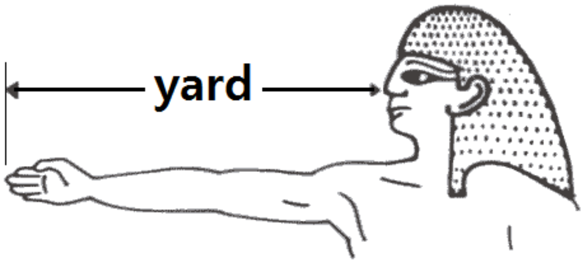
Chapter 4

Describing Physical Quantities

UNITS

- Metric system
  - cgs system: cm, gram, second
  - SI system (Systeme Internationale d'Unites)
- American engineering system
  - Based on cultural definitions from British history
  - e.g. a yard
    - the length from the king's nose to the tip of his middle finger on his fully-extended right arm



UNITS

Table 4.1 Base or Sample Units for Three Measurement Systems

System	Mass	Length	Time	Temperature
cgs	*g*	*cm*	*s*	Celsius
SI	*kg*	*m*	*s*	Kelvin
American	lb_m	*ft*	*s*	Fahrenheit

Conversion Factors

Acceleration	$1\ m/s^2 = 3.2808\ ft/s^2$	$1\ ft/s^2 = 0.3048\ m/s^2$
Area	$1\ cm^2 = 0.155\ in^2$	$1\ in^2 = 6.4516\ cm^2$
	$1\ m^2 = 10.764\ ft^2$	$1\ ft^2 = 0.092903\ m^2$
Density	$1\ g/cm^3 = 62.43\ lb_m/ft^3$	$1\ lb_m/ft^3 = 0.016019\ g/cm^3$
	$1\ kg/m^3 = 0.06243\ lb_m/ft^3$	$1\ lb_m/ft^3 = 16.019\ kg/m^3$
Energy	$1\ J = 0.7376\ ft\ lb_f$	$1\ ft\ lb_f = 1.3558\ J$
	$1\ J = 9.478 \times 10^{-4}\ Btu$	$1\ Btu = 1055.0\ J = 778.1\ ft\ lb_f$
	$1\ J = 2.778 \times 10^{-7}\ kW\ hr$	$1\ kW\ hr = 3.600 \times 10^6\ J$
	$1\ J = 10^7\ ergs$	$1\ hp\ s = 550\ ft\ lb_f$
	$1\ J = 0.2390\ cal$	
Force	$1\ N = 0.22481\ lb_f$	$1\ lb_f = 4.4482\ N$
	$1\ N = 10^5\ dynes$	
Length	$1\ cm = 0.3937\ in$	$1\ in = 2.540\ cm$
	$1\ m = 3.2808\ ft$	$1\ ft = 12\ in = 0.3048\ m$
	$1\ km = 0.6214\ mi$ (statute)	$1\ yd = 3\ ft$
	$1\ km = 0.5400\ nmi$ (nautical)	$1\ mi$ (statute) $= 1609\ m = 5280\ ft$
		$1\ nmi$ (nautical) $= 1.8520\ km$
Mass	$1\ g = 0.03527\ oz$	$1\ oz = 28.35\ g$
	$1\ kg = 2.2046\ lb_m$	$1\ lb_m = 16\ oz = 453.6\ g$
	$1\ metric\ ton = 1000\ kg = 2205\ lb_m$	$1\ ton = 2000\ lb_m = 907.2\ kg$

Conversion Factors

Power	$1\ W = 0.7376\ ft\ lb_f/s$	$1\ ft\ lb_f/s = 1.3558\ W$
	$1\ W = 9.478 \times 10^{-4}\ Btu/s$	$1\ Btu/s = 1055.0\ W = 778.1\ ft\ lb_f/s$
	$1\ W = 1.341 \times 10^{-3}\ hp$	$1\ hp = 745.7\ W = 550\ ft\ lb_f/s$
Pressure	$1\ Pa = 1.450 \times 10^{-4}\ lb_f/in^2\ (psi)$	$1\ lb_f/in^2 = 6894.8\ Pa$
	$1\ Torr = 1\ mm\ Hg\ (@\ 0°C)$	$1\ atm = 101{,}325\ Pa$
		$1\ atm = 760\ mm\ Hg\ (@\ 0°C)$
		$1\ atm = 14.696\ lb_f/in^2\ (psi)$
		$1\ atm = 33.9\ ft\ H_2O\ (@\ 4°C)$
Temperature	$T(°C) = 5/9\ [T(°F) - 32]$	$T(°F) = 1.8\ T(°C) + 32$
	$T(K) = T(°C) + 273.15$	$T(R) = T(°F) + 459.67$
		$T(R) = 1.80\ T(K)$
Viscosity	$1\ cp = 6.7197 \times 10^{-4}\ lb_m/ft\ s$	$1\ lb_m/ft\ s = 1488.2\ cp = 14.882\ Poise$
Volume	$1\ cm^3 = 1\ mL = 0.06102\ in^3$	$1\ in^3 = 16.387\ cm^3$
	$1\ m^3 = 35.3145\ ft^3$	$1\ ft^3 = 0.028317\ m^3$
	$1\ m^3 = 1000\ liters$	$1\ ft^3 = 7.4805\ gal$
	$1\ m^3 = 264.17\ gal$	$1\ ft^3 = 28.317\ liters$
	$1\ L = 0.26417\ gal$	$1\ gal = 3.785 \times 10^{-3}\ m^3 = 3.785\ L$
Volume Flow	$1\ m^3/s = 15{,}850\ gal/min$	$1\ gal/min = 6.309 \times 10^{-5}\ m^3/s$
		$1\ gal/min = 2.228 \times 10^{-3}\ ft^3/s$
		$1\ ft^3/s = 448.8\ gal/min$

