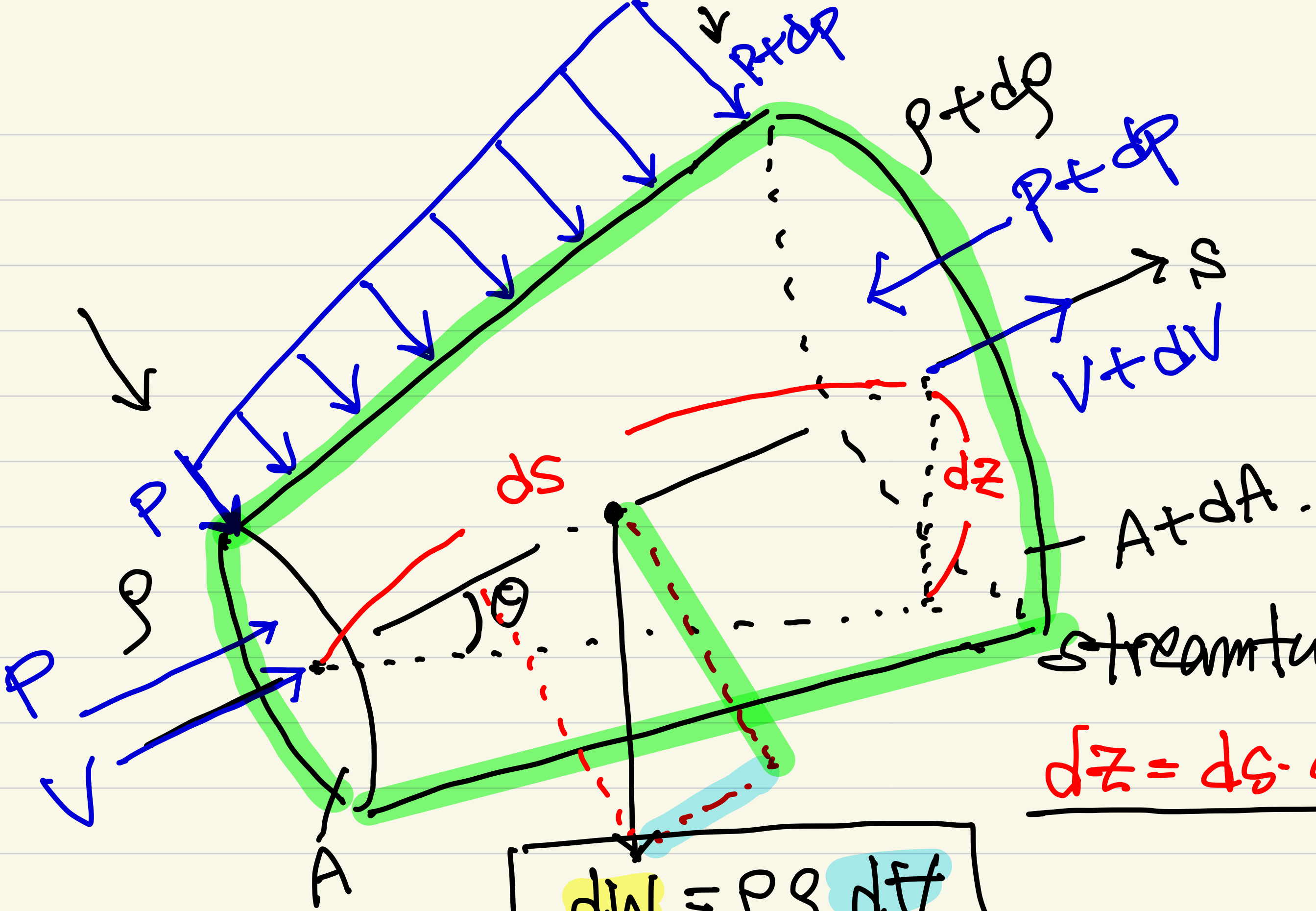


- $\tau = 0$ (frictionless)
- no heat, work.
- very small CV.

streamtube, CV, fixed.

$dz = ds \cdot \sin\theta$.

