$$\frac{d}{dt}B_{sys} = \boxed{\frac{d}{dt}\int_{CV} \beta\rho \, d\forall} + \boxed{\int_{CS} \beta\rho \,(\underline{V}_r \cdot \underline{n})\, dA}.$$

(RTT)

CV: Control volume

CS: Control surface.

flux.

$$\beta \equiv \frac{dB}{dm}.$$

3.3. Conservation of mass ($B = m$, $\beta = 1$)

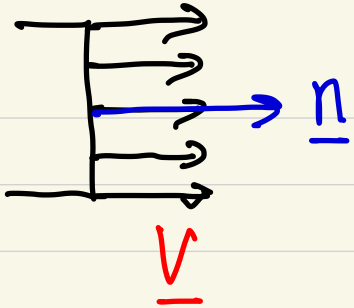
$$\left(\frac{dm}{dt}\right)_{sys} = \boxed{\frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho\,(\underline{V}_r \cdot \underline{n})\, dA = 0}$$

상대속도

for a fixed CV,

$$\int_{CV} \frac{\partial \rho}{\partial t}\, d\forall + \boxed{\int_{CS} \rho\,(\underline{V} \cdot \underline{n})\, dA} = 0.$$

for 1D inlet/outlet to CV. ($\underline{V} \,//\, \underline{n}$, uniform flow)




$$\int_{CV} \frac{\partial \rho}{\partial t} \, d\forall + \sum_{i} (\rho_i V_i A_i)_{out} - \sum_{i} (\rho_i V_i A_i)_{in}$$

# of outlet

# of inlet

for steady flow, $\frac{\partial}{\partial t} \simeq 0$.

$$\int_{CS} \rho (\underline{V_r} \cdot \underline{n}) \, dA = 0 \quad \text{or} \quad \sum_i (\rho_i V_i A_i)_{out} = \sum_i (\rho_i V_i A_i)_{in}$$

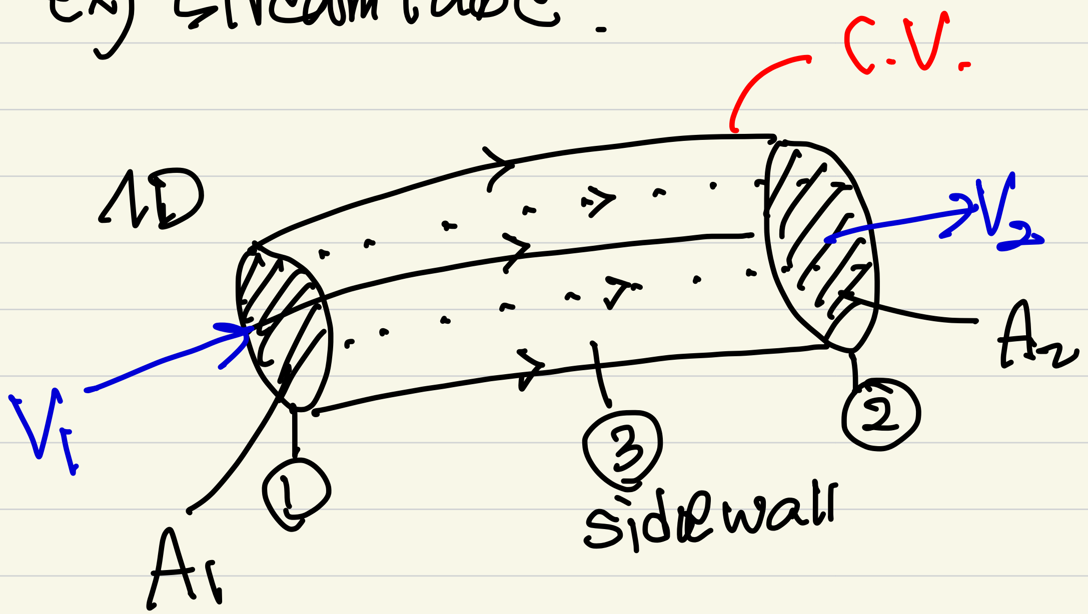
+ incompressible flow ($\rho$ = constant)

$$: \int_{CS} (\underline{V_r} \cdot \underline{n}) \, dA = 0. \quad \text{or} \quad \sum_i (V_i A_i)_{out} = \sum_i (V_i A_i)_{in}$$

$$\Downarrow$$

$$\sum_i Q_{i,out} = \sum_i Q_{i,in}$$

ex) Streamtube.