Conversion Factors

$$1\ ft = 12\ in, \quad 28\ in = ?\ ft$$

28 *in*	1 *ft*	= 2.333 *ft*
	12 *in*	

Moles

- One mole = Avogadro's number of particles
  (6.02×10^{23})
- Molecular weight (MW) of H_2O

2(1.01) + 16.00 = 18.02
H   O

- gmol (gram-mole)
  - 18 *g* water = 1 *gmol* water
- lbmol (pound-mole)
  - 18 lb_m water = 1 *lbmol*

Symbols

- m = mass
- m_A = mass of "A"
- n = the number of moles
- n_A = the number of moles of "A"
- MW_A = molecular weight of "A"

Combined Units

Table 4.2 Examples of Combined Units for Three Measurement Systems

System	cgs System	SI Systems	American System
density	g/cm^3	kg/m^3	lb_m/ft^3
velocity	cm/s	m/s	ft/s
acceleration	cm/s^2	m/s^2	ft/s^2
volumetric flow rate	cm^3/s	m^3/s	ft^3/s
mass flow rate	g/s	kg/s	lb_m/s
concentration	$gmol/L$*	$kgmol/m^3$	$lbmol/ft^3$

*often abbreviated *M* (i.e., molarity)

Force & Defined Units

- Newton's 2nd law

  $F = m\,a$

- Weight

  $F_{weight} = m\,g$

- lb_m "pound-mass"
- lb_f "pound-force"

Table 4.3 Gravitational Acceleration (at Sea Level) and Defined Units of Force

System	g	Defined Unit of Force
cgs	980.66 cm/s^2	1 *dyne* ≡ 1 $g\ cm/s^2$
SI	9.8066 m/s^2	1 *Newton (N)* ≡ 1 $kg\ m/s^2$
American	32.174 ft/s^2	1 *pound-force* (lb_f) ≡ 32.174 $lb_m\ ft/s^2$

Pressure & Defined Units

- Pressure
  - Force exerted per area
- *psi*

  "pound per square inch"

Table 4.4 Commonly Used Units of Pressure

System	Units of Pressure	Abbreviation	Defined and Equivalent Units
cgs	*Pascals*	*Pa*	$1\,Pa \equiv 1\,N/m^2 = 10\,g/cm\,s^2$
SI	*kiloPascals*	*kPa*	$1\,kPa \equiv 1000\,N/m^2 = 1000\,kg/m\,s^2$
American	lb_f/in^2	*psi*	$1\,lb_f/in^2 = 4633\,lb_m/ft\,s^2$

Symbols

- Density

$$\rho = \frac{m}{V}$$

- Flow rate
  - mass flow rate ($\dot{m}$)
  - molar flow rate ($\dot{n}$)
  - volumetric flow rate ($\dot{V}$)

$$\dot{m} = \rho \dot{V}$$

Mixture Composition

- Mole Concentration of A

$$c_A = \frac{moles\ of\ A}{volume\ of\ mixture} = \frac{n_A}{V}$$

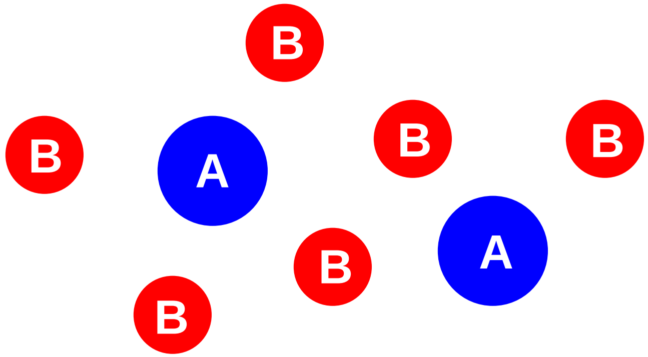
- Mass Fraction of A

$$x_A = \frac{mass\ of\ A}{mass\ of\ mixture} = \frac{m_A}{m}$$

- Mole Fraction of A

$$y_A = \frac{moles\ of\ A}{moles\ of\ mixture} = \frac{n_A}{n}$$

Mole Fraction & Mass Fraction



$$m_A = 2\ m_B$$

- Mole Fraction of A = 2/8 = 0.25
- Mass Fraction of A = 4/10 = 0.4

Mixture Composition

- Mass Percent of A

  (commonly expressed as *wt%*)

  $= 100\ x_A$

- Mole Percent of A

  $= 100\ y_A$

Dimensional Consistency

- Terms that are added together (or subtracted) must have the same units.

$$Q = ab + c^2$$

- Exponents must be unitless.
  - The units in the term *ab/c* must all cancel out to leave no units.

