Machine Design: Contact Mechanics

No materials are rigid, but just more stiff or less stiff;

->Issue of Stiffness during the contact

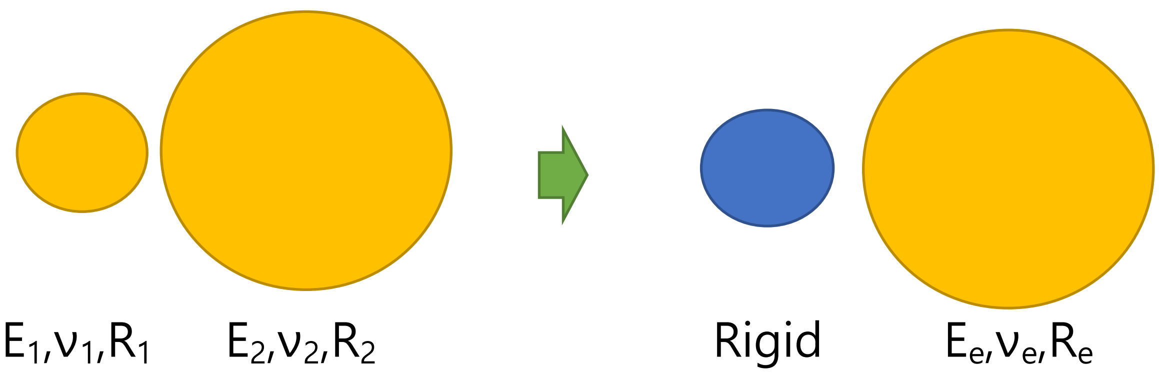
->Contact Mechanics based on Elasticity, that is,

Contact between Curved Surfaces, or Hertz Stress

Equivalent Contact System:

Elastic body 1, 2 contact

->A rigid sphere contacts elastic plane




Equivalent modulus of Elasticity: $E_e$

$1/E_e=(1-\nu_1^2)/E_1+(1-\nu_2^2)/E_2$

or $E_e=E/[2(1-\nu^2)]$ if 1,2 are the same material

Ex)$E_1$, $E_2$ are steel, $E_s$=200GPa

then $E_e=E_s/[2(1-0.3^2)]=0.549E_s$=109.0GPa

Equivalent Poissons' ratio:$\nu_e$

$1/\nu_e=[1/\nu_1+1/\nu_2]/2$, or $\nu_e=\nu_1=\nu_2$ if 1,2 are the same.

Equivalent radius of system, $R_e$

$1/R_e$

$=1/R_{1\ major}+1/R_{1\ minor}+1/R_{2\ major}+1/R_{2\ minor}$

where

Convex surface=+

Concave surface=-

Flat surface=$\infty$

Ex1) $R_1=R_2=R$=Radius of Ball,

$1/R_e=1/R+1/R+1/R+1/R$ $\therefore R_e=R/4$

Ex2) $R_1=R$, $R_2=\infty$(Plane)

$1/R_e=1/R+1/R+1/\infty+1/\infty=2/R \therefore R_e=R/2$

Radius of equivalent circular contact, a

$a=[3FR_e/(2E_e)]^{1/3}$

Ex)$R_1=25mm$, $R_2=\infty$(plane), F=10N≒1Kgf

$R_e=R/2=0.0125$, $E_e=109Gpa$,

thus

$a=[3(10)(0.0125)/(2*109E9)]^{1/3}=0.120[mm]=120[um]$

Deflection of the contact system, δ

$\delta=0.5(1/Re)^{1/3}(3F/2Ee)^{2/3}$ eq(1)

$=0.5(1/0.0125)^{1/3}(3(10)/(2*109E9))^{2/3}$

$=0.574[um]$

Stiffness of Contact System, K

Partially differentiate eq(1) w.r.t F;

$\partial\delta/\partial F=(0.5)(1/Re)^{1/3}(3/2)^{-1/3}Ee^{-2/3}F^{-1/3}$

Thus Stiffness, $K=\partial F/\partial\delta=1/[\partial\delta/\partial F]$

$\therefore \partial\delta/\partial F=(0.5)(1/0.0125)^{1/3}(3/2)^{-1/3}(2*109E9)^{-2/3}(10)^{-1/3}$

