

4. Control Rod Worth Measurement (Reactivity Measurement)

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1. Positive Period Method



Point Kinetics Equation

- The reactor kinetics equations based on one-point reactor approximation with one-energy-group theory are as follows:

$$\frac{dn(t)}{dt} = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \sum_{i=1}^6 \lambda_i C_i(t), \quad \text{----- (1a)}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_{i,eff}}{\Lambda} n(t) - \lambda_i C_i(t) \quad (i = 1, 2, \dots, 6), \quad \text{----- (1b)}$$

where $n(t)$ = neutron density or total neutron population,

$C_i(t)$ = i -th group delayed neutron precursor density,

λ_i = decay constant of the i -th group delayed neutron precursor,

Λ = prompt neutron generation time, which is the prompt neutron lifetime l divided by k_{eff} ,

β_{eff} = effective delayed neutron fraction

$\beta_{i,eff}$ = effective delayed neutron fraction of i -th delayed neutron precursor group

Inhour Equation

- All of the coefficients in Eq. (1) are physical constants, in practice, except the reactivity, which can be changed by variation of a operation parameter.
- In the case where reactivity does not vary, the system is a “constant coefficient” differential equation system, and its solution can be found by merely seeking exponential solutions of the form

$$\begin{aligned} n(t) &= a \cdot \exp(\omega t), \\ C_i(t) &= b_i \cdot \exp(\omega t) \quad (i = 1, 2, \dots, 6), \end{aligned} \quad \text{----- (2)}$$

where ω , a , and b_i are constants.

- Insertion of Eq. (2) into Eq. (1) gives

$$\omega a \cancel{\exp(\omega t)} = \frac{\rho - \beta_{\text{eff}}}{\Lambda} a \cancel{\exp(\omega t)} + \sum_{i=1}^6 \lambda_i b_i \cancel{\exp(\omega t)}, \quad \text{----- (3a)}$$

$$\omega b_i \cancel{\exp(\omega t)} = \frac{\beta_{i,\text{eff}}}{\Lambda} a \cancel{\exp(\omega t)} - \lambda_i b_i \cancel{\exp(\omega t)} \quad (i = 1, 2, \dots, 6). \quad \text{----- (3b)}$$

Inhour Equation (Contd.)

- By substituting b_i derived from Eq. (3b) into Eq. (3a), we can obtain a characteristic equation as

$$\rho = \omega \left[\Lambda_{eff} + \sum_{i=1}^6 \frac{\beta_{i,eff}}{\lambda_i + \omega} \right] \quad \dots\dots\dots (4a)$$

- Because $\Lambda_{eff} = l/k_{eff}$, Eq. (4a) can be expressed as

$$\rho = \omega \left[-l \left(1 - \frac{1}{k_{eff}} \right) + l + \sum_{i=1}^6 \frac{\beta_{i,eff}}{\lambda_i + \omega} \right]$$

$$\Rightarrow \rho = \frac{\omega l}{\omega l + 1} + \frac{\omega}{\omega l + 1} \sum_{i=1}^6 \frac{\beta_{i,eff}}{\lambda_i + \omega} \quad \dots\dots\dots (4b)$$

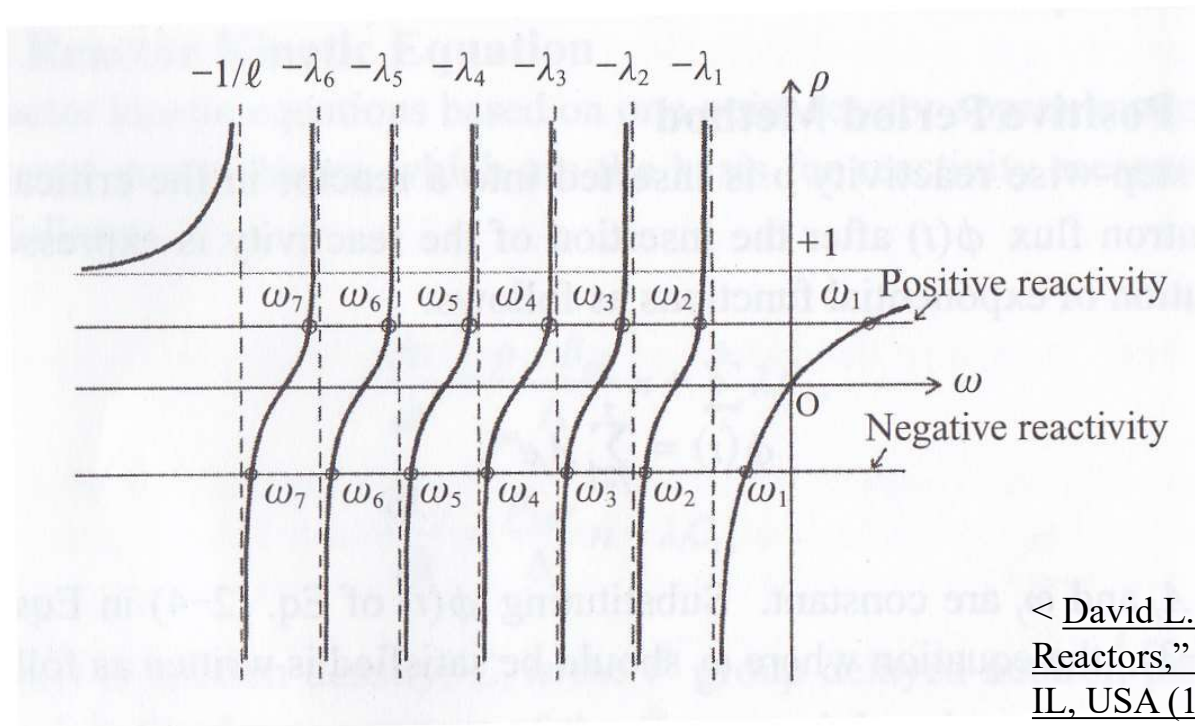
- Eq. (4) is called the “inhour equation” because it gives a quantity that can be expressed in hour^{-1} (inverse hour).

Inhour Equation (Contd.)

- When step-wise reactivity ρ is inserted into a reactor in the critical state, the neutron level $n(t)$ after the insertion of reactivity can be expressed as a summation of exponential functions as follows:

$$n(t) = \sum_{j=1}^7 A_j \exp(\omega_j t) \quad \dots\dots\dots (5)$$

where ω_j should satisfy Eq. (4).



< David L. Hetrick, "Dynamics of Nuclear Reactors," American Nuclear Society, Inc., IL, USA (1993) >

Positive Period Method

- From the figure of < Relation between reactivity ρ and ω >, when positive reactivity ρ is inserted into the core, only one positive ω_j is obtained and the rest of six ω_j are negative values; this means that after the terms with negative ω_j components of Eq. (5) vanish, the reactor power increases as:

$$n(t) \cong A_1 \exp(\omega_1 t) \quad \text{..... (6)}$$

- The inverse of ω_1 , T , is known as the stable reactor period (or, merely, the period), which satisfies the following equation:

$$\rho = \frac{\omega l}{\omega l + 1} + \frac{\omega}{\omega l + 1} \sum_{i=1}^6 \frac{\beta_i}{\lambda_i + \omega}$$

$$\Rightarrow \rho = \frac{l}{l + T} + \frac{T}{l + T} \sum_{i=1}^6 \frac{\beta_i}{\lambda_i T + 1} \quad \text{..... (7)}$$

- For measuring positive reactivity, the reactivity ρ is calculated from Eq. (7) using a measurement of the positive period T .

