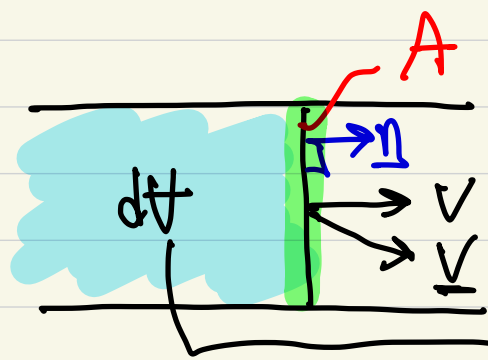
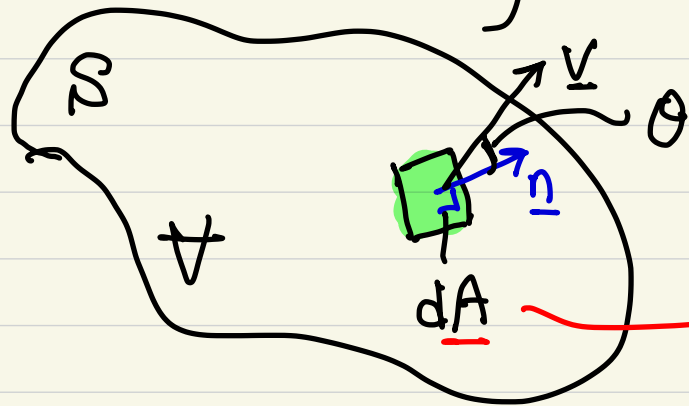


\* Volume and mass flow rate.



$$dV = (\underline{v} \cdot \underline{n}) \cdot A \cdot dt$$

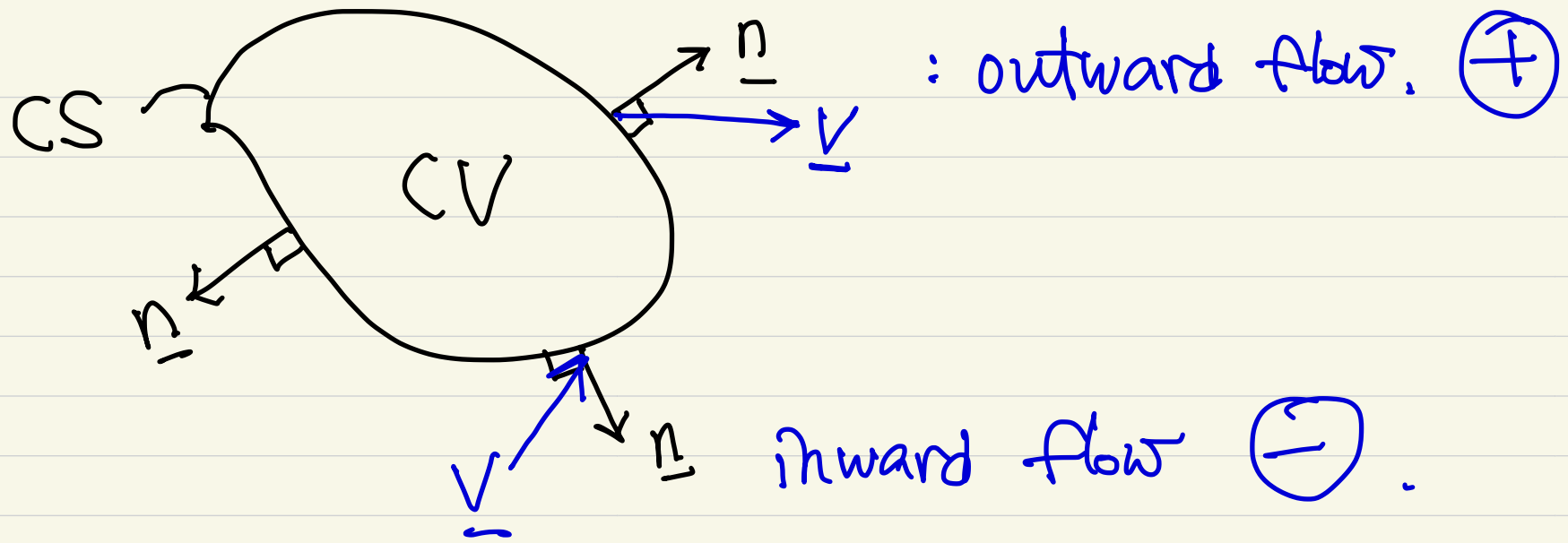
↓ arbitrary shape.