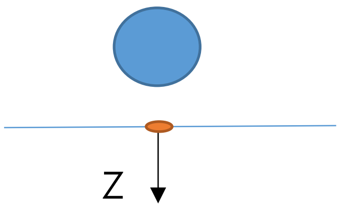
$=2.41E\text{-}8$ m/N

Stiffness$=1/[\partial\delta/\partial F]=41.49$ MN/m=41.49N/um

<u>Contact Stress by Hertz</u>

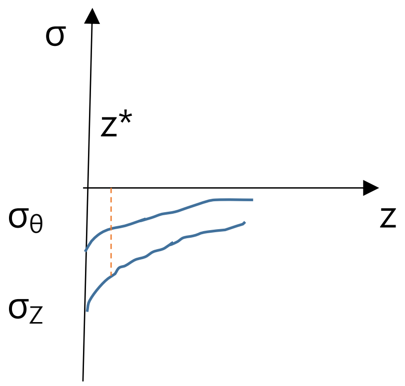
$q=1/\pi(1/R_e)^{2/3}(3E_e^2F/2)^{1/3}=aE_e/(\pi R_e)$

$=(120E\text{-}6)(109E9)/(3.14*0.0125)=333$ [MPa]




$\sigma_z=q[-1+z^3/(a^2+z^2)^{3/2}]$

$\sigma_r=\sigma_\theta=0.5q[-(1+2\nu)+(1+\nu)z/(a^2+z^2)^{1/2}-z^3/(a^2+z^2)^{3/2}]$




Shear stress, $\tau=(\sigma_\theta-\sigma_z)/2$

$=(0.5q)[(1-2\nu)/2+(1+\nu)z/(a^2+z^2)^{1/2}-3z^3/[2(a^2+z^2)^{3/2}]]$

Maximum shear stress, $\tau_{max}$, can be obtained from $d\tau/dz=0$,

$\therefore \tau_{max}=0.5q[(1-2\nu)/2+(2\sqrt{2}/9)(1+\nu)^{3/2}] \fallingdotseq q/3$

At $z=z^*=a\sqrt{2(1+\nu)}/\sqrt{(7-2\nu)}=(120E-6)\sqrt{(2.6/6.4)}=76.5$[um]

This result is well matching with the observation that fracture practically occurs at the submerged depth location.

This is also the reason the surface layer coating such as anodizing is widely used for the aluminum material.

Yield occurs when $\tau_{max}=q_{max}/3=\sigma_Y/2$, where $\sigma_Y$=Yield Strength

Fracture occurs when $\tau_{max}=q_{max}/3=\sigma_T/2$, where $\sigma_T$=Tensile Strength

Thus allowable maximum hertz stress, $q_{max}$

$q_{max}=3\sigma_T/2$ for metal

$q_{max}=3\sigma_T/(1-2\nu)$ for brittle material

For SUS304, $\sigma_T=826MPa$, $\sigma_Y=595MPa$

Force F such that fracture occurs;

$q_T=a_T Ee/(\pi Re)=3\sigma_T/2$

$\therefore a_T=(3\sigma_T/2)\pi Re/Ee$

$=(1.5)(826E6)(3.14)(0.0125)/(109E9)$

$=446E-6[m]=446[um]$

Depth location at fracture, $z_T=a_T\sqrt{2(1+\nu)}/\sqrt{(7-2\nu)}$

$\fallingdotseq 284[um]$

$F_T=2Ee/(3Re)a_T^3=2(109E9)/[3*0.0125]*(446)^3E-18$

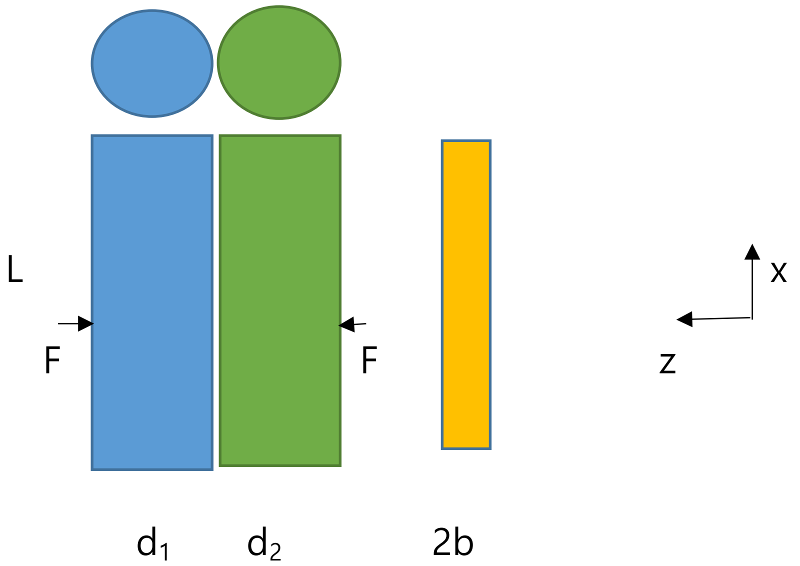
$=515[N]$

$\delta_T=0.5(1/Re)^{1/3}(3F_T/2Ee)^{2/3}=(0.5)(1/0.0125)^{1/3}(3*515/218E9)^{2/3}$