2. Rod Drop Method



Extrapolation Method

- When the reactor power is maintained at a constant value for a certain period of time, the neutron density and the delayed neutron precursor density are constants as n_0 and C_{i0} , respectively.
- From Eq. (1b), the static density of precursor group i can be obtained as

$$\frac{dC_i(t)}{dt} \stackrel{=0}{=} \frac{\beta_{i,eff}}{\Lambda} n(t) - \lambda_i C_i(t) \quad (i = 1, 2, \dots, 6)$$

$$\Rightarrow C_{i0} = \frac{\beta_{i,eff}}{\Lambda \lambda_i} n_0 \quad (i = 1, 2, \dots, 6) \quad \dots\dots\dots (8)$$

$$\Rightarrow \sum_{i=1}^6 \lambda_i C_{i0} = \sum_{i=1}^6 \frac{\beta_{i,eff}}{\Lambda} n_0 = \frac{\beta_{eff}}{\Lambda} n_0 \quad \dots\dots\dots (9)$$

- Assuming that **the delayed neutron precursor densities is unchanged** from these static values **during a very short time after a control rod of reactivity worth of ρ is suddenly inserted**, Eq. (1a) for this state can be written as

$$\frac{dn(t)}{dt} = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \frac{\beta_{eff}}{\Lambda} n_0 \quad \dots\dots\dots (10)$$

Extrapolation Method (Contd.)

$$\frac{dn(t)}{dt} = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \frac{\beta_{eff}}{\Lambda} n_0 \quad \text{..... (10)}$$

- One can easily solve Eq. (10) using the initial condition that $n_0 = n_0$ as

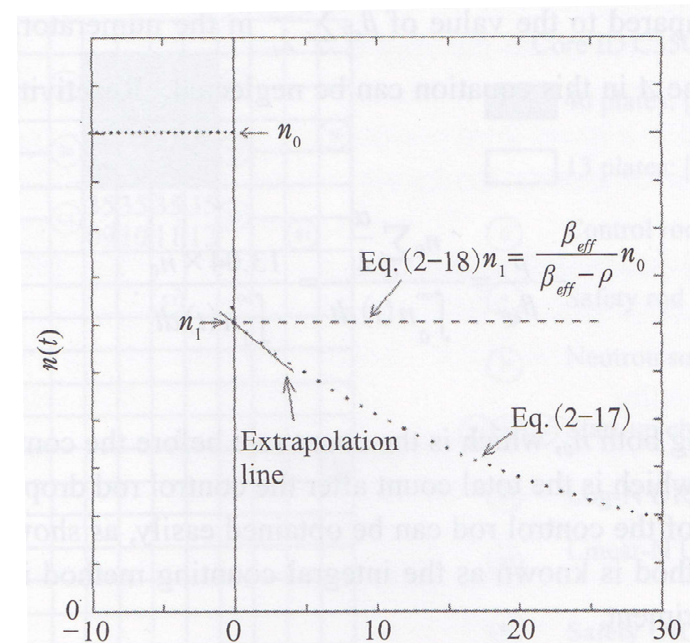
$$n(t) = \frac{\beta_{eff}}{\beta_{eff} - \rho} n_0 - \frac{\rho n_0}{\beta_{eff} - \rho} \exp\left(-\frac{\beta_{eff} - \rho}{\Lambda} t\right) \quad \text{..... (11)}$$

- Because the second term in the right side of Eq. (11) decreases immediately, $n(t)$ becomes immediately after the control rod insertion into

$$n_1 = \frac{\beta_{eff}}{\beta_{eff} - \rho} n_0 \quad \text{..... (12)}$$

- Therefore reactivity can be obtained by

$$\rho = \frac{n_1 - n_0}{n_1} \beta_{eff} \quad \text{..... (13)}$$



T. Misawa, H. Unesaki and C. Pyeon, *Nuclear Reactor Physics Experiments*, Kyoto University Press (2010).

Integral Counting Method

- If one solves Eqs. (1a) and (1b) by the Laplace transform method, the following equation is obtained

$$C_{i0} = \frac{\beta_{i,eff}}{\Lambda \lambda_i} n_0 \quad \text{..... (8)}$$

$$N(s) = \frac{\Lambda \left(n_0 + \sum_{i=1}^6 \frac{\lambda_i C_{i0}}{s + \lambda_i} \right)}{s\Lambda - \rho + \sum_{i=1}^6 \left(\frac{\beta_{i,eff} \omega}{s + \lambda_i} \right)} = \frac{n_0 \left(\Lambda + \sum_{i=1}^6 \frac{\beta_{i,eff}}{s + \lambda_i} \right)}{s\Lambda - \rho + \sum_{i=1}^6 \left(\frac{\beta_{i,eff} s}{s + \lambda_i} \right)} \quad \text{..... (14)}$$

- Using the following well-known characteristic of the Laplace transform:

$$\lim_{s \rightarrow 0} N(s) = \lim_{s \rightarrow 0} \int_0^{\infty} e^{-st} n(t) dt = \int_0^{\infty} n(t) dt \quad \text{..... (15)}$$

Eq. (14) can be expressed as

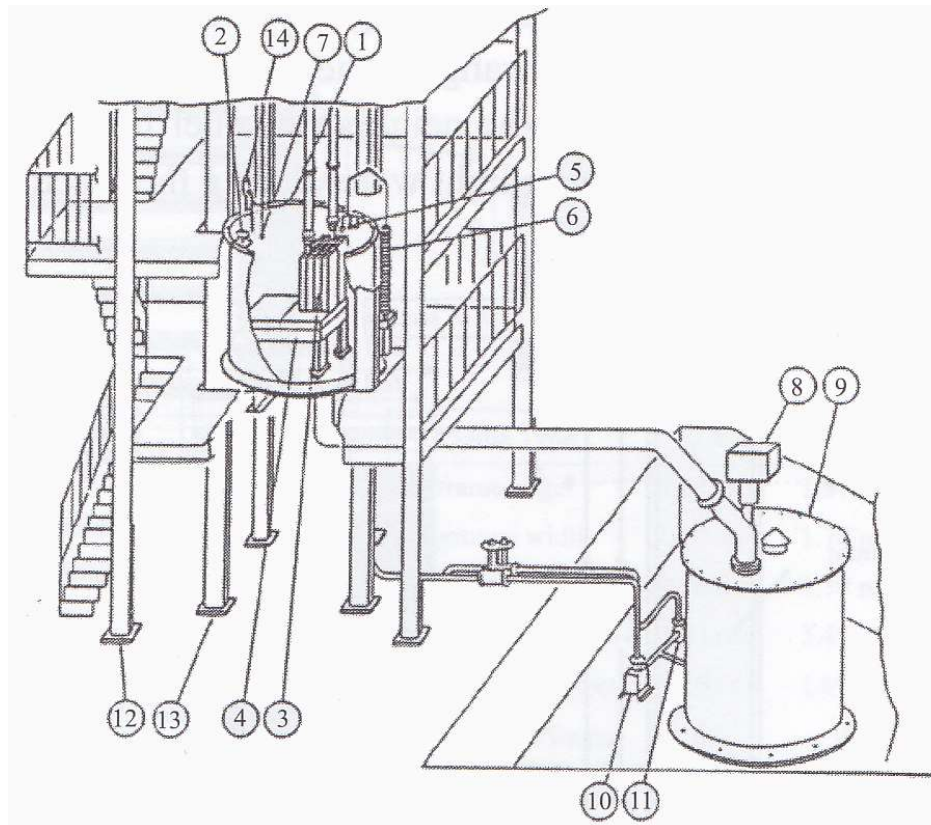
$$\int_0^{\infty} n(t) dt = \lim_{s \rightarrow 0} \frac{n_0 \left(\Lambda + \sum_{i=1}^6 \frac{\beta_{i,eff}}{s + \lambda_i} \right)}{s\Lambda - \rho + \sum_{i=1}^6 \left(\frac{\beta_{i,eff} s}{s + \lambda_i} \right)} = -\frac{n_0}{\rho} \left(\Lambda + \sum_{i=1}^6 \frac{\beta_{i,eff}}{\lambda_i} \right) \quad \text{..... (16)}$$