$$dV = (\underline{v} \cdot \underline{n}) dA \cdot dt$$

$$\frac{dV}{dt} = dQ = (\underline{v} \cdot \underline{n}) dA$$

Volume flow rate.  $\therefore Q = \int dQ = \int_S (\underline{v} \cdot \underline{n}) dA = \int_S v_n dA$ .



• mass flow rate:  $d\dot{m} = \rho dQ = \rho (\underline{v} \cdot \underline{n}) dA$ .

$$\therefore \dot{m} = \frac{dm}{dt} = \int_S \rho (\underline{v} \cdot \underline{n}) dA = \int_S \rho v_n dA$$

if  $\rho = \text{const}$  (incompressible flow,  $\theta \ll \frac{2}{\gamma} \frac{v}{c}$ )  
 ( $Ma < 0.2 - 0.3$ ).

$$\dot{m} = \rho Q$$

if  $\rho, V$  are constant,  $\dot{m} = \rho V A$ .

\* average velocity (bulk velocity) passing through a surface.

$$\therefore V_{\text{avg}} \equiv \frac{Q}{A} = \left( \int_S (\underline{V} \cdot \underline{n}) dA \right) / \left( \int_S dA \right)$$

$$\text{or, } Q = V_{\text{avg}} \cdot A.$$

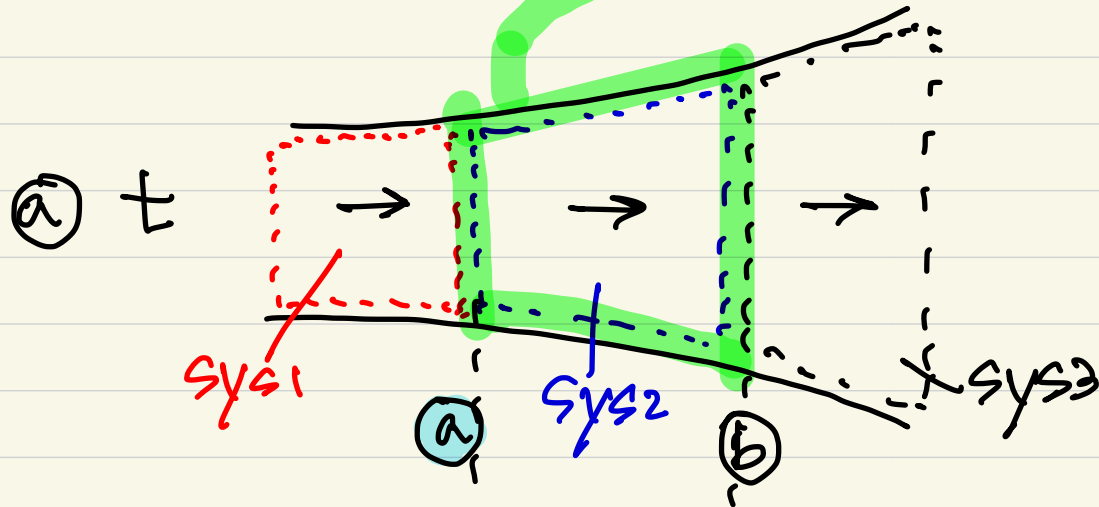
### 3.2. Reynolds Transport theorem.

system analysis  $\leftarrow$   $\Downarrow$   $\rightarrow$  Control volume analysis  
(individual mass) (specific area, volume)

- Lagrangian
- moving coordinate.

