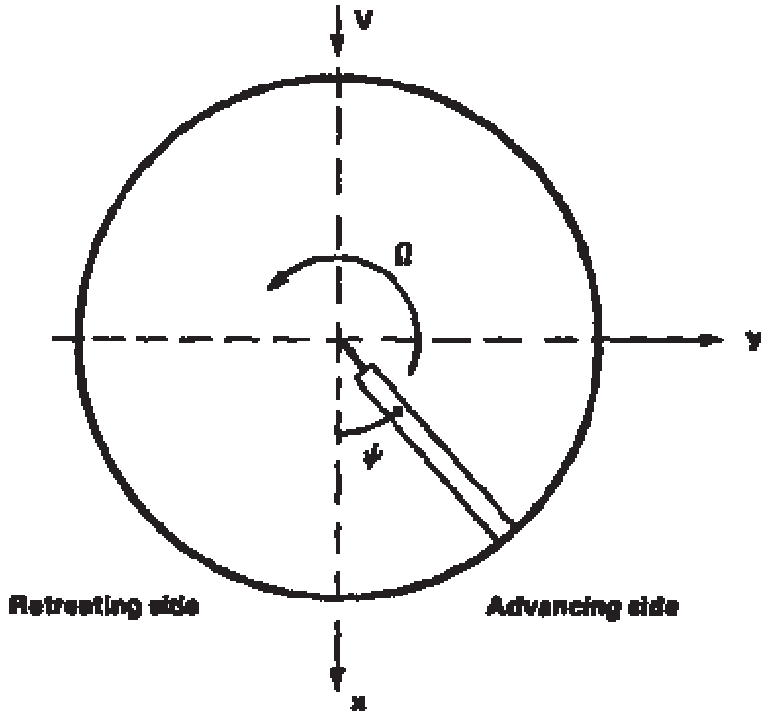


Typically, the value of κ_x is positive and the value of κ_y is negative. This means that the induced inflow is larger at the rear of the disk and on the retreating side. At higher velocities,

$$\lambda_i \simeq \lambda_m(1 + \frac{r}{R}\cos\psi)$$

The classical vortex theory gives an estimate of parameters κ_x and κ_y. There are a number of estimates available for these parameters. A popular one is given by Drees (1949)

$$\kappa_x = \frac{4}{3}[(1 - 1.8\mu^2)\sqrt{1 + (\lambda/\mu)^2} - \lambda/\mu]$$

$$\kappa_y = -2\mu$$

Dynamic inflow in forward flight is lot more involved than the hover case. A simple dynamic inflow model is to assume a perturbation to the induced inflow of the following form

$$\Delta\lambda_i = \lambda_u + \lambda_{1c}\frac{r}{R}\cos\psi + \lambda_{1s}\frac{r}{R}\sin\psi$$

where λ_u represents the uniform perturbation component and λ_{1c} and λ_{1s} represent the linear varying components in longitudinal and lateral directions over the rotor disk. These dynamic inflow components are related to the perturbation forces on the rotor disk, namely rotor thrust C_T, pitching moment C_{M_y} and rolling moment C_{M_x}. The perturbation forces are obtained from total forces after subtracting the steady forces. A simple form of relationships between dynamic inflow components and perturbation forces is obtained using actuator disk theory.

$$\begin{bmatrix} \tau_T & \dot{\lambda}_u \\ \tau_M & \dot{\lambda}_c \\ \tau_M & \dot{\lambda}_s \end{bmatrix} + \begin{bmatrix} \lambda_u \\ \lambda_{1c} \\ \lambda_{1s} \end{bmatrix} = \begin{bmatrix} \frac{1}{2(\lambda_0+\sqrt{\mu^2+\lambda_0^2})} & 0 & 0 \\ 0 & \frac{2}{\sqrt{\mu^2+\lambda_0^2}} & 0 \\ 0 & 0 & \frac{2}{\sqrt{\mu^2+\lambda_0^2}} \end{bmatrix} \begin{bmatrix} \Delta C_T \\ -\Delta C_{M_y} \\ \Delta C_{M_x} \end{bmatrix}$$

where τ_T and τ_M are time lags in seconds and these are approximately taken as

$$\tau_T = \frac{.42}{\mu\Omega}$$

$$\tau_M = \frac{.22}{\mu\Omega}$$

The λ_0 is the rotor steady inflow ($\lambda_m + \mu \tan\alpha$) and the Ω is the rotational speed (rad/sec).

An alternate form of dynamic inflow is given by Pitt and Peters (1980).

$$[M]\begin{bmatrix} \dot{\lambda}_u \\ \dot{\lambda}_c \\ \dot{\lambda}_s \end{bmatrix} + [L]^{-1}\begin{bmatrix} \lambda_u \\ \lambda_c \\ \lambda_s \end{bmatrix} = \begin{bmatrix} c_T \\ -C_{M_y} \\ C_{M_x} \end{bmatrix}$$

The matrices M and L are of size 3 x 3. There are many forms of these matrices; typically for simple momentum theory, these matrices are diagonal, for other theories they can be fully populated. One of the popular form of these matrices is

$$[L] = \frac{1}{c_v}\begin{bmatrix} \frac{1}{2} & 0 & \frac{15\pi}{64}\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} \\ 0 & \frac{-4}{1+\sin\alpha} & 0 \\ \frac{15\pi}{64}\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} & 0 & -\frac{-4}{1+\sin\alpha} \end{bmatrix}$$

$$[M] = \begin{bmatrix} \frac{128}{75\pi} & 0 & 0 \\ 0 & \frac{-16}{45\pi} & 0 \\ 0 & 0 & \frac{-16}{45\pi} \end{bmatrix}$$

where c_v is a mass-flow parameter

$$c_v = \frac{\mu^2 + \lambda(\lambda + \lambda_i)}{\sqrt{\mu^2 + \lambda^2}}$$

and α is the rotor disk tilt wrt free stream.

For dynamic analysis of the blade, the dynamic inflow components are treaded as additional degrees of freedom. The dynamic inflow models are well suited for aero-elastic stability calculations. For loads prediction a free wake based unsteady lifting line model or detailed CFD analysis is prefered. These are discussed in the next section.

Questions

Justify the following:

- The unsteady forces are more involved for a rotary wing than a fixed wing.
- Show the similarities and differences between the basic fluid mechanics equations and the basic structural mechanics equations.
- Virtual aerodynamic forces play an important role in the pitch dynamics of the blade.
- In a wind tunnel testing, the unsteady aerodynamic forces on a two-dimensional wing model were measured for a pure pitch motion as well as for a pure vertical vibratory motion. The discrepancy in the two sets of results was observed for identical angle of attack perturbation.
- The neglecting of the effect of shed wake and other unsteady aerodynamic forces on the analysis of rotor performance is quite justified, but for higher frequency vibrations one cannot ignore these forces.
- During the forward flight mode, there is a continuous stretching and compressing of the vorticity in the shed wake.
- One has to be very careful to include the effect of shed vorticity for higher harmonic vibrations.
- Is there any difference between the induced velocities calculated using the momentum theory and the lifting line theory?
- For blade aeroelastic analysis (flap-lag), quasi-steady aerodynamics is widely used.
- The shed wake plays a mroe important role in hovering flight than the forward flight.
- There are differences between the thin airfoil theory, lifting line theory, lifting surface theory and the rotor shed wake modeling.
- The larger the reversed flow region on the retreating side of the rotor, the more the vibration.
- In a circulation-controlled rotor blade, the steady lift is primarily caused by blowing circulation, causing the aerodynamic center to be close to the half-chord position. To reduce the aerodynamic moment, the elastic axis is positioned at half-chord, but this may result in an unstable torsional motion (single degree flutter).
- The Theodoresen function C(k) is referred to as a feedback parameter of blade motion.
- There is no Kutta condition for circulation control airfoils.
- Dynamic inflow modeling is an approximate representation for unsteady rotor forces.

Bibliography

[1] John, D. Anderson, Modern Compressible Flow, 1990, McGraw-Hill.

[2] Karamcheti, K., Principles of Ideal-Fluid Aerodynamics, Robert E. Krieger Publishing Co., 1980, Chapter 17 and 19.

[3] AShley, H. and Landahl, M., Aerodynamics of Wings and Bodies, Addison-Wesley Publishing Company, Inc., 1965.

[4] Donovan, A. F., and Lawrance, H. R., Aerodynamic Components of Aircraft At High Speeds, Princeton University Press, 1957. Volume VII. High Speed Aerodynamics and Jet Propulsion. Part F, Non-Steady Wing Characteristics by I. E. Garrick, pp. 658–778.

[5] Lamb, Sir Horace, Hydrodynamics, Sixth Edition, Dover Publications, New York, 1945.

[6] Milne-Thompson, L. M., Theoretical Hydrodynamics, Fourth Edition, Macmillan, New York, 1960.

[7] Thwaites, B., editor, Incompressible Aerodynamics, Oxford University Press, Oxford, 1960.

[8] Leishman, J. G., *Priciples of Helicopter Aerodynamics*, Cambridge University Press, New York, 2000

[9] Beddoes, T. S., "Representation of Airfoil Behaviours," *Vertica* Vol. 7, No. 2, pp. 183-197, 1983.

[10] T.S.Beddoes, "Practical Computation of Unsteady Lift," Proceedings of the 7th European Rotorcraft Forum, Sept. 1982, see also *Vertica*, Vol. 8, No. 1, 1984, pp. 55-71.

[11] Leishman, J.G., "Validation of Approximate Indicial Aerodynamic Functions for Two-Dimensional Subsonic Flow," *Journal of Aircraft*, Vol. 25, No. 1, October 1, 1988.

[12] Leishman, J. G. and Beddoes, T. S., "A Semi-Empirical Model for Dynamic Stall," *Journal of the American Helicopter Society*, July 1989, pp. 3-17.

[13] Leishman, J. G., "Modeling Sweep Effects on Dynamic Stall," *Journal of the American Helicopter Society*, July 1989, pp. 18-29.

[14] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison-Wesley Publishing Co., Reading, Mass., 1955.

[15] Lomax, H., Heaslet, M. A., Fuller, F. B., and Sludder, L., "Two and Three Dimensional Unsteady Lift Problems in High Speed Flight," NACA Report 1077, 1952.