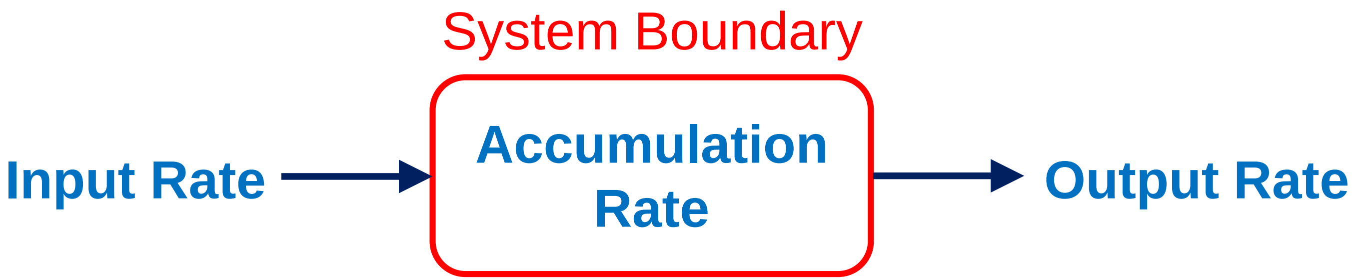
$$y = x^{ab/c}$$

Chapter 5

Material Balances

Conservation of Total Mass

- Total mass is conserved.



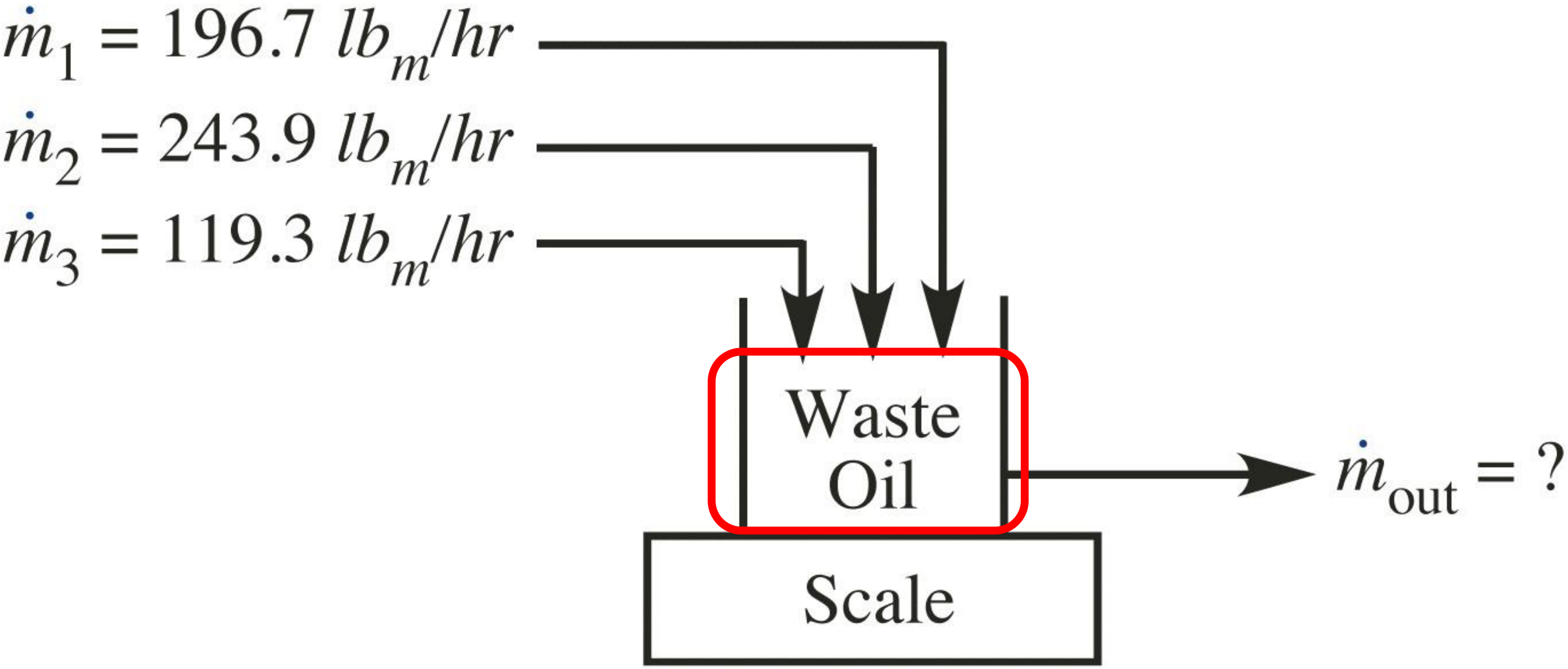
Accumulation Rate = Input Rate – Output Rate

- When acc. rate = 0, steady-state.

