Chapter 7

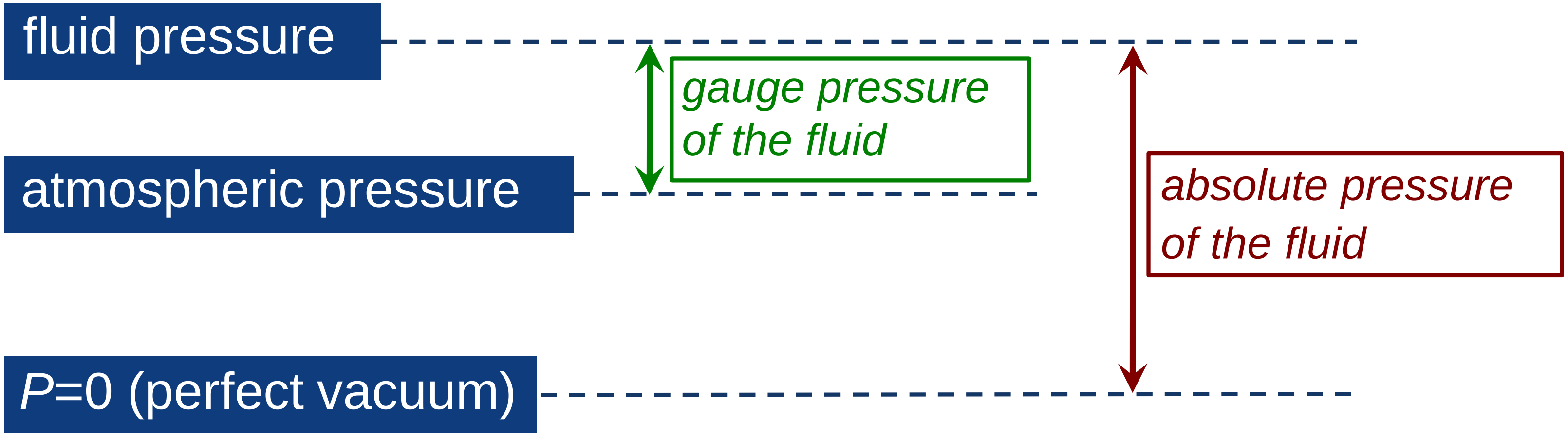
Fluid Flow

What is a fluid?

- Gas
  - loosely associated molecules that are not close together and that travel through space for long distances (many times larger than the molecular diameter) before colliding with each other
- Liquid
  - Molecules that are very close together (on the same order as their molecular diameter) and that are in collision with each other very frequently as they move around each other

The Concept of Pressure

- Absolute pressure
- Gauge pressure
  - Absolute pressure – Atmospheric pressure



Example 7.1

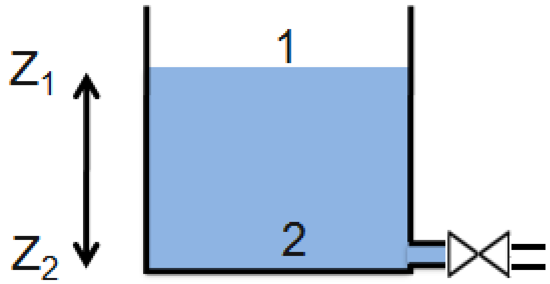
- Absolute pressure

  = Gauge pressure + Atmospheric pressure

  = 34.0 *psig* + 14.2 *psia* (usually 14.7 *psia*)

  = 48.2 *psia*
- *pounds* (lb_f) *per square inch* (*psi*)
  - *psia*
  - *psig*
- 1 *atm* = 14.7 *psi* = 760 *mmHg* = 101,300 *Pa*

Non-flowing (stagnant) Fluids



$$P_2 - P_1 = \rho\, g\, (z_1 - z_2)$$

ρ is fluid density

z is distance ***UPWARD***

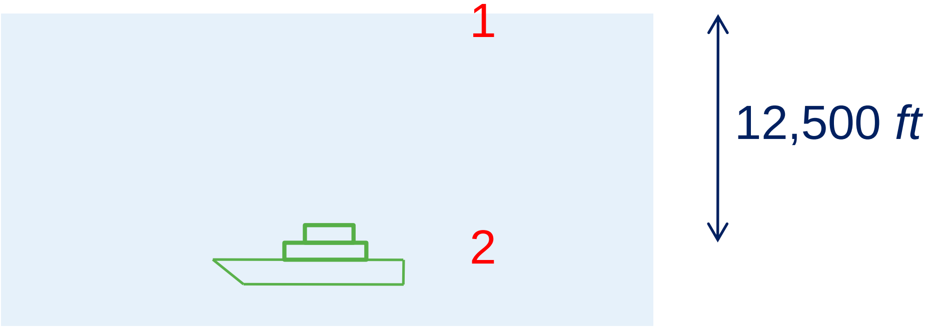
1 and *2* are locations in the liquid

Example

$$P_2 - P_1 = \rho\, g\, (z_1 - z_2)$$

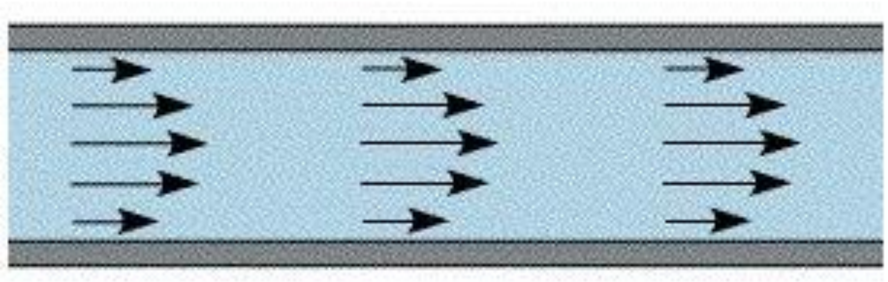
Example: The Titanic sank in 12,500 *ft*. What is the pressure (in *psi*) where she lies?