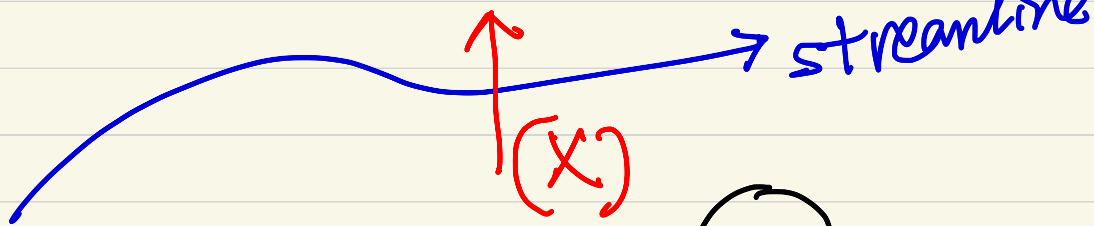



steady, incompressible, fixed

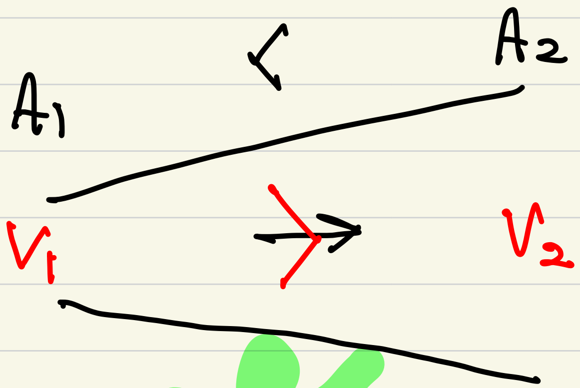
$V_1$ is known, $V_2 = ?$.