[16] Liiva, J., "Unsteady Aerodynamic and Stall Effects on Helicopter Rotor Blade Airfoil Sections," *Journal of Aircraft*, Vol. 6, (1), January-February 1969, pp. 46–51.

[17] Carta, F. O., “Unsteady Normal Force on an Airfoil in a Periodically Stalled Inlet Flow,” *Journal of Aircraft*, Vol. 4, (5), September-October 1967, pp. 416–421.

[18] Gormont, R. E., “A Mathematical Model of Unsteady Aerodynamics and Radial Flow for Application to Helicopter Rotors,” USA AVLABS TR 72-67, May 1970.

[19] Johnson, W., “The Response and Airloading of Helicopter Rotor Blades Due to Dynamic Stall,” ASRL TR 130-1, May 1970

[20] Petot, D., “Differential Equation Modeling of Dynamic Stall,” La Recherche Aerospatiale, Number 1989-5 (Corrections dated October 1990).

[21] Truong, V. K., “A 2-D Dynamic Stall Model Based on a Hopf Bifurcation,” Nineteenth European Rotorcraft Forum, Marseiles, France, September, 1998.

[22] Johnson, W., “Rotorcraft Aerodynamics Models for a Comprehensive Analysis,” 54th Annual Forum of the American Helicopter Society, Washington DC, May 20-22, 1998.

[23] Johnson, W., Helicopter Theory, Princeton University Press, 1980, Chapter 10 and 11.

[24] Woods, L. C., *The Theory of Subsonic Plane Flow*, Cambridge University Press, 1961.

[25] J. Weissinger, “The Lift Distribution of Swept-Back Wings,” National Advisory Committee for Aeronautics, Technical Memorandum No. 1120, 1942.

[26] Johnson, W., “Application of Unsteady Airfoil Theory to Rotary Wings,” Journal of Aircraft, Vol. 17, No. 4, April 1980, pp. 285-286.

[27] Pitt, D.M., and Peters, D. A., “Theoretical Prediction of Dynamic-inflow Derivatives,” Vertica, Vol. 5, 1981, pp. 21-34.

[28] Johnson, W., “A Comprehensive Analytical Model of Rotorcraft Aerodynamics and Dynamics, Part I: Analysis Development,” NASA TM 81182, June 1980.

Chapter 5

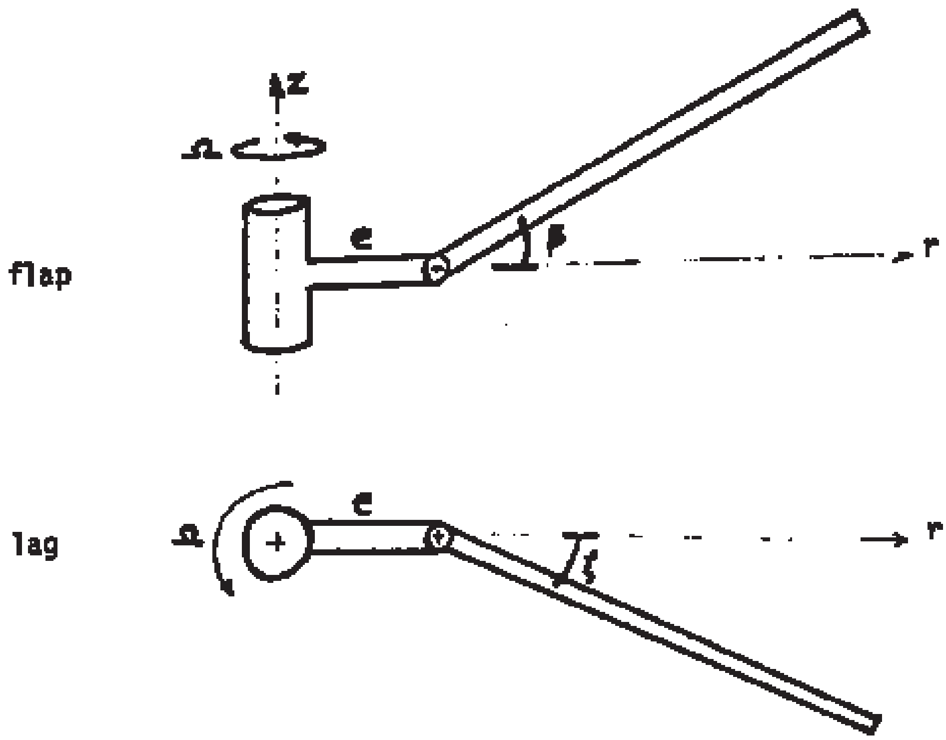
Aeroelastic Stability in Hover

Flutter is a dynamic aeroelastic instability caused by the interaction of aerodynamic, elastic and inertial forces. Flutter is self-sustained oscillations which are totally different from resonance or forced response oscillations. For flutter instability, the external forcing is not needed. The objective of this chapter is to understand the basic principles of blade flutter in hovering flight. The equations of motion are simple as compared to the forward flight case, where many periodic terms are present. The time and response solutions for forward flight are very involved as compared to those of hovering flight. For hover, it is relatively easy to determine the equilibrium position of the blade and then to determine a linearized stability analysis. There are many types of flutter. Two of the most important types of blade flutter are flap-lag and pitch-flap. The designer has to be very careful with these instabilities and has to establish the safety margin for critical flight conditions. These aeroelastic instablities will be investigated in detail for simple blade configurations, with two-degree-of-freedom models. One can interpret these results for complex configurations and then refine these analyses.

5.1 Flag-Lag Flutter

This aeroelastic instability is unique with rotor blades, and does not take place in fixed wings. The flap and lag modes participate in causing this instability, of course, with the inclusion of unsteady aerodynamic forces. The flap mode alone is highly damped because of aerodynamic damping. The lag mode alone is a low damped mode, but does not become unstable. The flap and lag modes together are coupled, and the couplings are due to the Coriolis forces and aerodynamic forces. There is no likelihood of blade flutter if the aerodynamic forces are neglected. Again, there is no likelihood of flutter if Coriolis forces are neglected. Hence, for blade flutter, both the aerodynamic forces and the Coriolis forces play an important role.

To understand the phenomena, a simple blade configuration is studied in hovering flight. The blade is assumed rigid and it undergoes two degrees of motion, flap and lag motions about hinges. The flap and lag hinges are coincided and are offset by a distance e from the rotation axis. Also, there are bending springs at the hinges to obtain desired flap and lag frequencies.



The equations of motion in nondimensional form are,

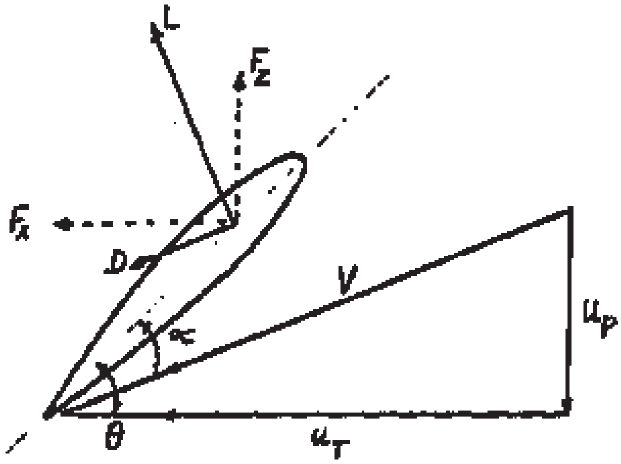
$$\text{Flap:} \quad \overset{**}{\beta} + \nu_\beta^2 \beta - 2\beta_0 \overset{*}{\zeta} = \gamma \overline{M}_\beta$$

$$\text{Lag:} \quad \overset{**}{\zeta} + \nu_\zeta^2 \zeta + 2\frac{\omega_{\zeta 0}}{\Omega} \zeta_L \overset{*}{\zeta} + 2\beta_0 \overset{*}{\beta} = \gamma \overline{M}_\zeta \tag{5.1}$$

These are perturbation equations. The ν_β and ν_ζ are rotating flap and lag frequencies, the ζ_L is the viscous damping coefficient due to lag damper, and β_0 is the steady flap deflection due to centrifugal and aerodynamic forces.

$$\nu_\beta^2 \beta_0 = \gamma \overline{M}_{\beta 0} + \frac{\omega_{\beta 0}^2}{\Omega^2} \beta_p$$

where β_p is a precone angle. The $\overline{M}_\beta$ and $\overline{M}_\zeta$ are perturbation aerodynamic moments about flap and lag hinges and γ is the Lock number. The $\omega_{\beta 0}$ and $\omega_{\zeta 0}$ are non-rotating flap and lag frequencies. Sometimes the viscous damping ratio of mechanical lag damper is defined with respect to the rotating lag frequency. Then the third term in Eq(5.1) becomes $2\zeta_L \nu_\zeta \overset{*}{\zeta}$. Let us derive aerodynamic forces.