Note: the density of sea water is ~64.3 lb_m/ft^3

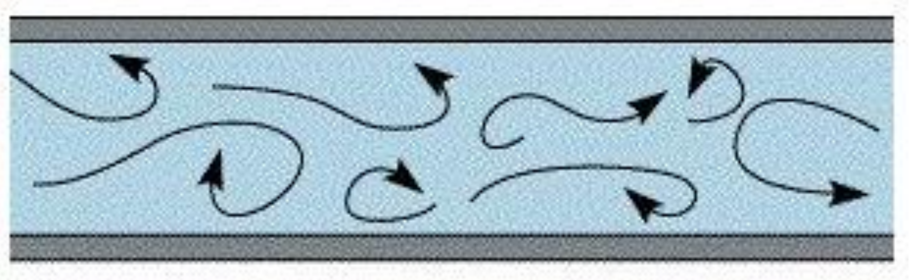


Principles of Fluid Flow

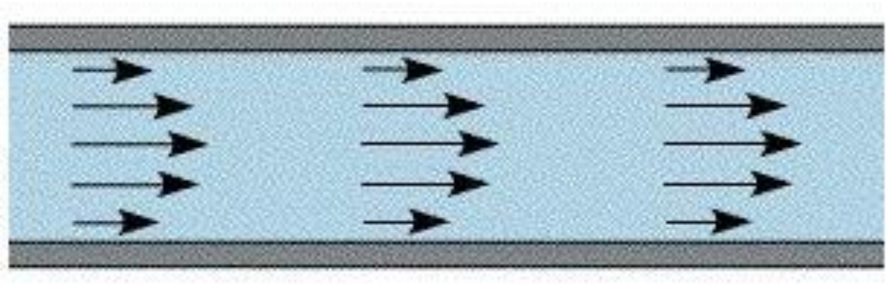
- Laminar flow



- Turbulent flow



Principles of Fluid Flow



- Average velocity (v_{avg})
  - Volumetric flow rate

$$\dot{V} = v_{avg} A_{cs}$$

A_{cs} : cross-sectional area

  - Mass flow rate

$$\rho \dot{V} = \dot{m} = \rho v_{avg} A_{cs}$$

Mechanical Energy Equation

For steady-state incompressible flow
(in the unit of energy per mass of fluid)

$$\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^{2}+gz\right)_{out}-\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^{2}+gz\right)_{in}=w_s-w_f$$

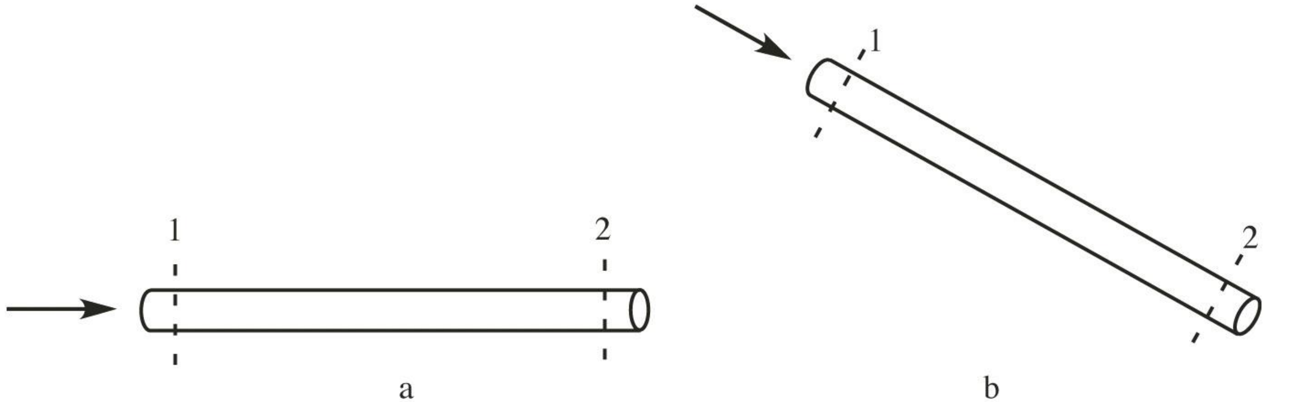


Figure 7.5

Mechanical Energy

- Kinetic energy
  - K.E.: $\frac{1}{2} m(v^2)_{avg}$
  - K.E. per mass: $\frac{1}{2} (v^2)_{avg} = \frac{1}{2} \alpha (v_{avg})^2$
  - α : a conversion factor from $(v_{avg})^2$ to $(v^2)_{avg}$
    - Can be assumed to equal 1.0
- Potential energy
  - P.E.: mgz
  - P.E. per mass: gz
- Energy associated with pressure
  - P : force/area, ρ : mass/volume
  - P/ ρ : energy/mass

Work and Friction

- Work (w_s)
  - This kind of work is called “shaft work”.
  - Positive when work is done on the fluid (e.g., by a pump)
  - Negative when the fluid does work on its environment (e.g., in a turbine)
- Friction (w_f)
  - Always positive

Mechanical Energy Equation

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = w_s - w_f$$

Increase in fluid mechanical energy
(pressure + kinetic energy + potential energy)