$$\int_{CS} \rho (\underline{V} \cdot \underline{n})\, dA = 0.$$

$CS \to 1 + 2 + 3$



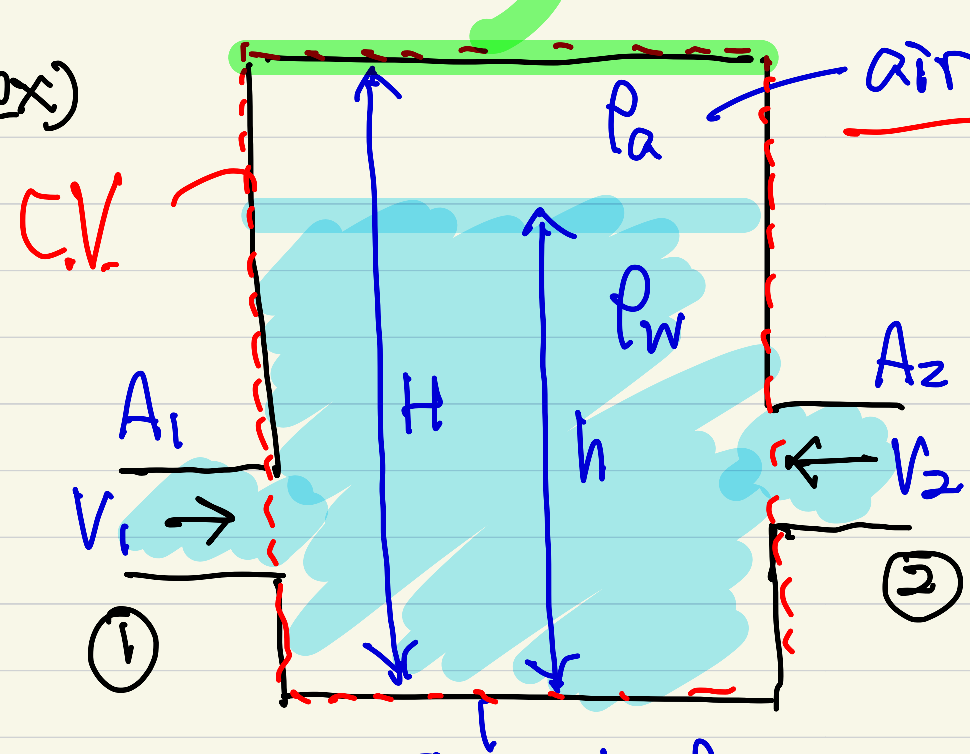

$$\dot{m} = \cancel{\rho_1} V_1 A_1 = \cancel{\rho_2} V_2 A_2 \Rightarrow V_2 = \boxed{\frac{A_1}{A_2}} V_1.$$




mass conservation

= continuity

ex)




$A_t$ : tank area.

air is trapped on top.

$$\frac{dh}{dt} = ?$$

$$\frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho(\underline{V}\cdot\underline{n})\, dA = 0.$$

$$\frac{d}{dt}\left[\rho_w A_t h + \underbrace{\cancel{\rho_a A_t (H-h)}}_{0}\right]$$

$$= \rho_w A_t \frac{dh}{dt}.$$

$$-\rho_1 V_1 A_1 - \rho_2 V_2 A_2 = -\rho_w V_1 A_1 - \rho_w V_2 A_2.$$

$$\therefore \frac{dh}{dt} = \frac{V_1 A_1 + V_2 A_2}{A_t}.$$

· How about the case of opened top ?

$$\underline{\frac{d}{dt}\left[\rho_w A_t h + \rho_a A_t (H-h)\right]} - \rho_w V_1 A_1 - \rho_w V_2 A_2 + \rho_a V_a A_t = 0$$

$$= \rho_w A_t \frac{dh}{dt} - \cancel{\rho_a A_t \frac{dh}{dt}} - \rho_w V_1 A_1 - \rho_w V_2 A_2 + \cancel{\rho_a \frac{dh}{dt} A_t} = 0$$

$$\therefore \frac{dh}{dt} = \frac{A_1 V_1 + A_2 V_2}{A_t}$$

## 3.4. Conservation of (Linear) momentum.

in RTT, let $B \equiv m\underline{V}$, $\beta \equiv \frac{dB}{dm} = \underline{V}$.

$$\frac{d}{dt}(m\underline{V})_{sys} = \boxed{\frac{d}{dt}\int_{CV} \rho \underline{V}\, d\forall + \int_{CS} \rho \underline{V}(\underline{V}_r \cdot \underline{n})\, dA} = \Sigma \underline{F}.$$

① $\underline{V}$ : defined in inertial reference frame (관성좌표계)
→ non-moving.

② $\Sigma \underline{F}$ : body force, surface force.

③ vector relation → three components.

- for a fixed CV. ($\underline{V}_r \equiv \underline{V}$).

$$\Sigma \underline{F} = \frac{d}{dt}\int_{CV} \rho \underline{V}\, d\forall + \int_{CS} \rho \underline{V} (\underline{V} \cdot \underline{n})\, dA.$$

* momentum flux term, $\underline{\dot{M}}$

(analogy to mass flux, $\dot{m}$) → $\int_{CS} \rho (\underline{V} \cdot \underline{n})\, dA.$

$$\underline{\dot{M}}_{CS} = \int_{CS} \rho \underline{V} (\underline{V} \cdot \underline{n})\, dA.$$

$(\underline{V} \cdot \underline{n})$:
- $+$ : outward flow
- $-$ : inward flow.

· for 1D outlet/inlet to CV.