Quasisteady theory is used to obtain the aerodynamic forces. The u_p and u_T are flow velocity components. Forces per unit span are

$$F_z \simeq L = \frac{1}{2}\rho ac(u_T^2\theta - u_p u_T)$$

$$F_x \simeq L\frac{u_p}{u_T} + D = \frac{1}{2}\rho ac\left(\frac{c_d}{a}u_T^2 + u_p u_T\theta - u_p^2\right)$$

Perturbations

$$F_z = \frac{1}{2}\rho ac\{\delta u_T(2u_T\theta - u_p) + \delta u_p(-u_T) + \delta\theta(u_T^2)\}$$

$$F_x = \frac{1}{2}\rho ac\{\delta u_T(2\frac{c_d}{a}u_T + u_p\theta) + \delta u_p(u_T\theta - 2u_p) + \delta\theta(u_p u_T)\}$$

Let us examine flow components

Steady:

$$u_T = \Omega r \qquad \frac{u_T}{\Omega R} = x$$

$$u_p = \Omega\lambda R \qquad \frac{u_p}{\Omega R} = \lambda$$

Perturbation:

$$\delta u_T = -r\dot{\zeta} \qquad \frac{\delta u_T}{\Omega R} = -x\overset{*}{\zeta}$$

$$\delta u_p = r\dot{\beta} \qquad \frac{\delta u_p}{\Omega R} = x\overset{*}{\beta}$$

$$\delta\theta = -k_{p_\beta}\beta - k_{p_\zeta}\zeta$$

where x = r/R, and λ is the wake induced inflow parameter at the rotor disk. The k_{p_β} and k_{p_ζ} are pitch-flap and pitch-lag coupling terms.

$$\overline{M}_\beta = \frac{1}{\rho ac\Omega^2 R^4}\int_e^R F_z(r-e)\,dr$$

For making analysis simple, the effect of e is neglected in the derivation of aerodynamic forces. This is, however, not a bad assumption.

$$\overline{M}_\beta = \frac{1}{\rho ac\Omega^2 R^4}\int_0^R F_z r\,dr$$

$$\overline{M}_\zeta = \frac{1}{\rho ac\Omega^2 R^4}\int_0^R F_x r\,dr$$

$$(\overline{M}_\beta)_{steady} = \frac{1}{2}\int_0^1 \left\{\left(\frac{u_T}{\Omega R}\right)^2\theta - \left(\frac{u_p}{\Omega R}\right)\left(\frac{u_T}{\Omega R}\right)\right\} x\,dx$$

$$= \frac{\theta}{8} - \frac{\lambda}{6}$$

It is assumed that the pitch θ as well as inflow ratio γ are uniform along the length of the blade. The $(\overline{M}_\zeta)_{steady}$ is not important since it is negligible, because the drag force is much smaller than the lift force.

$$(\overline{M}_\beta)_{perturbation} = \frac{1}{\rho ac\Omega^2 R^4}\int_0^R \delta F_z r\,dr$$

$$= \frac{1}{2}\int_0^1 \left\{\frac{\delta u_T}{\Omega R}\left(2\frac{u_T}{\Omega R}\theta - \frac{u_p}{\Omega R}\right) - \frac{\delta u_p}{\Omega R}\frac{u_T}{\Omega R} + \delta\theta\left(\frac{u_T}{\Omega R}\right)^2\right\} x\,dx$$

$$= -\overset{*}{\zeta}\left(\frac{\theta}{4} - \frac{\lambda}{6}\right) - \frac{\overset{*}{\beta}}{8} - \frac{1}{8}(k_{p_\beta}\beta + k_{p_\zeta}\zeta)$$

$$(\overline{M}_\zeta)_{perturbation} = \frac{1}{\rho ac\Omega^2 R^4}\int_0^R \delta F_x r\,dr$$

$$= \frac{1}{2}\int_0^1 \left\{\frac{\delta u_T}{\Omega R}\left(2\frac{c_d}{a}\frac{u_T}{\Omega R} + \frac{u_p}{\Omega R}\theta\right) + \frac{\delta u_p}{\Omega R}\left(\frac{u_T}{\Omega R}\theta - 2\frac{u_p}{\Omega R}\right) + \delta\theta\left(\frac{u_T}{\Omega R}\right)^2\right\} x\,dx$$

$$= -\overset{*}{\zeta}\left(\frac{1}{4}\frac{c_d}{a} + \frac{\lambda\theta}{6}\right) + \overset{*}{\beta}\left(\frac{\theta}{8} - \frac{\lambda}{3}\right) - \left(k_{p_\beta}\beta + k_{p_\zeta}\zeta\right)\frac{\lambda}{6}$$

Steady solution

$$\beta_0 = \frac{\gamma}{\nu^2_{\beta_e}}\left(\frac{\theta}{8} - \frac{\lambda}{6}\right) + \frac{\omega^2_{\beta 0}}{\Omega^2}\beta_p\frac{1}{\nu^2_{\beta_e}} \tag{5.2}$$

$$\zeta_0 \simeq 0$$

where

$$\nu^2_{\beta_e} = \nu^2_\beta + \frac{\gamma}{8}k_{p_\beta}$$

Perturbation Equations

Flap

$$\overset{**}{\beta} + \frac{\gamma}{8}\overset{*}{\beta} + (\nu^2_\beta + \frac{\gamma}{8}k_{p_\beta})\beta + \left[-2\beta_0 + \gamma\left(\frac{\theta}{4} - \frac{\lambda}{6}\right)\right]\overset{*}{\zeta} + k_{p_\zeta}\frac{\gamma}{8}\zeta = 0$$

Lag

$$\overset{**}{\zeta} + \left[2\frac{\omega_{\zeta 0}}{\Omega}\zeta_L + \gamma\left(\frac{c_d}{4a} + \frac{\lambda\theta}{6}\right)\right]\overset{*}{\zeta} + \left(\nu^2_\zeta + \frac{\gamma}{6}k_{p_\zeta}\lambda\right)\zeta + \left[2\beta_0 - \gamma\left(\frac{\theta}{8} - \frac{\lambda}{3}\right)\right]\overset{*}{\beta} + \frac{\gamma}{6}k_{p_\beta}\lambda\beta = 0$$

Rewriting these equations

$$\begin{bmatrix} \overset{**}{\beta} \\ \overset{**}{\zeta} \end{bmatrix} + \begin{bmatrix} \frac{\gamma}{8} & -2\beta_0 + \gamma\left(\frac{\theta}{4} - \frac{\lambda}{6}\right) \\ 2\beta_0 - \gamma\left(\frac{\theta}{8} - \frac{\lambda}{3}\right) & 2\frac{\omega_{\zeta 0}}{\Omega}\zeta_L + \gamma\left(\frac{c_d}{4a} + \frac{\lambda\theta}{6}\right) \end{bmatrix} \begin{bmatrix} \overset{*}{\beta} \\ \overset{*}{\zeta} \end{bmatrix}$$

$$+ \begin{bmatrix} \nu_\beta^2 + \frac{\gamma}{8}k_{p_\beta} & \frac{\gamma}{8}k_{p_\zeta} \\ \frac{\gamma}{6}k_{p_\beta}\lambda & \nu_\zeta^2 + \frac{\gamma}{6}k_{p_\zeta}\lambda \end{bmatrix} \begin{bmatrix} \beta \\ \zeta \end{bmatrix} = 0 \tag{5.3}$$

For the solution of the above equations (5.3), one needs trim solution i.e., λ and θ. It is assumed that the wake induced inflow λ for hover is uniform along the length of the blade. For constant pitch θ, there may be some variation in λ, but is not considered here. Using the simple momentum theory,

$$\lambda = k_h\sqrt{\frac{c_T}{2}} \tag{5.4}$$

where c_T is the thrust coefficient and k_h is an empirical factor to cover tip losses and nonuniform distribution, typically 1.15. Comparing thrust obtained using momentum theory and blade element theory, one gets

$$\theta = \frac{6c_T}{\sigma a} + \frac{3}{2}\lambda \tag{5.5}$$

Two simple ways to calculate the solution on Eqs. (5.3) are determinant expansion and the eigenvalue solution.

I. Determinant Expansion

Assume flap and lag displacements as

$$\beta(\psi) = \beta e^{s\psi}$$

$$\zeta(\psi) = \zeta e^{s\psi}$$

Eqs. (5.3) becomes

$$\begin{bmatrix} s^2 + \frac{\gamma}{8}s + \nu_\beta^2 + \frac{\gamma}{8}k_{p_\beta} & (-2\beta_0 + \frac{\gamma\theta}{4} - \frac{\gamma\lambda}{6})s + \frac{\gamma}{8}k_{p_\zeta} \\ (2\beta_0 - \frac{\gamma\theta}{8} + \frac{\gamma\lambda}{3})s + \frac{\gamma}{6}k_{p_\beta} & \begin{matrix} s^2 + (\frac{2\omega_{\zeta 0}}{\Omega}\zeta_L + \frac{\gamma}{4}\frac{c_d}{a} + \frac{\gamma\lambda}{6}\theta)s \\ +\nu_\zeta^2 + \frac{\gamma}{6}k_{p_\zeta}\lambda \end{matrix} \end{bmatrix} \begin{bmatrix} \beta \\ \zeta \end{bmatrix} = 0$$

For a nontrivial solution, the determinant of the matrix is zero. This results into

$$As^4 + Bs^3 + Cs^2 + Ds + E = 0 \tag{5.6}$$

where

$$A = 1$$

$$B = \frac{\gamma}{8} + 2\frac{\omega_{\zeta 0}}{\Omega}\zeta_L + \frac{\gamma}{4}\frac{c_d}{a} + \frac{\gamma\lambda}{6}\theta$$

$$\begin{aligned} C = &\tfrac{\gamma}{8}(2\tfrac{\omega_{\zeta 0}}{\Omega}\zeta_L + \tfrac{\gamma}{4}\tfrac{c_d}{a} + \tfrac{\gamma\lambda}{6}\theta) + \nu_\beta^2 + \tfrac{\gamma}{8}k_{p_\beta} + \nu^2 + \tfrac{\gamma}{6}k_{p_\zeta}\lambda \\ &-(-2\beta_0 + \tfrac{\gamma\theta}{4} - \tfrac{\gamma\lambda}{6})(2\beta_0 - \tfrac{\gamma\theta}{8} + \tfrac{\gamma\lambda}{3}) \end{aligned} \tag{5.7}$$

$$D = (\nu_\beta^2 + \frac{\gamma}{8}k_{p_\beta})(\frac{2\omega_{\zeta 0}}{\Omega}\zeta_L + \frac{\gamma}{4}\frac{c_d}{a} + \frac{\gamma\lambda}{6}\theta) + \frac{\gamma}{8}(\nu_\zeta^2 + \frac{\gamma}{6}k_{p_\zeta}\lambda)$$
$$-\frac{\gamma}{8}k_{p_\zeta}(2\beta_0 - \frac{\gamma\theta}{8} + \frac{\gamma\lambda}{3}) - \frac{\gamma}{6}k_{p_\beta}\lambda(-2\beta + \frac{\gamma\theta}{4} - \frac{\gamma\lambda}{4})$$
$$E = (\nu_\beta^2 + \frac{\gamma}{8}k_{p_\beta})(\nu_\zeta^2 + \frac{\gamma}{6}k_{p_\zeta}\lambda) - (\frac{\gamma}{8}k_{p_\zeta})(\frac{\gamma}{6}k_{p_\beta}\lambda)$$

At the critical flutter condition, the system damping becomes zero.

$$s = i\omega$$

Substituting in Eq. (5.6),

$$A\omega^4 - iB\omega^3 - C\omega^2 + iD\omega + E = 0$$

Setting separately the real and imaginary parts to be zero, gives,

$$A\omega^4 - C\omega^2 + E = 0$$

$$-B\omega^3 + D\omega = 0$$

The second equation gives

$$\omega^2 = \frac{D}{B}$$

Substituting in the first equation

$$A(\frac{D}{B})^2 - C(\frac{D}{B}) + E = 0$$

or

$$AD^2 - CDB + EB^2 = 0$$

This is the condition for instability. It is called as Routh's stability criteria.

