion

A free(=unconstrained) body has 6 Degree of Freedoms (6 DOFs) in Space, that means any unconstrained body has 6 DOFs in 3D space.

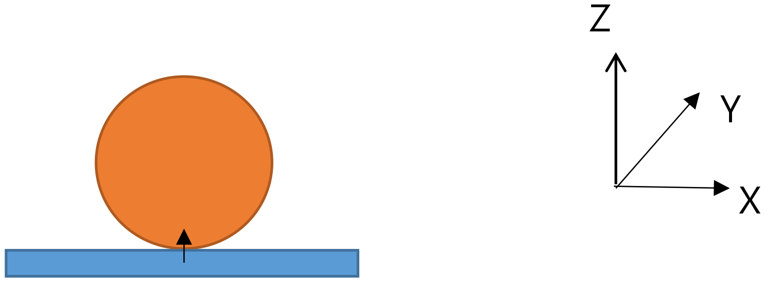
: 3 translational DOFs (δx, δy, δz)

+ 3 rotational DOFs (θx, θy, θz)



DOFs=(δx,δy,δz,θx,θy,θz)=(O,O,O,O,O,O)=6

1. When the ball is constrained in the Z direction by a plane perpendicular with one point contact, i.e. C=1, then z translation motion is constrained, thus the DOFs of ball is 5 (D=5)

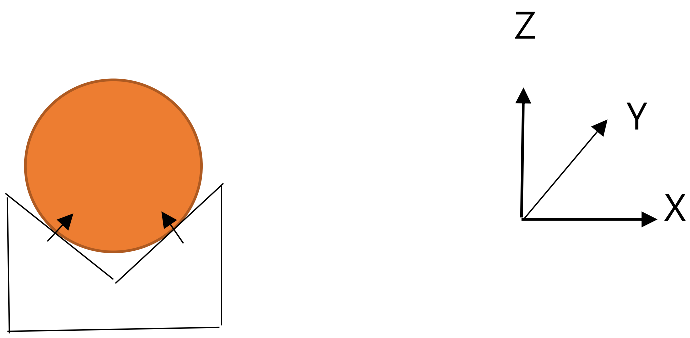


DOFs=$(\delta x, \delta y, \delta z, \theta x, \theta y, \theta z)$=(O,O,X,O,O,O)=5

;where X indicates the constrained motion.

This provides a very useful configuration for Flat slide, such as motion on a plane.

2. When the ball is constrained in Z, X directions by two planes forming V block with two points contacts, i.e. C=2, then X and Z translation are constrained, and the DOFs of ball is 4 (D=4)



DOFs=$(\delta x,\delta y,\delta z,\theta x,\theta y,\theta z)$=(X,O,X,O,O,O)=4

This provides a very useful configuration for V slide, such as motion on a V slot.

3. When the ball is constrained in Z, X, Y directions by three planes forming the trihedral hole with three points contacts, i.e. C=3, then X, Y, Z translation are constrained, and the DOFs of ball is 3 (D=3)

DOFs=(δx,δy,δz,θx,θy,θz)=(X,X,X,O,O,O)=3

This is very useful configuration for <u>Ultra high positional repeatability at one point</u>, or clamp such as "Kelvin Clamp"

Any <u>non-repeatability</u> at the point may be from the elastic deformation at contacts.

Kinematic Constraint:

Let C be the number of constraints, and D be the number of DOFs. When C+D=6, then this situation is such that *'The body is Kinematically Constrained'* and the constraints become the *Kinematic Constraints*.

And,

Under-constrained if C+D<6, and

Over-constrained or Redundant constraints if C+D>6

Two lemmas for the fundamental kinematic design principle

First Lemma: Motions should match the DOFs allowed

The motion of machine elements should match well with the DOFs allowed to the machine elements by proper arrangements of actuation-constraints configuration. Detail arrangements for DOFs and motion will be explained.

