2021 Fall

"Phase Transformation in Materials"

10.25.2021

Eun Soo Park

Office: 33-313

Telephone: 880-7221

Email: espark@snu.ac.kr

Office hours: by an appointment

Chapter 3 Crystal Interfaces and Microstructure

- 1) Interfacial Free Energy
- 2) Solid/Vapor Interfaces
- 3) Boundaries in Single-Phase Solids
- 4) Interphase Interfaces in Solid (α/β)
- 5) Interface migration

Contents for previous class

3) Boundaries in Single-Phase Solids

(Pulling force per unit area of boundary)

Thermally Activated Migration of Grain Boundaries:

Metastable equilibrium of grain boundary (Balances of 1) boundary E + 2) surface tension)

- \rightarrow real curvature ($\Delta P \rightarrow \Delta G$: Gibbs Thomson Eq.) \rightarrow F = $2\gamma/r = \Delta G/V_m$ (by curvature)
- → Grain coarsening at high T annealing
- Kinetics of Grain Growth
 - Grain boundary migration (v) by thermally activated atomic jump

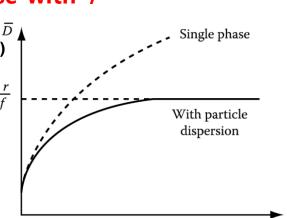
Boundary velocity
$$v = \frac{A_2 n_1 v_1 V_m^2}{N_a RT} \exp\left(-\frac{\Delta G^a}{RT}\right) \frac{\Delta G}{V_m} \frac{v \sim \Delta G/V_m \text{ driving force}}{\Rightarrow F = \Delta G/V_m}$$

M: mobility = velocity under unit driving force $\sim \exp(-1/T)$

rate of grain growth $d\underline{D}/dt\sim 1/\underline{D}$, exponentially increase with $\mathcal T$

- \rightarrow D = $k't^n$ (Experimental: n << 1/2, ½ at pure metals or high Temp.)
- Mobility of GB~ affected by both type of boundaries $\frac{r}{\sqrt{r}}$ and GB segregation or 2nd phase precipitation

Ex) Effect of second-phase particle - Zener Pinning



Contents for previous class

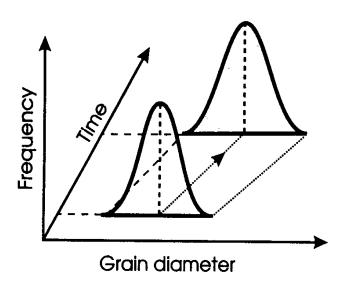
- Grain Growth
 - Normal grain growth

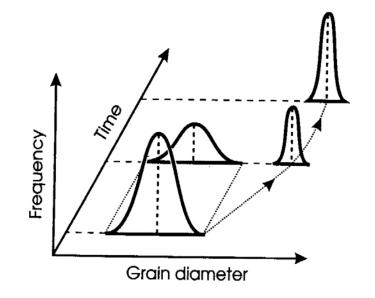


Abnormal grain growth

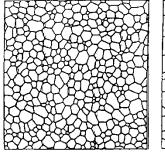
(high mobility of special GBs

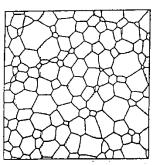
→ development of recrystallization textures)

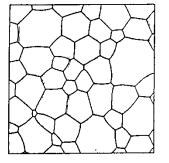


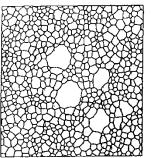


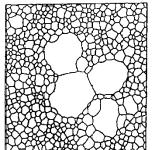
< Bimodal Size distribution >

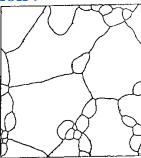












Contents for today's class

- Interphase Interfaces in Solid (α/β)
 - Types of interphase interfaces in solid (α/β)
 - Second-Phase Shape

$$\sum A_i \gamma_i + \Delta G_S = minimum$$

Interface Energy Effects

Coherent / Semi-coherent / incoherent

Misfit Strain Effects

- Coherency Loss
- Interface migration

Q: What kind of interphase interfaces in solid (α/β) exist?

- = coherent/ semi-coherent/ incoherent/ complex semi-coherent
- → different interfacial free energy, γ

3.4 Interphase Interfaces in Solids

Interphase boundary

- different two phases : different crystal structure or different composition

coherent, semicoherent incoherent

3.4.1 Coherent interfaces

Disregarding chemical species, if the interfacial plane has the same atomic configuration in both phases,

Perfect atomic matching at interface

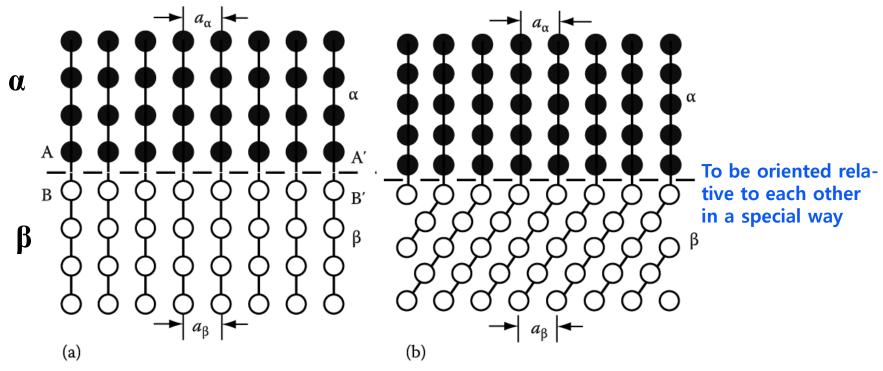
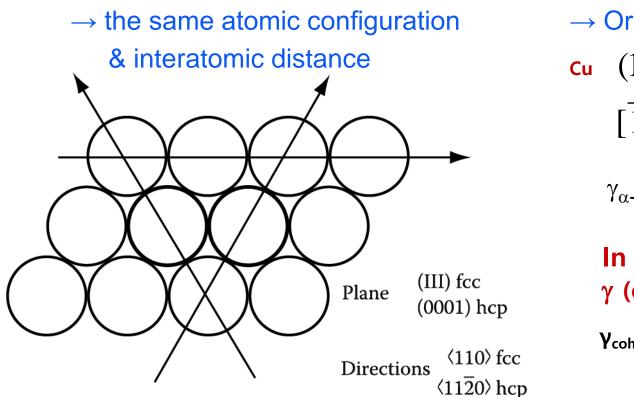


Fig. 3.32 Strain-free coherent interfaces. (a) Each crystal has a different chemical composition but the same crystal structure. (b) The two phases have different lattices

3.4.1 Coherent interfaces

Which plane and direction will be coherent between FCC and HCP?