- Eulerian. C.V.  $\downarrow$
- fixed coordinate

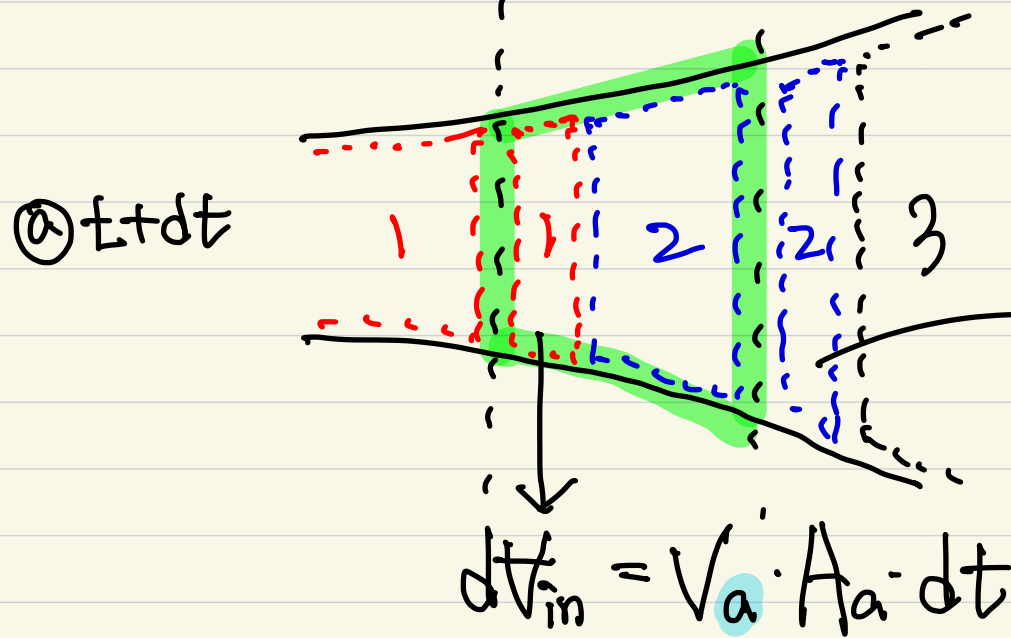
- 1D fixed CV.



$\beta$  : flow property.

$$\beta \equiv \frac{dB}{dm} \text{ (intensive value)}$$

$$(dB = \beta \cdot dm)$$



$$dV_{out} = V_b \cdot A_b \cdot dt$$

$$dV_{in} = V_a \cdot A_a \cdot dt$$

- total amount of "B" in the CV.

$$\therefore \underline{B_{cv}} = \int_{cv} dB = \int_{cv} \beta \cdot \underline{dm} = \int_{cv} \beta \rho \underline{dV}$$

$$\frac{d}{dt} B_{cv} = \frac{B_{cv}(t+dt) - B_{cv}(t)}{dt}$$

$$= \frac{1}{dt} \left[ \underline{B_2(t+dt)} + \underline{\beta \rho dV_{in}} - \underline{\beta \rho dV_{out}} - \underline{B_2(t)} \right]$$

$$= \frac{1}{dt} \left[ B_2(t+dt) - B_2(t) \right] - (\beta \rho V A)_{out} + (\beta \rho V A)_{in}$$