Second Lemma: Redundant constraints

"When a rigid body is constrained by more than the necessary minimum number of constraints, the redundant constraints will cause strain which results in distortion, wear, and higher cost than necessary to achieve a specified precision"

by Lord Kelvin and Clark Maxwell

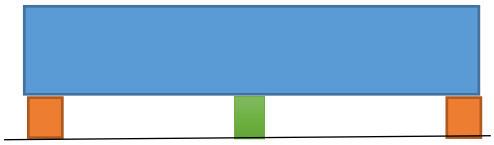
Kinematic (minimum) constraints vs. Redundant constraints

1. Beam supports

Beam with 2 supports

Kinematic Constraints

Beam with 3 supports



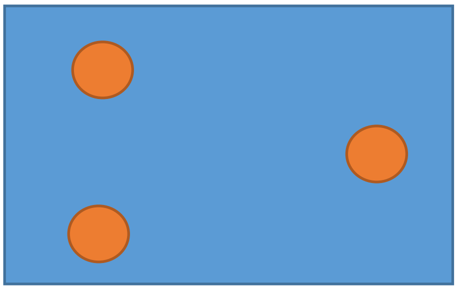
Redundant Constraints

Beam with 2 supports satisfy the minimum number of supports required (=2, in this case) for the kinematic supporting of beam on the plane. (Statically determinate in this case)

But, the third support in the middle (green color) may generate possible distortion, strain. In order to prevent this situation, it should be manufactured precisely such that its height is within tolerance from the line connecting the end supports, requiring extra effort and cost. (Statically Indeterminate in this case)

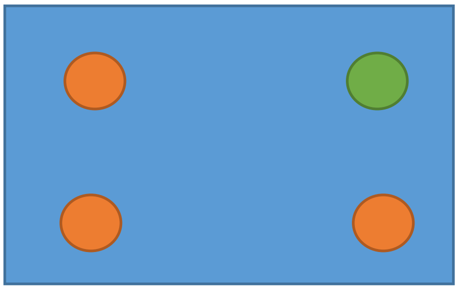
2. Foundation Legs

3 Legs Foundation



Kinematic Constraints

4 Legs Foundation



Redundant Constraints

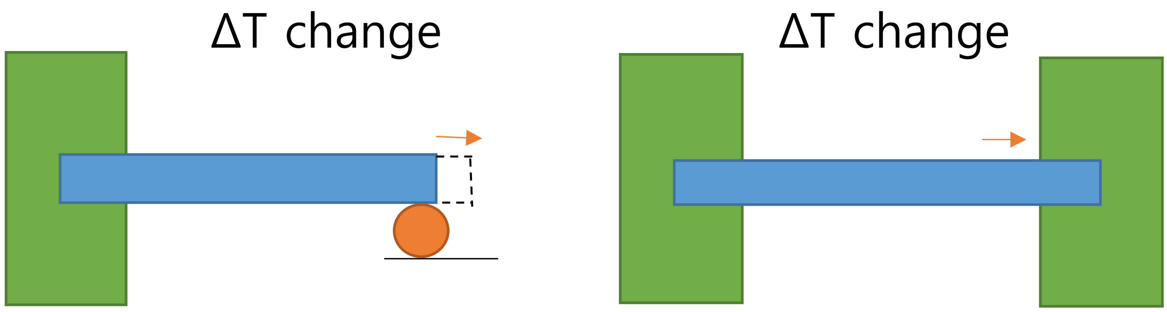
The 3 Legs foundation satisfy the minimum number of supports required (=3, in this case) for the kinematic foundation of machine on the plane.

But, the fourth leg (green color) of the redundant constraints of the 4 legs foundation may generate possible distortion, strain. In order to prevent this situation, it should be manufactured precisely such that its height is within tolerance from the plane connecting the existing 3 legs, thus requiring extra effort and cost.

The redundant constraints generally require higher cost and higher accurate components than the kinematic(optimum) constraints.

Also, redundant constraints may not accommodate small dimensional changes due to such as thermal deformation without creating internal stress or strain.

Kinematic constraints vs. Redundant constraints



<u>Kinematic constraints:</u>

A 10m steel structure of 12ppm/°C thermal expansion coefficient is under 50°C change, then it causes about 6mm thermal expansion;

$\Delta L=\alpha L \Delta T=12(10)(50)=6[mm]$

And it can be accommodated by the roll support constraint, thus no internal stress is occurred.