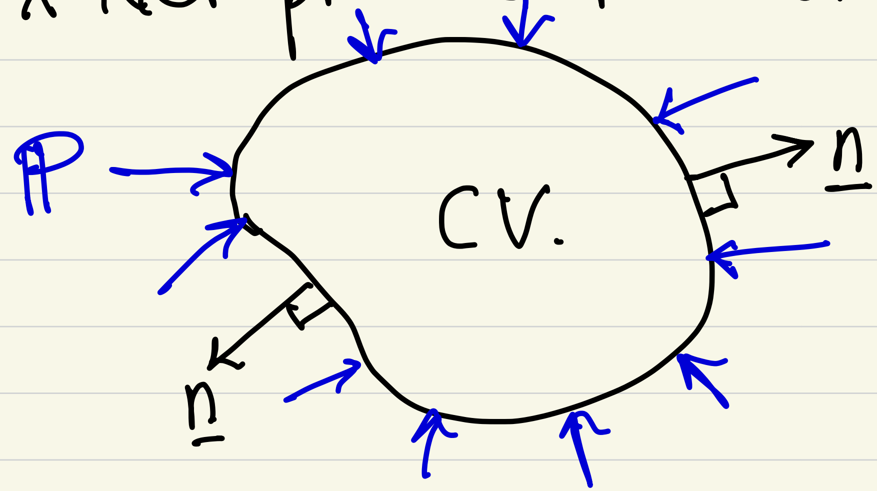
$$\underline{\dot{M}}_{cs} = \rho_i \underline{V}_i (V_{ni} \cdot A_i) = \underline{V}_i (\underbrace{\rho_i V_{ni} A_i}_{\dot{m}_i}) = \underline{V}_i \dot{m}_i$$

$\Downarrow$

mtm conservation eq. becomes

$$\sum \underline{F} = \frac{d}{dt}\int_{CV} \rho \underline{V} \, d\forall + \sum_i (\dot{m}_i \underline{V}_i)_{out} - \sum_i (\dot{m}_i \underline{V}_i)_{in}.$$

* Net pressure force on a closed CS.