Input Rate = Output Rate

Ex. 5.1. Three different streams deliver contaminated oil to a waste oil tank. At what mass flow rate must the oil be withdrawn to maintain a constant scale reading?

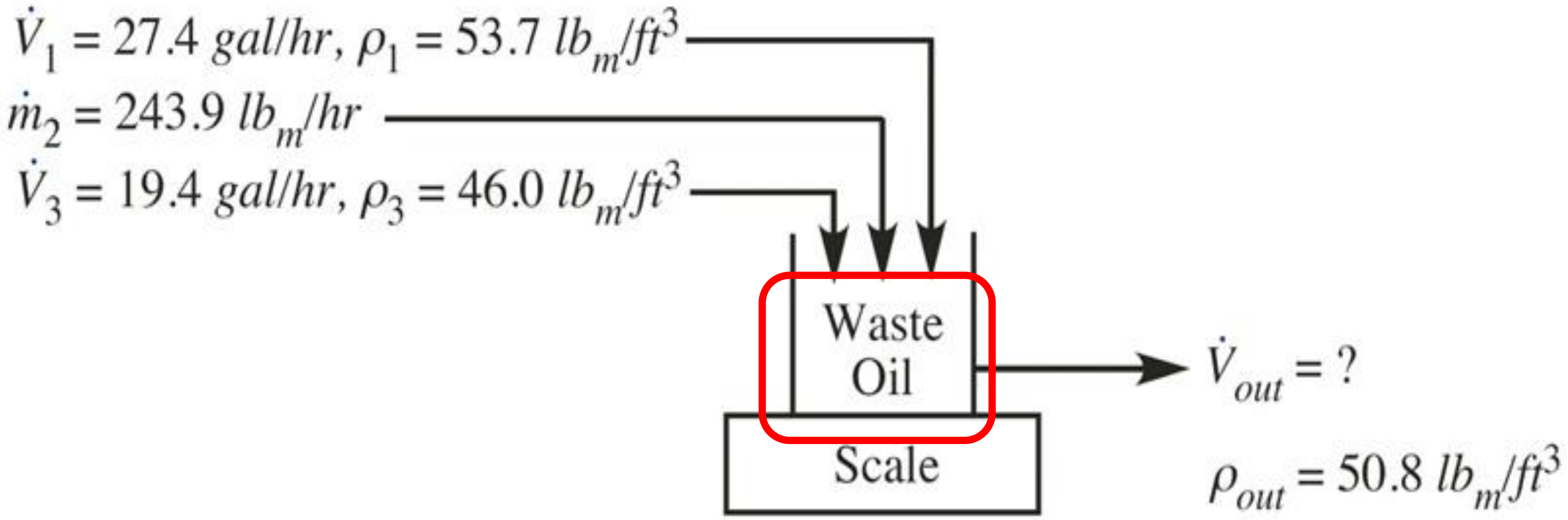
(at steady-state)



Example 5.1

Ex. 5.2. At what volumetric flow rate must the oil be withdrawn to maintain a constant scale reading?

(at steady-state)



Example 5.2

Total Mass Balance

At steady-state

$$\sum_{\substack{inlet \\ streams}} \dot{m}_{stream} = \sum_{\substack{outlet \\ streams}} \dot{m}_{stream}$$

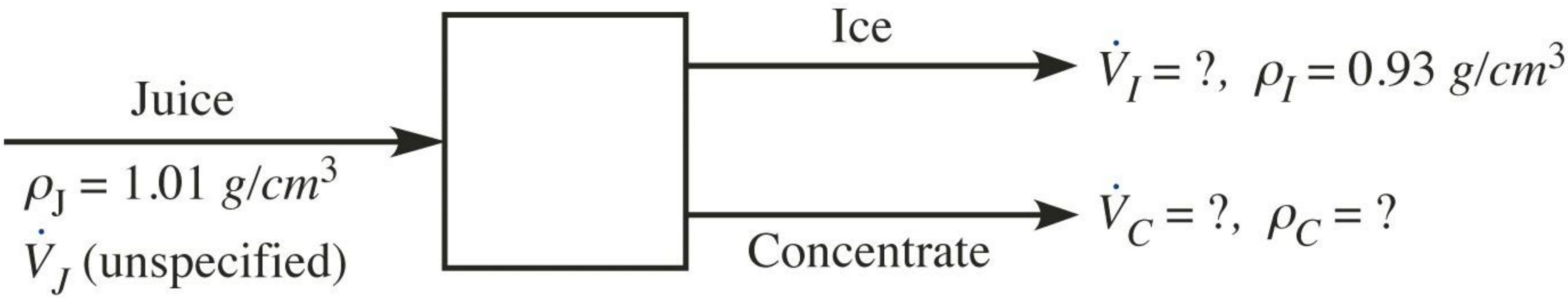
Steps for analyzing material balance problems