Redundant constraints:

The temperature change causes thermal strain internally and causes compressive stress or thermal stress, σ_T, accordingly;

$$\sigma_T = E\alpha\Delta T = (200E9)(12E\text{-}6)(50) = 120MPa$$

≒1/3Y of Yield Stress of Steel caused by the internal strain. Very small margin is left for the load induced stress afterwards; this is neither a safe design, nor desirable, thus should to be avoided.

Another advantage of kinematic design is that higher accuracy of positioning can be achieved without high dimensional accuracy of elements involved (Third Lemma)

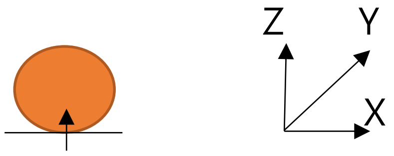
(*Redundant double Vee-guide may cause trapping unless it is precisely manufactured in tolerance.)

Constraints, DOFs, Motion with Configuration

The following figure shows the constraints, DOFs, motions with configuration. It will be very useful for developing ideas and insights for practical kinematic designs for specific applications

1 Constraint: C=1, D=5

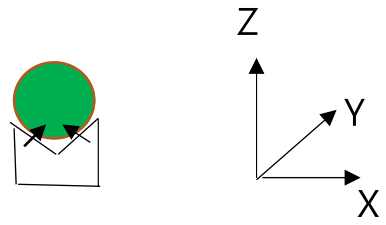
1) Ball on Flat(or plane)



DOFs=$(\delta x,\delta y,\delta z,\theta x,\theta y,\theta z)$=(O,O,X,O,O,O)=5

2 Constraints: C=2, D=4

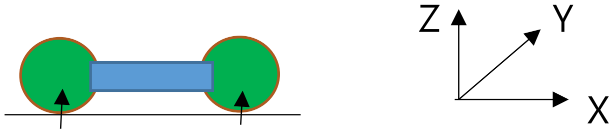
1) Ball on Groove (or 'ball on V'):



DOFs=(δx,δy,δz,θx,θy,θz)=(X,O,X,O,O,O)=4

2)Two balls link on Flat (Plane):

Two balls rigidly linked and resting on a Flat

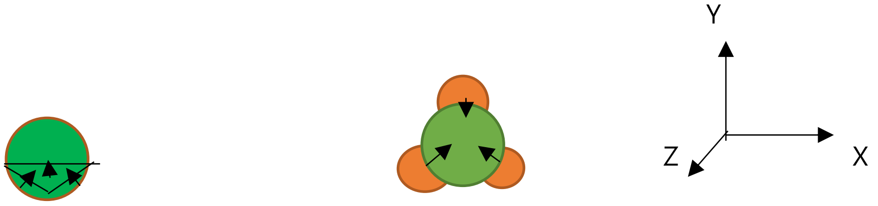


DOFs=(δx,δy,δz,θx,θy,θz)=(O,O,X,O,X,O)=4

3 Constraints: C=3, D=3

1) Ball on the trihedral hole, or

Ball on the three-balls-nest



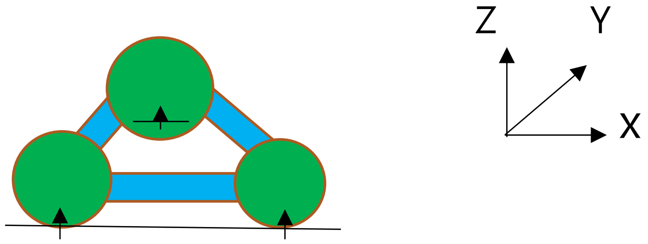
trihedral hole (Side View)

three-balls-nest (Top View)

DOFs=(δx,δy,δz,θx,θy,θz)=(X,X,X,O,O,O)=3

2) Three Balls link on Flat

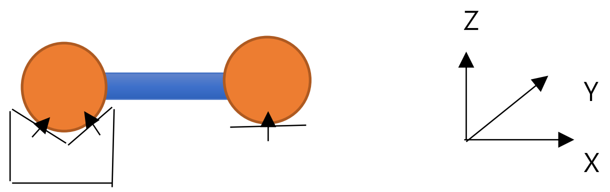
:Three rigidly linked balls that suitably (not on straight line) distributed on Flat