Positive when work is done on the fluid

Always positive

Note 1: each grouping of variables has units of energy per mass of fluid. To cast the equation in terms of "**Power**" (energy/time), multiply all terms by mass flow rate

energy/mass x mass/time = energy/time

Special Case: No Friction or Shaft Work

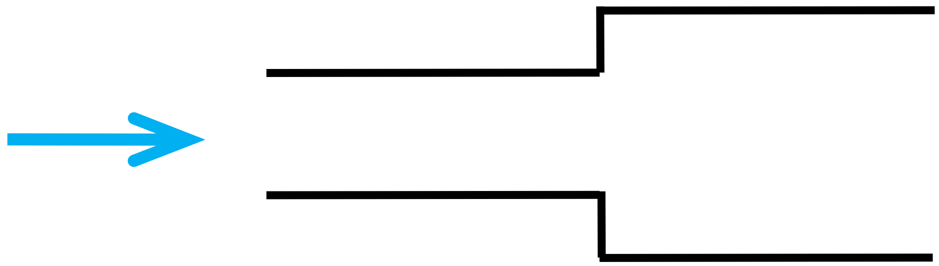
$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

Called the "Bernoulli Equation" after Daniel Bernoulli, a 19th Century fluid mechanics expert

For no work or friction

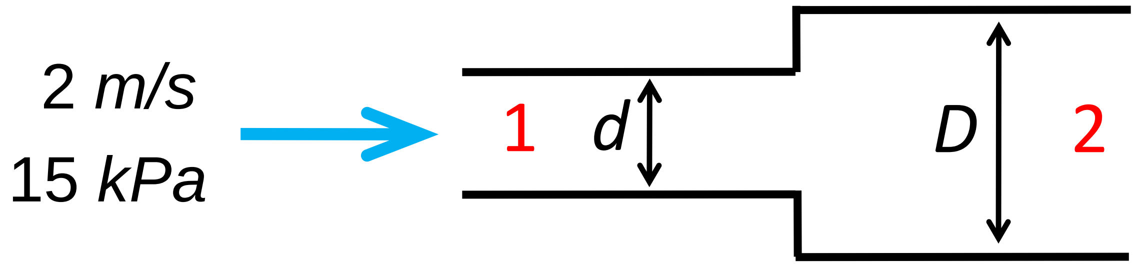
$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

What happens to the pressure in a horizontal pipe when it expands to a larger diameter?



Which form(s) of energy is (are) decreasing, and which is (are) increasing?

Example: Liquid Flow in an Expanding Pipe



a. What is the average velocity in the larger pipe?

$$v_{avg,2} = v_{avg,1}\,A_1/A_2 = v_{avg,1}\,d_1^{\,2}/D_2^{\,2} = v_{avg,1}/4 = 0.5\ m/s$$

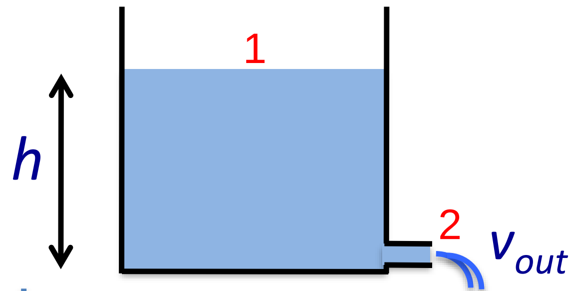
b. What is the pressure in the larger pipe?

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\,\alpha\,(v_2^{\,2} - v_1^{\,2}) + g(z_2 - z_1) = 0$$

$$P_2 - P_1 = \frac{1}{2}\,\rho\,\alpha\,(v_1^{\,2} - v_2^{\,2})$$ The Pressure Increases!

Example: An Emptying Tank

Liquid in an open tank flows out through a small outlet near the bottom of the tank. Friction is negligible. What is the outlet velocity as a function of the height of the liquid in the tank?



$$P_1 = P_2 = 0$$

$$v_1 = 0 \qquad v_2 = v_{out}$$

$$z_2 - z_1 = -h$$

$$\alpha \approx 1$$

$$\frac{\cancelto{0}{P_2} - \cancelto{0}{P_1}}{\rho} + \frac{1}{2}\cancelto{1}{\alpha}(v_2^2 - \cancelto{0}{v_1^2}) + g(\overset{-h}{z_2 - z_1}) = 0$$

$$\frac{1}{2}v_{out}^2 - gh = 0$$

$$\boxed{v_{out} = \sqrt{2gh}}$$

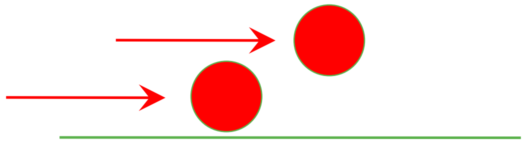
Torricelli's Equation

The Effects of Fluid Friction

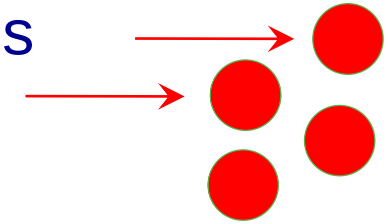
The mechanical energy equation says that friction (w_f) causes mechanical energy to decrease.

Friction is produced in flowing fluid, because fluid molecules...