- Draw a diagram
- Write all known quantities
- Identify and assign symbols to all unknown quantities
- Select a basis if needed (if no flow rates are known)
- Determine the appropriate set of equations (# of equations = # of unknown)
- Solve algebraically and then numerically

Ex. 5.3. Your company uses a process to concentrate orange juice by freeze drying. What is the density of the concentrate?

Given conditions

$$\dot{V}_C = 0.25\,\dot{V}_J \qquad \dot{V}_I = 0.7\,\frac{\rho_J \dot{V}_J}{\rho_I}$$



Example 5.3

Conservation of Total Mass

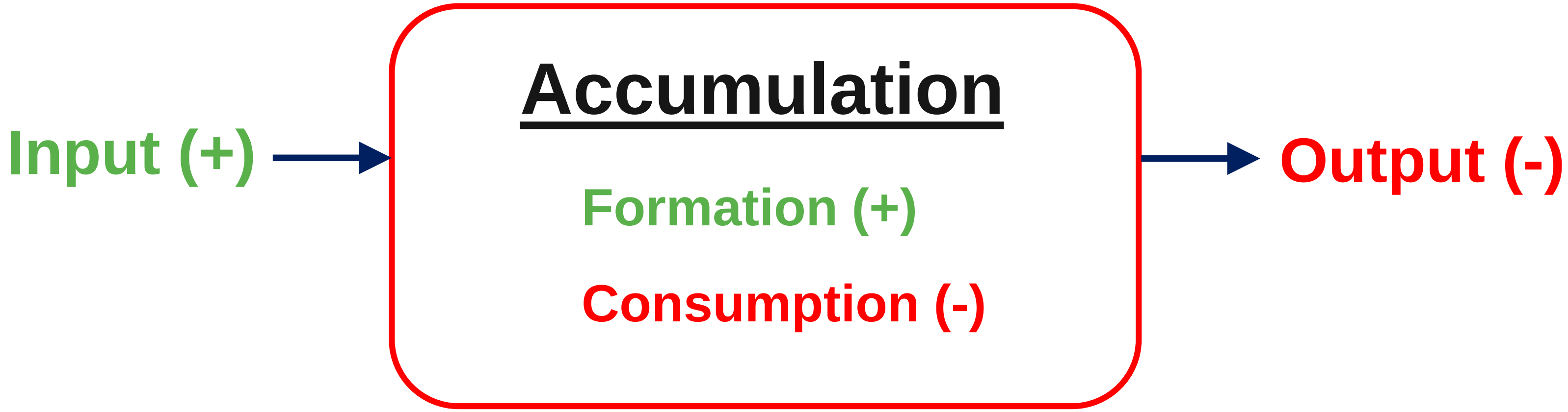
- Unlike total mass, total moles are not always conserved.

$$A + B \rightarrow C$$

- Mass Balance
  - NOT Mole Balance
  - NOT Concentration Balance
  - NOT Volume Balance

Material Balances with Reactions

- Chemical Reactions
  - Formation (= additional input)
  - Consumption (= additional output)