**Positive Period Method Experiment
in Kyoto Univ. Critical Assembly**



ITRC Experiments for KUCA C-core with D₂O Tank

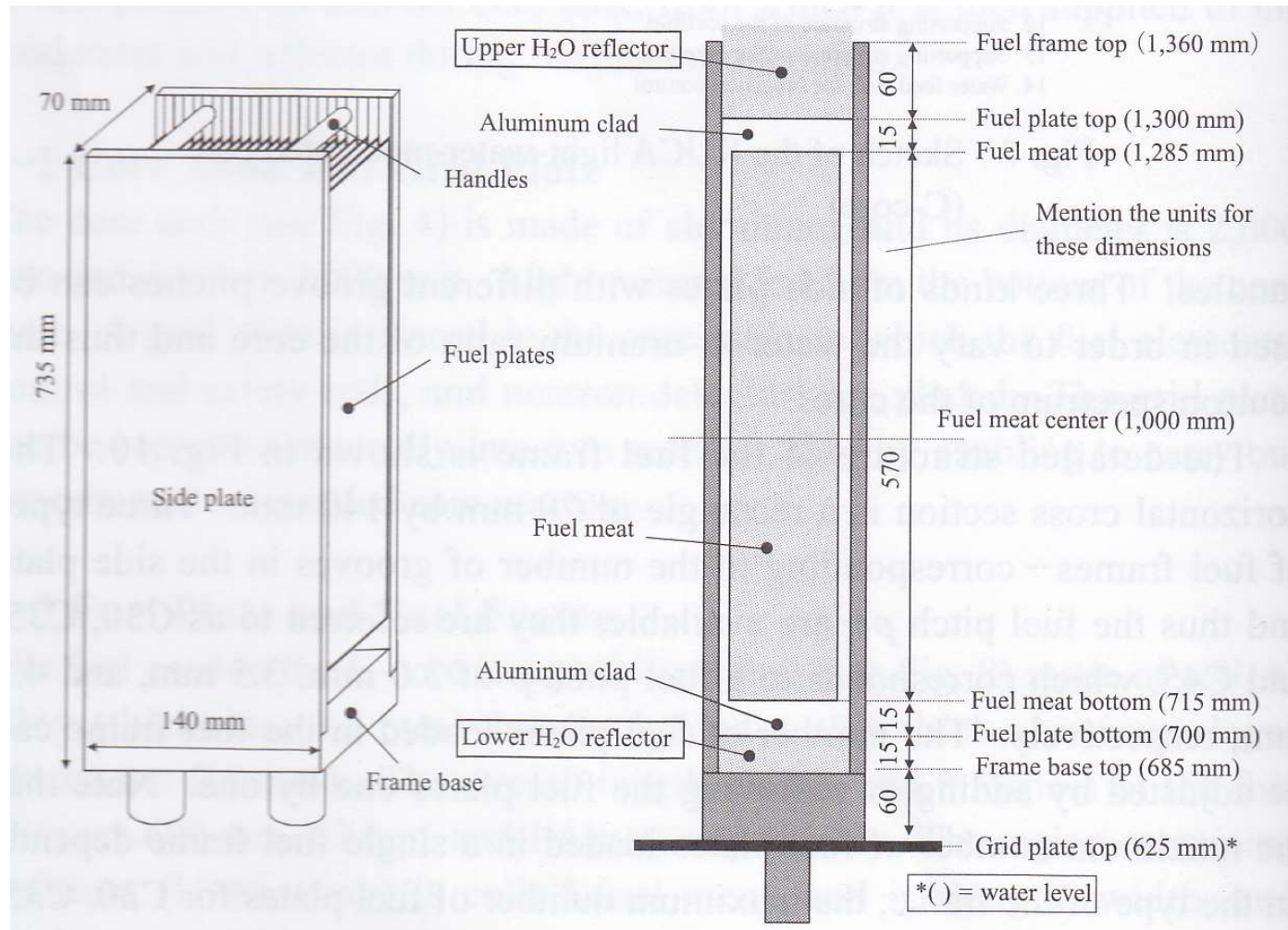
- Experiment for C-core w/ D2O Tank: Aug. 28-31, 2012
Experiment for C-core w/o D2O Tank: Jan. 14-18, 2013
- Reactor: Kyoto University Critical Assembly of Kyoto University Research Reactor Institute
- Participants: C. H. Pyeon, H. J. Shim, S. H. Choi, B. K. Jeon, E. H. Ryu



1. Core tank
2. Handl for the core separation mechanism
3. Core assembly
4. Core separation mechanism
5. Float switch to detect the core overflow
6. Overflow tube
7. Water level switch
8. Dump valve
9. Dump tank
10. High-flow feed pump
11. Low-flow feed pump
12. Supporting structure of the scaffold
13. Supporting structure of the core tank
14. Water feed tank for brecision control

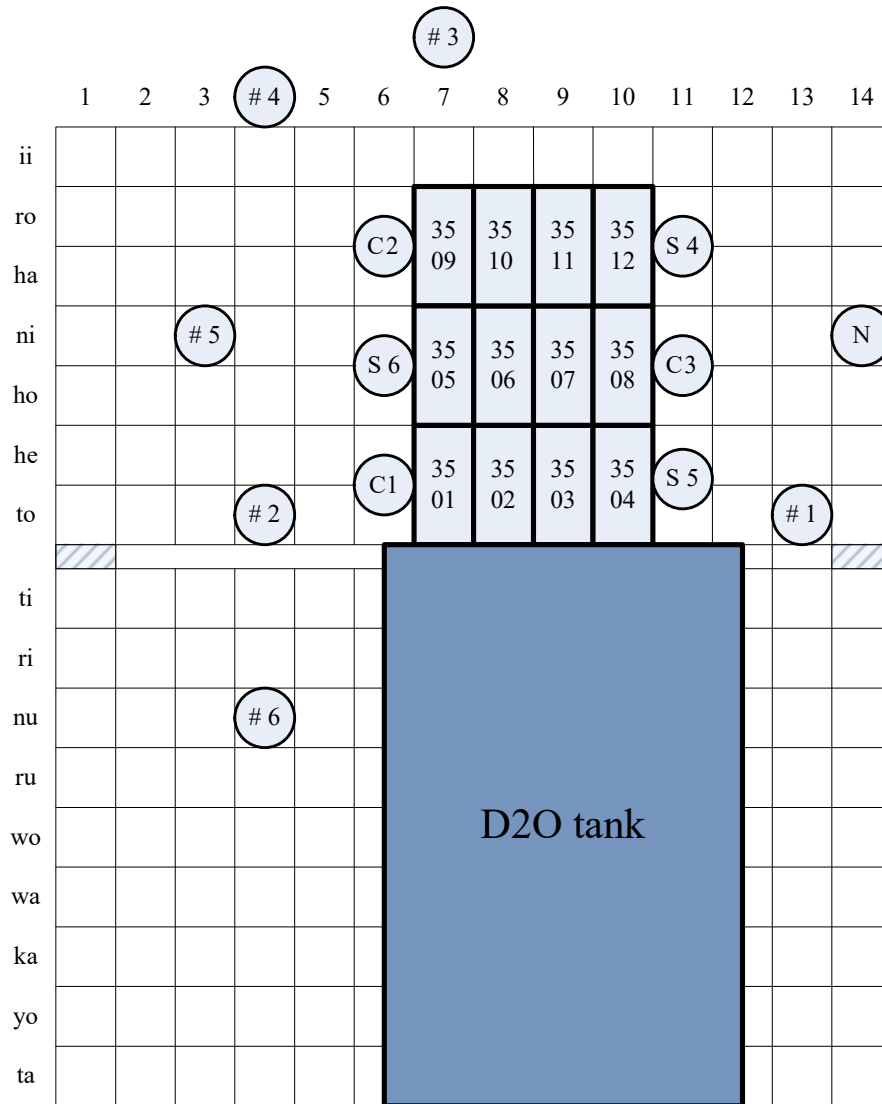
T. Misawa, H. Unesaki and C. Pyeon,
Nuclear Reactor Physics Experiments, Kyoto
University Press (2010).