$=7.95[um]$

# Line contact between Objects

(for semi-kinematic design)




Ee is similarly defined, and contact width=2b

where $b=[2Fd_1d_2/[\pi LEe(d_1+d_2)]]^{1/2}$

Maximum contact pressure, q

$q=2F/\pi bL=(4/\pi)Pm$,

where Pm=mean pressure=F/2bL

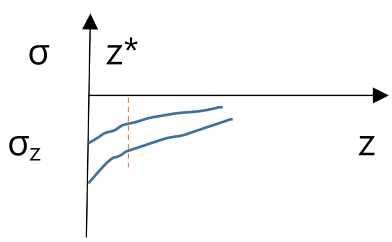
$\sigma_z=-q[b^2/(b^2+z^2)]^{1/2}$

$\sigma_x=-2q\eta[(1+z^2/b^2)^{1/2}-z/b]$, η=friction coeff.

$\sigma_y=-q[(2-b^2/(b^2+z^2))(1+z^2/b^2)^{1/2}-2z/b]$

$\tau_{yx}=(\sigma_y-\sigma_x)/2$

$\tau_{zx}=(\sigma_z-\sigma_x)/2$



$\tau_{zy}=(\sigma_z-\sigma_y)/2$

$\tau_{zy}=(\sigma_z-\sigma_y)/2$ is observed as max. at z*

Max shear stress is $\tau_{zy}≒0.3q$ at z*/b=0.786

At fracture, $\tau_{zy}=\sigma_T/2$

$\therefore 0.3q=\sigma_T/2$ and $q=\sigma_T/0.6$

Force F such that fracture occurs

$d_1=d_2=0.05$[m], L=0.05m

$q_T=\sigma_T/0.6=826/0.6=1376$ [MPa]$=2F/\pi bL$

$\therefore b=2F/\pi qL=[2Fd_1d_2/\pi LEe(d_1+d_2)]^{1/2}$, and L=0.05[m]

$\therefore F=(\pi/2)q^2Ld_1d_2/Ee(d_1+d_2)$

$=(3.14/2)(1376E6)^2(0.05)(0.05)(0.05)/[(0.1)109E9]$

=34089=34.089[KN]

$\therefore b=2F/\pi qL=2(34089)/3.14/1376E6/0.05=315.6$[um]

Max shear stress is observed at z*/b=0.786

$\therefore z^*=(0.786)(315.6)=248$[um]

Tangential stiffness of the contact surface

With No slip

From Mindlin, under no slip

$\delta_{tan} = F_{tan}(2-\nu)(1+\nu)/(4aE)$

Thus Tangential stiffness, $K_{tan}$ is

$K_{tan} = 1/[\partial\delta_{tan}/\partial F_{tan}]$

$= 4aE/[(2-\nu)(1+\nu)]$ without slip at interface

With slip

Assuming $a'/a = b'/b = [1 - F_{tan}/\mu F]^{1/2}$

where a′, b′ are of the semi axis of the inner ellipse, inside of which there is no slip, F is the normal force.

Deresiewicz gives

$\delta_{tan} = 3\mu F(2-\nu)(1+\nu)[1-(1-F_{tan}/\mu F)^{2/3}]\Phi/(8aE)$

where $\Phi = 0.2\text{-}2.7$ depending on the material constants,

or $\Phi = 1$ for spherical contact ($a=b$)

Thus the tangential stiffness can be obtained as,

$K_{tan}=1/[\partial\delta_{tan}/\partial F_{tan}]= 4aE(1-F_{tan}/\mu F)^{1/3}/[(2-\nu)(1+\nu)]$

(If $F_{tan}=0$, this is the same as no slip condition between the interface)

HW) Design Homework (2 weeks)

Design a Kelvin Clamp plate to support 100N (about 10Kgf) central vertical force, choose proper material and dimension for the clamp, which is to handle the wafer stage of 12 inch. Verify the design values by calculating the stiffness and deflections along each DOFs.