Solution Procedure

Given the rotor characteristics in terms of coefficients

$$\gamma,\ \sigma,\ a,\ c_d,\ k_{p_\beta},\ k_{p_\zeta},\ \zeta_L,\ \nu_\beta,\ \nu_\zeta,\ \beta_p,\ \omega_{\beta_0}/\Omega,\ \omega_{\zeta_0}/\Omega$$

Step 1. Calculate the trim solution. For a given $\frac{c_T}{\sigma}$ calculate λ and θ using Eqs.(5.4) and (5.5).
Step 2. Calculate the steady flap detection β_0 using Eq. (5.2).
Step 3. Calculate the constants A, B, C, D, E for Eqs. (5.7).
Step 4. Calculate $R = AD^2 - BCD + B^2E$.

If the remainder R is zero it gives critical condition. For a non-zero value of R, select a new $\frac{c_T}{\sigma}$ and repeat Steps 1-4. Vary $\frac{c_T}{\sigma}$ till R changes sign. Take finer steps of $\frac{c_T}{\sigma}$ to get the critical value at which R is nearly zero.

Note that Eq.(7) can also be solved using any standard subroutine for polynomial equation. The solution will give complex roots, the real part represents damping and the imaginary part presents the damping of the mode.

II. Eigen Analysis

The perturbation equations of motion (5.3) can be interpreted in a standard spring-mass-damper form.

$$[M]\left\{\begin{array}{c}\overset{**}{\beta}\\ \overset{**}{\zeta}\end{array}\right\}+[C]\left\{\begin{array}{c}\overset{*}{\beta}\\ \overset{*}{\zeta}\end{array}\right\}+[K]\left\{\begin{array}{c}\beta\\ \zeta\end{array}\right\}=0 \tag{5.8}$$

The matrices C and K are not symmetric. These can be transformed into a first order system (2.14) and solved as an eigenvalue problem. This will give two complex conjugate pairs, i.e., four eigenvalues. A typical eigenvalue λ will be

$$\lambda = \lambda_{\text{real}} + i\lambda_{\text{imaginary}}$$

(Note this eigenvalue λ is totally different from inflow λ.) The real part of the eigenvalue represents damping of the mode and the imaginary part represents the frequency of the mode.

If any one of the eigenvalue has a positive real part, the blade is unstable. The $\lambda_R = 0$ gives the stability boundary. Also note that if the frequency becomes zero in the unstable region, it represents static divergence condition.

This figure shows the flap-lag flutter stability boundaries as a function of thrust level obtained for a rotor blade in hovering flight. The following rotor parameters have been used for calculations

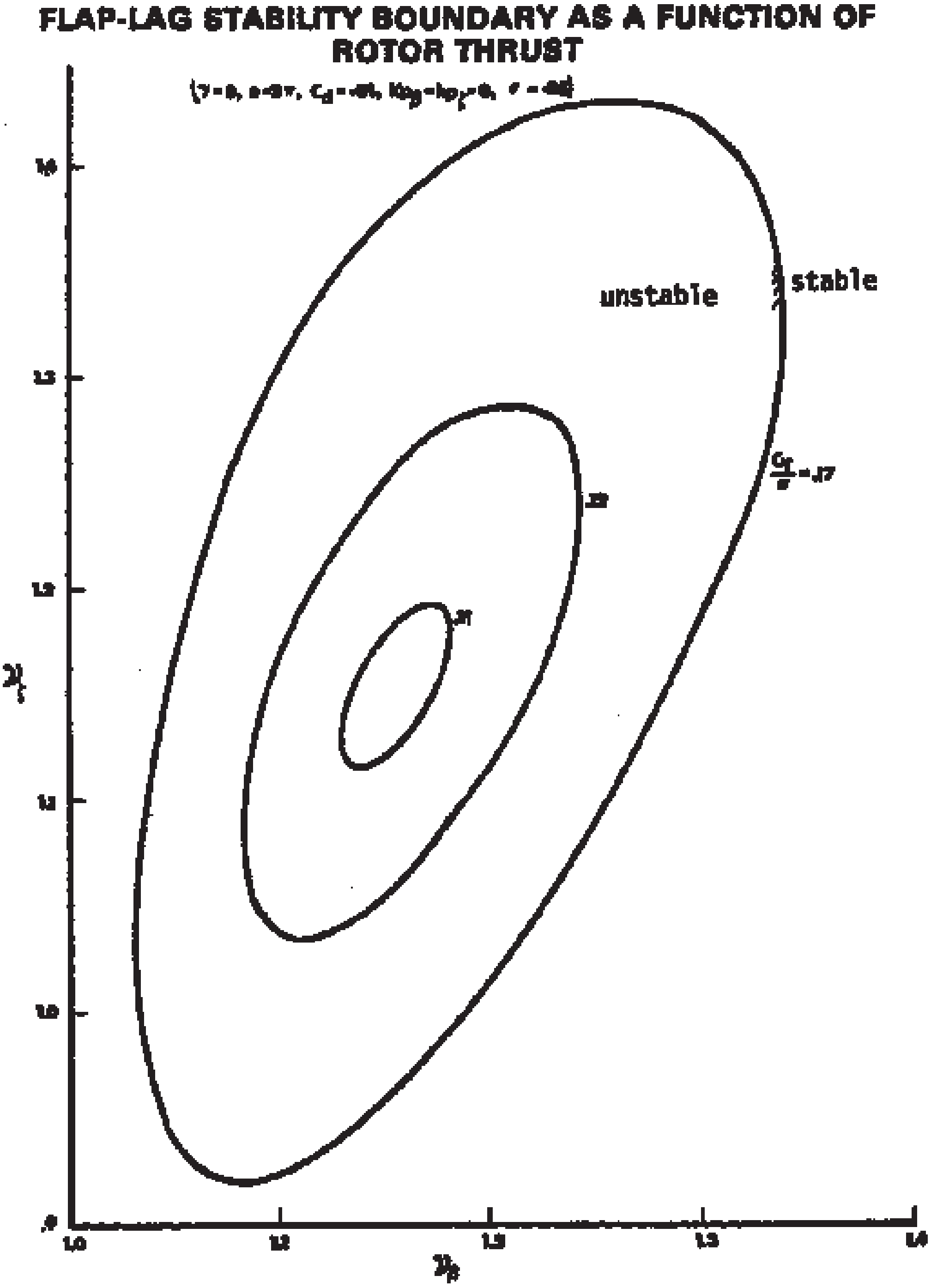
$\gamma = 8.0 \qquad \sigma = .05 \qquad k_{p_\beta} = k_{p_\zeta} = 0$

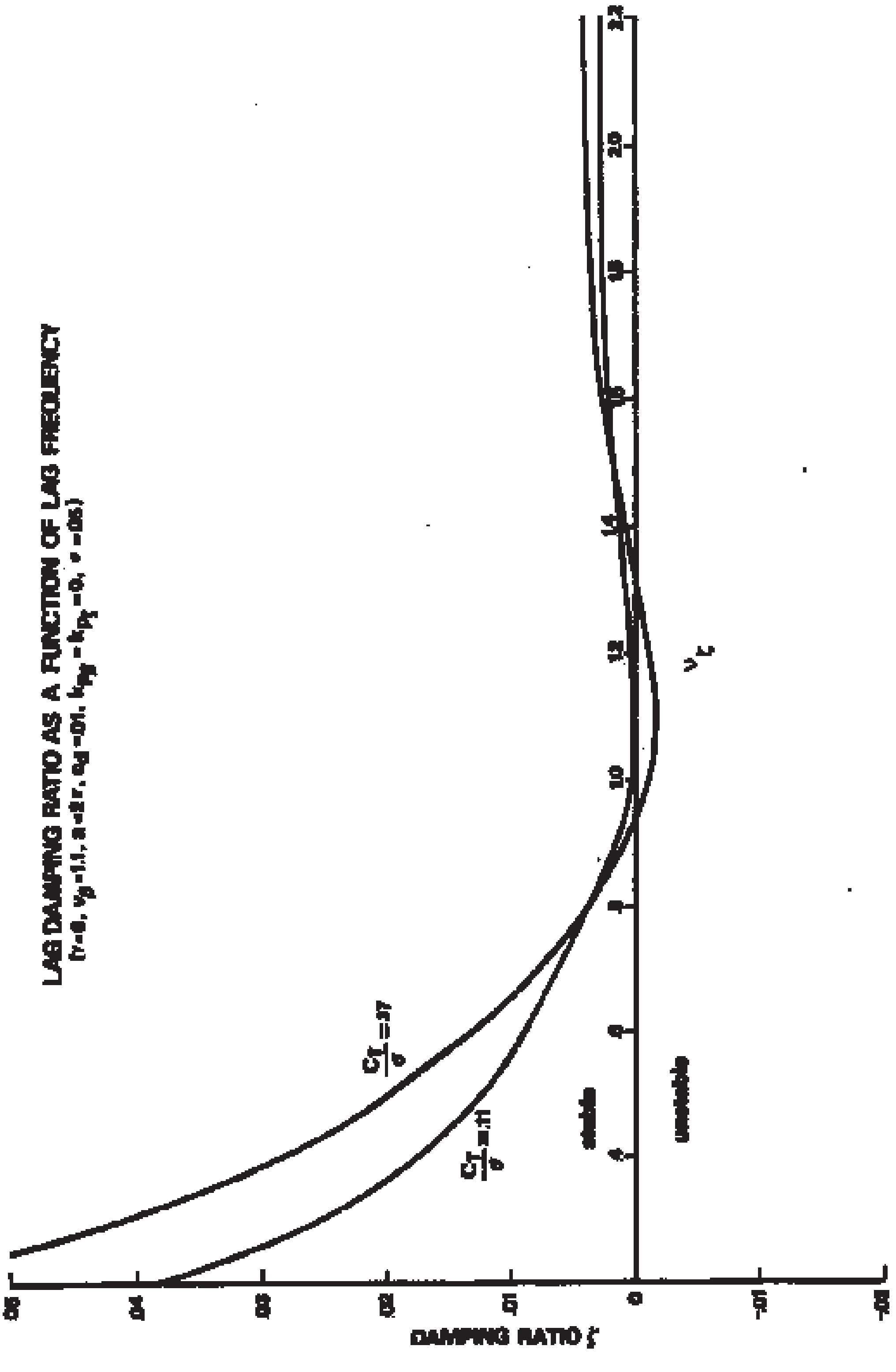
$\beta_p = 0 \qquad \text{a} = 2\pi \qquad c_d = .01 \qquad \zeta_L = 0$

The flutter boundary is calculated using the determinant expansion. For a particular thrust level represented by $\frac{c_T}{\sigma}$, the blade is unstable inside the elliptic graph. The less damped lag mode becomes unstable and the damping ratio of the lag mode is plotted on the next figure for a fixed flap frequency, and for varying lag frequencies. These results are obtained using eigen-analysis. It is interesting to note that the lag mode instability is soft in nature and can be easily stabilized with the inclusion of a small amount of structural damping in the lag mode. Flap-lag flutter is an instability of lag mode which occurs at lag frequency. Because of low reduced frequencies, the unsteady effects play less important role here. The application of quasisteady aerodynamics theory appears adequate to predict flap-lag flutter. For hingeless rotors with no pitch-flap or pitch-lag coupling or flap-lag structural coupling, the critical condition for flap-lag stability occurs with zero precone and

flap frequency = lag frequency = 1.15/rev

The rotor is stable for a flap frequency of less than 1/rev or greater than 1.4/rev.