DOFs=(δx,δy,δz,θx,θy,θz)=(O,O,X,X,X,O)=3

3)Two balls link on V and Flat

:Two balls rigidly linked, one on V-Groove and other on Flat

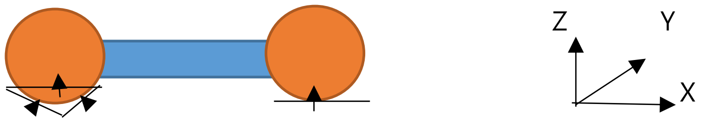


DOFs=(δx,δy,δz,θx,θy,θz)=(X,O,X,O,X,O)=3

<u>4 Constraints: C=4, D=2</u>

1)Two balls link on trihedral hole and on Flat

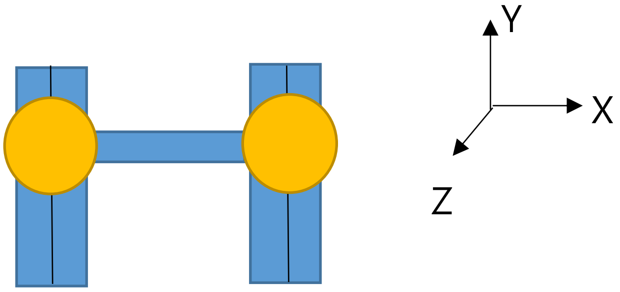
Two rigidly linked balls, one ball resting on trihedral hole and other on Flat



DOFs=(δx,δy,δz,θx,θy,θz)=(X,X,X,O,X,O)=2

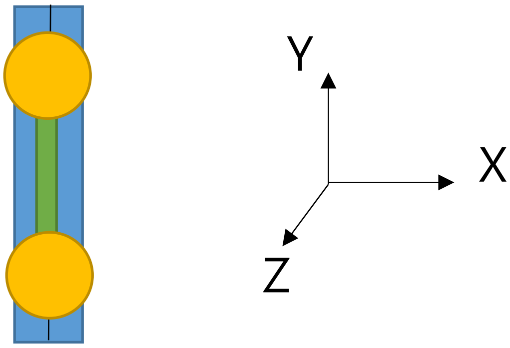
2) Two balls link both on V-groove: C=4, D=2