- Flow past solid boundaries



- Flow past other fluid molecules



Friction in liquid flow through horizontal constant-diameter pipe:

$v_1 = v_2$

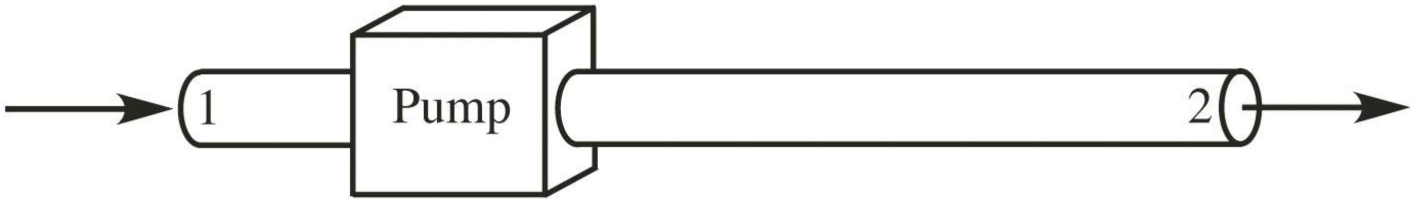
$z_2 - z_1 = 0$

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(\overset{0}{\cancel{v_2^2 - v_1^2}}) + g(\overset{0}{\cancel{z_2 - z_1}}) = -w_f$$

$$P_2 = P_1 - \rho w_f$$

Friction in liquid pipe flow *reduces pressure* (not velocity)

Pumps



Example 7.8

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha\left(\cancelto{0}{v_2^2 - v_1^2}\right) + g\left(\cancelto{0}{z_2 - z_1}\right) = w_s - w_f$$

$$P_2 = P_1 + \rho w_{pump} - \rho w_f$$

Pumps

work per mass

$$Power = \frac{work}{time} = \dot{m}\, w_s = \rho \dot{V}\, w_s$$

$$Pump\ Efficiency = \frac{Power\ delivered\ to\ the\ fluid}{Power\ to\ operate\ the\ pump}$$

Turbines

- The calculated power
  - Power extracted from the fluid using a perfect turbine
  - Actual power delivered by the turbine is smaller than that value. (friction loss, mechanical inefficiencies, etc.)

$$\textit{Turbine Efficiency} = \frac{\textit{Power delivered by the turbine}}{\textit{Power extracted from the fluid}}$$

Chapter 8

Mass Transfer

Mass Transfer

- Molecular Diffusion
  - Concentration difference
- Mass Convection
  - By bulk fluid flow

Molecular Diffusion

- Random movement (in liquids, called Brownian motion)
- Molecules of one species (A) moving through a stationary medium of another species(B)

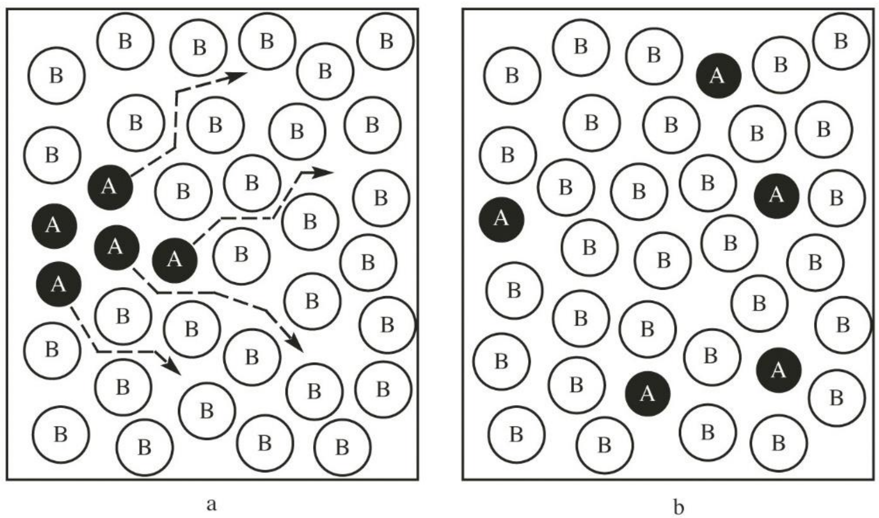
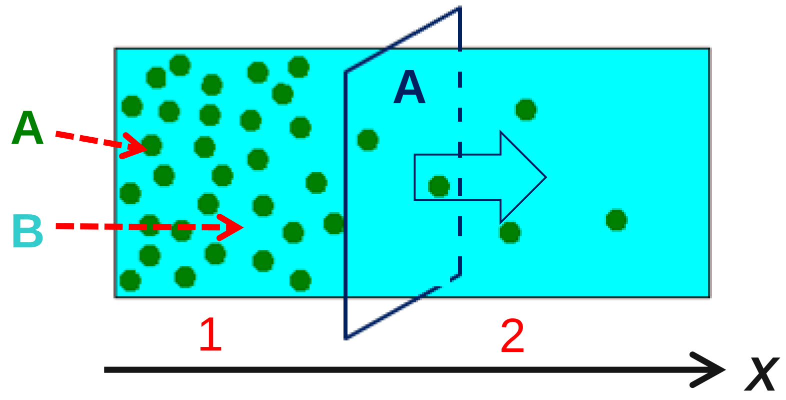


Figure 8.1

Fick's Law



("A" diffusing through "B")

$$\dot{N}_A = -D_{AB}\, A\, \frac{c_{A,2} - c_{A,1}}{x_2 - x_1}$$

$\dot{N}_A$ = moles of "A" transferred per time from "1" to "2"

D_{AB} = "diffusivity" of "A" diffusing through "B"

A = area through which diffusion occurs (cross-section)

Fick's Law

$$\dot{N}_A = -D_{AB}A\frac{c_{A,2} - c_{A,1}}{x_2 - x_1}$$

Transfer rate = Driving force / Resistance

Analogy with Ohm's Law

$$I = \frac{V}{R} \qquad \dot{N}_A = \frac{c_{A,1} - c_{A,2}}{R} = \frac{c_{A,1} - c_{A,2}}{\left(\frac{x_2 - x_1}{D_{AB}A}\right)}$$