Fuel Frame



T. Misawa, H. Unesaki
and C. Pyeon, *Nuclear
Reactor Physics
Experiments*, Kyoto
University Press (2010).

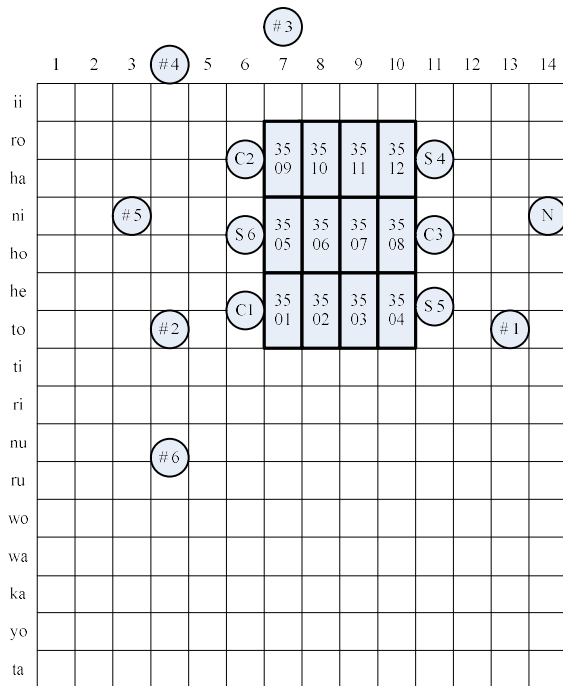
C35R80D2O Configuration



Frame	Num. of plates
3501~3508	40
3509~3512	16
Total	384

- (C) Control rod
- (#5) Linear-N UIC #5
- (S) Safety rod
- (#6) Safety UIC #6
- (N) Neutron source(Am-Be)
- [Blue Box] D2O tank
- (#n) Start-up channels FC (n=1 to 3)
- [Hatched Box] 26.2mm spacer
- (#4) Log-N UIC #4

Core Configurations

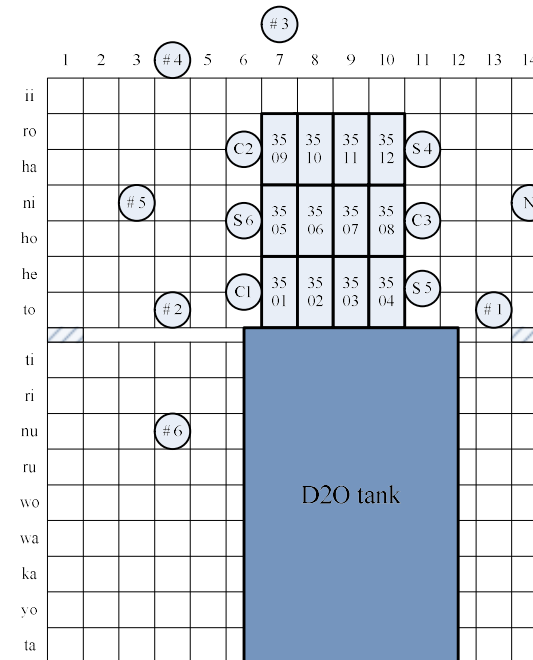


- Control rod
- Safety rod
- Neutron source(Am-Be)
- Start-up channels FC (n=1 to 3)
- Log-N UIC #4
- Linear-N UIC #5
- Safety UIC #6
- D2O tank
- 26.2mm spacer

C35G0(4)

(Light water moderated)

Fuel Frame	Num. of plates
3501~3508	40
3509~3512	27
Total	428

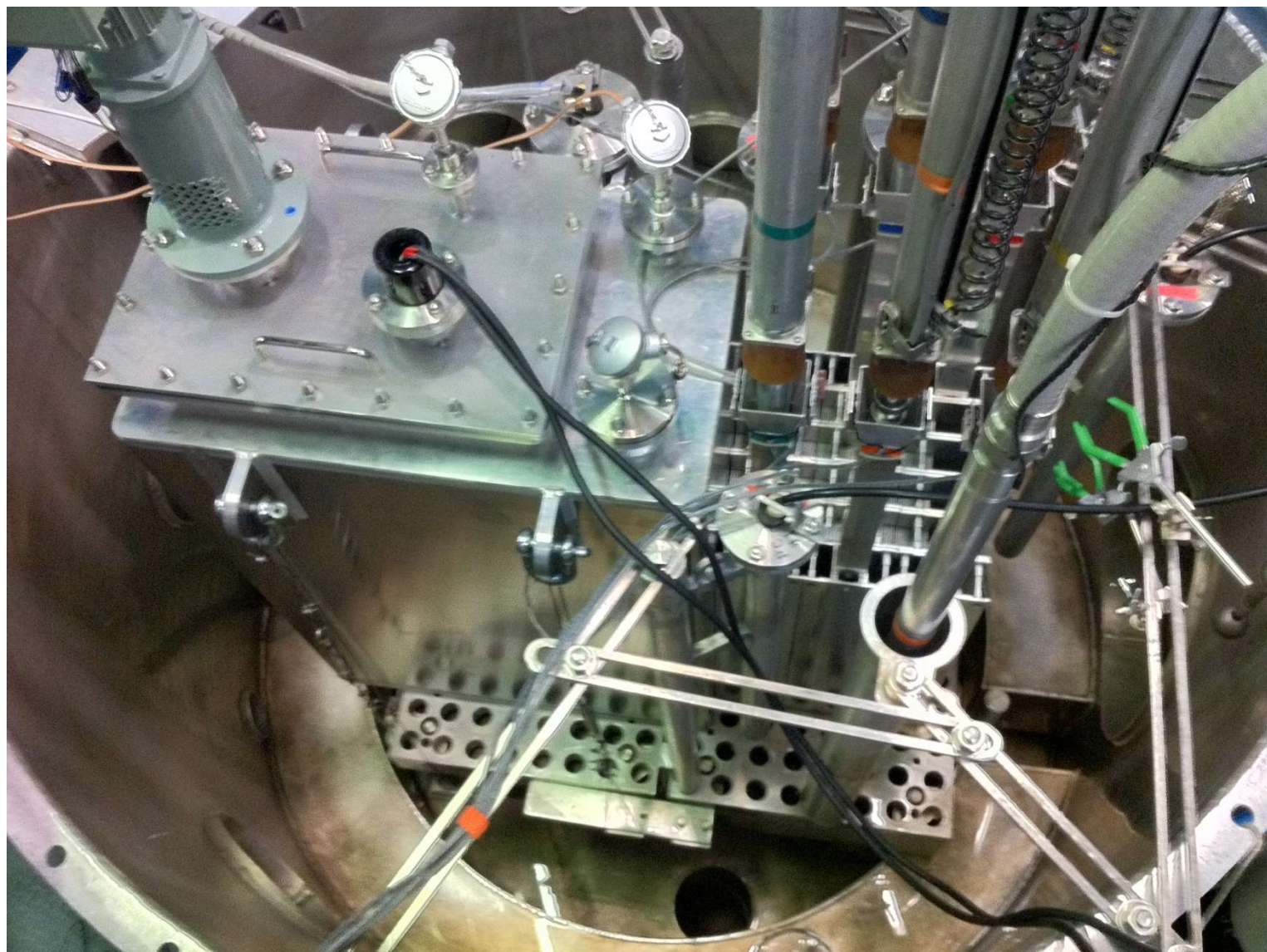


C35R80D2O(4)

(Light water moderated with D₂O tank)

Fuel Frame	Num. of plates
3501~3508	40
3509~3512	16
Total	384

C35R80D2O Layout



Procedure of Experiments

- Conditions: The experiments were conducted in the power-level less than 0.01 W.
- Procedures
 - ① The desired temperature of light water is mainly achieved in the dump tank by heaters or radiator. Then the heated or cool-downed water is fed into the core tank.
 - ② The criticality was adjusted by using the C3 rod in the condition that the other control rods and all the safety rods were withdrawn to the upper limit at the desired temperature condition:

< C35R80D2O(4) >

Case	CT 5 ^{a)}	CT 6 ^{a)}	DT center ^{b)}	DT upper ^{b)}
27	26.7	26.7	27.3	27.5
38	38.1	38.1	38.0	38.0
40	40.1	40.2	39.9	40.2
45	44.9	45.0	44.9	45.1
50	50.4	50.4	50.1	50.3
55	55.2	55.1	54.6	54.8
58	58.5	58.4	58.2	58.6

< C35G0(4) >

Case	CT 5 ^{a)}	CT 6 ^{a)}
19	19.3	19.2
27	27.0	27.0
30	30.1	30.1
40	39.4	39.4
45	45.3	45.2
50	49.2	49.2
55	53.2	53.3
60	61.0	61.0

- a) The CT 5 and 6 are the light water temperatures of lower and upper positions, respectively.
 b) The DT center and upper are temperatures of the heavy water in the D₂O tank.