$\boxed{dW = \rho g \, d\forall}$ → Vol. of CV ($\doteq A \cdot ds$)

- mass conservation.

$$\frac{d}{dt}\int_{CV} \rho \, d\forall + \underbrace{\dot{m}_{out} - \dot{m}_{in}}_{} = 0.$$

$\dot{m}_{out} = \dot{m}_{in} + d\dot{m}$

미소변화량 → $d\dot{m} = d(\rho A V)$

$$\rightarrow \frac{\partial \rho}{\partial t} \cdot d\forall + d\dot{m} = 0.$$

$$d\dot{m} = d(\rho A V) = -\frac{\partial \rho}{\partial t} \cdot d\forall \simeq -\frac{\partial \rho}{\partial t} \cdot A \cdot ds.$$

- mtm conservation (in s-direction)

$$\sum F_s = \frac{\partial}{\partial t}\int_{CV} \rho V_s \, d\forall + \int_{CS} \rho V_s (\underline{V} \cdot \underline{n}) dA.$$

① gravity : $dF_{s,grav} = -dW \cdot \sin\theta$

$$= -\rho g A \cdot ds \cdot \sin\theta.$$

② pressure : $dF_{s,press} = P \cdot A - (P + dp)(A + dA) + (p + \frac{1}{2}dp) dA.$

$$\simeq -A \cdot dp.$$

③ ~~viscous stress.~~

$$\frac{\partial}{\partial t}\int_{CV} \rho\, V_s \, d\forall \simeq \frac{\partial}{\partial t}(\rho V_s)\cdot d\forall \simeq \frac{\partial}{\partial t}(\rho V_s)\cdot A\, ds$$

$$\int_{CS} \rho V_s (\underline{V}\cdot\underline{n})\, dA = (\dot{m}+d\dot{m})(V_s+dV_s) - \dot{m}V_s$$

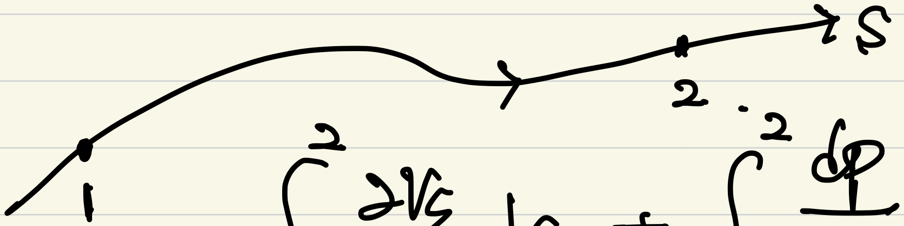
$$= \dot{m}\, dV_s + V_s\cdot d\dot{m}$$

$$\Rightarrow -\rho g A \underbrace{ds\cdot \sin\theta}_{=dz} - A\,dp = \underbrace{\frac{\partial}{\partial t}(\rho V_s) A\,ds}_{=\cancel{\frac{\partial \rho}{\partial t} V_s A\cdot ds} + \frac{\partial V_s}{\partial t}\cdot \rho\cdot A\cdot ds} + \dot{m}\,dV_s + \underbrace{V_s\cdot d\dot{m}}_{=\cancel{-\frac{\partial \rho}{\partial t} A ds\cdot V_s}}$$

$$\left(-A\,dp - \rho g A\,dz = \frac{\partial V_s}{\partial t}\rho A\,ds + \rho V_s\cdot A\cdot dV_s\right) \Big/ \rho A.$$

$$\Rightarrow \boxed{\frac{\partial V_s}{\partial t} ds + \frac{dp}{\rho} + V_s\cdot dV_s + g\,dz = 0}$$

Bernoulli eq. for unsteady frictionless flow along a streamline.



$$\int_1^2 \frac{\partial V_s}{\partial t}\, ds + \int_1^2 \frac{dp}{\rho} + \frac{1}{2}\left(V_2^2 - V_1^2\right) + g\left(z_2 - z_1\right) = 0$$

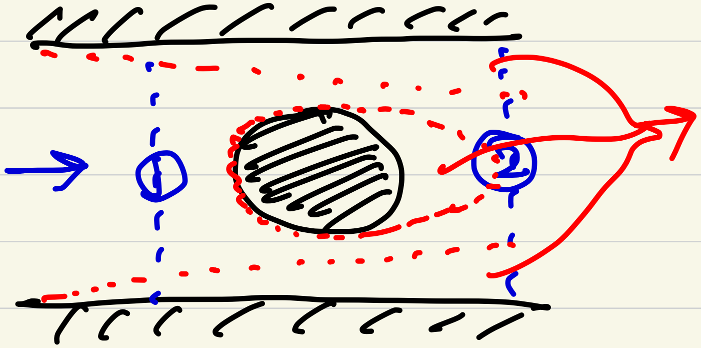
· for steady, incompressible flow

$$\frac{1}{\rho}\left(P_2 - P_1\right) + \frac{1}{2}\left(V_2^2 - V_1^2\right) + g\left(z_2 - z_1\right) = 0.$$