: Interphase interface will make lowest energy and thereby the lowest nucleation barrier ex) hcp silicon-rich κ phase in fcc copper-rich α matrix of Cu-Si alloy



Orientation relation

Cu $(111)_{\alpha}$ //(0001)_{κ} Si $[\overline{1}10]_{\alpha}$ //[11 $\overline{2}0$]_{κ} $\gamma_{\alpha-\kappa}$ of Cu-Si ~ 1 mJm⁻²

In general, γ (coherent) ~ 200 mJm⁻²

Ycoherent = Ystructure + Ychemical

Ychemical

 γ (coherent) = γ_{ch}

hcp/ fcc interface: only one plane that can form a coherent interface

Fig. 3.33 The close-packed plane and directions in fcc and hcp structures.

When the atomic spacing in the interface is not identical between the adjacent phase, what would happen?

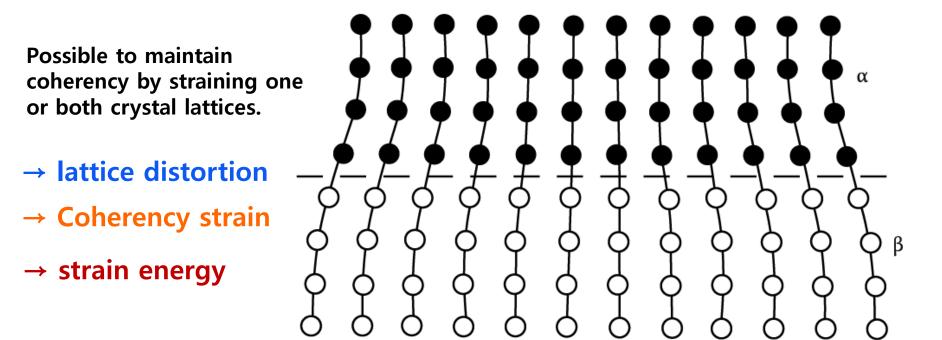


Fig. 3.34 A coherent interface with slight mismatch leads to coherency strains in the adjoining lattices.

The strains associated with a coherent interface raise the total energy of the system.

If coherency strain energy is sufficiently large, → "misfit dislocations"

→ semi-coherent interface

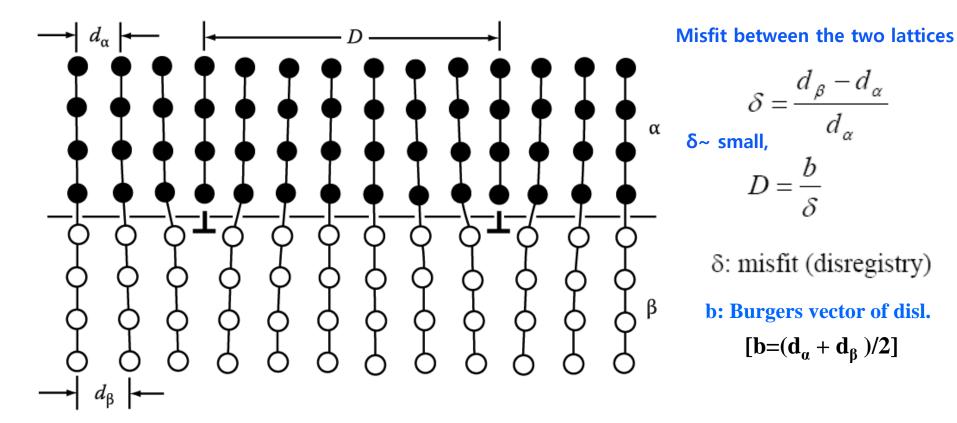


Fig. 3.35 A semi-coherent interface. The misfit parallel to the interface is accommodated by a series of edge dislocations.

(2) Semicoherent interfaces

$$\mathbf{d}_{\alpha} < \mathbf{d}_{\beta}$$

$$\delta = (d_{\beta} - d_{\alpha})/d_{\alpha}$$
: misfit
 $\rightarrow D$ vs. δ vs. n

$$(n+1) d_{\alpha} = n d_{\beta} = D$$

$$\delta = (d_{\beta}/d_{\alpha}) - 1, (d_{\beta}/d_{\alpha}) = 1 + 1/n = 1 + \delta$$
 $\rightarrow \delta = 1/n$

$$D = d_{\beta} / \delta \approx b / \delta [b = (d_{\alpha} + d_{\beta})/2]$$

δ~ small,

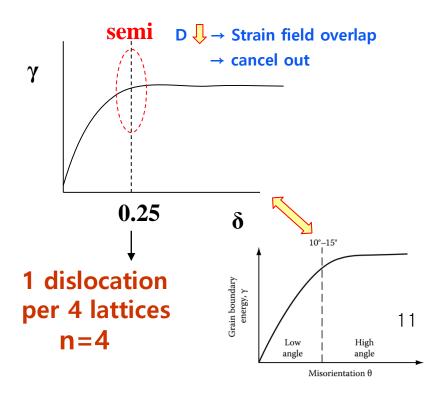
Burgers vector of dislocation

$$\gamma$$
(semicoherent) = $\gamma_{ch} + \gamma_{st}$

 $\gamma_{st} \rightarrow$ due to structural distortions caused by the misfit dislocations

$$\gamma_{
m st} \propto \delta$$
 for small δ

In general, γ (semicoherent) ~ 200~500 mJm⁻²



3) Incoherent Interfaces ~ high angle grain boudnary

- 1) $\delta > 0.25$ No possibility of good matching across the interface
- 2) different crystal structure (in general)

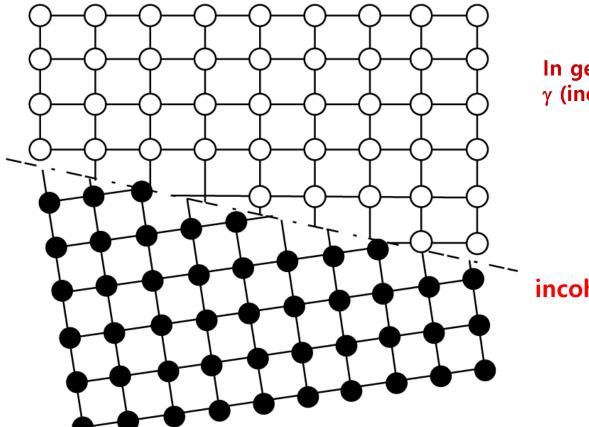
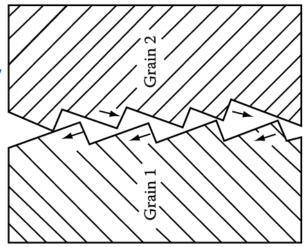


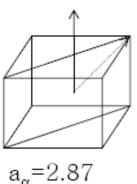
Fig. 3.37 An incoherent interface.

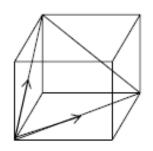


In general, γ (incoherent) ~ 500~1000 mJm⁻²

incoherent

4) Complex Semicoherent Interfaces





If bcc α is precipitated from fcc γ , which interface is expected?

Which orientation would make the <u>lowest interface energy</u>?