- ③ The reactivity worth of the inserted rod was measured by the **positive period method**.

Positive Period Method

- The reactivity worth of the inserted C3 rod was measured by withdrawing the rod and timing the doubling time. Then from the double time t_2 , the reactor period t_e can be calculated by

$$t_e = t_2 / \ln 2$$



Positive Period Method Exp. in KUCA (Cont.)

- From a measured doubling time T_2 , the e -folding time can be calculated by

$$e^{\omega T_2} = 2$$

$$\Rightarrow \omega = \frac{\ln 2}{T_2}$$

$$\Rightarrow T = \frac{T_2}{\ln 2}$$

KUCA反応度計算シート

炉心名称 _____

日付 _____ Run. No. _____

$\beta_{\text{eff}} = 7.611 \times 10^{-3}$ $\ell = 4.92 \times 10^{-5}$

i	a_i	λ_i (sec ⁻¹)	$\frac{a_i}{1 + \lambda_i T}$ ($T = 66.89$ sec)
1	0.033	0.0124	0.0180
2	0.219	0.0305	0.0720
3	0.196	0.111	0.0233
4	0.395	0.301	0.0187
5	0.115	1.14	0.00149
6	0.042	3.01	0.000208
$\sum_{i=1}^6 \frac{a_i}{1 + \lambda_i T}$			0.1337
$\sum_{i=1}^6 \frac{a_i}{1 + \lambda_i T} \times \beta_{\text{eff}}$			0.00101757
$\frac{\ell}{T + \ell}$			7.35×10^{-7}
$\frac{T}{T + \ell}$			
$\frac{\ell}{T + \ell} + \frac{T}{T + \ell} \left(\sum_{i=1}^6 \frac{a_i}{1 + \lambda_i T} \times \beta_{\text{eff}} \right)$ ($\Delta k/k$)			1.018×10^{-3}

ITRC Estimation

- From the excess reactivities measured at the different temperatures T_1 and T_2 , the ITRC at the mid-temperature point, α_{iso} , can be directly calculated by

$$\alpha_{\text{iso}} \left(\frac{T_1 + T_2}{2} \right) \cong \frac{\rho_{\text{ex}}(T_2) - \rho_{\text{ex}}(T_1)}{T_2 - T_1}$$

- To enhance the smoothness of the ITRC estimations, the excess reactivity can be fitted to a quadratic curve [1,2]:

$$\rho_{\text{ex}}(T) \cong aT^2 + bT + c$$

where a , b , and c are fitting constants.

- Then α_{iso} can be determined by

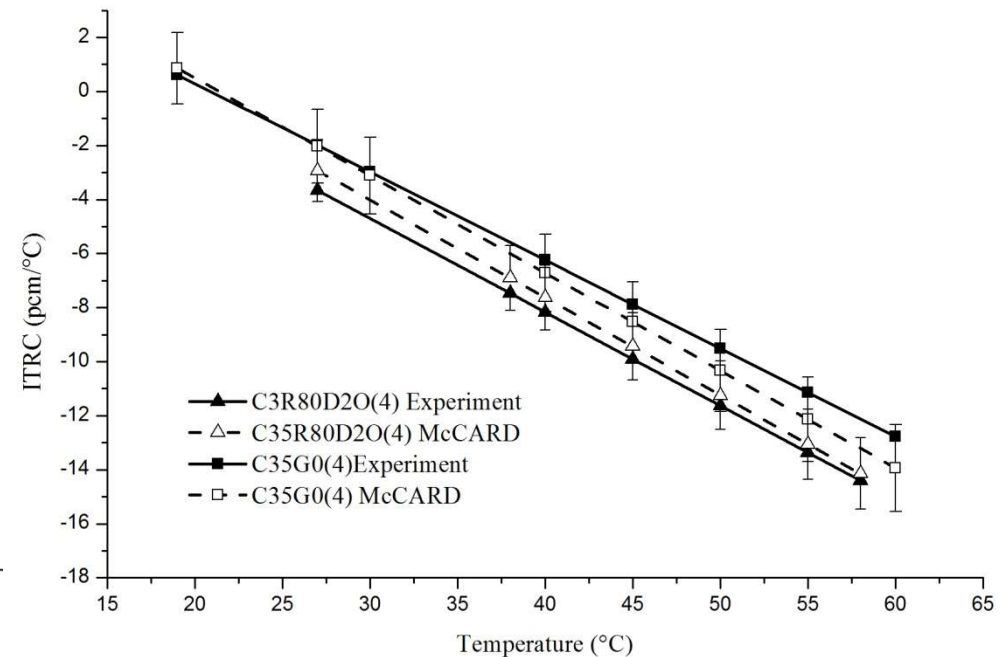
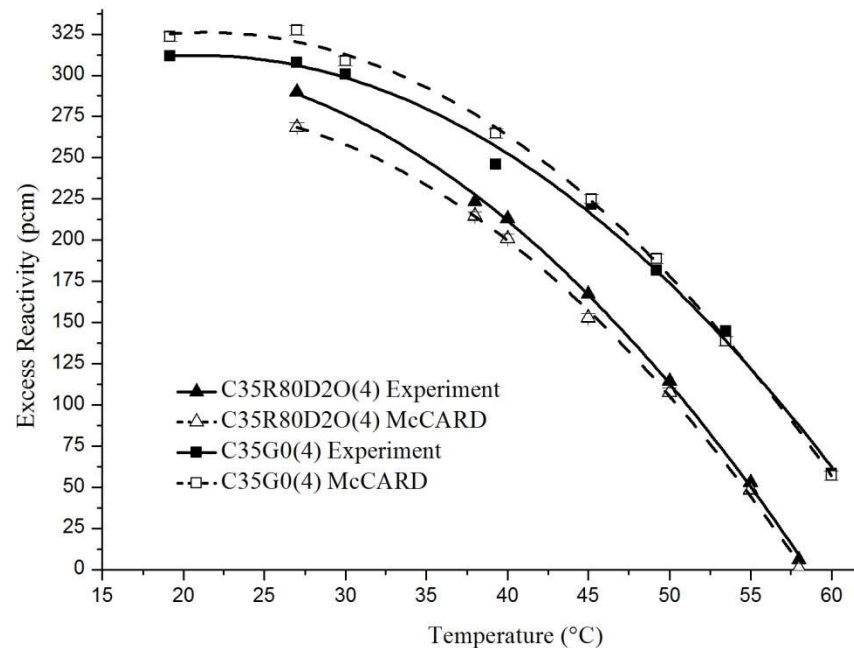
$$\alpha_{\text{iso}}(T) \cong 2aT + b$$

[1] M. Mori, S. Shiroya and K. Kanda, "Temperature Coefficient of Reactivity in Light-Water Moderated and Reflected Cores Loaded with Highly-Enriched-Uranium Fuel," *J. Nucl. Sci. Technol.*, **24**[8], 653, 1987.

[2] S. Shiroya, *et al.*, "Experimental Study on Temperature Coefficient of Reactivity in Light-Water-Moderated and Heavy-Water-Reflected Cylindrical Core Loaded with Highly-Enriched-Uranium or Medium-Enriched-Uranium Fuel," *J. Nucl. Sci. Technol.*, **32**[11], 1081, 1995.