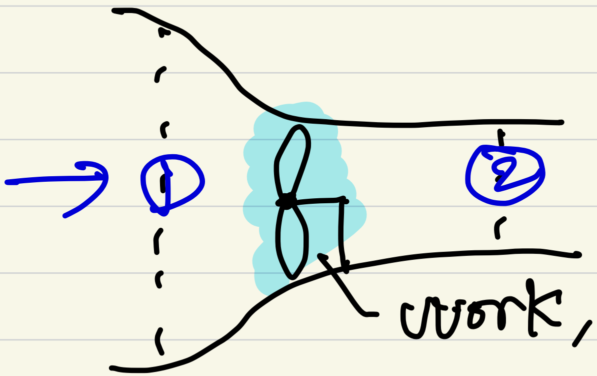
$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 + g z_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + g z_2 .$$

(Bernoulli eq. for steady, frictionless, incompressible flow along a streamline).

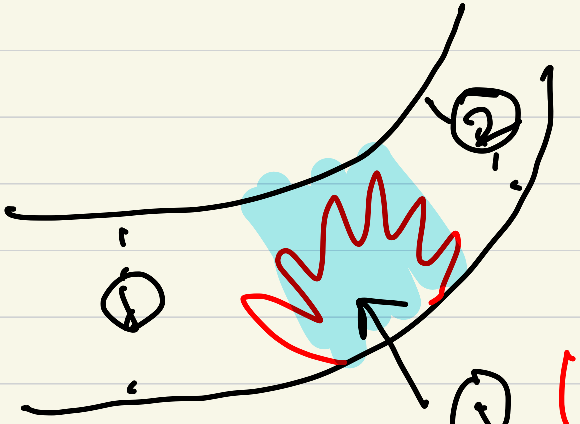
. We assumed no heat, no work, no friction.



viscous stress (friction)



work, viscous effect.



Q (heat).

$$\Rightarrow \frac{P_1}{\rho} + \frac{1}{2}V_1^2 + g z_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + g z_2 + \underbrace{W_s}_{\text{shaft work}} + \underbrace{W_f}_{\text{friction loss}} - \underbrace{W_q}_{\text{heat}}$$

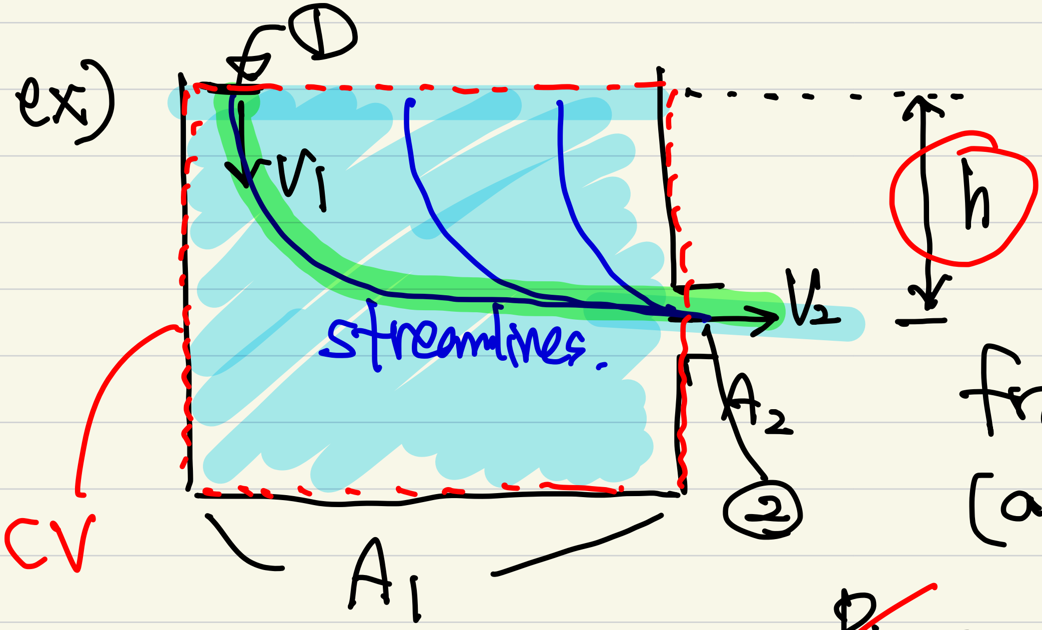
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \underbrace{h_s}_{\text{work head}} + h_f - \underbrace{h_q}_{\text{heat head}}$$

friction head
dimension of length.

for ideal conditions.