For fcc and bcc crystals ~ closest-pack planes in each phase almost parallel to each other

Nishiyama-Wasserman (N-W) Relationship

$$(110)_{bcc} //(111)_{fcc}, [001]_{bcc} //[\overline{1}01]_{fcc}$$

[001] _a= 2.87

Kurdjumov-Sachs (K-S) Relationships

$$(110)_{bcc} //(111)_{fcc}, [1\overline{1}1]_{bcc} //[0\overline{1}1]_{fcc}$$

(The only difference between these two is a rotation in the closest-packed planes of 5.26°.)

Complex Semicoherent Interfaces

Semicoherent interface observed at boundaries formed by low-index planes. (atom pattern and spacing are almost equal.)

N-W relationship

Good fit is restricted to small diamond-shaped areas that only contain ~8% of the orientation relationship.

A similar situation can be shown to exist for the K-S orientation relationship.

⇒ But, impossible to form a large interfaces

→ Incoherent interface

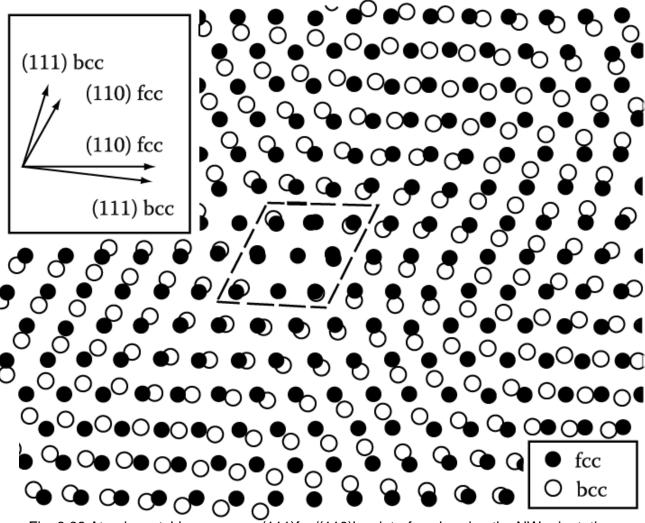
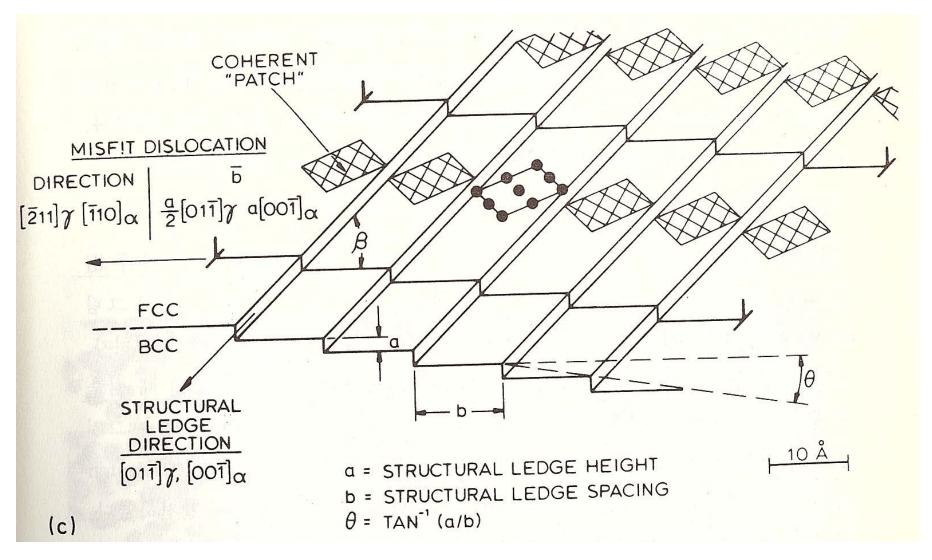


Fig. 3.38 Atomic matching across a (111)fcc/(110)bcc interface bearing the NW orientation relationship for lattice parameters closely corresponding to the case of fcc and bcc iron.

Complex Semicoherent Interfaces



The degree of coherency can, however, be greatly increased if a macroscopically irrational interface is formed. The detailed structure of such interfaces is, however, uncertain due to their complex nature.

3.4 Interphase Interfaces in Solids

Interphase boundary - different two phases : different crystal structure different composition

incoherent

- 1) $\delta > 0.25$ No possibility of good matching across the interface
- 2) different crystal structure (in general)

 γ (incoherent) ~ 500~1000 mJM⁻²

Complex Semicoherent Interfaces

Nishiyama-Wasserman (N-W) Relationship Kurdjumov-Sachs (K-S) Relationships

(The only difference between these two is a rotation in the closest-packed planes of 5.26°.)

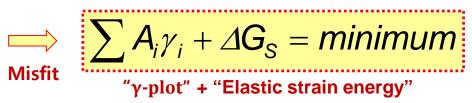
The degree of coherency can, however, be greatly increased if a macroscopically irrational interface is formed.

 δ =4: 1 dislocation per 4 lattices

Q: How is the second-phase shape determined?

If misfit is small, **Equilibrium shape of a coherent** precipitate or zone can only be predicted from the "γ-plot"

$$\sum A_i \gamma_i$$



Lowest total interfacial free energy by optimizing the shape of the precipitate and its orientation relationship

Fully coherent precipitates

 γ_{ch}

different composition

γ_{ch} + Lattice misfit

Coherency strain energy

Incoherent inclusions

 γ_{ch} + Volume Misfit $\Delta = \frac{\Delta V}{V}$

Chemical and structural interfacial E

- (a) Precipitate shapes : $\sum A_i \gamma_i \downarrow$
- (b) Calculation of misfit strain energy

3.4.2 Second-Phase Shape: Interfacial Energy Effects

How is the second-phase shape determined? $\sum A_i \gamma_i = minimum$

$$\sum A_i \gamma_i = minimum$$

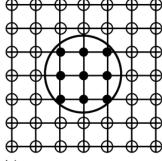
Lowest total interfacial free energy by optimizing the shape of the precipitate and its orientation relationship

A. Fully Coherent Precipitates

(G.P. Zone)

- If α , β have the same structure & a similar lattice parameter
- Happens during early stage of many precipitation hardening

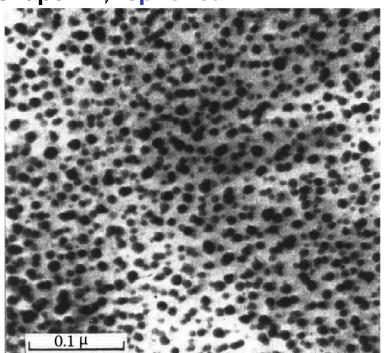
A zone with no misfit \oplus



GP(Guinier- Preston) Zone in Al – Ag Alloys

$$\varepsilon_a = \frac{r_A - r_B}{r_A} = 0.7\%$$

→ negligible contribution to the total free energy



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(b) Ag-rich GP zones (Dia. ~10 nm) in Al-4at% Ag alloy

B. Partially Coherent Precipitates

- α , β have different structure and one plane which provide close match
- Coherent or Semi-coherent in one Plane;
 Disc Shape (also plate, lath, needle-like shapes are possible)

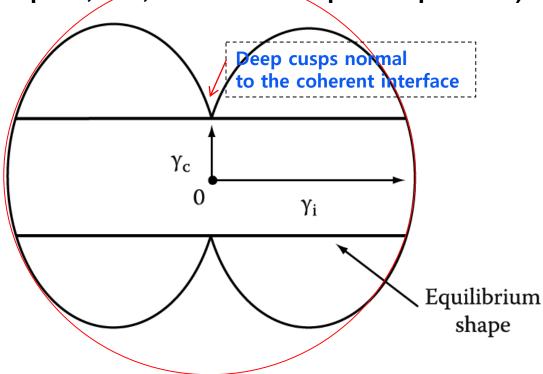


Fig. 3.40 A section through a **γ-plot** for a precipitate showing one coherent or semi-coherent interface, together with the equilibrium shape (a disc).