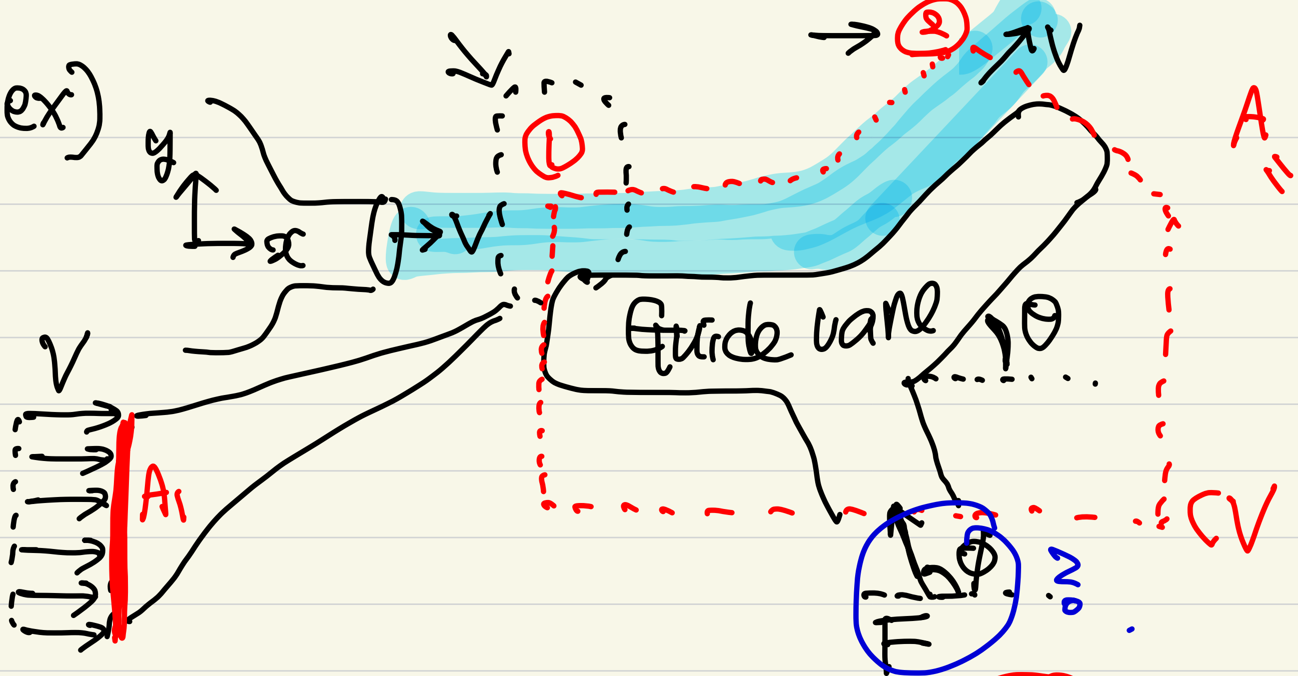



$$\underline{F}_{press} = \int_{CS} P \cdot (-\underline{n}) \, dA = \int_{CS} \underbrace{(P - P_a)}_{\text{gage pressure}} (-\underline{n}) \, dA.$$

$$\left( \because \; \underline{\int_{CS} \mathbb{P} \cdot (-\underline{n})\, dA} - \underline{\int_{CS} P_a \cdot (-\underline{n})\, dA} \right.$$

$$\left. 0 = P_a \int_{CS} (-\underline{n})\, dA = 0 \right)$$

ex)




A

$A_1 = A_2$

steady,

Pa everywhere.

→ V is maintained.

frictionless.

· mass conservation : $\dot{m}_1 = \dot{m}_2 = \rho V A = \dot{m}$

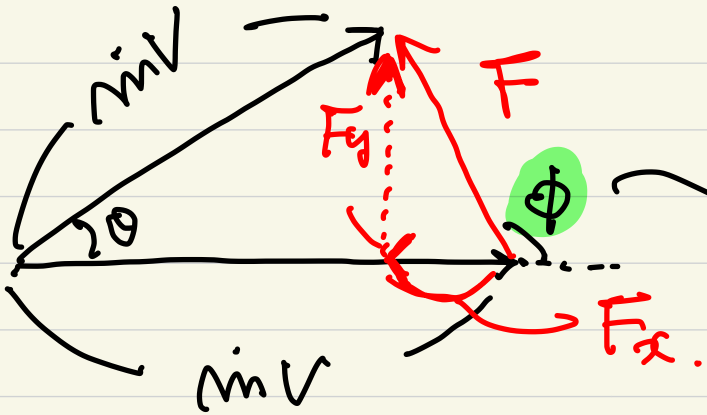
· mtm 〃 : $\underline{\Sigma F} = \cancel{\frac{d}{dt}\int_{CV} \rho \underline{V}\, d\forall}^{\,0} + \underbrace{\int_{CS} \rho \underline{V} (\underline{V}\cdot\underline{n})\, dA}$.

$= -\dot{m}_1 \underline{V_1} + \dot{m}_2 \underline{V_2}$

$\underline{F} = -\dot{m} V \hat{i} + \dot{m} V (\cos\theta \hat{i} + \sin\theta \hat{j})$

$\quad = \dot{m} V (\cos\theta - 1)\hat{i} + \dot{m} V \sin\theta \hat{j}$.

* $F_x = \dot{m}V(\cos\theta - 1)$, $F_y = \dot{m}V\sin\theta$.