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant} = h_0 \text{ (total head)}$$

ex)



steady, frictionless.
$A_1 \gg A_2$, $V_2 = ?$.

from Bernoulli eq.
(along the streamline).

$$\cancel{\frac{p_1}{\rho}} + \frac{1}{2}V_1^2 + gz_1 = \cancel{\frac{p_2}{\rho}} + \frac{1}{2}V_2^2 + gz_2.$$

$z_1 - z_2 = h.$

$$\Rightarrow \frac{1}{2}V_1^2 + gh = \frac{1}{2}V_2^2$$

· from continuity. (mass conservation).

$$\cancel{\rho} A_1 V_1 = \cancel{\rho} A_2 V_2 . \rightarrow V_1 = \frac{A_2}{A_1} V_2$$

$$\therefore V_2^2 = \frac{2gh}{\cancel{1-(A_2/A_1)^2}} \simeq 2gh, \quad V_2 \simeq \sqrt{2gh}.$$

- How about the unsteady condition?

$$h = h(t), \quad V_2 = V_2(t).$$

$$\frac{\partial V_s}{\partial t} ds + \frac{dp}{\rho} + V_s dV_s + g\,dz = 0.$$

$$\rightarrow \int_1^2 \frac{\partial V_s}{\partial t} ds + \frac{1}{\rho}(\cancel{P_2 - P_1})_{\to 0} + \frac{1}{2}(V_2^2 - V_1^2) + g(\underbrace{z_2 - z_1}_{h}) = 0.$$

$$\simeq \int_1^2 \frac{\partial V_1}{\partial t} ds \doteqdot \frac{\partial V_1}{\partial t} \cdot h \qquad + \text{ mass conservation } (A_1 V_1 = A_2 V_2).$$

$$\Rightarrow \left\{ \begin{array}{l} 2h\dfrac{A_2}{A_1}\cdot\dfrac{dV_2}{dt} + V_2^2\left(1-\dfrac{A_2^2}{A_1^2}\right) = 2gh \\[2ex] \dfrac{dh}{dt} = -V_1 = -\dfrac{A_2}{A_1}V_2. \end{array} \right.$$

Coupled.

3.6. Angular momentum Theorem.

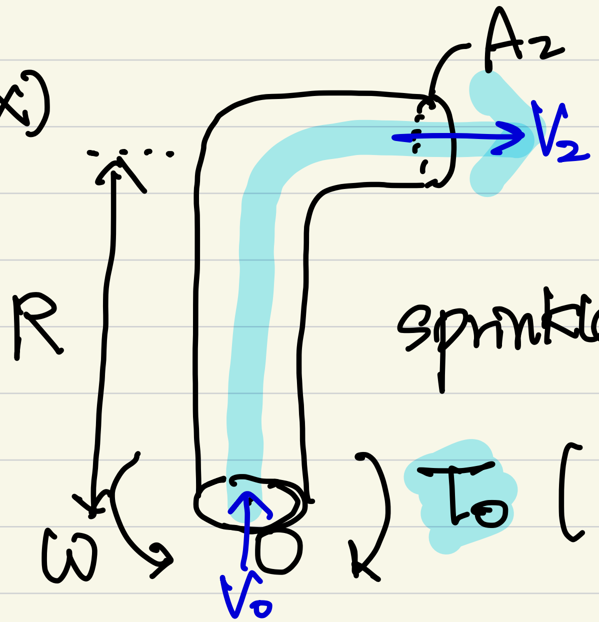
in $\underline{\underline{\text{RTT}}}$, $\underline{B} = \underset{\substack{\uparrow \\ \text{angular momentum vector.}}}{\underline{H}_o} = \int_{sys} (\underline{r}\times\underline{V})\,dm.$

$\underline{\beta} = \dfrac{dB}{dm} = \underline{r}\times\underline{V}.$

$$\underbrace{\left.\frac{dH_o}{dt}\right|_{sys}}_{\shortparallel} = \frac{d}{dt}\int_{cv}\rho(\underline{r}\times\underline{V})\,d\forall + \int_{cs}(\underline{r}\times\underline{V})\rho(\underline{V_r}\cdot\underline{n})\,dA.$$

$$\underbrace{\sum M_o}_{\text{moment}} = \sum (\underline{r} \times \underline{F})_o .$$

ex)



Top view

steady, $\omega = ?$.

sprinkler arm.