What molecular variables affect D_{AB}?

molecular size, shape, charge, temperature

Diffusion in Contact Lens

- “Hard lenses” (polymethylmethacrylate)
  - physically uncomfortable
  - inadequate oxygen diffusion (irritation, inflammation)
- “Soft lenses” (hydrocarbon hydrogels)
  - physically more comfortable
  - inadequate oxygen diffusion
- “Oxygen permeable” (siloxane)
  - physically uncomfortable
  - better oxygen diffusion
- Latest (siloxane hydrogels)
  - physically more comfortable
  - better oxygen diffusion

Mass Convection

- Flow-enhanced transfer of one species moving through another species.

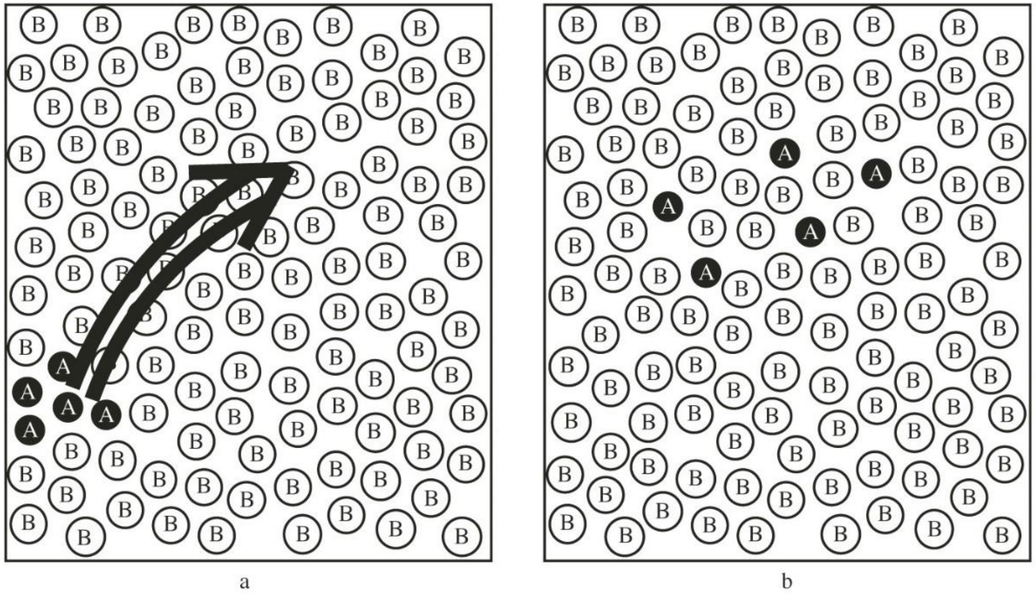


Figure 8.3

Mass Transfer across Phase Boundaries

- Mass convection + Molecular diffusion
  - Mass convection >> Molecular diffusion
- Phase boundaries
  - Liquid/Gas, Solid/Liquid, Liquid/Liquid