Precipitate shapes observed in practice

~ not equilibrium shape why? 1) misfit strain E effects ~ ignored.

through a γ -plot 2) different growth rates depending on directions

$hcp \gamma' Precipitates in Al - 4\%Ag Alloys \rightarrow plate$

Semicoherent broad face parallel to the $\{111\}_{\alpha}$ matrix planes (usual hcp/fcc orientation relationship)

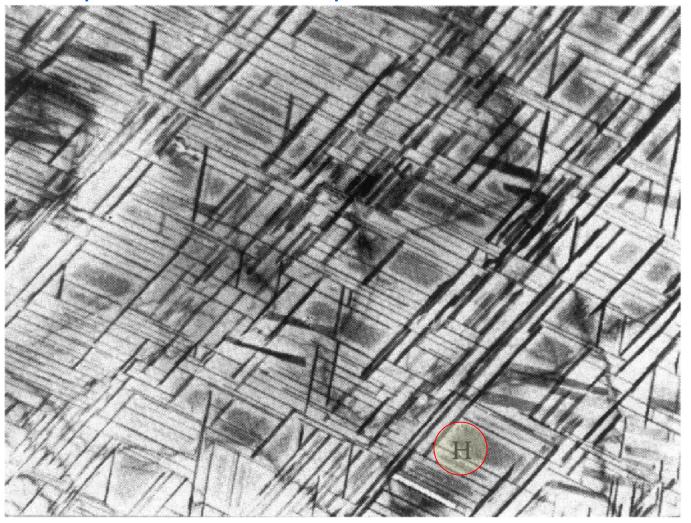
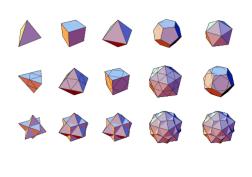


Fig. 3. 42 Electron micrograph showing the Widmanstatten morphology of γ' precipitates in an Al-4 atomic % Ag alloy. GP zones can be seen between the γ' e.g. at H (x 7000).

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C. Incoherent precipitates

- when α , β have completely different structure \implies Incoherent interfaces or When the two lattices are in a random orientation
- Interface energy is high for all plane \implies spherical shape with smoothly curved interface
- Polyhedral shapes: certain crystallographic planes of the inclusion lie at cusps in the γ-plot

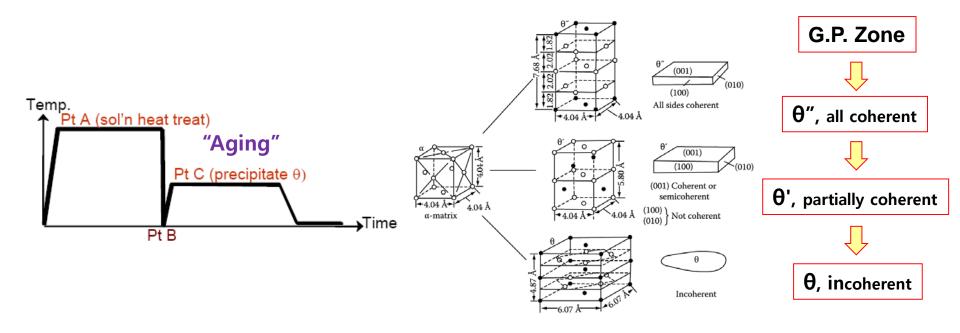




 θ phase in Al-Cu alloys

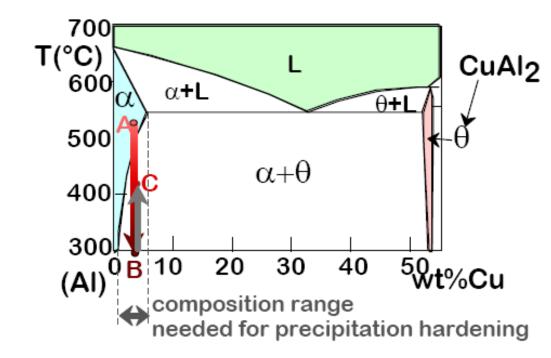
Q: Example of Second-Phase Shape

precipitates from solid solution in Al-Cu alloys

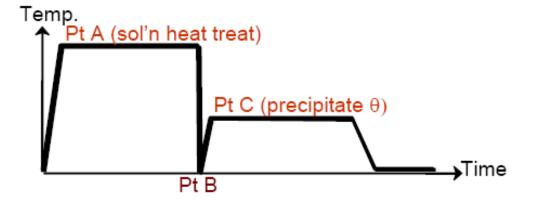


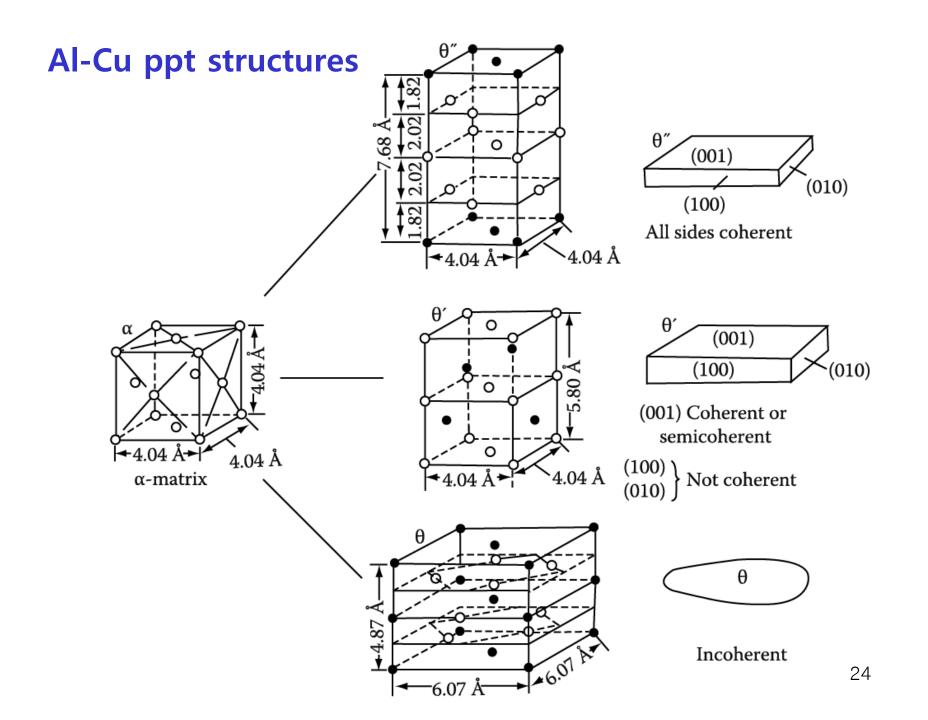
Precipitation Hardening

- Ex: Al-Cu system
- Procedure:
- Pt A: solution heat treat (get a solid solution)
- Pt B: quench to room temp.
- Pt C: reheat to nucleate small θ crystals within α crystals.