T_o (retarding torque due to friction, e.g. bearing)

$$\begin{cases} Q = V_0 A_2 \quad \text{or} \quad V_0 = Q/A_2 . \\ V_2 = V_0 - R\omega . \end{cases}$$

$$(+\circlearrowright) \sum M_o = T_o = \underline{\dot{m}} (\underline{r} \times \underline{V})_{out} - \cancel{\dot{m} (\underline{r} \times \underline{V})_{in}}^{\,0} .$$

$$= \rho Q R V_2 = \rho Q R (V_0 - R\omega) .$$

$$\therefore \omega = \frac{V_0}{R} - \frac{T_0}{\rho Q R^2}.$$

3.7 Energy equation.

$$B = E, \quad \beta = \frac{dB}{dm} = e = e_{internal} + e_{kinetic} + e_{potential} + \cdots$$

<u>RTT</u>

$$\Rightarrow \frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \rho e \, (\underline{V_r} \cdot \underline{n}) \, dA.$$

(heat transfer, applied FM, ...)

Ch. 4 Differential relations for fluid flow.

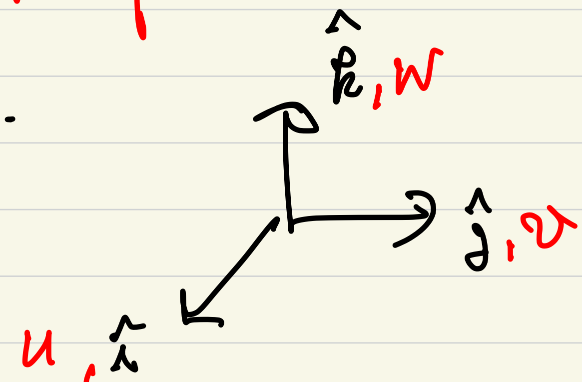
↳ infinitesimally small CV.

→ differential eq. (+ boundary / initial conditions)

⇓

Solutions ← velocity, pressure, Temperature,

4.1. Acceleration field of a fluid.



$$\underline{V} = u\hat{i} + v\hat{j} + w\hat{k}.$$

$u = u(x, y, z, t)$.

$v = v(x, y, z, t)$.

$w = w(x, y, z, t)$.

$$\underline{a} = \frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}}{\partial t} + \frac{\partial \underline{V}}{\partial x}\overset{u}{\frac{\partial x}{\partial t}} + \frac{\partial \underline{V}}{\partial y}\overset{v}{\frac{\partial y}{\partial t}} + \frac{\partial \underline{V}}{\partial z}\overset{w}{\frac{\partial z}{\partial t}}.$$

↳ material (total) derivative.

$$= \frac{\partial \underline{V}}{\partial t} + \boxed{u\frac{\partial \underline{V}}{\partial x} + v\frac{\partial \underline{V}}{\partial y} + w\frac{\partial \underline{V}}{\partial z}}.$$

$\underline{V} = (u, v, w)$

$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$\underline{V}\cdot\nabla = \left(u\frac{\partial}{\partial x}, v\frac{\partial}{\partial y}, w\frac{\partial}{\partial z}\right)$