Comparison of ITRCs

- Comparison of the excess reactivities and ITRC for the two core configurations



- ✓ The quadratic polynomial is observed fairly well fit in the excess reactivity according to the system temperature.
- ✓ The ITRCs calculated from $\rho_{\text{ex}}^{\text{McCARD}}$ agree with those from $\rho_{\text{ex}}^{\text{exp.}}$ within 95% confidence intervals.

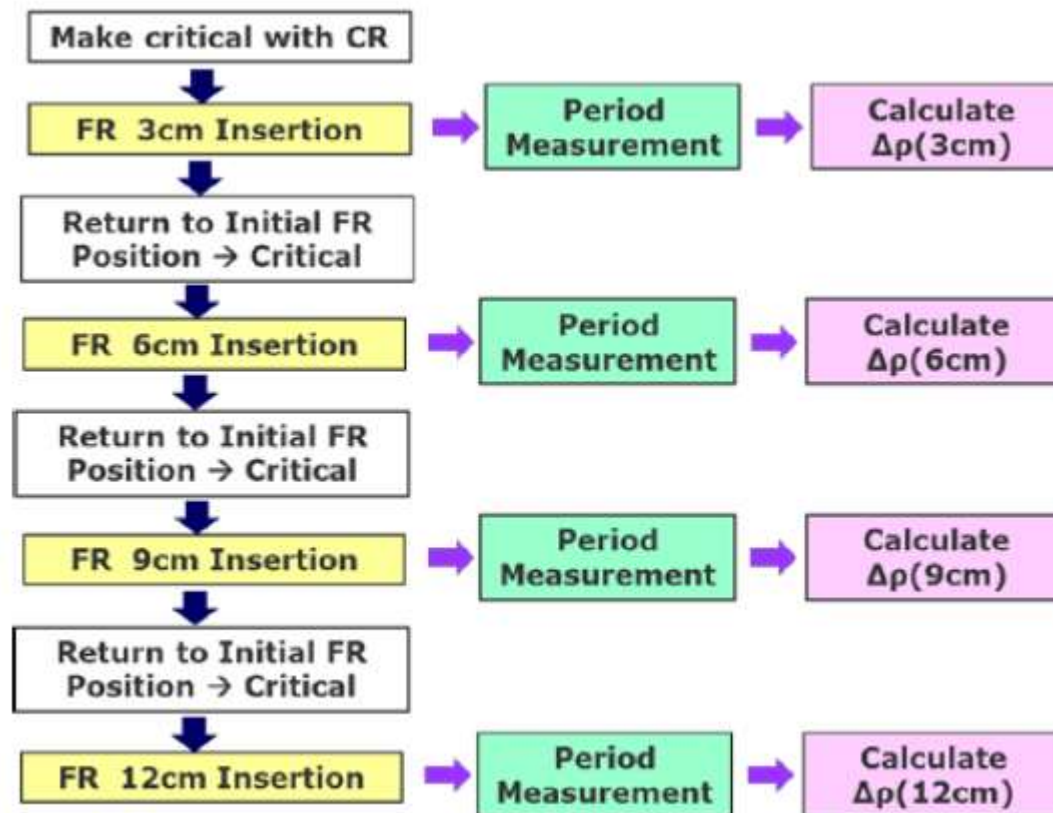
Control Rod Worth Measurement at AGN-201K



Method #1: Positive Period Method

- For the method #1 of positive period method, a fine control rod (FR) is inserted from 0 to the certain level as shown in the following figure at each step.

What was the critical mass of ^{235}U [g]?



Myung Hyun Kim, Reactor Experiment, Reactor Research & Education Center, Kyung Hee University (2018).

Method #1: Procedure of Positive Period Method

1. Student operator should read all reactor conditions and write down at the worksheet first. They are temperatures at various locations, source position, neutron and gamma radiation dose in the reactor hall. Once steady state condition is confirmed, he should be ready to measure the time for power increase by **1.5 times**.
2. SRO (as experiment instructor) asked a student commander to remind the whole procedures.
3. According to the request of student commander, RO is ready to move FR as his/her request. Then RO move the FR quickly in order to simulate step-wise movement. When he finished, he gives sign of start.
4. After the time of start sign, each student should measure twice the time of power increase by 1.5 times. Then he gives a sign of finish to commander. During this period commander should read period and reactivity from DMC and DDRCS.
5. When a commander recognize all measurement was done twice, he gives order to **power decrease with FR and CR to make a reactor to the original steady state**. The positions of FR and CR should be the same as the initial condition. Power level should be the same with the initial condition.
6. The procedure mentioned above should be repeated until FR is inserted upto the full length.

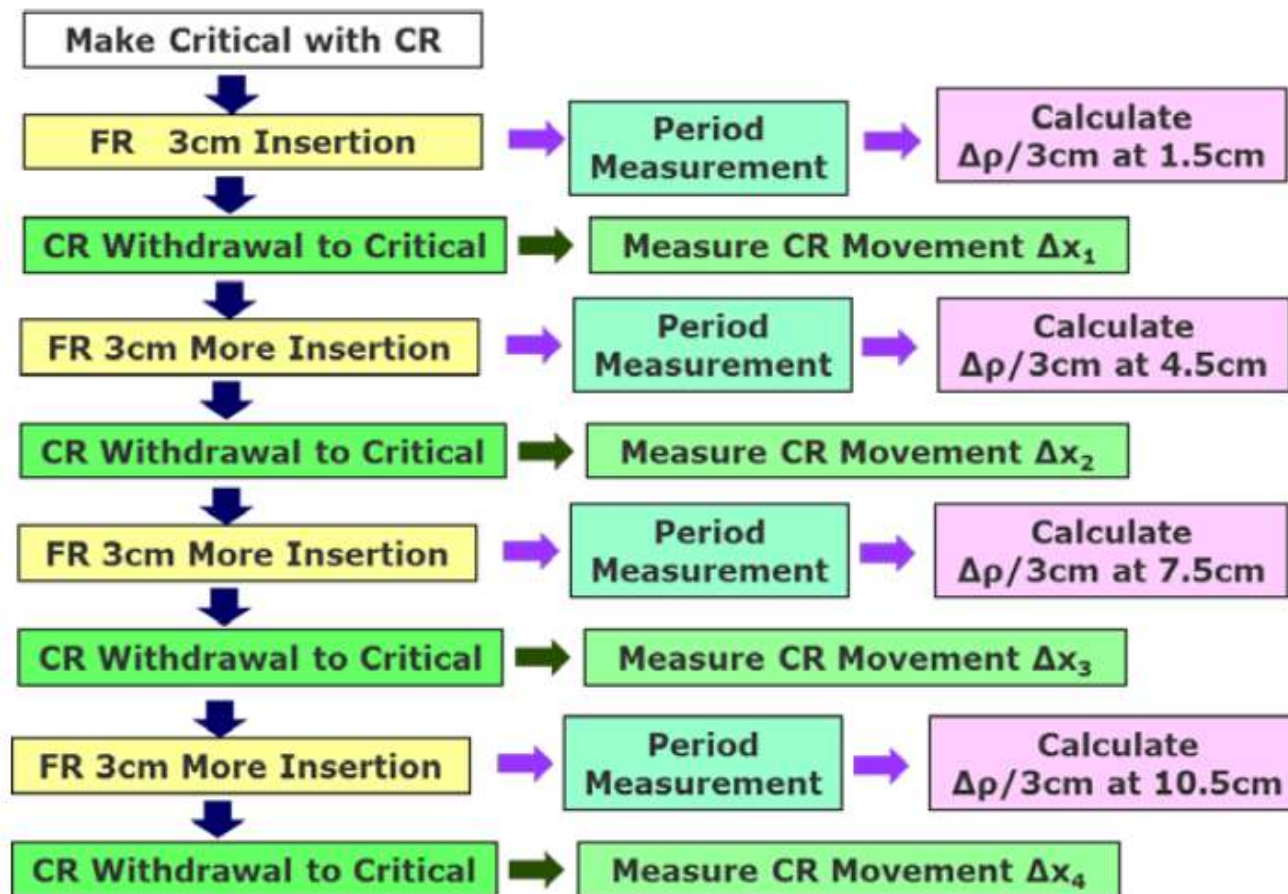
Worksheet of Method 1

<1> Positive Period Method

#	Time	Rod Position		Channel #__		Channel #__		Reactivity Record from DDRCS
		CR	FR	Time for Increase by 1.5 Times	Period $T = \frac{\Delta t}{\ln 1.5}$	Time for Increase by 1.5 Times	Period $T = \frac{\Delta t}{\ln 1.5}$	
Initial Criticality			0	-	-	-	-	
1	X	"	4cm	1)		1)		
				2)		2)		
				Avg.=		Avg.=		
Recriticality			0	-	-	-	-	
2	X	"	7cm	1)		1)		
				2)		2)		
				Avg.=		Avg.=		
Recriticality			0	-	-	-	-	
3	X	"	10cm	1)		1)		
				2)		2)		
				Avg.=		Avg.=		
Recriticality			0	-	-	-	-	