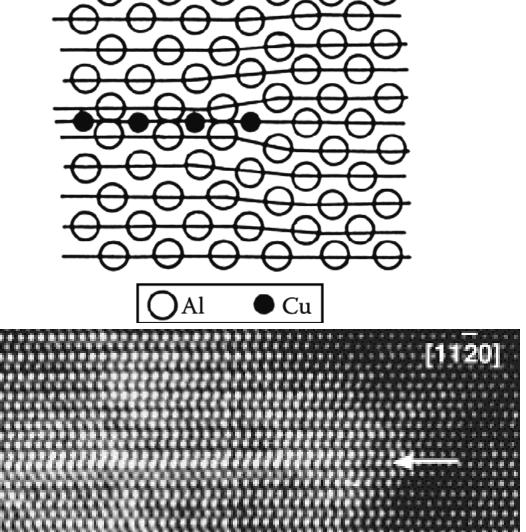
$$\alpha + \theta \rightarrow \text{Heat} (\sim 550^{\circ}\text{C}) \rightarrow \text{Quench} (0^{\circ}\text{C}) \rightarrow \alpha \text{ (ssss)} \rightarrow \text{Heat/age} (\sim 150^{\circ}\text{C}) \alpha + \theta_{ppt}$$





Al-Cu ppt structures

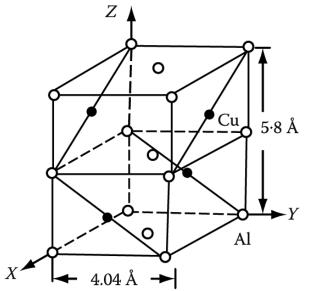
GP zone structure

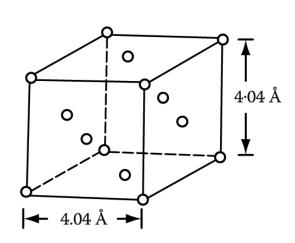


(a) Bright-field TEM image showing G.P. zones, and (b) HRTEM image of a G.P. zone formed on a single $(0\ 0\ 0\ 1)_{\alpha}$ plane. Electron beam is parallel to in both (a) and (b).

θ' Phase Al-Cu Alloys

Semicoherent broad face parallel to the $\{100\}_{\alpha}$ matrix planes (habit plane)





- (a) The unit cell of the Θ' precipitate in Al-Cu alloys
- (b) The unit cell of the matrix

Orientation relationship between α and θ '

$$(001)_{\theta'}$$
 // $(001)_{\alpha}$

$$(001)_{\theta'}$$
 // $(001)_{\alpha}$ $[100]_{\theta'}$ // $[100]_{\alpha}$

Cubic symmetry of the Al-rich matrix (α) ~ many possible orientations for the precipitate plates within any given grain

S phase in Al-Cu-Mg alloys ; Lath shape

Widmanstätten morphology

 β' phase in Al-Mg-Si alloys ; Needle shape

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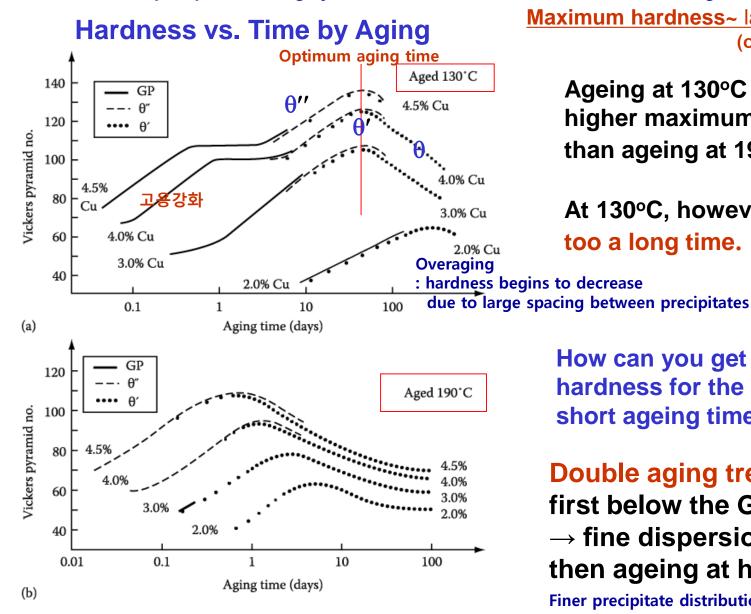
θ phase in Al-Cu alloys (Al₂Cu)



- Polyhedral shapes: certain crystallographic planes of the inclusion lie at cusps 27 in the γ-plot

5.5.4. Age Hardening

Transition phase precipitation → great improvement in the mechanical properties Coherent precipitates → highly strained matrix → dislocations~forced during deformation



Maximum hardness~ largest fraction of θ'' (coherent precipitates)

Ageing at 130°C produces higher maximum hardness than ageing at 190°C.

At 130°C, however, it takes too a long time.

How can you get the high hardness for the relatively short ageing time?

Double aging treatment first below the GP zone solvus → fine dispersion of GP zones then ageing at higher T.

Finer precipitate distribution

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Fig. 5. 37 Hardness vs. time for various Al-Cu alloys at (a) 130 °C (b) 190 °C

Precipitates on Grain Boundaries

Formation of a second-phase particle at the interfaces with two differently oriented grains

- 1) incoherent interfaces with both grains
- 2) a coherent or semi-coherent interface with one grain and an incoherent interface with the other,
- 3) coherent or semi-coherent interface with both grains

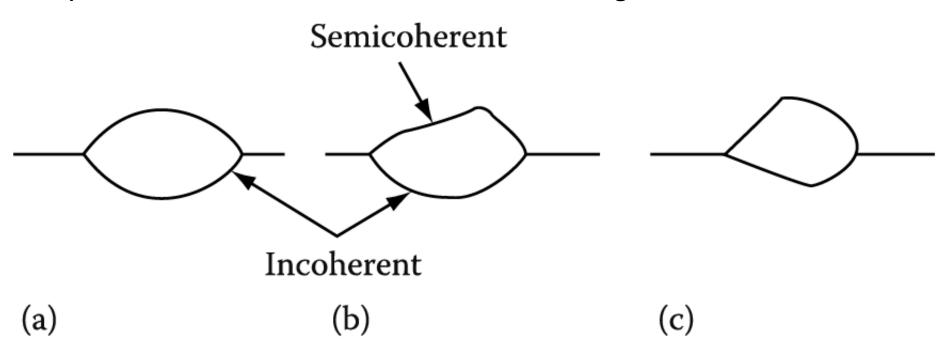


Fig. 3. 45 Possible morphologies for grain boundary precipitates. Incoherent interfaces smoothly curved. Coherent or semicoherent interface plannar.