Two rigidly linked balls both resting on V groove



Top view of two balls on two V-Grooves

DOFs=(δx,δy,δz,θx,θy,θz)=(X,O,X,O,X,X)=2

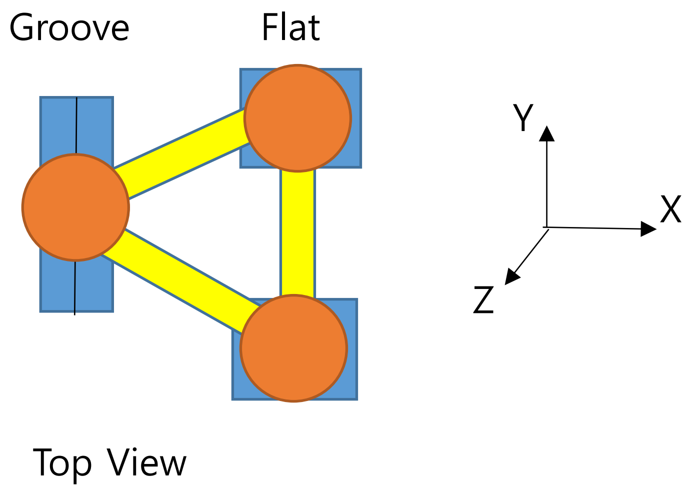


Top view on two balls on one V-Groove

DOFs=(δx,δy,δz,θx,θy,θz)=(X,O,X,X,O,X)=2

3) Three balls link

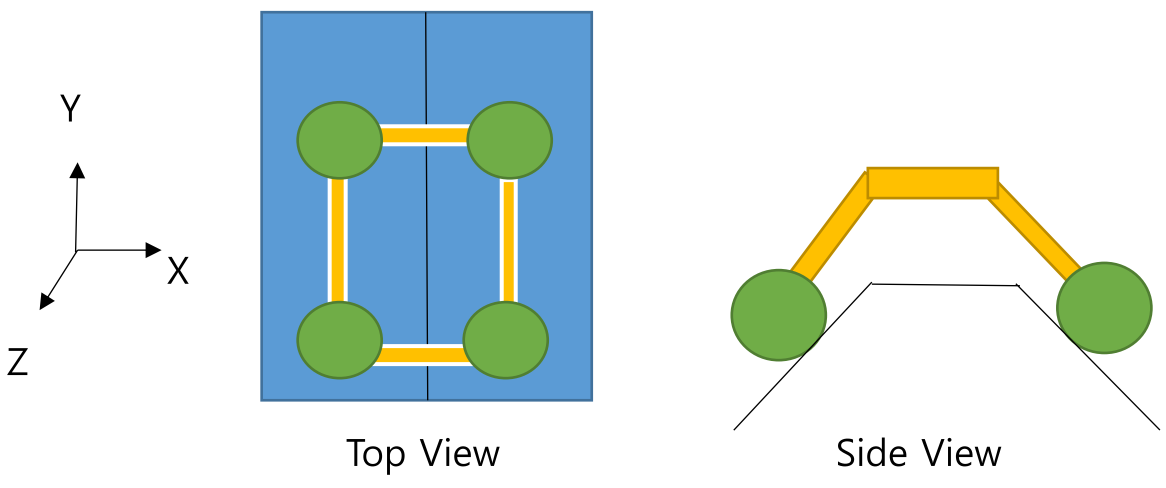
:Two on Flat and one on V, suitably distributed, not on single line



DOFs=(δx,δy,δz,θx,θy,θz)=(X,O,X,X,X,O)=2

4) Four balls link on two inclined Planes

:Four rigidly linked balls on suitable distributed on two inclined planes

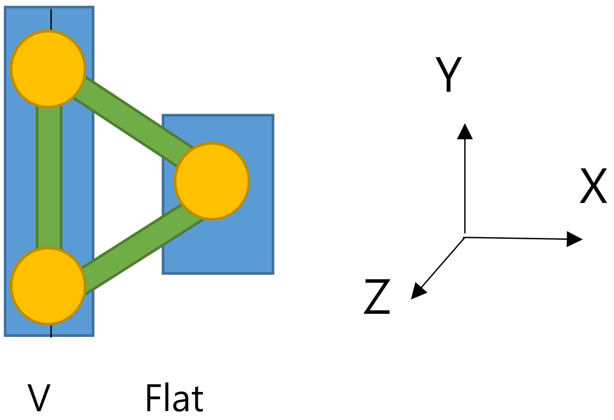


DOFs=(δx,δy,δz,θx,θy,θz)=(X,O,X,X,X,O)=2

<u>5 Constraints: C=5, D=1</u>

(1) Three Balls link with two on V and one on Flat

:Three rigidly linked balls; two balls on V-Groove, one ball on Plane

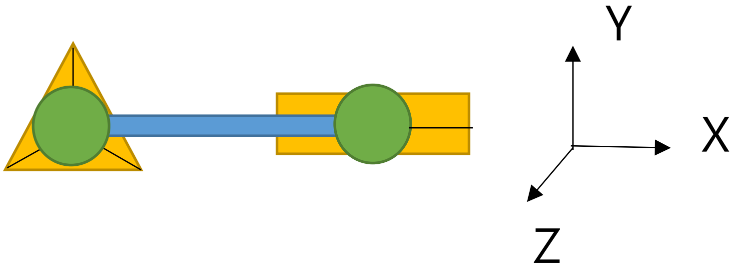


Top View of Three Linked Balls

DOF=(δx,δy,δz,θx,θy,θz)=(X,O,X,X,X,X)=1

This V and Flat configuration is very useful for the kinematic design of 1 DOF axis.

(2) Two linked balls; one on trihedral hole, other one on V groove



Top View of Two linked Balls on Trihedral Hole and V

DOF=(δx,δy,δz,θx,θy,θz)=(X,X,X,O,X,X)=1

This configuration can be useful for the spindle structures design.