FLAP-LAG STABILITY BOUNDARY AS A FUNCTION OF
ROTOR THRUST
unstable
stable



5.1.1 Comment on Flap-Lag Flutter

Some general remarks are made on flap-lag flutter.

1. Articulated Rotor

Assume the blade with $k_{p_\beta} = k_{p_\zeta} = 0$ and with no hinge offset, e = 0. Gives $\nu_\beta = 1$. The steady solution is given in Eq. (5.2),

$$\beta_0 = \frac{\gamma\theta}{8} - \frac{\gamma\lambda}{6}$$

Let us examine the perturbation equations in Eq. (5.3),

coupling term in flap equation $= (-2\beta_0 + \frac{\gamma\theta}{4} - \frac{\gamma\lambda}{6})\overset{*}{\zeta}$

$$= \frac{\gamma\lambda}{6}\overset{*}{\zeta}$$

This term is small and can be neglected. This nearly uncouples the flap equation from the lag equation. It has been seen that the articulated blade with zero hinge offset, the blade is stable from the aeroelastic stability. However, the coupling term in the lag equation plays an important role for the determination of vibration and loads.

2. Ideal Precone

   An ideal precone is the initial blade coning setting so that at the operating condition, the moments due to centrifugal force and aerodynamic force balance out, result in

$$\beta_0 \approx \beta_p$$

   From steady solution, Eq.(5.2)

$$\beta_0 = \frac{1}{\nu_\beta^2}\left[\frac{\omega_{\beta 0}^2}{\Omega^2}\beta_p + \frac{\gamma\theta}{8} - \frac{\gamma\lambda}{6}\right] \qquad \text{(assume } k_{p_beta} = k_{p_\zeta} = 0\text{)}$$

$$\nu_\beta^2 = 1 + \frac{3}{2}\frac{e}{R} + \frac{\omega_{\beta 0}^2}{\Omega^2}$$

   This gives

$$\beta_p = \frac{\gamma}{1 + \frac{3}{2}e}\left(\frac{\theta}{8} - \frac{\lambda}{6}\right)$$

   and this does not depend on nonrotating flap frequency. Again the flap mode gets nearly uncoupled from lag mode resulting in a stable blade from flap-lag flutter instability. Otherwise, precone can be destabilizing.

3. Thrust Level

   Flap-lag flutter is a high thrust phenomena. To achieve a high thrust level in hover, a high collective pitch is required. Also the inflow λ is higher for a higher thrust level. The result of this all is that the coupling terms particular in the lag equation becomes larger with higher thrust. At zero thrust, the coupling is minimum and the blade is free from aeroelastic instability.

4. Elastic Coupling

   If the section principal axes do not lie along the flap and lag axes, then the flap and lag equations get coupled structurally due to elastic coupling. The elastic coupling allows the transfer of kinetic energy from weakly damped lag mode to well damped flap mode. The soft lag rotors get stabilized with small coupling. The stiff lag rotors on the other hand get destabilized with the small coupling term but generally become stabilized with the large coupling term.

5. Matched Stiffness Rotors

   For matched stiffness rotors, the flap bending stiffness EI_x is equal to the lag stiffness EI_z. This is generally achieved through a circular cross-section at the root of the blade. This means that the nonrotating flap and lag frequencies are equal. Not shown so far, but it results in uncoupling, the bending and torsion equations structurally. This means that there is a less influence of torsion on flap-lag stability. Thus the matched stiffness condition stabilizes the blades.

6. Structural Damping

   The flap mode is highly damped because of aerodynamic damping. The structural damping in the flap mode is unimportant. On the other hand, the lag mode is weakly damped and the flag-lap mode is instability of the lag mode. This instability is soft in nature and can stabilized damping in the lag mode. The other possibility is to add a mechanical lag damper at the root hinge to stabilize the blade.

7. Hinge Sequence

   The earlier analysis is made for lag hinge followed by flap hinge outboard, other possibility is flap hinge followed by lag hinge outboard. The changed hinge sequence will introduce some extra nonlinear terms, in particular, in the aerodynamic forces. The flap-lag aeroelastic stability is very sensitive to small terms so the results can be somewhat different due to a change in hinge sequence (Kaza & Kvaternik.)

8. Pitch-Flap Coupling k_{p_β}

   Pitch-flap coupling, due to torsion dynamics or kinematic coupling is introduced in the two-degree-of-freedom problem by assuming a feather motion of the form

   $$\Delta\theta = -k_{p_\beta}\beta$$

   The pitch-flap coupling k_{p_β} is positive for flap up/pitch down motion. The positive value of pitch-flap coupling raises the flap frequency. However, its influence on flap-lag stability is small.

9. Pitch-Lag Coupling k_{p_ζ}

   Pitch-lag coupling due to torsion dynamics or kinematic coupling is introduced in two-degree model by assuming a feather motion of the form

   $$\Delta\theta = -k_{p_\zeta}\zeta$$

   The pitch-lag k_{p_ζ} is positive for lag back-pitch down motion. Generally, a negative coupling is stabilizing analysis.

10. Quasistatic Torsion Model

    For low to moderate torsional frequencies one has to include torsion degree of motion for stability analysis. For high torsional frequencies (typically $\nu_\theta > 5$) the feathering inertia and damping terms are generally small and these do not influence the flap-lag instability. Either one can drop the torsion effect all together or one can include approximately the stiffness terms through a quasistatic torsion model assumption. Let us look at the flap-lag torsion equations.

    $$[M]\left\{\begin{array}{c}\ddot\beta\\ \ddot\zeta\\ \ddot\theta\end{array}\right\} + [C]\left\{\begin{array}{c}\dot\beta\\ \dot\zeta\\ \dot\theta\end{array}\right\} + [k]\left\{\begin{array}{c}\beta\\ \zeta\\ \theta\end{array}\right\} = 0 \tag{5.9}$$

keeping only static stiffness terms in the torsion equation, one gets

$$k_{31}\beta + k_{32}\zeta + k_{33}\theta = 0$$

$$\theta = -\frac{k_{31}}{k_{33}}\beta - \frac{k_{32}}{k_{33}}\zeta \tag{5.10}$$

Replace θ from the flap and lag equations using above expression. Again this results into a two-degree system. The effective pitch-flap and pitch-lag coupling terms due to torsion mode have been retained. These coupling terms are primarily caused by aerodynamic forces.

11. Stall

    The flap-lag instability generally takes place at a high pitch setting, which also means high angle of attack. There is the likehood of getting into stalled flow for part of the rotor blade. At stall, there is a loss of flap damping because of the reduced lift-curve slope.

12. Compressibility

    Near the tip of the blade, there is a high speed region, and sometimes there can be transonic flow conditions. The compressibility effects are important near the tip, because of larger dynamic pressure there. Also due to transonic conditions, there is a shift in aerodynamic center from 1/4-chord to 1/2-chord, resulting in a large torsional moment. Also there is a large increase in the drag force. The compressibility effects can be quite destabilizing.

Ex. In a circulating controlled rotor blade, the aerodynamic characteristics are functions of geometric angle as well as blowing,

$$c_l = c_1\alpha + c_2 c_\mu$$

$$c_d = c_{d_0} + d_1 c_\mu$$

Using quasisteady aerodynamics, derive the equations of motion for blade flap-lag aeroelastic stability in hover.