Precipitates on Grain Boundaries

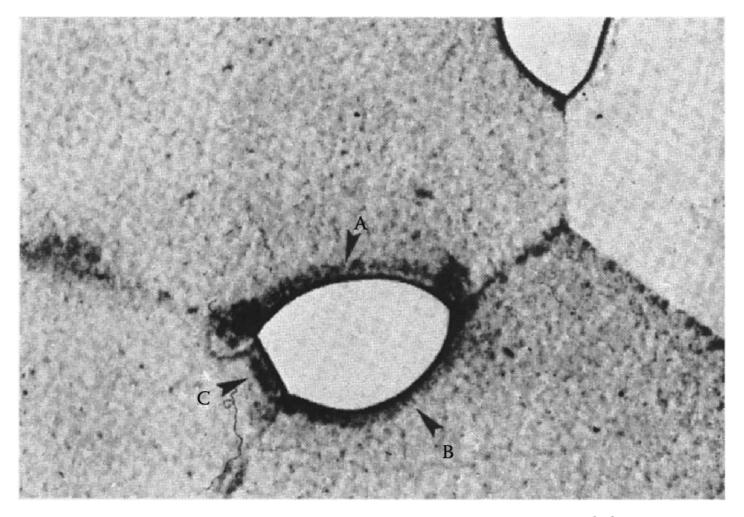
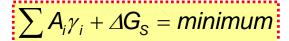
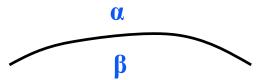


Fig. 3. 46 An α precipitate at a grain boundary triple point in an α – β Cu-In alloy. Interfaces A and B are incoherent while C is semicoherent (x 310).

3.4 Interphase Interfaces in Solids (α/β) $\sum A_i \gamma_i + \Delta G_s = minimum$



1) Interphase boundary - different two phases : different crystal structure or different composition



Coherent/ Semicoherent/ Incoherent Complex Semicoherent

Fully coherent precipitates

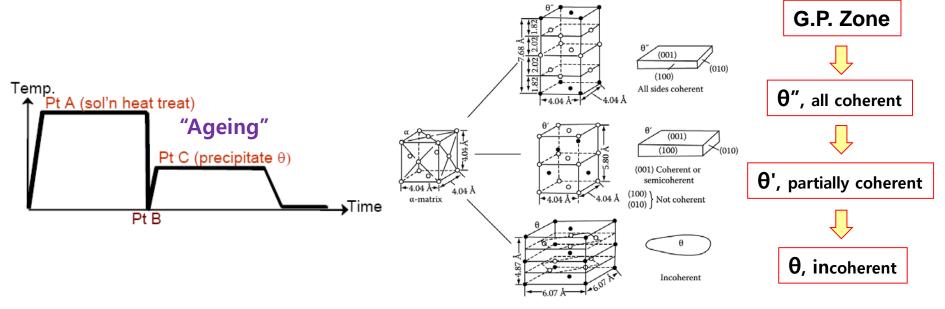
 γ_{ch} different composition

γ_{ch} + Lattice misfit **Coherency strain energy**

Incoherent inclusions

 γ_{ch} + Volume Misfit $\Delta = \frac{\Delta V}{V}$ Chemical and structural interfac

2) Second-Phase Shape: precipitate from solid solution in Al-Cu alloys



Q: How is the second-phase shape determined?

$$\sum A_i \gamma_i + \Delta G_{S} = minimum$$

+ misfit strain E γ – plot

Lowest total interfacial free energy by optimizing the shape of the precipitate and its orientation relationship

Fully coherent precipitates

 γ_{ch}

different composition

Coherency strain energy

Incoherent inclusions

 $\gamma_{ch} + Lattice misfit$ $\gamma_{ch} + Volume Misfit <math>\Delta = \frac{\Delta V}{V}$

Chemical and structural interfacial E

- (a) Precipitate shapes
- (b) Calculation of misfit strain energy

3.4.3. Second-Phase Shape: Misfit Strain Effects

If misfit is small, precipitate or zone can only be predicted from the "γ-plot"





If misfit is small, Equilibrium shape of a coherent precipitate or zone can only be predicted from the "
$$\gamma$$
-plot"
$$\sum A_i \gamma_i = \sum A_i \gamma_i + \Delta G_S = \min \sum A_i \gamma_i + \Delta G_S = \Delta G_S = \min \sum A_i \gamma_i + \Delta G_S = \Delta G_S$$

A. Fully Coherent Precipitates

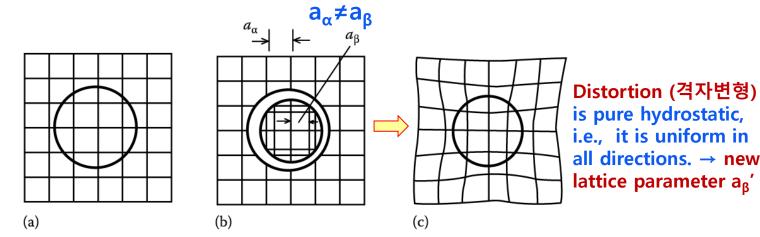


Fig. 3. 47 The origin of coherency strains. The number of lattice points in the hole is conserved.

Unconstrained Misfit

Constrained Misfit

In practice, different elastic constants $E_{\beta} \neq E_{\alpha} \rightarrow 0.5\delta \leq \varepsilon \leq \delta$

if thin disc-type precipitate,

In situ misfit is no longer equal in all directions

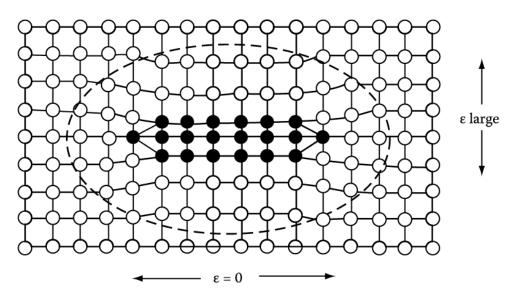


Fig. 3. 48 For a coherent thin disc there is little misfit parallel to the plane of the disc. Maximum misfit is perpendicular to the disc. \rightarrow reduction in coherency strain E

* Total elastic energy (ΔG_s) depends on the "shape" and "elastic properties" of both matrix and inclusion.