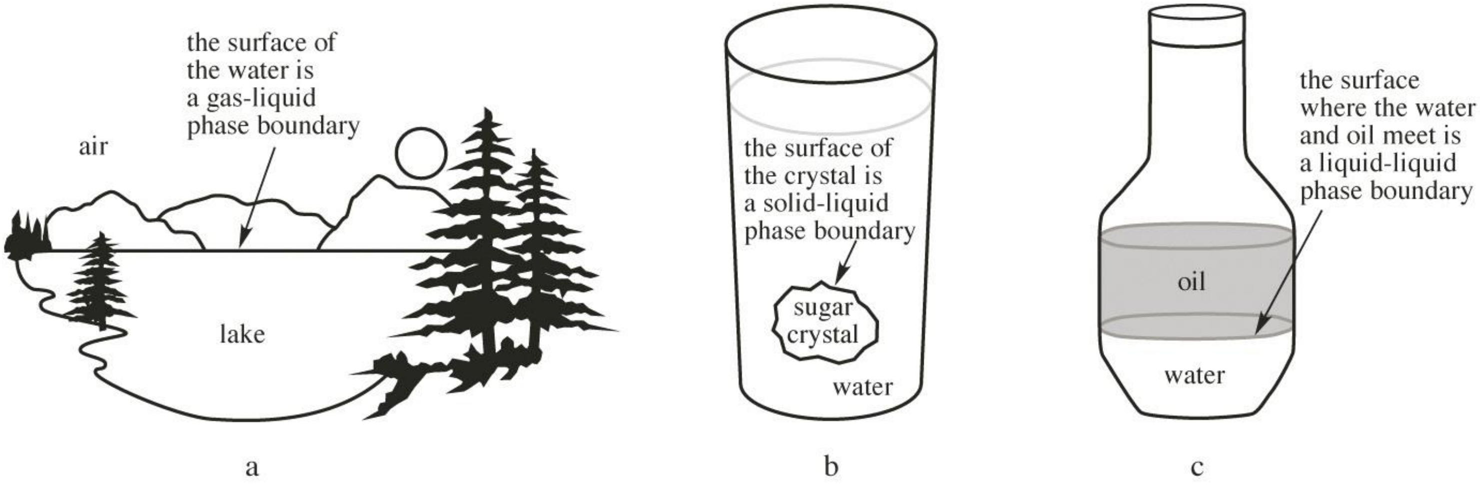


Figure 8.4

Mass Transfer across Phase Boundaries

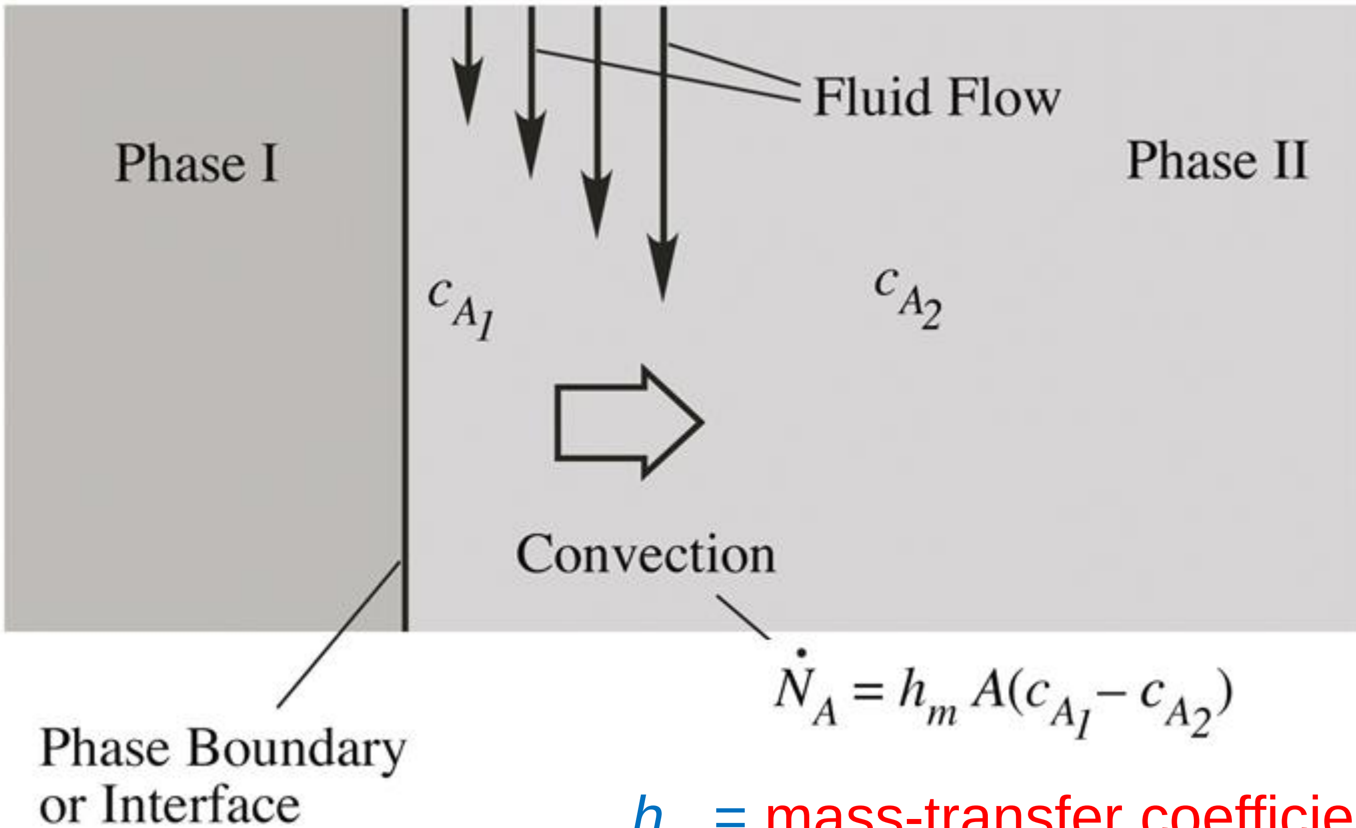


Figure 8.5

h_m = mass-transfer coefficient

c_{A1} = concentration of “A” at the phase boundary in phase II

Mass Transfer across Phase Boundaries

$$\dot{N}_A = h_m A(c_{A,1}-c_{A,2})$$

Analogy with Ohm's Law

$$I = \frac{V}{R} \qquad \dot{N}_A = \frac{c_{A,1}-c_{A,2}}{R} = \frac{c_{A,1}-c_{A,2}}{\left(\frac{1}{h_m A}\right)}$$

What variables affect h_m?

- *flow patterns (depends on geometry, etc.)*
- *molecular size*
- *molecular shape*
- *molecular charge*
- *temperature*

Ex. 8.1. The level of a lake drops throughout the summer due to water evaporation.

(a) How much volume will the lake lose per day to to evaporation?

(b) How long will it take for the water level to drop 1m?

conc. of water at the water surface	$1.0 \times 10^{-3} kgmol / m^3$
conc. of water in the wind	$0.4 \times 10^{-3} kgmol / m^3$
area of the lake	$1.7 mi^2$
mass transfer coefficient	$0.012 m / s$
density of the lake water	$1000 kg / m^3$