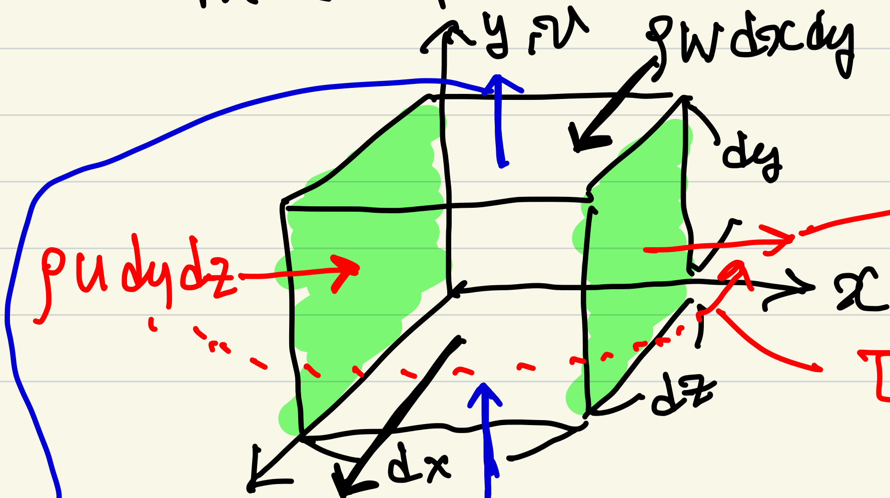
$$\Rightarrow = \frac{\partial \underline{V}}{\partial t} + \boxed{(\underline{V}\cdot\nabla)\underline{V}}.$$

local acceleration. (@ steady, zero)

convective acceleration.

4.2 Differential eq. of mass conservation.

· in cartesian coordinate.



Taylor series expansion.

$$0 = \frac{dm}{dt} = \underline{\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall} + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in}$$

$$\simeq \frac{\partial \rho}{\partial t} \cdot dx\,dy\,dz$$

(*)

z, w $\quad \rho v \, dz \, dx$

$$\rho w \, dx \, dy + \frac{\partial}{\partial z}(\rho w \, dx \, dy) \, dz \qquad \rho u \, dy \, dz + \frac{\partial}{\partial x}(\rho u \, dy \, dz) \, dx + \cancel{HOT} \to 0$$

$$\rho v \, dz \, dx + \frac{\partial}{\partial y}(\rho v \, dz \, dx) \, dy .$$

$$\circledast : 0 = \frac{\partial \rho}{\partial t} dx \, dy \, dz$$
$$+ \cancel{\rho u \, dy \, dz} + \frac{\partial}{\partial x}(\rho u \, dy \, dz) \, dx$$
$$+ \cancel{\rho v \, dx \, dz} + \frac{\partial}{\partial y}(\rho v \, dx \, dz) \, dy$$
$$+ \cancel{\rho w \, dx \, dy} + \frac{\partial}{\partial z}(\rho w \, dx \, dy) \, dz$$
$$- \cancel{\rho u \, dy \, dz} - \cancel{\rho v \, dx \, dz} - \cancel{\rho w \, dx \, dy}$$

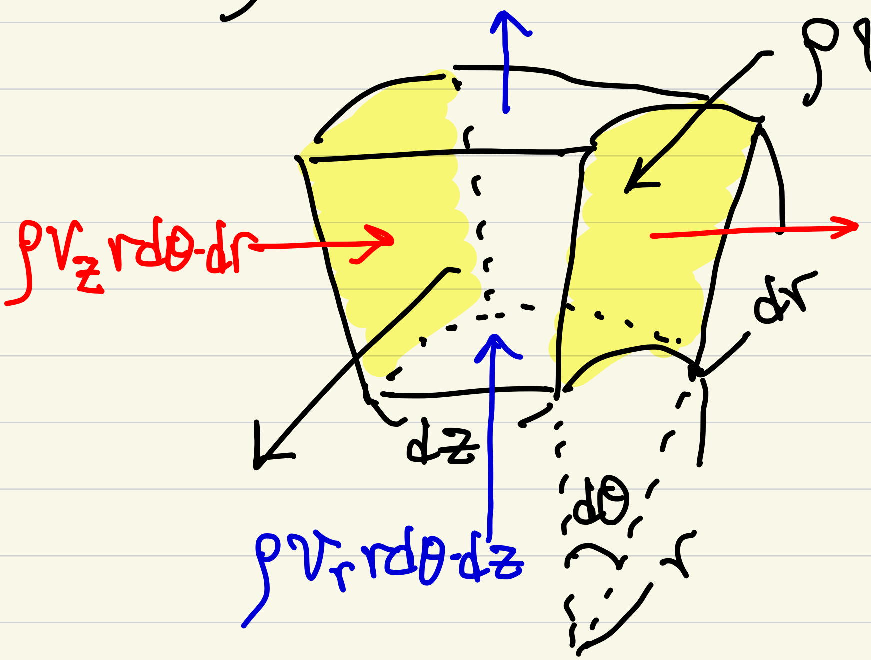
$$\therefore \boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