Acc. rate = In. rate – Out. rate + Form. rate – Con. rate

Material Balances with Reactions

Acc = In – Out + Form – Con

Accumulation rate

= Input rate – Output rate

+ Formation rate – Consumption rate

$$\frac{d(\quad)}{dt} =$$

Material Balances with Reactions

At steady-state

~~Accumulation rate~~ → 0

= Input rate – Output rate

+ Formation rate – Consumption rate

Input rate+ Formation rate

= Output rate + Consumption rate

Material Balances for Multiple Species

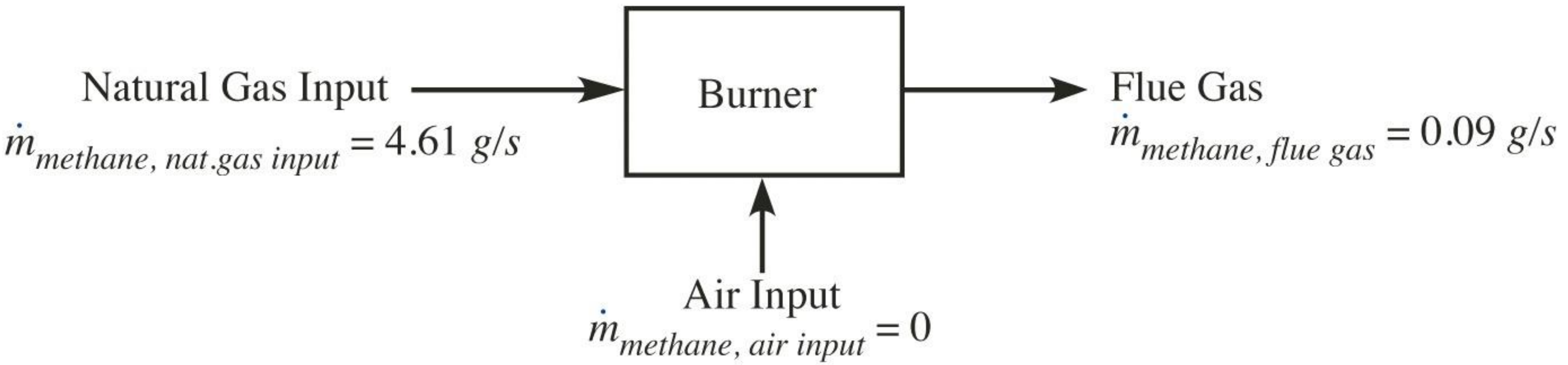
Mass of Species A at steady-state:

$$\sum_{\substack{inlet\\streams}} \dot{m}_{A,stream} + R_{formation,A} = \sum_{\substack{outlet\\streams}} \dot{m}_{A,stream} + R_{consumption,A}$$

$$\dot{m}_A = x_A \dot{m} = MW_A \dot{n}_A = MW_A y_A \dot{n} = MW_A c_A \dot{V}$$

$$\dot{n}_A = \frac{\dot{m}_A}{MW_A} = \frac{x_A \dot{m}}{MW_A} = y_A \dot{n} = c_A \dot{V}$$

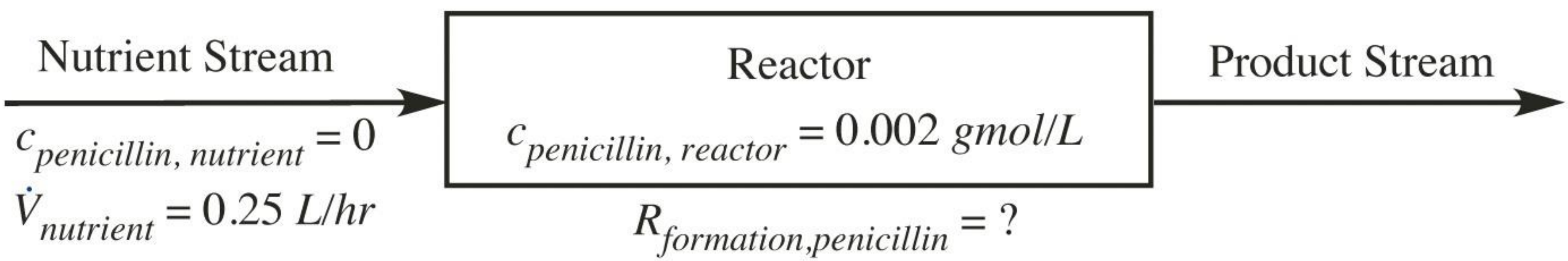
Ex. 5.4. Natural gas, which is essentially pure methane, undergoes steady-state combustion by injecting it into a small burner into which air is also injected. At what rate is the methane being burned (consumed)?



Example 5.4

Material balances with formation or consumption where chemical reaction stoichiometry is not given