Multi-Step Mass Transfer

- Membrane Separation

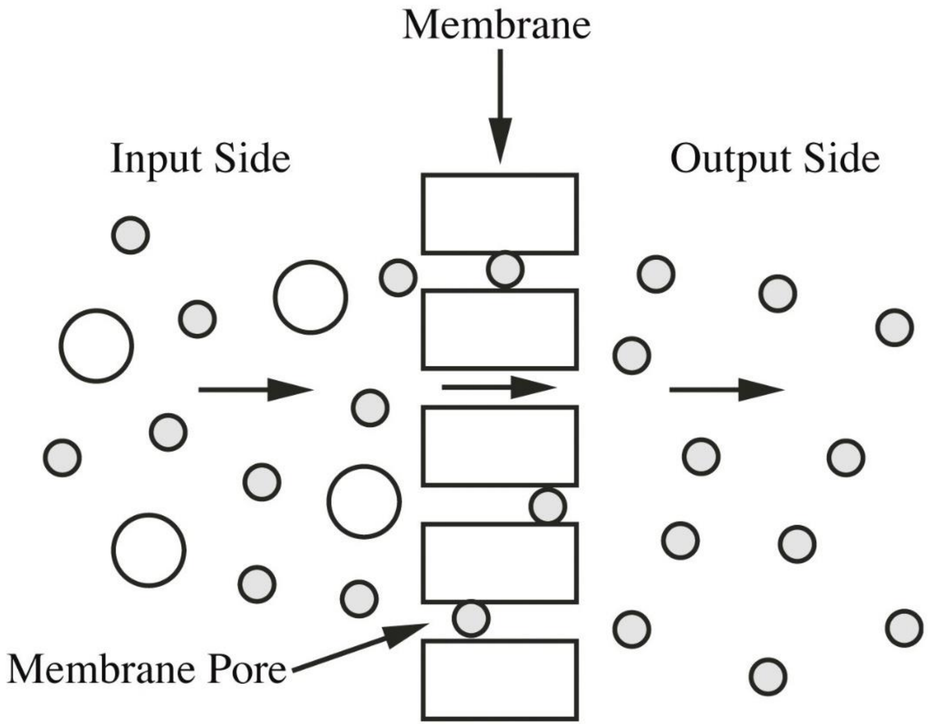


Figure 8.6

Membrane Separation

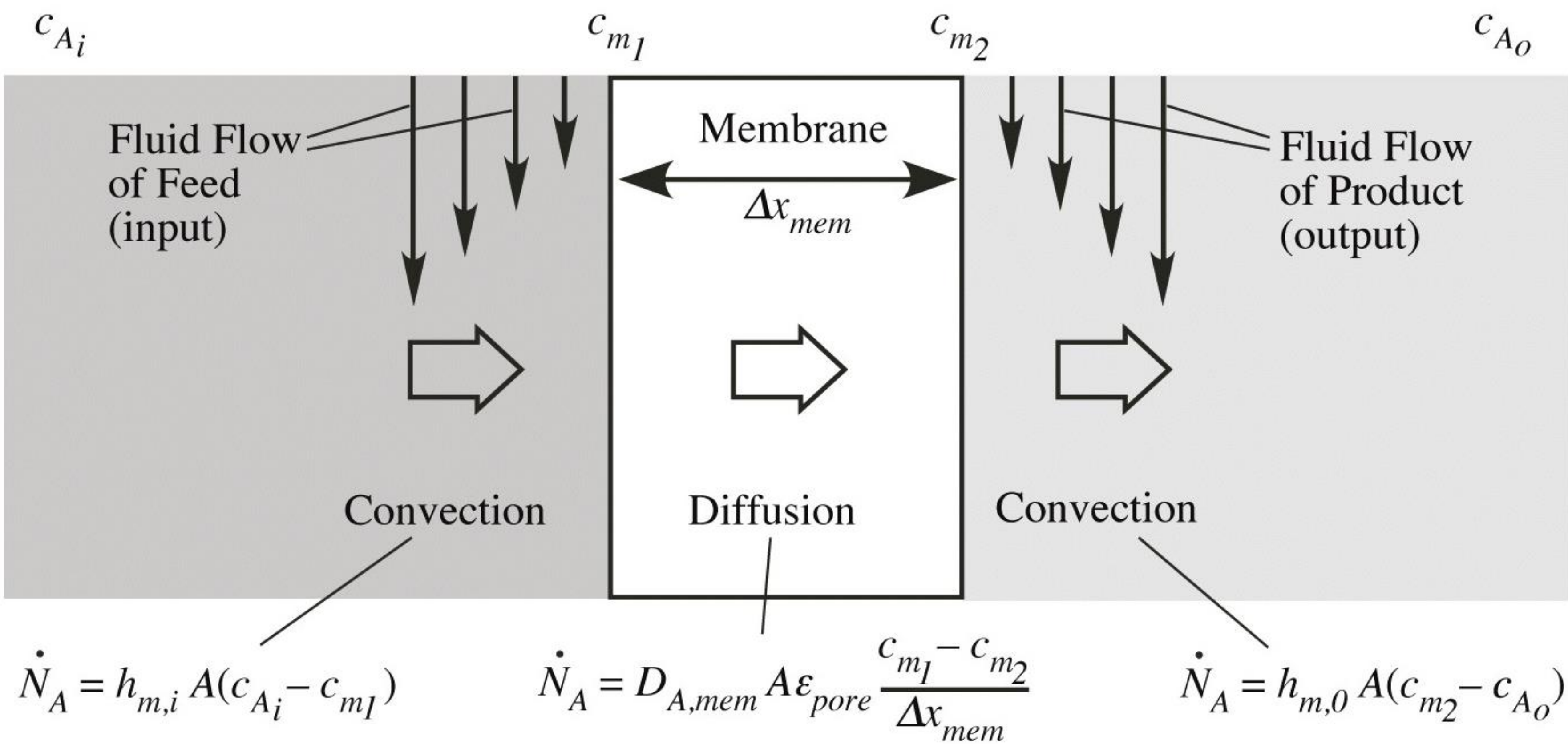
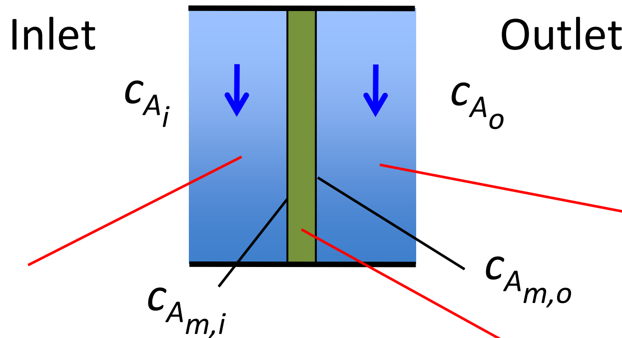