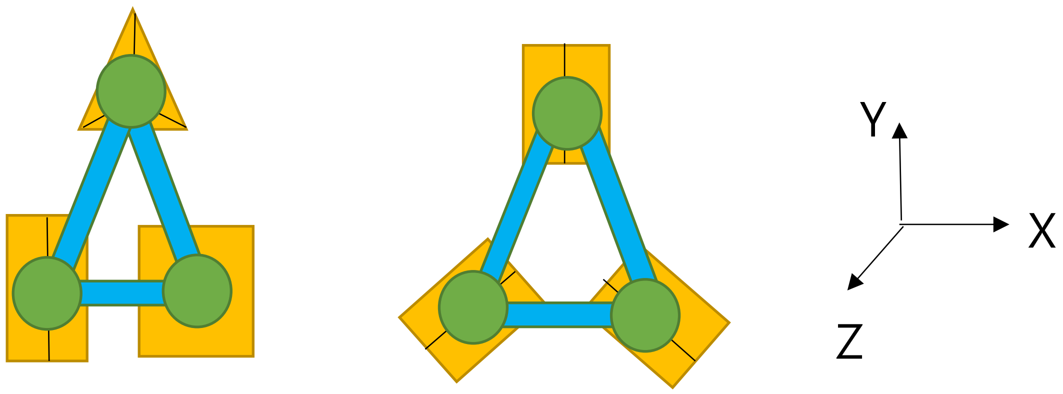
6 Constraints: C=6, D=0, or Clamp

All 6 DOFs are constrained, this case is called as 'Clamp'.

Three linked Balls;

(1) First ball on trihedral hole, second ball on V, and third ball on Flat, or

(2) Three balls on three V's



(1) (2)

Top view of Kelvin Clamp

(1) Trihedral hole-V-Flat configuration

(2) V-V-V configuration

DOF=(δx,δy,δz,θx,θy,θz)=(X,X,X,X,X,X)=0

This Kelvin clamp is very much useful for applications of precise positioning, or positioning repeatability, giving zero repeatability in theory (rigid body), and much less than sub-micrometer repeatability in practical application (under elasticity).

<u>Stability for V-V-V configuration of Kelvin Clamp</u>

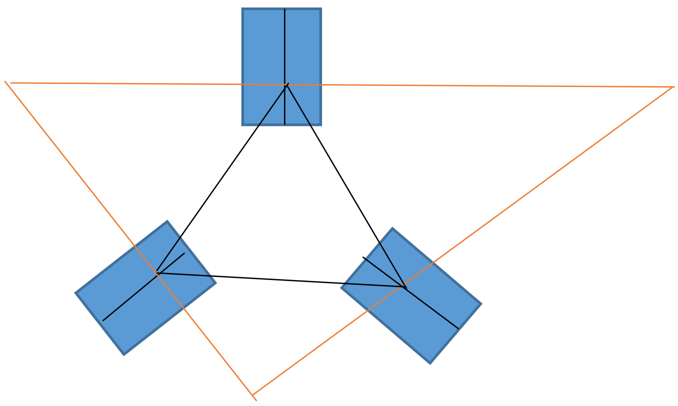
Direction of Constraint

=Direction of force vectors at constraints(Red Line)

Constraint triangle, or coupling triangle

= Triangle formed by constraints location(Black Line)

<u>Stable Kelvin Clamp</u>



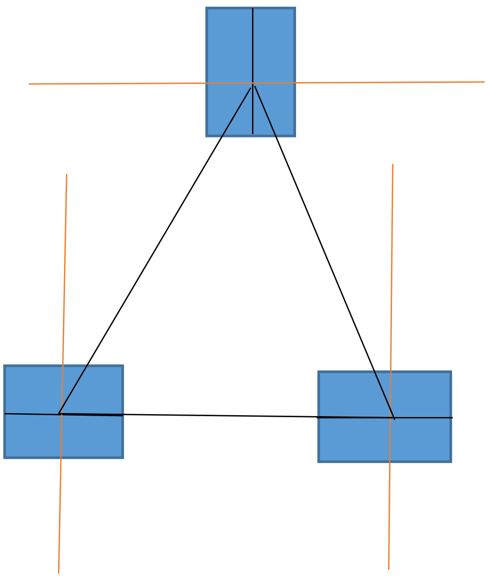
: Direction of Constraint

: Constraint triangle or coupling triangle

∴Stability condition: Normal to the Direction of Constraint must bisect the angles of constraints triangle