&
$$E_{\beta} = E_{\alpha}$$

GP Zone Shape

 $\Delta Gs\!\rightarrow$ independent of the shape of the precipitate

$$\Delta G_S = 4\mu \delta^2 \cdot V \qquad \text{(If } v=1/3\text{)}$$

here, μ = shear modulus of the matrix,

V= volume of the unconstrained hole in the matrix Elastically Anisotropic Materials & $E_{\beta} \neq E_{\alpha}$ ΔGs^{min} : if inclusion is hard-sphere/ soft-disc shape $\Delta Gs\!\rightarrow$ dependent of the shape of the precipitate

Atom radius (A) Al: 1.43 Ag: 1.44 Zn: 1.38 Cu: 1.28

- + 0.7% - 3.5% - 10.5% Zone Misfit (δ)

 $\sum A_i \gamma_i + \Delta G_S = minimum$

Equilibrium shape sphere sphere Interfacial E effect dominant

disc strain E effect dominant

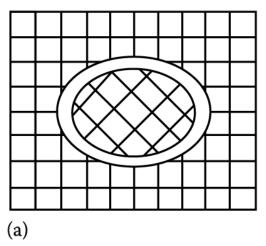
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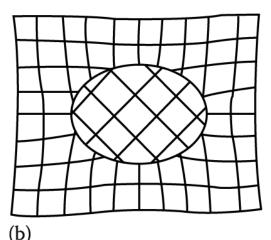
B. Incoherent Inclusions

Lattice sites are not conserved. \rightarrow no coherency strain, ΔG_s

But, misfit strain still arise if the inclusion is the wrong size.

 δ (lattice misfit) $\rightarrow \Delta$ (volume misfit)





Volume Misfit
$$\Delta = \frac{\Delta V}{V}$$

Ex) coherent spherical inclusion: $\Delta=3\delta$

#of lattice sites within the hole is not preserved for incoherent inclusion (no lattice matching)

For spheroidal Inclusions
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
 For Elliptical Inclusions $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$$

For a homogeneous incompressible inclusion in an isotropic matrix

등방성 기지내 균질 비압축성 개재물

 $\Delta G_S = \frac{2}{3} \mu \Delta^2 \cdot V \cdot f(c/a)$

μ: the shear modulus of the matrix

1) The elastic strain energy is proportional to the square of the volume misfit Δ^2 .

2) Shape effect for misfit strain $E \sim function f(c/a)$

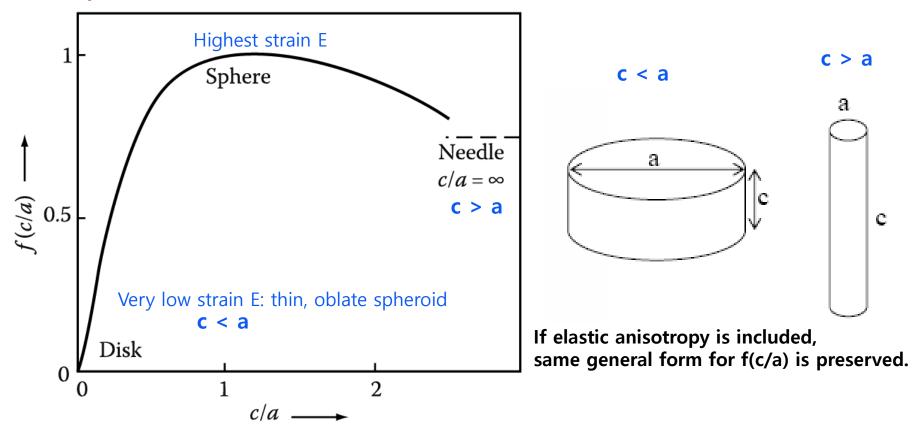


Fig. 3. 50 The variation of misfit strain energy with ellipsoid shape, f(c/a).

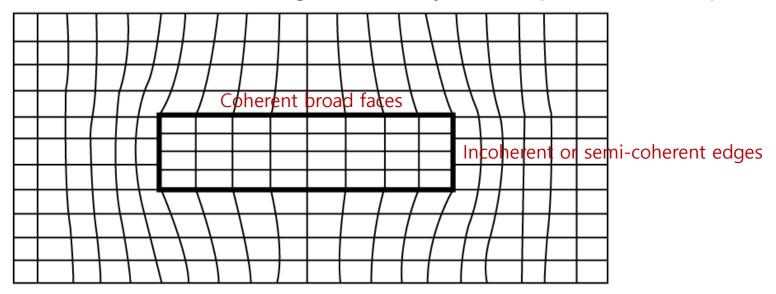
$$\Delta G_s = \frac{2}{3} \mu \Delta^2 \cdot V \cdot f(c/a) \qquad \Delta = \frac{V_\beta - V_\alpha}{V_\alpha} \approx 3\delta \text{ for sphere}$$
Elastic strain E Precipitate shape effect $\neq 3\delta$ for disc or needle

* Equil. Shape of an incoherent inclusion: an oblate spheroid with c/a value that balances the opposing effects of interfacial E and strain E

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C. Plate-like precipitates

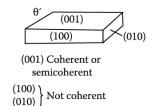
Misfit across the broad faces → large coherency strains parallel to the plate



In situ misfit across the broad faces increases with increasing plate thickness

- --- greater strains the matrix and higher shear stresses at the corners of the plates
- --- energetically favorable for the broad faces to become semi-coherent
- → the precipitate behaves as an incoherent inclusion with comparatively little

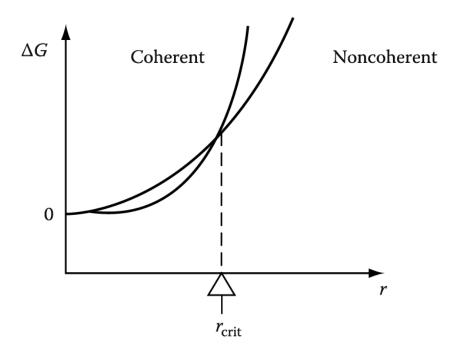
misfit strain E, ex) θ ' phase in Al-Cu alloy



Q: Which state produces the lowest total E for a spherical precipitate?

"Coherency loss"

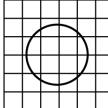
If a coherent precipitate grows, during aging for example, It should lose coherency when it exceeds r_{crit} .



Coherency Loss Precipitates with coherent interfaces=low interfacial E + coherency strain E Precipitates with non-coherent interfaces=higher interfacial E

If a coherent precipitate grows, it should lose coherency to maintain minimum interfacial free E.

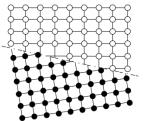
$$\Delta G(coherent) = 4\mu\delta^{2} \cdot \frac{4}{3}\pi r^{3} + 4\pi r^{2} \cdot \gamma_{ch} \iff \Delta G(non-coherent) = 4\pi r^{2} \cdot (\gamma_{ch} + \gamma_{st})$$



Coherency strain energy

Chemical interfacial E

Eq. 3.39



Chemical and structural interfacial E

$$\frac{4}{3}\pi r^{3} (4\pi\mu \delta^{2}) + 4\pi r^{2} (\gamma_{ch}) = 4\pi r^{2} (\gamma_{st} + \gamma_{ch})$$

coherent

$$\Delta G_s$$
-relaxed

$$\therefore r_{crit} = \frac{3 \cdot \gamma_{st}}{4\mu \, \delta^2}$$

for small δ , $\gamma_{st} \propto \delta$

(semi-coherent interface)

$$\approx \frac{1}{\delta} (\delta = (\mathbf{d}_{\beta} - \mathbf{d}_{\alpha}) / \mathbf{d}_{\alpha} : \mathbf{misfit})$$

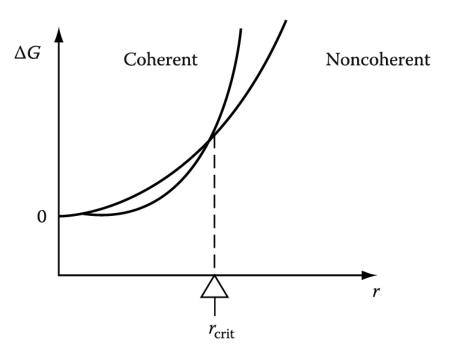


Fig. 3. 52 The total energy of matrix + precipitate vs. precipitate radius for spherical coherent and non-coherent (semicoherent of incoherent) precipitates.