Method #2: Compensation Method

- For the method #2 of compensation method, a FR is inserted by certain distance at every step after achieving the critical state.



Myung Hyun Kim, Reactor Experiment, Reactor Research & Education Center, Kyung Hee University (2018).

Method #2: Procedure of Compensation Method

1. Student operator should read all reactor conditions and write down at the worksheet first. They are temperatures at various locations, source position, neutron and gamma radiation dose in the reactor hall. Once steady state condition is confirmed, he should be ready to measure the time for power increase by 1.5 times.
2. SRO (as experiment instructor) asked a student commander to remind the whole procedures.
3. According to the request of student commander, RO is ready to move FR as his (her) request. Then RO move the FR quickly in order to simulate step-wise movement. When he finished, he gives sign of start.
4. After the time of start sign, each student should measure twice the time of power increase by 1.5 times. Then he gives a sign of finish to commander. During this period commander should read period and reactivity from DMC and DDRCS.
5. When a commander recognize all measurement was done twice, he gives order to **power decrease with only CR to make a reactor to the original steady state**. The positions of FR should not be changed after movement but the position of CR should be changed to make reactor critical at the initial steady-state condition.
6. The procedure mentioned above should be repeated until FR is inserted upto the full length.

Worksheet of Method 2

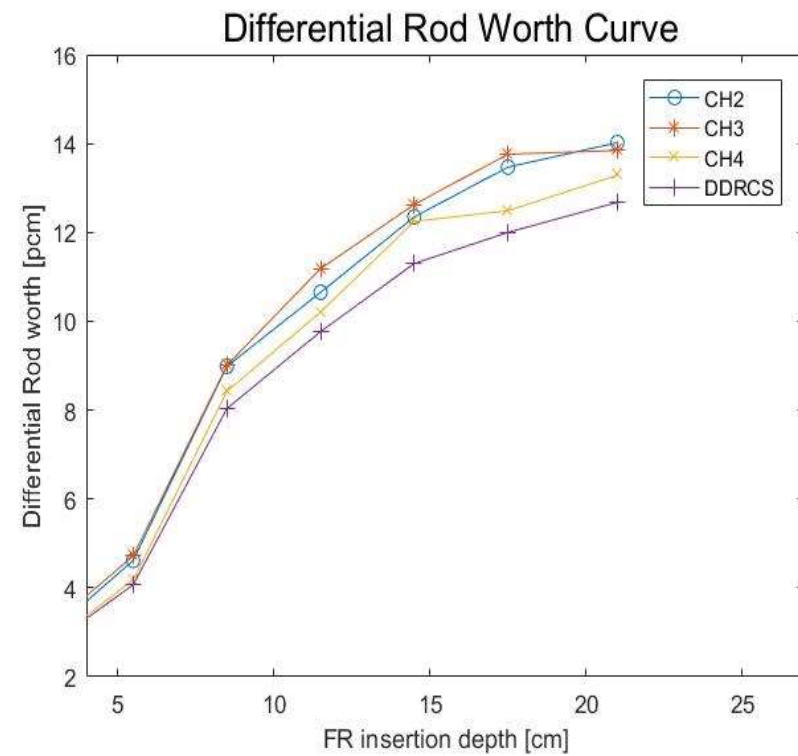
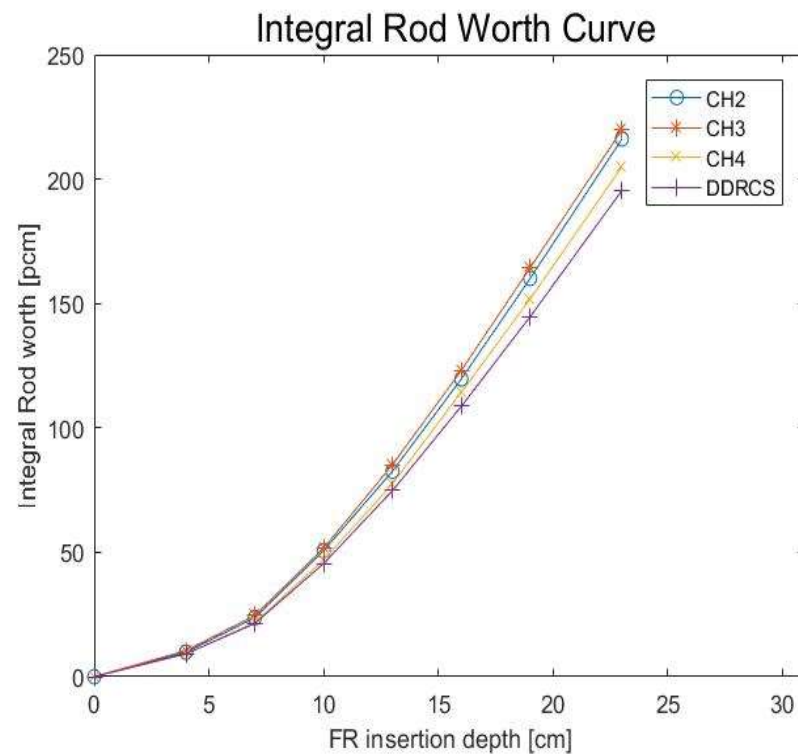
<2> Compensation Method

#	Time	Rod Position		Channel #__		Channel #__		Reactivity Record from DDRCS
		CR	FR	Time for Increase by 1.5 Times	Period $T = \frac{\Delta t}{\ln 1.5}$	Time for Increase by 1.5 Times	Period $T = \frac{\Delta t}{\ln 1.5}$	
Initial Criticality			0	-	-	-	-	
1	X	"	4cm	1)		1)		
				2)		2)		
				Avg.=		Avg.=		
Recriticality		↓	"	-	-	-	-	
2	X	"	7cm	1)		1)		
				2)		2)		
				Avg.=		Avg.=		
Recriticality		↓	"	-	-	-	-	
3	X	"	10cm	1)		1)		
				2)		2)		
				Avg.=		Avg.=		
Recriticality		↓	"	-	-	-	-	