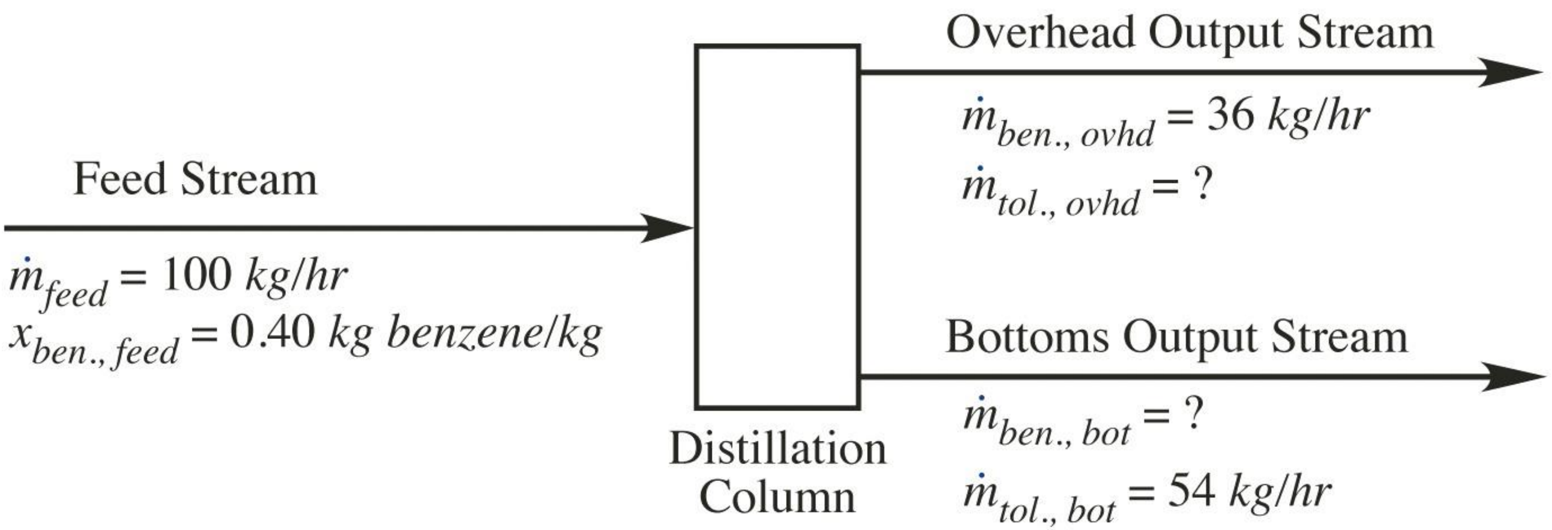
Ex. 5.5. Penicillin is produced in reactors containing the bacteria of the species Penicillium chrysogenum. A nutrient stream (containing no penicillin) is fed to a 10L reactor containing the bacteria. A product stream containing penicillin leaves the reactor (the bacteria stays in the reactor, and the penicillin concentration in the product stream is the same as inside the reactor). The densities of the nutrient and product stream can be assumed to be equal. What is the production rate of penicillin? The molecular weight of penicillin is 334.4.



Example 5.5

Material balances with no formation or consumption

Ex. 5.6. Benzene and toluene are partially separated using a distillation column. The feed stream of 100kg/hr contains benzene at a mass fraction of 0.4, with the balance being toluene. In the overhead output stream, the benzene flow rate is 36kg/hr, and in the bottoms output stream, the tolune flow rate is 54kg/hr. What are the toluene flow rate in the overhead output stream, and the benzene flow rate in the bottoms output stream?



Example 5.6

Material balances with formation/consumption where chemical reaction stoichiometry is given

More convenient to use mole balances

$$\sum_{\substack{input\\streams}} \dot{n}_{A,in} + r_{formation,A} = \sum_{\substack{output\\streams}} \dot{n}_{A,out} + r_{consumption,A}$$

$$\nu_A A + \nu_B B \rightarrow \nu_C C + \nu_D D$$

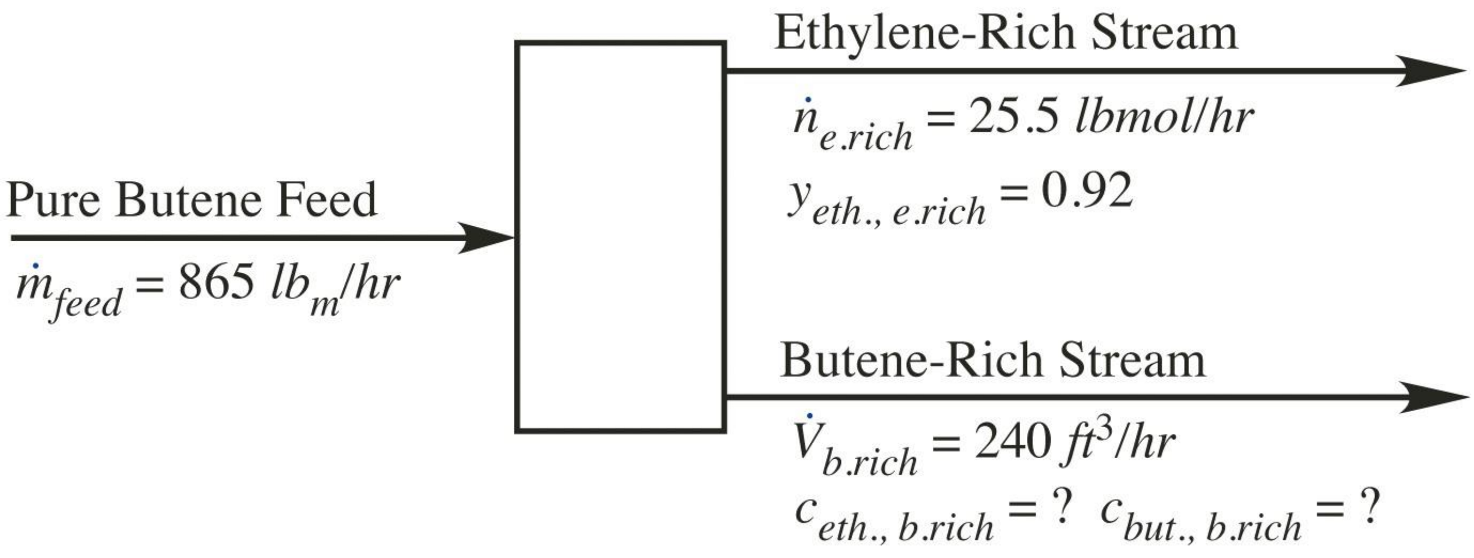
$$\frac{r_{consumption,B}}{r_{consumption,A}} = \frac{\nu_B}{\nu_A}, \quad \frac{r_{formation,C}}{r_{consumption,A}} = \frac{\nu_C}{\nu_A}, \quad \frac{r_{formation,D}}{r_{consumption,A}} = \frac{\nu_D}{\nu_A}$$