: continuity eq. (연속방정식)

$\Downarrow$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{V}) = 0 \leftarrow$$

in cylindrical coordinate (r, θ, z).



$$\Rightarrow \frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r \rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0.$$

- steady flow $\left(\frac{\partial}{\partial t}(\cdot) = 0\right)$: $\nabla \cdot (\rho \underline{V}) = 0$.
- incompressible flow (ρ = constant) : $\nabla \cdot \underline{V} = 0$.

$$\Downarrow$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

$$\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0\right)$$

condition for incompressibility:

$$\frac{\partial}{\partial x}(\rho u) = \rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} \Rightarrow \left|\rho \frac{\partial u}{\partial x}\right| \gg \left|u \frac{\partial\rho}{\partial x}\right|$$

미소변화량

$$\left|\rho \frac{\delta u}{\cancel{\delta x}}\right| \gg \left|u\frac{\delta\rho}{\cancel{\delta x}}\right| \rightarrow \left|\frac{\delta u}{u}\right| \gg \left|\frac{\delta\rho}{\rho}\right| \quad ①$$

· speed of sound, (a).

$$a^2 \equiv \frac{\partial p}{\partial \rho} \ (\text{ch. 9}) \rightarrow \delta p \simeq a^2 \cdot \delta\rho .$$

Bernoulli eq.: $\frac{dp}{\rho} + V\,dV = 0$.

대입.

$$a^2\,\delta\rho \simeq -\rho V\,dV \quad ②$$

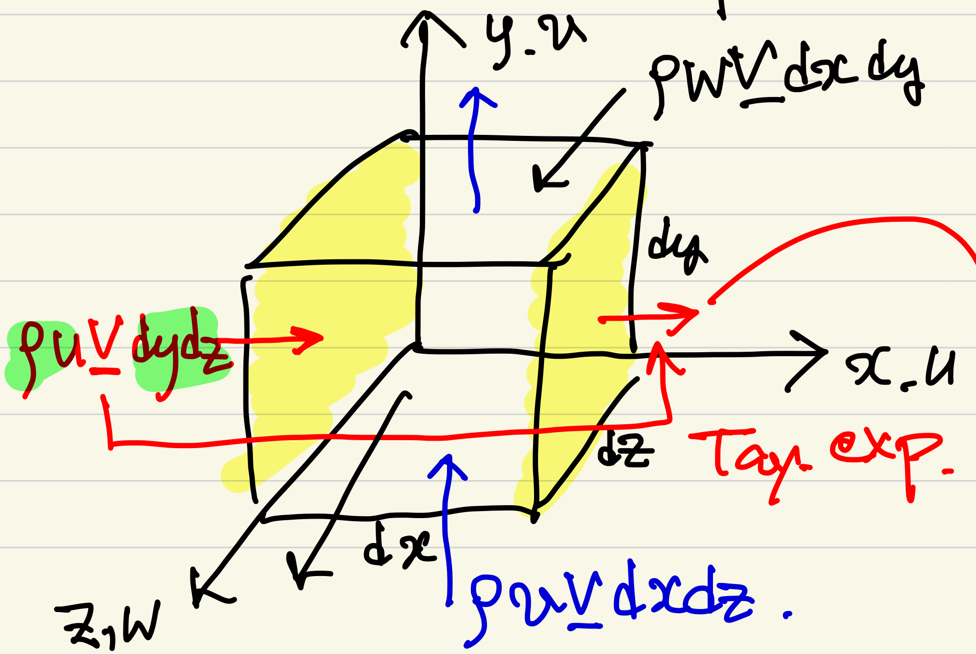
$$①+②:\ \left|\frac{\delta\rho}{\rho}\right| = \left|\frac{V}{a^2}\cancel{\delta V}\right| \ll \left|\frac{\cancel{\delta V}}{V}\right|.$$

$$\therefore\ \frac{V^2}{a^2} \ll 1. \quad \frac{V}{a}\ (\equiv Ma) \ll 1.$$

if $Ma < 0.3$, incompressible in practical.