Example of Experimental Results

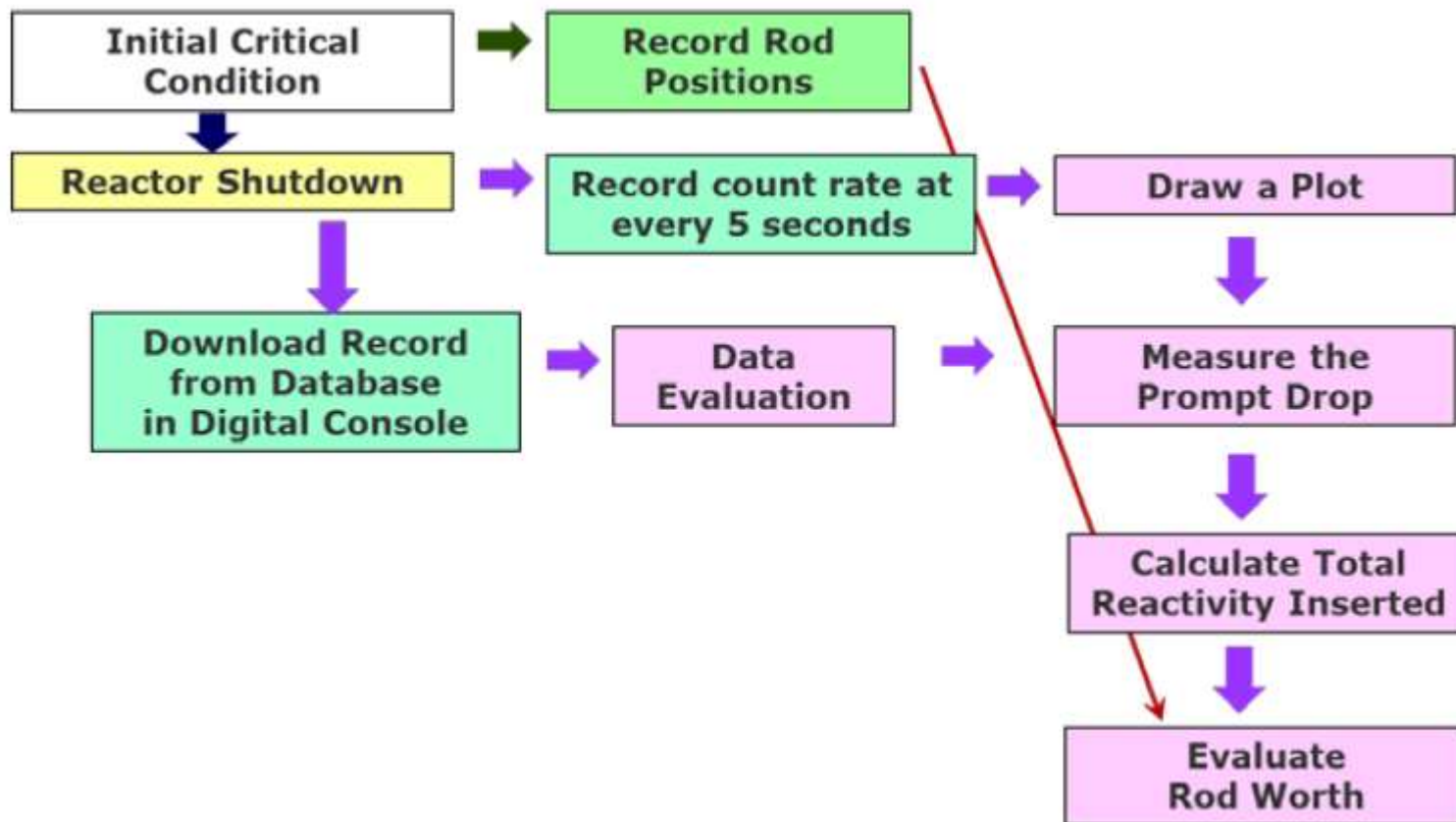
#	Time	Rod Position		Channel 2						Channel 3						Channel 4						DDRCs					
		CR	FR	분	초	초 환산	T	Reactivity (pcm)	#REF!	#REF!	분	초	초 환산	T	Reactivity (pcm)	#REF!	#REF!	분	초	초 환산	T	Reactivity (pcm)	#REF!	#REF!	Reactivity (pcm)		
Initial Criticality	14:47	21.87	0																					0	0		
1		21.87	4	5	39.00	339		#REF!	#REF!	5	3.00	303		#REF!	#REF!	5	55	355		#REF!	#REF!						
				5	27.50	327.5				5	3.00	303				5	54	354									
				5	21.00	321				5	20.00	320															
				Avg=	329.1667	811.82		0.00010		Avg=	308.67	761.27		0.0001		Avg=	354.5	874.305		0.00009						9.2	9.2
Recriticality	15:08	21.67	4																								
2		21.67	7	3	50.00	230		#REF!	#REF!	3	37.00	217		#REF!	#REF!	4	13	253		#REF!	#REF!						
				3	50.00	230				3	41.50	221.5				4	18	258									
				3	50.00	230				3	56.00	236															
				Avg=	230	567.25		0.000138		Avg=	224.83	554.51		0.00014		Avg=	255.5	630.141		0.000125						12.2	21.4
Recriticality	15:28	21.47	7																								
3		21.47	10	1	54.50	114.5		#REF!	#REF!	1	50.00	110		#REF!	#REF!	1	59.5	119.5		#REF!	#REF!						
				1	53.50	113.5				1	51.00	111				2	0.5	120.5									
				1	47.00	107				1	53.00	113															
				Avg=	111.6667	275.404		0.000269		Avg=	111.33	274.58		0.00027		Avg=	120	295.956		0.000252						24.1	45.5
Recriticality	15:36	21.05	10																								
4		21.05	13	1	33.50	93.5		#REF!	#REF!	1	29.00	89		#REF!	#REF!	1	39	99		#REF!	#REF!						
				1	32.00	92				1	29.00	89				1	34.5	94.5									
				1	31.00	91				1	23.50	83.5															
				Avg=	92.16667	227.311		0.000319		Avg=	87.167	214.98		0.00034		Avg=	96.75	238.615		0.000306						29.3	74.8
Recriticality	15:47	20.58	13																								
5		20.58	16	1	15.50	75.5		#REF!	#REF!	1	16.00	76		#REF!	#REF!	1	18.5	78.5		#REF!	#REF!						
				1	20.00	80				1	15.50	75.5				1	18.5	78.5									
				1	18.50	78.5				1	15.00	75						0									
				Avg=	77.75	191.755		0.00037		Avg=	75.75	186.82		0.00038		Avg=	78.5	193.605		0.000367						33.9	108.7
Recriticality	15:56	20.05	16																								
6		20.05	19	1	11.00	71		#REF!	#REF!	1	7.50	67.5		#REF!	#REF!	1	18.5	78.5		#REF!	#REF!						
				1	9.50	69.5				1	9.50	69.5				1	15	75									
				1	10.50	70.5				1	8.50																
				Avg=	70.25	173.258		0.000403		Avg=	68.5	168.94		0.00041		Avg=	76.75	189.289		0.000374						36	144.7
Recriticality	16:04	19.48	19																								
7		19.48	23		47.50	47.5		#REF!	#REF!		49.00	49		#REF!	#REF!		50	50		#REF!	#REF!						
					47.00	47					47.00	47					51	51									
					47.50	47.5					46.00																
				Avg=	47.25	116.533		0.00056		Avg=	48	118.38		0.00055		Avg=	50.5	124.548		0.000531						50.7	195.4

Example of Rod Worth Graphs



Method #3: Rod Drop Method

- Reactor is scrammed at the time of zero. All students should read and record at every 5 seconds. General procedure is shown in the following figure.



Myung Hyun Kim, Reactor Experiment, Reactor Research & Education Center, Kyung Hee University (2018).

Worksheet of Method 3

<3> Rod Drop Experiment

#	Time (sec)	Rod Position		Channel #__	Channel #__
		CR	FR		
Initial Criticality					
1	Rx Shutdown n0				
2	5	0 cm	0 cm		
3	10	0 cm	0 cm		
4	15	0 cm	0 cm		
5	20	0 cm	0 cm		
6	25	0 cm	0 cm		
7	30	0 cm	0 cm		
8	35	0 cm	0 cm		
9	40	0 cm	0 cm		
10	45	0 cm	0 cm		
11	50	0 cm	0 cm		
12	55	0 cm	0 cm		
13	60	0 cm	0 cm		
14	65	0 cm	0 cm		
15	70	0 cm	0 cm		

$$\rho = \frac{n_1 - n_0}{n_1} \beta_{eff}$$

$$\int_0^{\infty} n(t) dt = -\frac{n_0}{\rho} \left(\Lambda + \sum_{i=1}^6 \frac{\beta_{i,eff}}{\lambda_i} \right)$$