$$\rightarrow F = \sqrt{F_x^2 + F_y^2}$$

$$= \dot{m}V\sqrt{\sin^2\theta + (\cos\theta - 1)^2}$$

$$= 2\dot{m}V\sin\frac{\theta}{2}.$$

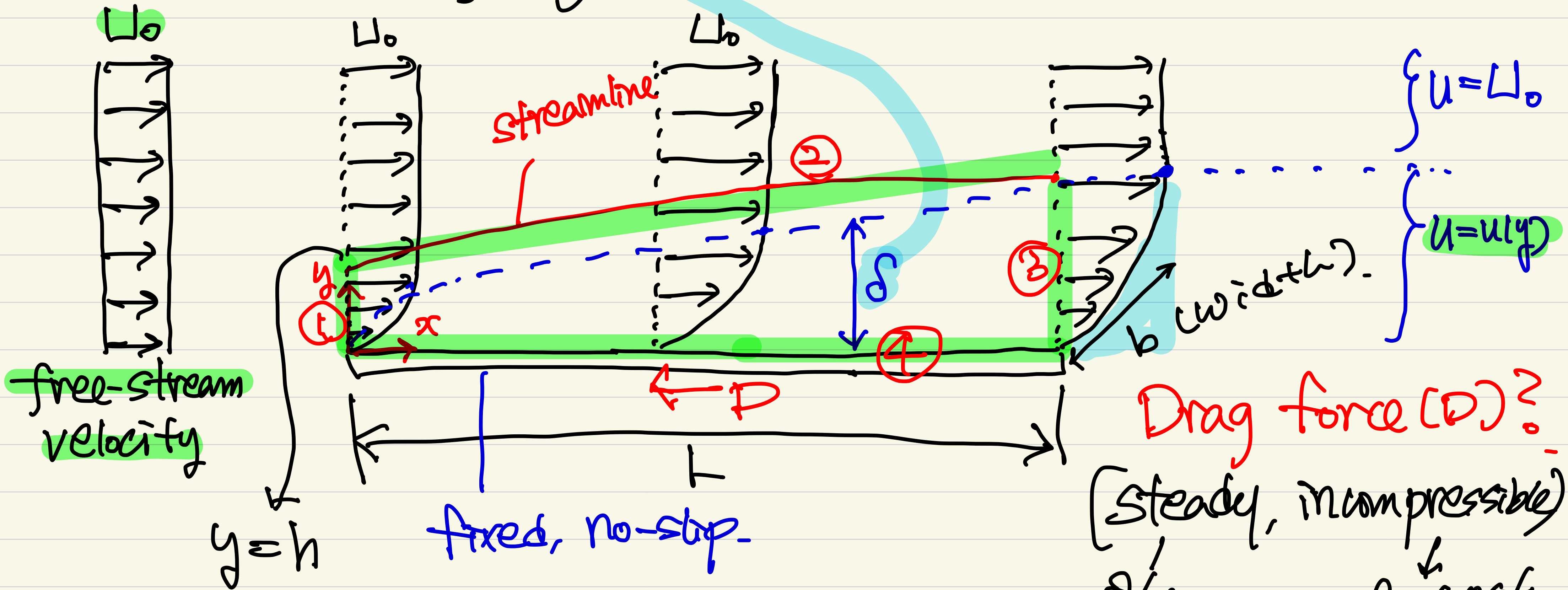
$$\tan(\pi - \phi) = \left|\frac{F_y}{F_x}\right|. \rightarrow \phi = \frac{1}{2}(\pi + \theta).$$

ⓐ $\theta = \pi$. $F = 2\dot{m}V$, $\phi = \pi$.

ⓐ $\theta \ll 1$, $F \simeq 2\dot{m}V \cdot \frac{\theta}{2} = \dot{m}V\theta$. $\phi \simeq \frac{\pi}{2}$. (lift force).

ex) Drag force on a flat plate due to boundary-layer. (경계층. Ch.6.7)

$$\tau = \mu \frac{du}{dy}$$




Drag force (D)?

[steady, incompressible]

$\partial / \partial t = 0$ $\qquad$ $\rho = \text{const}$.

· mass conservation.