(for air, $V \lesssim 100\ m/s$,
liquid, mostly)

4.3 Differential eq. of linear momentum.



$$\Sigma \underline{F} = \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho \underline{V}\, d\forall} + \underbrace{\int_{CS} \rho \underline{V}(\underline{V}_r \cdot \underline{n})\,dA}.$$

$$\simeq \frac{\partial}{\partial t}(\rho \underline{V})\,dxdydz$$

$$\rho u \underline{V}\,dydz + \frac{\partial}{\partial x}(\rho u \underline{V}\,dydz)\cdot dx.$$

$$\therefore \Sigma \underline{F} = dx\,dy\,dz\left[\frac{\partial}{\partial t}(\rho \underline{V}) + \frac{\partial}{\partial x}(\rho u \underline{V}) + \frac{\partial}{\partial y}(\rho v \underline{V}) + \frac{\partial}{\partial z}(\rho w \underline{V})\right]$$

$$= \underline{V}\left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right)$$

$$+ \left(\rho\frac{\partial V}{\partial t} + \rho u\frac{\partial V}{\partial x} + \rho v\frac{\partial V}{\partial y} + \rho w\frac{\partial V}{\partial z}\right)$$

$$\Sigma \underline{F} = \rho\left(\frac{\partial \underline{V}}{\partial t} + (\underline{V}\cdot\nabla)\underline{V}\right) dx\,dy\,dz.$$

$$= \rho\frac{DV}{Dt}\,dx\,dy\,dz.$$

$\frac{D}{Dt}$: material derivative.

$\underline{F} = ?$ body force, surface force.

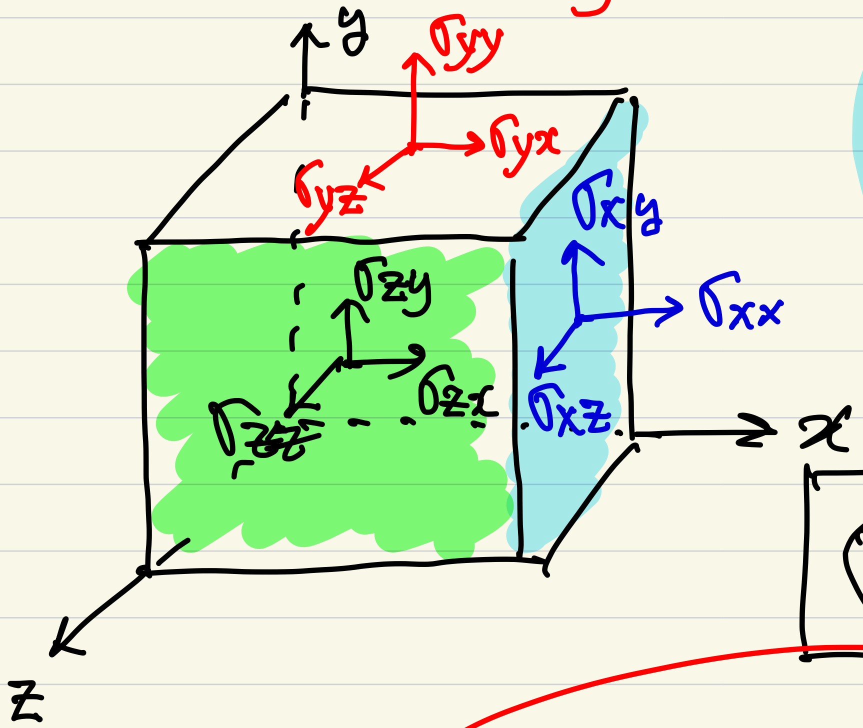
· gravity : $d\underline{F}_{grav} = \rho \underline{g}\,dx\,dy\,dz$, $\underline{g} = -g\hat{k}$

$$= -\rho g \hat{k}\, dx\,dy\,dz.$$

· magnetic force, electric potential,

stresses on the sides of CV. : σ_{ij}

stress in j-direction on a face normal to i-axis



· $p \equiv -\frac{1}{3}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})$.

hydrostatic pressure

@ non-zero velocity cond.

$$\boxed{\sigma_{ij} = -p\,\delta_{ij} + \tau_{ij}}$$

$(i, j = x, y, z)$

hydrostatic pressure

viscous stress.

(kronecker delta) $\delta_{ij} = \begin{cases} 1 & (i=j) \\ 0 & (i \neq j) \end{cases}$

$$\sigma_{ij} = \begin{bmatrix} -P+\tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & -P+\tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & -P+\tau_{zz} \end{bmatrix}.$$