Flap-Lag equations

$$\overset{**}{\beta} - \nu_\beta^2\beta - 2\beta_0 \overset{*}{\zeta} = \gamma\overline{M}_\beta + \frac{\omega_{\beta_0}^2}{\Omega^2}\beta_p$$

$$\overset{**}{\zeta} - \nu_\zeta^2\zeta - 2\beta_0 \overset{*}{\beta} + 2\zeta_L\frac{\omega_{\beta_0}}{\Omega}\zeta = \gamma\overline{M}_\zeta$$

$$F_Z = \frac{1}{2}\rho c(c_l u_T V - c_d u_p V)$$

$$\delta F_Z = \frac{1}{2}\rho c(\delta c_l u_T V + c_l\delta u_T V + c_l u_T\delta V - \delta c_d u_p V$$
$$-c_d\delta u_p V - c_d u_p\delta V)$$

$$\delta c_l = c_{l_\alpha}\delta\alpha + c_{l_\mu}\delta c_\mu$$

$$\delta c_d = c_{d_u}\delta c_\mu$$

$$\delta c_\mu = -2c_\mu\frac{\delta V}{V}$$

$$\delta V = \frac{1}{V}(u_T\delta u_T + u_p\delta u_p)$$

$$\alpha = \theta - \frac{u_p}{u_T}$$

$$\delta\alpha = \delta\theta - \frac{1}{V^2}(u_T\delta u_p - u_p\delta u_T)$$

$$\frac{\delta F_z}{\frac{1}{2}\rho c} = \delta u_T\left\{\frac{u_p u_T}{V}(c_1 + d_1 c_\mu - c_{d0}) + (\frac{u_T^2}{V} + V)(\theta - \frac{u_p}{u_T})c_1\right.$$

$$\left. -(\frac{u_T^2}{V} - V)c_2 c_\mu\right\}$$

$$+\delta u_p\left\{-\frac{u_T^2}{V}c_1 + \frac{u_p u_T}{V}c_1(\theta - \frac{u_p}{u_T}) - \frac{u_p u_T}{V}c_2 c_\mu\right.$$

$$\left. +\frac{u_p^2}{V}(d_1 c_\mu - c_{d_0}) - V(c_{d_0} + d_1 c_\mu)\right\}$$

$$+\delta\theta(c_1 u_T V)$$

$$F_x = \frac{1}{2}\rho c(c_l u_p V + c_d u_T V)$$

$$\frac{\delta F_x}{\frac{1}{2}\rho c} = \delta u_T\left\{c_1\frac{u_p^2}{V} + \frac{u_p u_T}{V}c_1(\theta - \frac{u_p}{u_T}) - \frac{u_p u_T}{V}c_2 c_\mu\right.$$

$$\left. +\frac{u_T^2}{V}(c_{d_0} - d_1 c_\mu) + V(c_{d_0} + d_1 c_\mu)\right\}$$

$$+\delta u_p\left\{-\frac{u_p u_T}{V}(c_1 + d_1 c_\mu - c_{d_0}) + (\frac{u_p^2}{V} + V)c_1(\theta - \frac{u_p}{u_T})\right.$$

$$\left. -\frac{u_p^2}{V}c_2 c_\mu\right\}$$

$$\delta\theta(c_1 u_p V)$$

For Hover

$$\frac{u_T}{\Omega R} = x, \qquad \frac{u_p}{\Omega R} = \lambda$$

$$\frac{\delta u_T}{\Omega R} = -x\overset{*}{\zeta},\ \frac{\delta u_p}{\Omega R} = x\overset{*}{\beta}\ \ \delta\theta = -k_{p_\beta}\beta - k_{p_\zeta}\zeta$$

$$V \approx u_T$$

$$\overline{M}_\beta = \frac{1}{\rho a c R^4\Omega^2}\int_0^R r\,\delta F_Z\,dr$$

$$\overline{M}_\zeta = \frac{1}{\rho a c R^4\Omega^2}\int_0^R r\,\delta F_x\,dr$$

Perturbation Equations are

Flap:

$$\overset{**}{\beta} + \gamma(\frac{1}{8} - \frac{1}{6}\lambda\theta - \frac{1}{8}\frac{c_{d_0}}{c_1} + \frac{1}{6}\frac{c_2}{c_1}c_\mu)\overset{*}{\beta} + (\nu_\beta^2 + \frac{\gamma}{8}k_{p_\beta})\beta$$

$$+\left\{-2\beta_0 + \gamma(\frac{\theta}{4} - \frac{\lambda}{6} - \frac{1}{8}\frac{c_2}{c_1}c_\mu + \frac{1}{6}\frac{d_1}{c_1}c_\mu - \frac{1}{6}\frac{c_{d_0}}{c_1})\right\}\overset{*}{\zeta} + \frac{1}{8}k_{p_\zeta}\zeta = 0$$

Lag:

$$\overset{**}{\zeta} + \left\{\frac{2\omega_{\zeta0}}{\Omega}\zeta_L + \gamma(\frac{c_{d_0}}{4c_1} + \frac{\lambda\theta}{6} - \frac{1}{6}\frac{c_2}{c_1}c_\mu\lambda)\right\}\overset{*}{\zeta} + (\nu_\zeta^2 + \frac{\gamma}{6}k_{p_\zeta}\lambda)\zeta$$

$$+\left\{2\beta_0 - \gamma(\frac{\theta}{8} - \frac{1}{6}\frac{d_1}{c_1}c_\mu + \frac{1}{6}\frac{c_{d0}}{c_1})\right\}\overset{*}{\beta} + \frac{\gamma}{6}k_{p_\beta}\lambda\beta = 0$$

5.2 Pitch-Flap Instabilities

The flap and torsion modes participate causing this instability. This flutter instability is also called as classical or conventional flutter because a similar type flutter is also called as classical or conventional flutter because a similar type flutter instability takes place in fixed wings. However, there are certain differences for rotor blade flutter from fixed wing flutter.

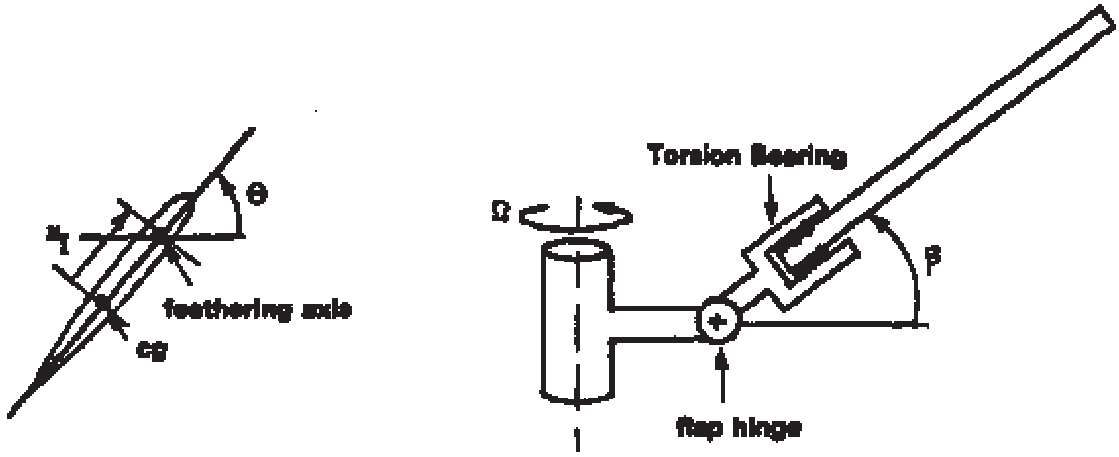
i) There is an important coupling due to centrifugal force if there is a cg offset from the elastic axis.

ii) There is a tennis racket effect in the torsion equation.

iii) Aerodynamic forces are more involved, in particular, returning wake can be important here.

iv) Periodic forces are present if forward flight is also considered.

The result of all this is that one may not be able to apply the fixed wing results here. Let us investigate this problem for a simple blade configuration with two degrees of motion, rigid flap about flap hinge and rigid pitch about pitch bearing. The torsion bearing is assumed to be located outboard of the flap hinge (which is typical). The study is carried out for hovering flight.



The equations of motion for uniform blades in nondimensional form are

$$\text{Flap:}\quad \overset{**}{\beta} + \nu_\beta^2 \beta - \frac{3}{2}\frac{X_I}{R}(\overset{**}{\theta} + \theta) = \gamma \overline{M}_\beta + \frac{\omega_{\beta 0}^2}{\Omega^2}\beta_p \tag{5.11}$$

$$\text{Pitch:}\quad I_{\underset{f}{*}}(\overset{**}{\theta} + \nu_\theta^2\theta + 2\frac{\omega_{\theta 0}}{\Omega}\zeta_\theta \overset{*}{\theta}) - \frac{3}{2}\frac{x_I}{R}(\overset{**}{\beta} + \beta) + K_{p_\beta}(\frac{W_{\theta 0}}{\Omega})^2 I_{\underset{f}{*}}\beta$$

$$= \gamma \overline{M}_\theta + I_{\underset{f}{*}}\frac{\omega_{\theta 0}^2}{\Omega^2}\theta_{con}$$

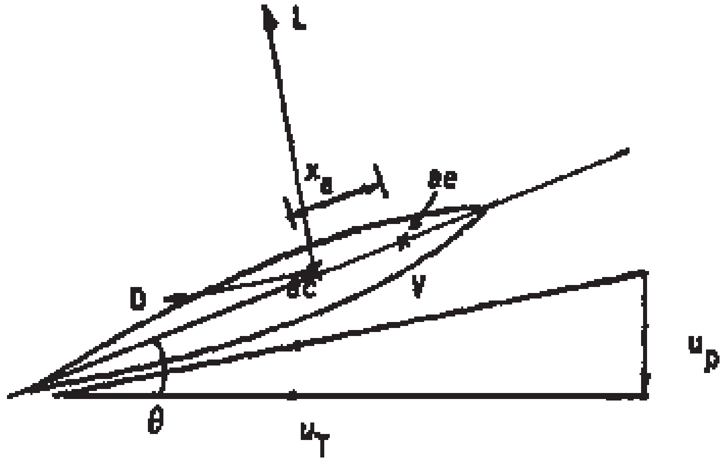
where $I_{\underset{f}{*}} = I_f/I_b$, the I_f is the feather moment of inertia and I_b is the flap moment of inertia. The $\omega_{\beta 0}$ and $\omega_{\theta 0}$ are the nonrotating flap and torsion frequencies. The ζ_θ is the viscous damping coefficient in the pitch mode with respect to nonrotating frequency and K_{p_β} is the pitch-flap coupling. The ν_β and ν_θ are respectively, the rotating natural frequencies of flap and torsion modes.

$$\nu_\beta^2 = 1 + \frac{3}{2}\frac{e}{R} + \frac{\omega_{\beta 0}^2}{\Omega^2}$$

$$\nu_\theta^2 = 1 + \frac{\omega_{\theta 0}^2}{\Omega^2}$$

where e is the offset for flap hinge. The θ_{con} is the control system command pitch.