$dA = b \cdot dy$

$$\cancel{\frac{d}{dt}\int_{CV} \rho \, d\forall}^{\,0} + \int_{CS} \rho (\underline{V} \cdot \underline{n}) \, dA = 0.$$

$$\Rightarrow \int_{①} dA + \cancel{\int_{②} dA} + \int_{③} dA + \cancel{\int_{④} dA}.$$

$$\therefore -\int_0^h \rho u_0 b\,dy + \int_0^\delta \rho u b\,dy = 0$$

$u = u(y)$

$$\boxed{\int_0^\delta u\,dy} = \int_0^h u_0\,dy = \boxed{U_0 h}.$$

☆ ☆

· mtm conservation.

steady

$$\sum F_x = -D = \cancel{\frac{d}{dt}\int_{CV} \rho \underline{V}\,d\forall} + \int_{CS} \rho u (\underline{V}\cdot\underline{n})\,dA.$$

① + ③.

$$= -\int_0^h \rho u_0 u_0 b\,dy + \int_0^\delta \rho u u b\,dy.$$

$$= -\rho u_0^2 b h + \int_0^\delta \rho u^2 b \, dy.$$

$$= -\rho U_0 b \int_0^\delta u \, dy + \int_0^\delta \rho u^2 b \, dy$$

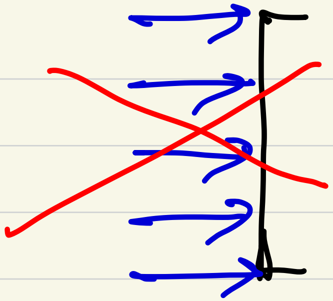
$$\therefore -D = \int_0^\delta (\rho u^2 b - \rho u U_0 b) \, dy = \rho b \int_0^\delta (u^2 - u U_0) \, dy.$$

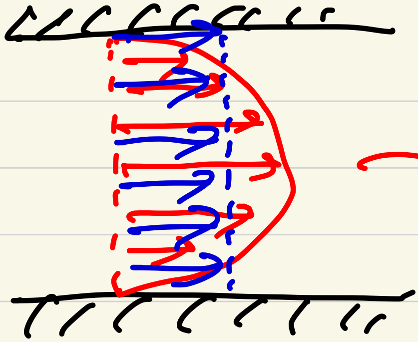
$$= \rho b \int_0^\delta u (u - U_0) \, dy$$

$$D = \rho b U_0^2 \int_0^\delta \frac{u}{U_0} \left(1 - \frac{u}{U_0}\right) dy.$$

* Momentum flux correction factor.

: correction for <u>non-uniform</u> velocity distribution





$\rightarrow$ flux.

correction factor.

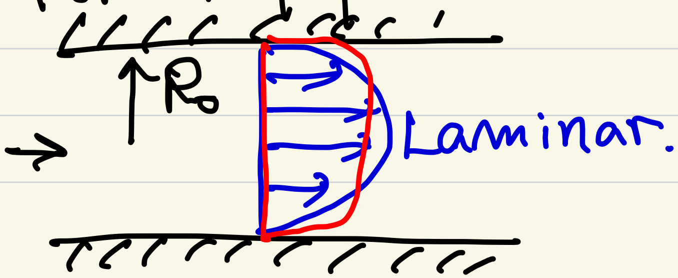
$$\int_{CS} \rho \underline{V}(\underline{V}\cdot\underline{n})\,dA \xrightarrow{\text{"1D"}} \boxed{\int \rho u^2\,dA = \beta \cdot \rho A \underline{V_{avg}^2}}$$

bulk velocity

$$\underline{V_{avg} = \frac{1}{A}\int_A u\,dA}$$

$$\therefore \boxed{\beta = \frac{1}{A}\int\left(\frac{u}{V_{avg}}\right)^2 dA}$$

ex) for a pipe flow (Ch. 6).




Lam: $u = u_0\left(1-\frac{r^2}{R_0^2}\right)$ $\rightarrow$ $\beta = \frac{4}{3}$

Turb: $u = u_0\left(1-\frac{r}{R}\right)^m$

$$\beta = \frac{(1+m)^2(2+m)^2}{2(1+2m)(2+2m)} \simeq 1.$$

3.5. Frictionless Flow : Bernoulli equation.

(neglecting the viscosity)

- assumption : $\tau = 0$. (frictionless).

no heat, no external work,

"A very small streamtube" as a control Volume.

↳ streamline.