Unstable or Marginally-Stable Kelvin Clamp

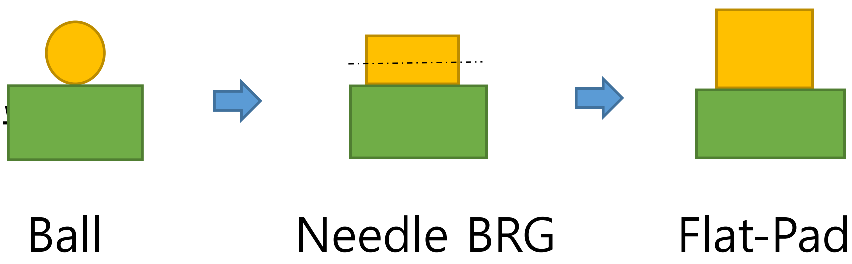
:Normal to the Direction of Constraint does not bisect the Constraint triangle



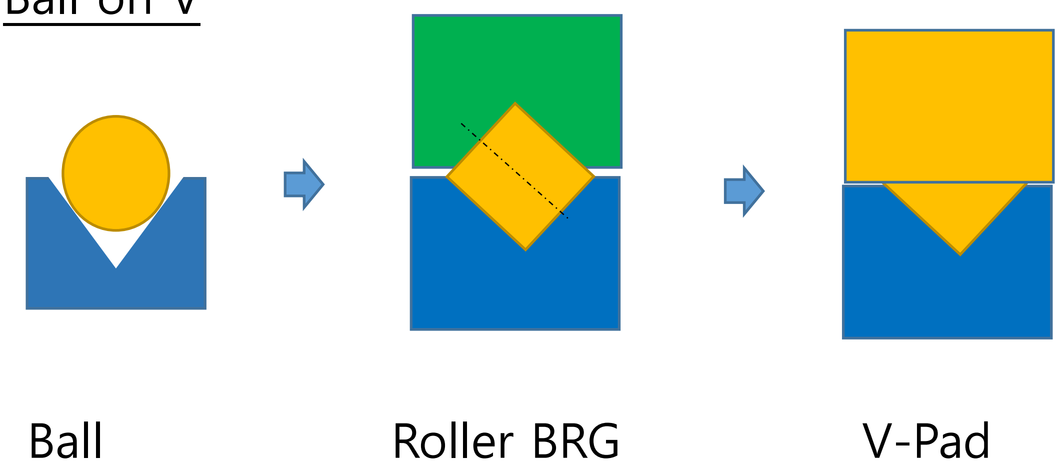
Semi-Kinematic or Pseudo-Kinematic Design

: For Higher Load Capacity thus Higher Stiffness, the Kinematic Design based on points contact can be developed into semi-kinematic design, or pseudo-kinematic design based on the line contact and area contact; where line consists of a series of points, area consist of a series of lines. Usually, kinematic design includes the semi-kinematic design, or pseudo-kinematic design.