Let us examine the aerodynamic forces



$$\overline{M}_\beta \approx \frac{1}{\rho ac\Omega^2 R^4}\int_0^R rL\,dr \tag{5.12}$$

$$\overline{M}_\theta \approx \frac{1}{\rho ac\Omega^2 R^4}\left[\int_0^R (-x_a)L\,dr + M_{Nc}\right]$$

where x_a is the chordwise offset of aerodynamic center from elastic axis, positive towards the trailing edge. The airfoils used for the rotor blades are generally symmetric and therefore the aerodynamic moment coefficient c_{mac} is zero. The M_{Nc} is the noncirculatory aerodynamic moment which is important for the pitch dynamics. The lift force per unit length is

(5.13)

where $\overline{C}(k)$ is a lift deficiency function and it depends on the reduced frequency $k = \frac{\omega b}{U}$. For simplifying the analysis, a representative value of $C(k)$ is taken at the 3/4-radius position and assumed constant for the blade. The perturbation aerodynamic force is

$$(\overline{M}_\beta)_{perturbation} = \frac{1}{\rho ac\Omega^2 R^4}\int_0^R \delta L\, r\,dr$$

$$\frac{1}{2}\int_0^1 \overline{C}(k)\left\{\frac{\delta u_T}{\Omega R}(2\frac{u_T}{\Omega R}\theta - \frac{u_p}{\Omega R}) - \frac{\delta u_p}{\Omega R}\frac{u_T}{\Omega R} + \delta\theta(\frac{u_T}{\Omega R})^2\right\} x\,dx$$

$$\overline{M}_\theta)_{perturbation} = -\frac{1}{2}\int_0^2 \frac{x_a}{R}\overline{C}(k)\left\{\frac{\delta u_T}{\Omega R}(2\frac{u_T}{\Omega R}\theta - \frac{u_p}{\Omega R}) - \frac{\delta u_p}{\Omega R}\frac{u_T}{\Omega R}\right.$$

$$\left. +\delta\theta(\frac{u_T}{\Omega R})^2\right\} dx + \overline{M}_{nc}$$

and

$$\overline{M}_{nc} = \frac{1}{\rho ac\Omega^2 R^4}\left\{\frac{1}{4}\pi\rho\Omega^2 c^3[r(\frac{1}{4} + \frac{x_a}{c})\ddot{\beta} - r(\frac{1}{2} + \frac{x_a}{c})\dot{\theta}\right.$$

$$-c(\frac{3}{32}+\frac{1}{2}\frac{x_a}{c})\ddot{\theta}]\Big\}$$

The steady and perturbation flow components are

$$\frac{u_T}{\Omega R}=x, \qquad \frac{u_p}{\Omega R}=\lambda$$

$$\frac{\delta u_T}{\Omega R}=0, \qquad \frac{\delta u_p}{\Omega R}=x\,\overset{*}{\beta}-\frac{c}{R}(\frac{1}{2}+\frac{x_a}{c})\,\overset{*}{\theta}$$

$$\delta\theta=\theta \text{ (elastic)}$$

Perturbation moments become

$$\overline{M}_\beta=\frac{1}{2}\overline{C}\left\{-\frac{1}{4}\overset{*}{\beta}+\frac{1}{3}\frac{c}{R}(\frac{1}{2}+\frac{x_a}{c})\,\overset{*}{\theta}+\frac{1}{4}\theta\right\}$$

$$\overline{M}_\theta=-\frac{1}{2}\overline{C}\frac{x_a}{R}\left\{-\frac{1}{3}\overset{*}{\beta}+\frac{1}{2}\frac{c}{R}(\frac{1}{2}+\frac{x_a}{c})\,\overset{*}{\theta}+\frac{1}{3}\theta\right\}-\frac{1}{16}(\frac{c}{R})^2(\frac{1}{2}+\frac{x_a}{c})\,\overset{*}{\theta} \tag{5.14}$$

The equations of motion (5.11) become

$$\begin{bmatrix} 1 & -\frac{3}{2}\frac{x_I}{R} \\ -\frac{3}{2}\frac{x_I}{R} & I_{\overset{*}{f}} \end{bmatrix}\begin{bmatrix} \overset{**}{\beta} \\ \overset{**}{\theta} \end{bmatrix}+\begin{bmatrix} \frac{\gamma}{8}\overline{C}(k) & -\frac{\gamma}{12}\frac{c}{R}(1+2\frac{x_a}{c})\overline{C}(k)-\frac{1}{24}\frac{C}{R} \\ -\frac{\gamma}{6}\frac{c}{R}\frac{x_a}{c}\overline{C}(k) & \begin{matrix}\frac{\gamma}{8}(\frac{c}{R})^2\frac{x_a}{c}(1+2\frac{x_a}{c})\overline{C}(k) \\ +\frac{1}{16}(\frac{c}{R})^2(\frac{1}{2}+\frac{x_a}{c})+2I_{\overset{*}{f}}\frac{\omega_{\theta 0}}{\Omega}\zeta_\theta\end{matrix} \end{bmatrix}\begin{bmatrix} \overset{*}{\beta} \\ \overset{*}{\theta} \end{bmatrix}$$

$$+\begin{bmatrix} \nu_\beta^2 & -\frac{\gamma}{8}\overline{C}(k)-\frac{3}{2}\frac{x_I}{R} \\ -\frac{3}{2}\frac{x_I}{R}k_{p_\beta} & I_{\overset{*}{f}}\nu_\theta^2+\frac{\gamma}{6}\frac{c}{R}\frac{x_a}{c}\overline{C}(k) \end{bmatrix}\begin{bmatrix} \beta \\ \theta \end{bmatrix}=0 \tag{5.15}$$

These are second order equations expressed in standard spring-mass-damper form. These can be solved many different ways. Two possible ways are:

(a) Expansion of the determinant

(b) Eigen Analysis

Two types of instabilities are possible

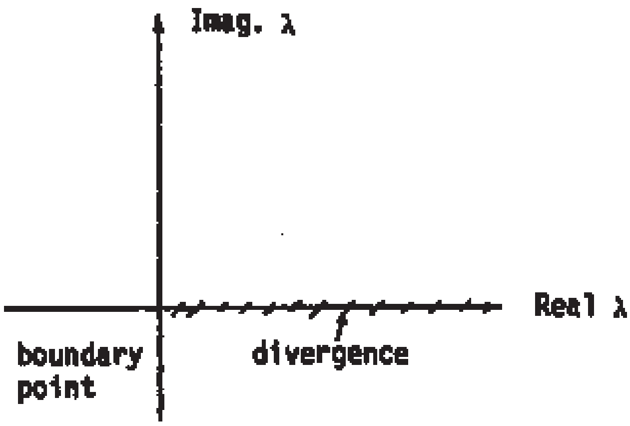
(a) Static instability (Divergence)

(b) Dynamic instability (Flutter)

Let us examine each one separately.

5.2.1 Pitch Divergence

At a particular operating condition, if a disturbance is given to the blade, the elastic pitch becomes larger and larger till the blade fails. This is a static instability and the dynamics of the blade does not play any role. It is only the pitch mode which becomes unstable. One can solve the governing pitch-flap equations (5.15) as an eigenvalue problem. This will result into two complex conjugates pairs.



eigen $\lambda = \lambda_{Real} + i\lambda_{Imag.}$

For divergence condition

$$\lambda_{Real} \geq 0$$

and

$$\lambda_{Imag.} = 0$$

Divergence is a zero frequency condition. For divergence, the acceleration and velocity terms are not important. Also, $\overline{C}(k) = 1$.

$$\begin{bmatrix} \nu_\beta^2 & -\frac{\gamma}{8} - \frac{3}{2}\frac{x_I}{R} \\ -\frac{3}{2}\frac{x_I}{R} & I_{\underset{f}{*}}\nu_\theta^2 + \frac{\gamma}{6}\frac{x_a}{R} \end{bmatrix} \begin{bmatrix} \beta \\ \theta \end{bmatrix} = 0$$

Setting the determinant to be zero, gives the critical condition

$$R = \nu_\beta^2(I_{\underset{f}{*}} - \nu_\theta^2 + \frac{\gamma}{6}\frac{x_a}{R}) - -\frac{3}{2}\frac{x_I}{R}(\frac{\gamma}{8} + \frac{3}{2}\frac{x_I}{R})$$

$= 0$ gives the critical condition.
If $R > 0$, the system is stable or

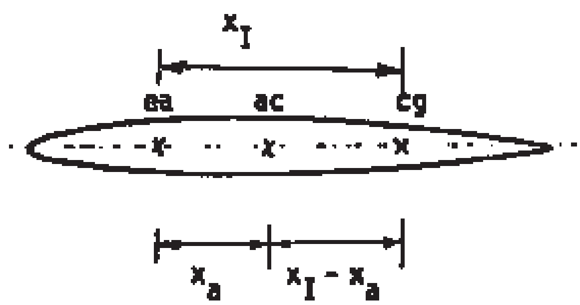
$$\frac{3}{16}\gamma\frac{x_I}{R} - \frac{\gamma}{6}\nu_\beta^2\frac{x_a}{R} < \nu_\beta^2 I_{\underset{f}{*}}\nu_\theta^2$$

$$\frac{x_I}{R} - \frac{8}{9}\nu_\beta^2\frac{x_a}{R} < \frac{16}{3\gamma}\nu_\beta^2 I_{\underset{f}{*}}\nu_\theta^2$$

$$\frac{8}{9}\nu_\beta^2 \approx 1$$

Thus for the stable blade from pitch divergence

$$\frac{x_I - x_a}{R} < \frac{16}{3\gamma}\nu_\beta^2 I_{\underset{f}{*}}\nu_\theta^2$$



The pitch divergence depends on the chordwise offset of the cg after aerodynamic center. The elastic axis location is unimportant for the divergence boundary.

Let us consider a typical rotor with the following properties

$$\gamma = 8 \qquad \nu_\beta = 1.1 \qquad I_{\underset{f}{*}} = .001 \qquad \frac{R}{c} = 10$$

For the unstable blade

$$\frac{x_a - x_I}{c} \geq \frac{R}{c} \times \frac{16}{3} \times \frac{1}{8} \times 1.21 \times \nu_\theta^2 \times 0.001 \approx .01\nu_\theta^2$$

The right hand side is a positive number. The aerodynamic center is quite close to 1/4-chord. Thus if cg falls on 1/4-chord or ahead of 1/4-chord, there is no possibility of pitch divergence.
There are two important parameters for pitch divergence

(a) cg offset from 1/4-chord

(b) torsional frequency $\frac{\omega_\theta}{\Omega}$

It should be kept in mind that for fixed wing, the divergence depends on aerodynamic center offset from the elastic axis. For the rotor blade, the elastic axis position is unimportant and divergence can happen even if the aerodynamic center and elastic axis are coincident. This is because in rotors, as blade twists, the lift increases and this increases steady flap deflection β, resulting in larger twisting action due to the centrifugal force component ($\beta \times CF$).

5.2.2 Flutter

The self-excited oscillations are caused by the coupling of pitch and flap modes. The flutter boundary is defined by the zero damping condition. This flutter does not depend on the thrust level and in fact, it can take place at zero thrust. The rotor trim is not required for the calculation of flutter speed.

