*Single Fluid Equation $\quad (n_i \approx n_e \approx n)$

$\rho \equiv n_i M_i + n_e m_e \approx n(M+m)$; mass density

$\sigma \equiv e(n_i - n_e)$; charge density $\neq 0$ (plasma approximation)

$$\vec{v} \equiv \frac{n_i M \vec{u}_i + n_e m \vec{u}_e}{n_i M + n_e m} = \frac{n_i M \vec{u}_i + n_e m \vec{u}_e}{\rho} \approx \frac{M\vec{u}_i + m\vec{u}_e}{M+m}$$; mass velocity

$$\vec{J} = n_i e \vec{u}_i - n_e e \vec{u}_e \approx ne(\vec{u}_i - \vec{u}_e)$$; current density

$$\vec{u}_i = \vec{u}_e + \frac{\vec{J}}{ne}, \quad (M+m)\vec{v} = M\vec{u}_i + m\vec{u}_e \rightarrow \vec{u}_e = -\frac{M}{m}\vec{u}_i + \frac{M+m}{m}\vec{v}$$

$$\therefore \vec{u}_i = -\frac{M}{m}\vec{u}_i + \frac{M+m}{m}\vec{v} + \frac{\vec{J}}{ne} \Rightarrow \vec{u}_i = \vec{v} + \frac{m}{m+M}\frac{\vec{J}}{ne} \approx \vec{v} + \frac{m}{M}\frac{\vec{J}}{ne}$$

as the same way $\vec{u}_e = \vec{v} - \frac{M}{M+m}\frac{\vec{J}}{ne} \approx \vec{v} - \frac{\vec{J}}{ne}$

$$\frac{\partial}{\partial t} n_i + \nabla\cdot(n_i \vec{u}_i) = 0 \quad \times M$$

$$\frac{\partial}{\partial t} n_e + \nabla\cdot(n_e \vec{u}_e) = 0 \quad \times m$$

$$\rightarrow \frac{\partial}{\partial t}(M n_i + m n_e) + \nabla\cdot(M n_i \vec{u}_i + m n_e \vec{u}_e) = 0$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\vec{v}) = 0}$$

$\times e$, $\times(-e)$

$$\rightarrow \frac{\partial}{\partial t}[e(n_i - n_e)] + \nabla\cdot(e n_i \vec{u}_i - e n_e \vec{u}_e) = 0 \Rightarrow \boxed{\frac{\partial}{\partial t}\sigma + \nabla\cdot\vec{j} = 0}$$ charge continuity equation

$$nM\frac{d\vec{u}_i}{dt} = ne(\vec{E} + \vec{u}_i \times \vec{B}) - \nabla P_i + \vec{R}_{ie}$$

$$nm\frac{d\vec{u}_e}{dt} = -ne(\vec{E} + \vec{u}_e \times \vec{B}) - \nabla P_e + \vec{R}_{ei}$$

$$\rightarrow n\frac{d}{dt}(M\vec{u}_i + m\vec{u}_e) = e(n_i - n_e)\vec{E} + e(n_i\vec{u}_i - n_e\vec{u}_e)\times\vec{B} - \nabla P$$

$$\Rightarrow n(M+m)\frac{d\vec{v}}{dt} = \boxed{\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{j}\times\vec{B} - \nabla P}$$

*Electron inertia neglected.

$$nm\frac{d\vec{u}_e}{dt} = 0 = -ne(\vec{E} + \vec{u}_e \times \vec{B}) - \nabla P_e + \vec{R}_{ei} \Rightarrow \vec{E} + \vec{u}_e \times \vec{B} = -\frac{\nabla P_e}{ne} + \frac{\vec{R}_{ei}}{ne}$$

$$\Rightarrow \vec{E} + \left(\vec{v} - \frac{\vec{j}}{ne}\right)\times\vec{B} = -\frac{\nabla P_e}{ne} - \frac{n m \nu_{ei}(\vec{u}_e - \vec{u}_i)}{ne}$$

$$\left(\begin{aligned} &\vec{u}_e = \vec{v} - \frac{\vec{j}}{ne} \\ &\vec{R}_{ei} = -nm\nu_{ei}(\vec{u}_e - \vec{u}_i) \\ &\quad = -n\eta ne^2(\vec{u}_e - \vec{u}_i) = \eta ne\vec{j} \end{aligned}\right)$$

$$\Rightarrow \boxed{\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j} + \frac{\vec{j}\times\vec{B} - \nabla P_e}{ne}}$$: generalized Ohm's Law.

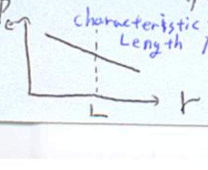
$$\frac{\nabla P_e / ne}{\vec{v}\times\vec{B}} \approx \frac{\frac{P_e}{Lne}}{v_{ti}B} \approx \frac{\frac{nT_e}{Lne}}{v_{ti}B} \approx \frac{\frac{Mv_{ti}^2}{Le}}{v_{ti}B} \approx \frac{\frac{Mv_{ti}}{Be}}{L} \approx \frac{\rho_i}{L} \ll 1$$

ion의 Lamor radius

assumption)

$\vec{v} = v_{ti}$ (thermal ion)

$T_i = T_e$ (temperature)



characteristic Length / $\nabla P_e \sim \frac{+P_e}{L}$

$$\vec{E} + \vec{v}\times\vec{B} = \eta\vec{j} + \frac{\cancel{\vec{j}\times\vec{B}}^{\,0} - \cancel{\nabla P_e}^{\,0}}{ne}$$ order낮음

∇P_e와 order 비슷

$\cancel{\eta\vec{j}}\rightarrow 0$ 인경우 = ideal MHD eq-n

$\varepsilon_0 = 0 \Leftrightarrow c \rightarrow \infty$

Single Fluid Equation

Complete Form (MHD eq-n)

Magneto hydro dynamics (자기유체역학)

$$\frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\vec{v}) = 0, \quad \cancel{\frac{\partial}{\partial t}\sigma}^{\,0} + \nabla\cdot\vec{j} = 0$$

$$\rho\frac{d\vec{v}}{dt} = \sigma\vec{E} + \vec{j}\times\vec{B} - \nabla P$$

$$\vec{E} + \vec{v}\times\vec{B} = \cancel{\eta\vec{j}}^{\,0}, \quad \frac{d}{dt}\left(\frac{\rho^\gamma}{P}\right) = 0$$

$$\cancel{\varepsilon_0}^{\,0}\nabla\cdot\vec{E} = \cancel{\sigma}^{\,0}, \quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0, \quad \nabla\times\vec{B} = \mu_0\vec{j} + \cancel{\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}^{\,0}$$