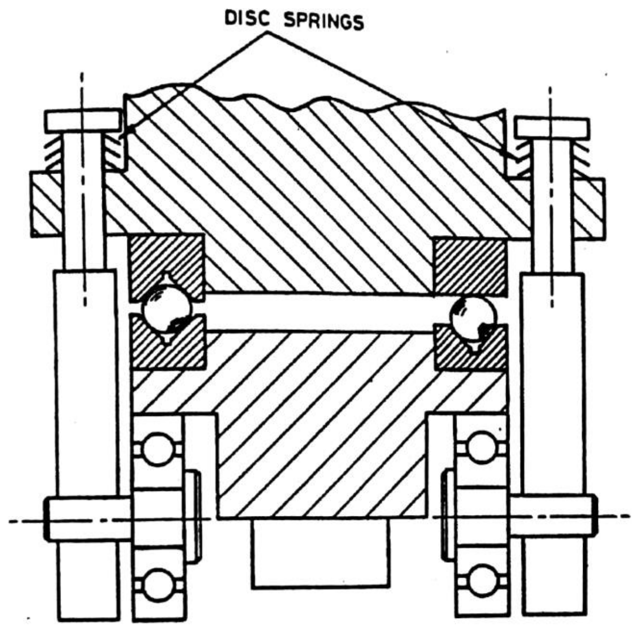
Ball on Flat



Ball on V

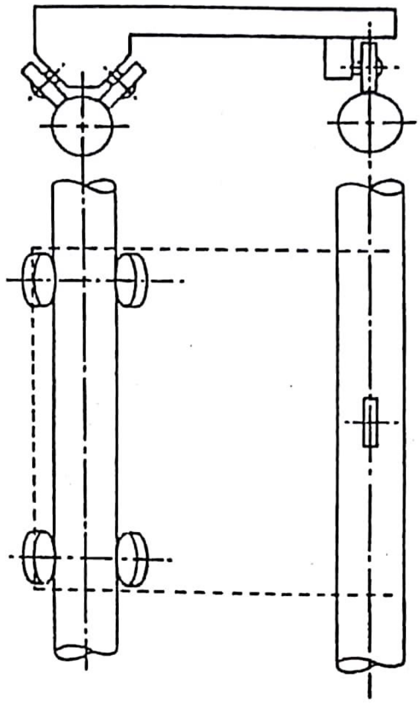


The followings show typical examples of kinematic design for machine elements.



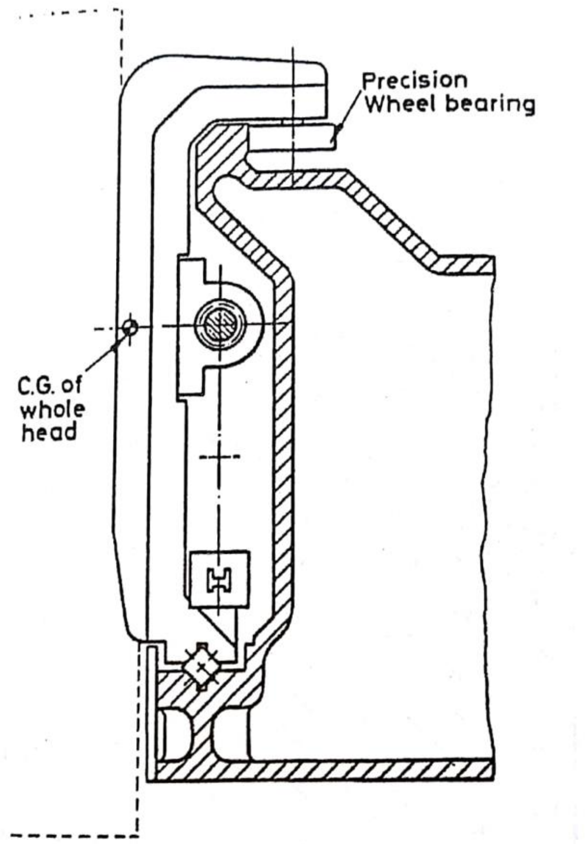
Mounting Configuration of Schneeberger bearing from McKeown

(source: D.Dornfeld's Precision Manufacturing)



Kinematic Slide with Ball Bearings as wheels from McKeown

(Source: Dornfeld’s Precision Manufacturing)



‘Hooked on” configuration of carriage guiding system from McKeown

(source: D.Dornfeld’s Precision Manufacturing)

Guidelines for Kinematic Design

1) Kinematic Design principles based on the rigid body assumption, provide very effective constraints or guides for motion, even under elastic compliance occurring.

2) The centre of gravity, and axis of rotation (or instantaneous centre) are better to be met, then undesirable motion such as rotation, can be minimized.

3) Kinematic design requiring less precise parts will give less manufacturing cost or higher precision with existing parts of the same precision.

4) Play or hysteresis must be minimized to achieve the highest precision in the kinematic design, via preloading, etc. Also, nesting forces are generally required for pressing against the constraints, by gravitational, elastic, magnetic, or other mechanical forces.

5) Points contact is ideal in the kinematic design, but large load can result in local deformation and wear, thus contact mechanics such as Hertz stress must be

considered. The semi-kinematic design principles, the elastic average design principles are preferred for larger load application.