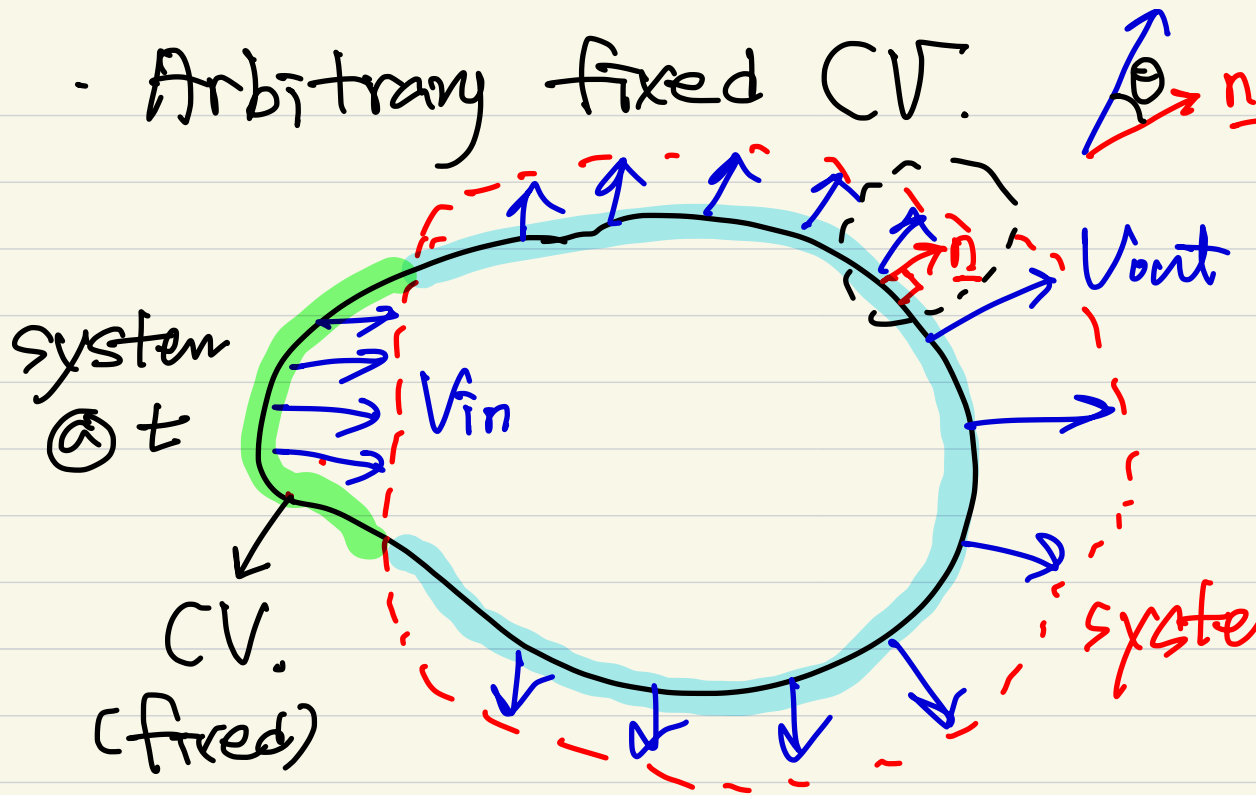
$$= \frac{d}{dt} B_{sys}$$

$$(dV = V \cdot A \cdot dt)$$

$$\therefore \frac{d}{dt} B_{sys} = \frac{d}{dt} \int_V \beta \rho dV + (\beta \rho V A)_{out} - (\beta \rho V A)_{in}$$

- 1D RTT.

Arbitrary fixed CV.



$$\begin{aligned}
 dB &= \beta \cdot dm. \\
 &= \beta \rho dV \\
 &= \beta \rho (\underline{V} \cdot \underline{n}) dA.
 \end{aligned}$$

system @ t + dt.

$$\frac{d}{dt} B_{\text{sys}} = \underbrace{\frac{d}{dt} \int_{CV} \beta \rho dV}_{\text{Lagrangian}} + \underbrace{\int_{CS} \beta \rho V \cos \theta \cdot dA_{\text{out}} - \int_{CS} \beta \rho V \cos \theta \cdot dA_{\text{in}}}_{\text{flux terms.}}$$

$$= \frac{d}{dt} \int_{CV} \beta \rho dV + \int_{CS} \beta \rho (\underline{V} \cdot \underline{n}) dA.$$

Eulerian.

if CV is moving at  $\underline{V}_s$ , relative velocity,  $\underline{V}_r = \underline{V} - \underline{V}_s$

$$\frac{d}{dt} B_{sys} = \frac{d}{dt} \int_{CV} \beta \rho dV + \int_{CS} \beta \rho (\underline{V}_r \cdot \underline{n}) dA$$

• mass :  $B = m$ ,  $\beta = \frac{dB}{dm} = 1$ .

momentum :  $B = m \underline{V}$ ,  $\beta = \frac{dB}{dm} = \underline{V}$ .

angular mom :  $B = (\underline{r} \times \underline{V}) m$ ,  $\beta = \underline{r} \times \underline{V}$

energy :  $B = E$ ,  $\beta = e$ .