Figure 8.7

Membrane Separation



$$\dot{N}_A = h_{m_i} A(c_{A_i} - c_{A_{m,i}})$$

$$\dot{N}_A = h_{m_o} A(c_{A_{m,o}} - c_{A_o})$$

$$\dot{N}_A = D_{A,m} A \varepsilon_{pore} \frac{c_{A_{m,i}} - c_{A_{m,o}}}{\Delta x_m}$$

Solving for the concentration differences

$$c_{A_i} - c_{A_{m,i}} = \frac{\dot{N}_A}{h_{m_i} A}$$

$$c_{A_{m,i}} - c_{A_{m,o}} = \frac{\dot{N}_A \Delta x_m}{D_{A,m} A \varepsilon_{pore}}$$

$$c_{A_{m,o}} - c_{A_o} = \frac{\dot{N}_A}{h_{m_o} A}$$

Summing these…

$$c_{A_i} - c_{A_o} = \dot{N}_A \left(\frac{1}{h_{m_i} A} + \frac{\Delta x_m}{D_{A,m}\, A\, \varepsilon_{pore}} + \frac{1}{h_{m_o} A} \right)$$

Membrane Separation

$$\dot{N}_A = \frac{c_{A_i} - c_{A_o}}{\dfrac{1}{h_{m_i} A} + \dfrac{\Delta x_m}{D_{A,m} A\, \varepsilon_{pore}} + \dfrac{1}{h_{m_o} A}} = \frac{\textit{overall driving force}}{\sum \textit{resistances}}$$

Concept: Limiting Resistance

$$\text{Total Resistance} = \frac{1}{h_{m_i} A} + \frac{\Delta x_m}{D_{A,m} A\, \varepsilon_{pore}} + \frac{1}{h_{m_o} A}$$

= convection resistance on the inlet side + diffusion resistance in the membrane + convection resistance on the outlet side

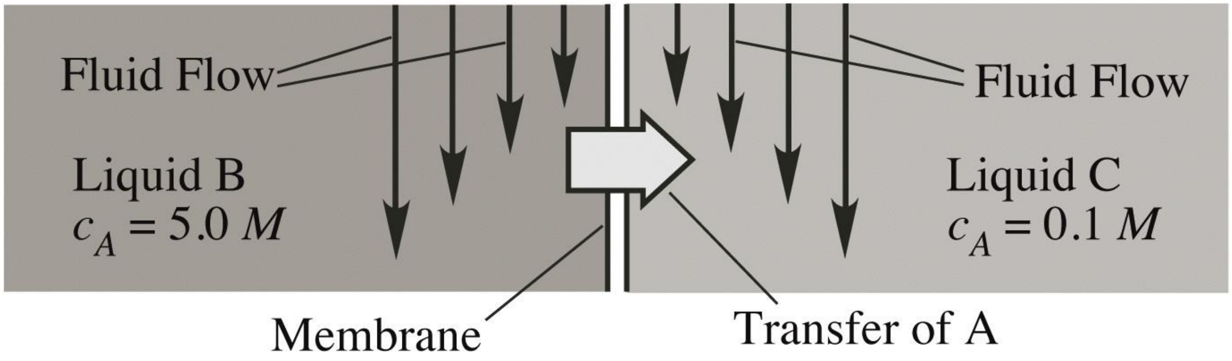
If one resistance >> the others, changing the others will not change the total resistance significantly

Ex. 8.2. Liquid B flows on one side of a membrane, and liquid C flows along the other side. Species A present in both liquids transfers from liquid B into liquid C.

(a) What is the transfer rate of A from B to C?

(b) Calculate the limiting resistance.

conc. of A in liquid B	$5.0M$
conc. of A in liquid C	$0.1M$
thickness of the membrane	$200\mu m$
diffusivity of A in the membrane	$1.0\times10^{-9} m^2/s$
area of membrane	$1m^2$
porosity of membrane	70%
mass transfer coefficient on side B	$7.0\times10^{-4} m/s$
mass transfer coefficient on side C	$3.0\times10^{-4} m/s$



Example 8.2

Ex. 8.3. In patient with severe kidney disease, urea must be removed from the blood with a hemodialyzer. In that device, the blood passes by special membranes through which urea can pass. A salt solution (dialysate) flows on the other side of the membrane to collect the urea and to maintain the desired concentration of vital salts in the blood.

(a) What is the initial removal rate of urea? (Note. This rate will decrease as the urea concentration in the blood decrease.)

(b) One might be tempted to try to increase the removal rate of urea by developing better hemodialyzer membrane. Is such an effort justified?

Blood side	
mass transfer coeff. for the urea	$0.0019 cm/s$
urea conc. within the dialyzer	$0.020 gmol/l$
Dialysate side	
mass transfer coeff. for the urea	$0.0011 cm/s$
urea conc. within the dialyzer	$0.003 gmol/l$
Membrane	
thickness	$0.0016 cm$
diffusivity of urea in the membrane	$1.8\times10^{-5} cm^2/s$
total membrane area	$1.2 m^2$
porosity	20%