$$r_{consumption,A} = X_A \sum_{\substack{all\,input\\streams}} \dot{n}_{A,in}$$

X_A: fractional conversion of A

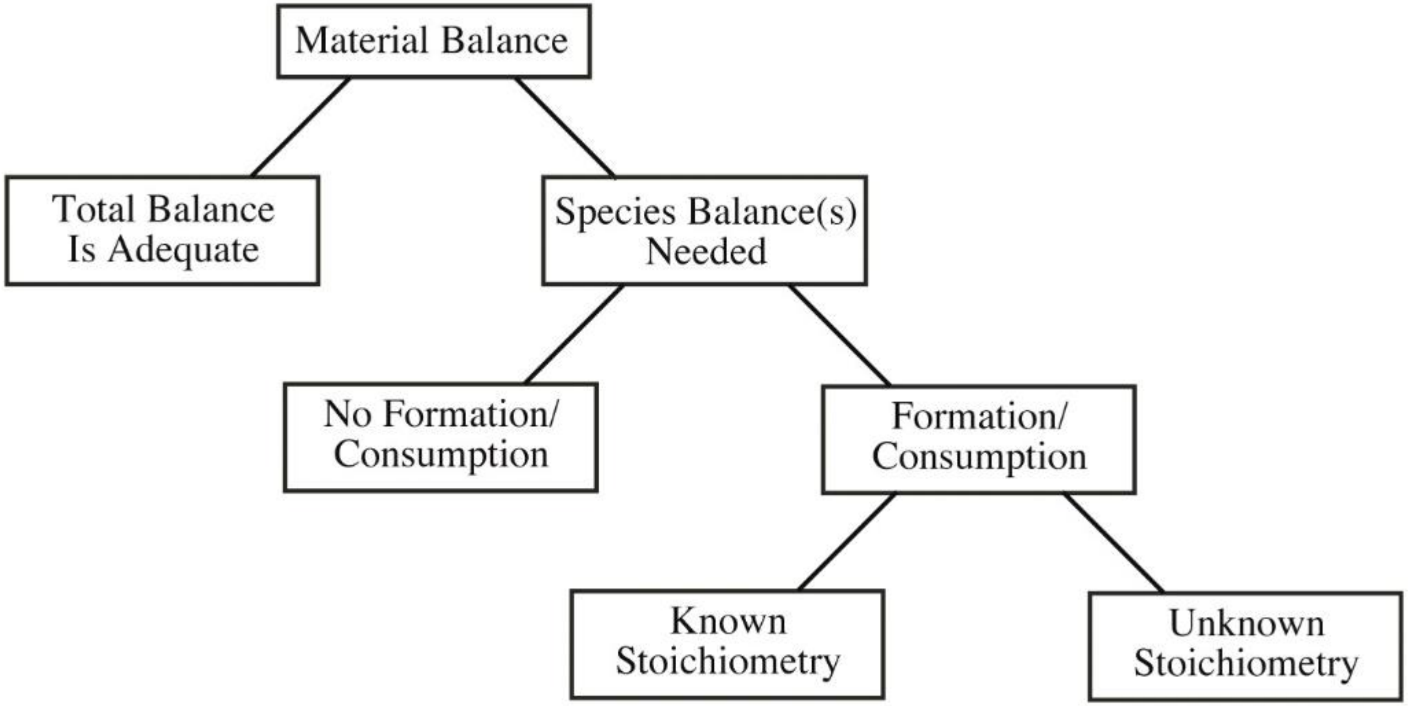
Ex. 5.7. You are designing a process to convert excess butane to ethylene using the reaction. Of the incoming butane, 84% is converted to ethylene. What will be the concentrations of ethylene and butane in the butane-rich outlet stream?

$$C_4H_8 \rightarrow 2C_2H_4$$



Example 5.7

Decision-tree diagram for solving material-balance problems



- Is species information required, or will a total balance suffice?
- If species information is required, are there formation/consumption terms?
- If there are formation/consumption terms, is the reaction stoichiometry known or unknown?