There are two simple ways to solve the dynamic pitch and flap equations (5.15), either the determinant expansion or as an eigenvalue problem. Let us discuss the first method.
Determinant Expansion:

Assume the perturbation motion as

$$\beta(\psi) = \beta e^{s\psi}$$

$$\theta(\psi) = \theta e^{s\psi}$$

Substituting in the governing Eq. (5.15),

$$\begin{bmatrix} s^2 + \frac{\gamma}{8}\overline{C}(k)s + \nu_\beta^2 & \begin{array}{c} -\frac{3}{2}\frac{x_I}{R}s^2 - \frac{\gamma}{12}\frac{c}{R}(1 + 2\frac{x_a}{c})\overline{C}(k)s \\ -\frac{\gamma}{8}\overline{C}(k) - \frac{3}{2}\frac{x_I}{R} \end{array} \\ \begin{array}{c} -\frac{3}{2}\frac{x_I}{R}s^2\frac{\gamma}{6}\frac{x_a}{R}\overline{C}(k)s \\ -\frac{3}{2}\frac{x_I}{R} \end{array} & \begin{array}{c} I_{\underset{f}{*}}s^2 + \frac{\gamma}{8}(\frac{c}{R})^2\frac{x_a}{c}(1 + 2\frac{x_a}{R}\overline{C}(k)s \\ +I_{\underset{f}{*}}\nu_\theta^2 + \frac{\gamma}{6}\frac{x_a}{R}\overline{C}(k) + 2I_{\underset{f}{*}}\frac{\omega_{\theta_0}}{\Omega}\zeta_\theta \end{array} \end{bmatrix} \begin{bmatrix} \beta \\ \theta \end{bmatrix} = 0$$

Expansion of the determinant will result in

$$As^4 + Bs^3 + Cs^2 + Ds + E = 0$$

Routh's stability criteria is

$$R = AD^2 - BCD + B^2E > 0 \text{ for stable system.}$$

Neglecting the shed wake effect. The above stability criteria can be put in an approximate form as

$$\frac{x_I - x_a}{R} < (\frac{c}{R})^2(\frac{1}{3\sqrt{2}} + \frac{\gamma}{48}) \text{ for stable blade}$$

Again it is the offset of cg behind the aerodynamic center that is important. If cg lies on the 1/4-chord or ahead of it, there is no likelihood of flutter.
Eigen Analysis

The governing equations (5.15) are second order equations and can be put in a standard spring-mass-damper form and these can be solved as an eigenvalue problem. The solution will give two complex conjugate pairs, i.e, four eigen-values.

Eigenvalue $\lambda = \lambda_{Real} + i\lambda_{Imag.}$

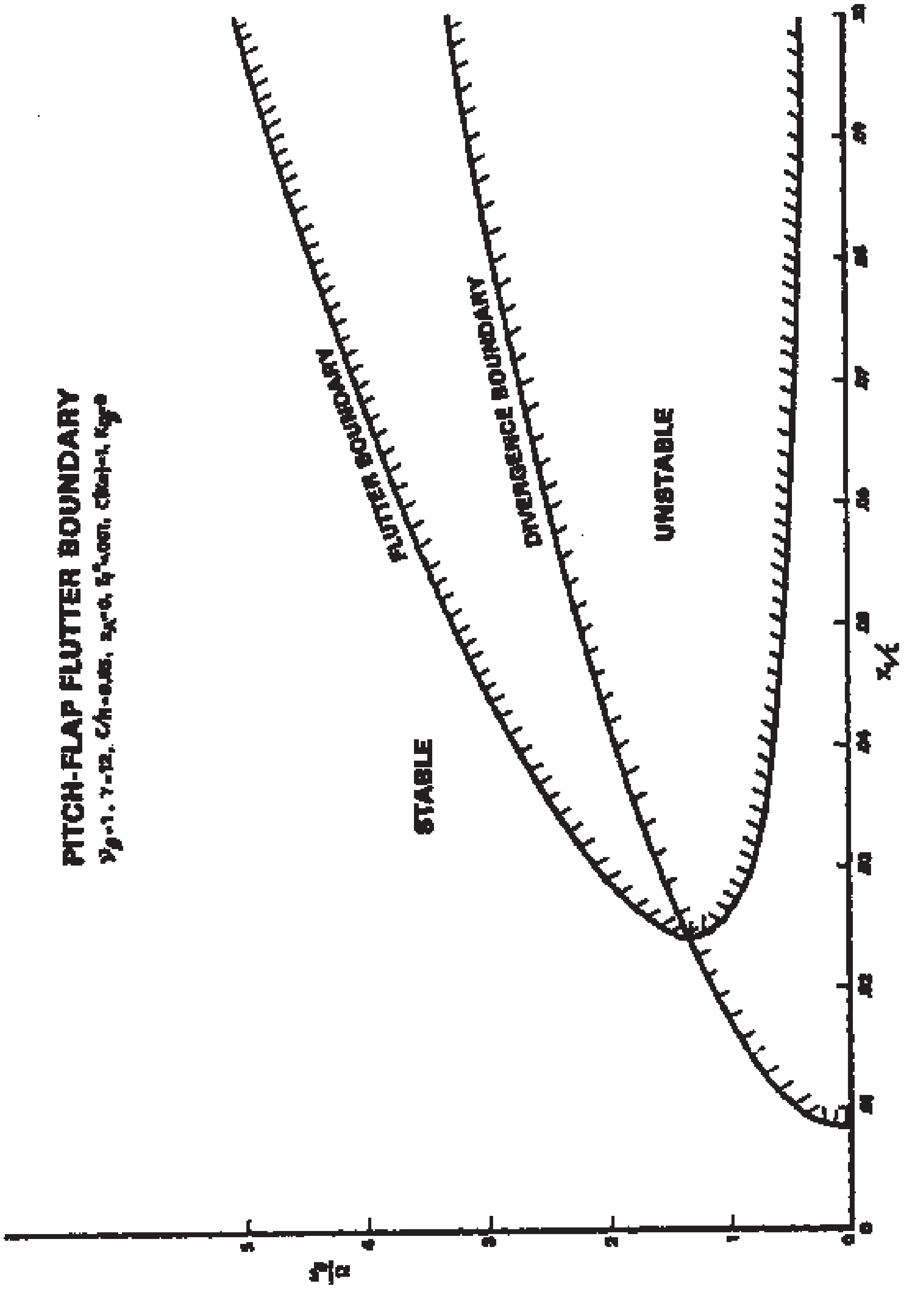
If any of the eigenvalues has a positive real part, that system is unstable. The flutter boundary is marked where the real part of the eigenvalue is zero.

Flutter stability is increased by

(a) Increasing the control stiffness, i.e. ω_θ.

(b) Reducing x_I, i.e. bringing cg closer to the elastic axis. A conservative approach is to keep cg on the aerodynamic center.

(c) Introducing damping in the torsion mode either through a mechanical damper or attaching damping tape on the control system.



Ex. The characteristics of an articulated rotor with 6% hinge offset are given as

$$\gamma = 8.0 \quad I^*_f = .001 \quad \frac{c}{R} = 20$$

The blade cg and elastic axis lies respectively at 35% and 20% of chord. If the nonrotating torsional frequency is 3 times the rotational speed.

(a) Find out whether the blade is stable from pitch divergence or not.

(b) If the elastic axis is brought to 25% chord position, you would like to find whether the blade can get into pitch-flap flutter. For simplicity assume C(k)=1 C(k) = 1

(a) For pitch divergence,

$$\frac{x_I - x_A}{R} < \frac{16}{3\gamma}\nu_\beta^2 I_f^* \nu_\theta^2 \text{ for stable blade}$$

$$\nu_\beta^2 = 1 + \frac{3}{2} \times .06 = 1.09$$

$$\text{RHS} = \frac{16}{3 \times 8} \times 1.09 \times .001 \times 10 = .0073$$

$$\text{LHS} = .10 \times \frac{1}{20} = .005$$

Blade is stable

(b) Flutter Eqs.

$$\begin{bmatrix} 1 & -.0075 \\ -.0075 & .001 \end{bmatrix} \begin{bmatrix} \beta \\ \theta \end{bmatrix} + \begin{bmatrix} 1 & -.033 \\ 0 & .000625 \end{bmatrix} \begin{bmatrix} \beta \\ \theta \end{bmatrix} + \begin{bmatrix} 1.09 & -1.0 \\ -.0075 & .01 \end{bmatrix} \begin{bmatrix} \beta \\ \theta \end{bmatrix} = 0$$

Determinant becomes

$$\begin{vmatrix} s^2 s + 1.09 & -.0075s^2 - 1.0 \\ -.0075s^2 - .0075 & -.001s^2 + .000625s + .01 \end{vmatrix} = 0$$

$$= As^4 + Bs^3 + Cs^2 + Ds + E$$

$$A = .00094 \qquad B = .00075 \qquad C = .0035$$
$$D = .00975E = .0034$$

$$R = AD^2 - BCD + B^2E = .66 \times 10^{-8}$$

Blade stable (marginal)

5.3 Flap-Lag-Torsion Flutter

The earlier two-degrees of freedom representation of flap-lag and pitch-flap blades is quite analogous to the fixed wing "typical section" analysis that treats a two-dimensional wing undergoing rigid body pitch and heave motions. An improvement over the two-degree model will be to introduce the third degree of motion. Thus, the blade undergoes rigid body flap, lag and feather rotations about hinges at the blade root, with hinge springs to obtain arbitrary natural frequencies. With this three-degree flap-lag-torsion model, both flap-lag and pitch-flap instabilities are covered. The equations of motion for this system are covered in 3.10 and the generic aerodynamic forces are defined in 4.6. This will result into three second order equations in terms of β, ζ, and θ (like Eqs. (5.3)) and these equations can be solved as an eigenvalue problem. Again the nature of the eigenvalues tells whether the system is stable or not. For a three-degree model, the hinge sequence is quite important, and the results can be quite different for different hinge sequences. The suitability of a particular hinge sequence depends on the physical configuration of the blade. For most blade configurations, a hinge sequence of inboard flap, followed by lag and then the torsion outboard, appears quite adequate. For analysis details, see Chopra (83). These simple two-degree and three-degree models help to