If a coherent precipitate grows, during aging for example, It should lose coherency when it exceeds r_{crit} .

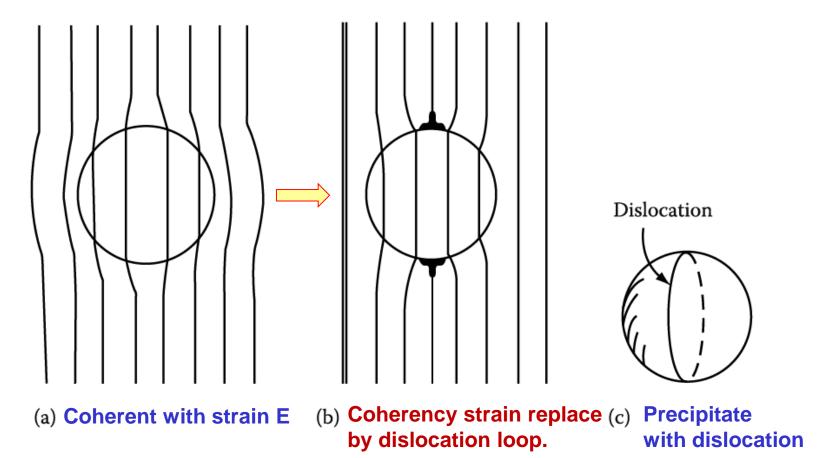
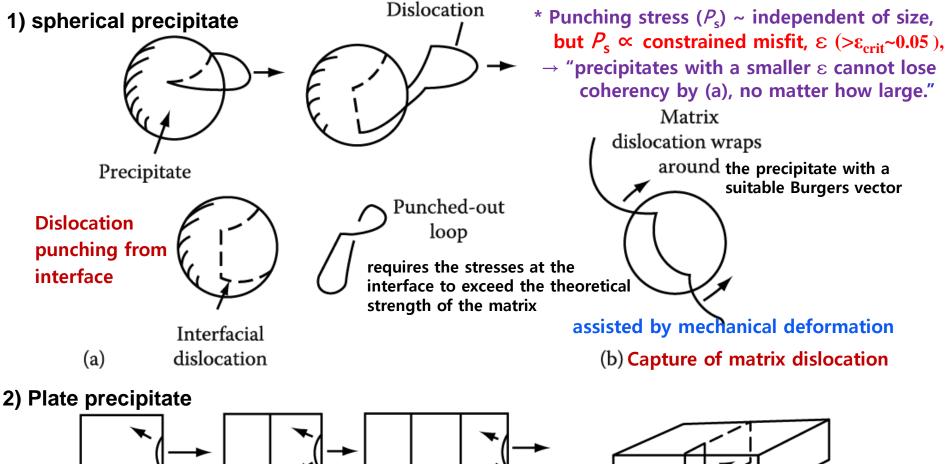


Fig. 3.53. Coherency loss for a spherical precipitate

In practice, this phenomena can be rather difficult to achieve.

→ Coherent precipitates are often found with sizes much larger than r_{crit}.

"Mechanisms for coherency loss": all require the precipitate to reach a larger size than r_{crit}



Misfit

dislocation

Constant inter D spacing

Nucleation of dislocation at the edge → maintain a roughly constant inter-dislocation spacing during plate lengthening

New dislocation

as plate

lengthens

High stress at the edges

(c)



Nucleation of D loops within the precipitate

(d) Vacancies can be attracted to coherent interfaces and 'condense' to form a prismatic dislocation loop which can expand across the precipitate

Contents for previous class

3.4 Interphase Interfaces in Solids

Interphase boundary - different two phases : different crystal structure different composition

Coherent, Perfect atomic matching at interface $\gamma \text{ (coherent)} = \gamma_{ch} \qquad \gamma \text{ (coherent)} \sim 200 \text{ mJM}^{-2}$ Semicoherent $\gamma \text{ (semicoherent)} = \gamma_{ch} + \gamma_{st} \qquad \gamma \text{ semi } \text{ of } \text{ of$

incoherent

- 1) $\delta > 0.25$ No possibility of good matching across the interface
- 2) different crystal structure (in general)

 γ (incoherent) ~ 500~1000 mJM⁻²

Complex Semicoherent Interfaces

Nishiyama-Wasserman (N-W) Relationship Kurdjumov-Sachs (K-S) Relationships

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(The only difference between these two is a rotation in the closest-packed planes of 5.26°.)

3.4 Interphase Interfaces in Solids

$$\sum A_i \gamma_i + \Delta G_S = minimum$$

Lowest total interfacial free energy

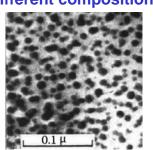
by optimizing the shape of the precipitate and its orientation relationship

Fully coherent precipitates

 γ_{ch}



different composition





$$AC = A u S^2 \cdot V$$
 (15 $v=4.12$)

Fully coherent precipitates

$$\gamma_{ch}$$
 + Volume Misfit $\Delta = \frac{\Delta V}{V}$

Chemical and structural interfacial E

$$\Delta G_S = 4\mu\delta^2 \cdot V$$
 (If v=1/3) $\iff \Delta G_S = \frac{2}{3}\mu\Delta^2 \cdot V \cdot f(c/a)$

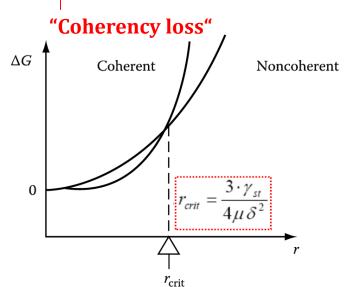
Incoherent inclusions

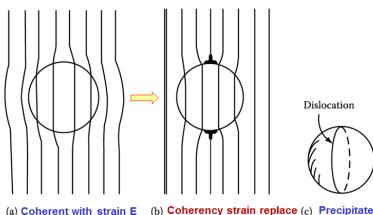
$$\Delta G(coherent) = 4\mu\delta^2 \cdot \frac{4}{3}\pi r^3 + 4\pi r^2 \cdot \gamma_{ch}$$

$\Delta G(non-coherent) = 4\pi r^{2} \cdot (\gamma_{ch} + \gamma_{st})$



Incoherent inclusions





by dislocation loop.

