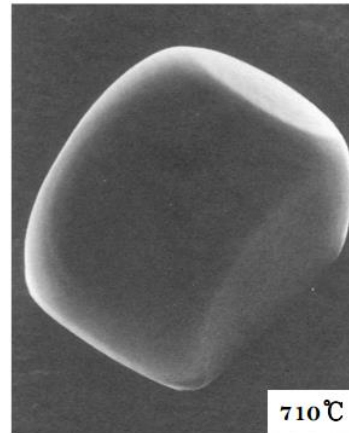
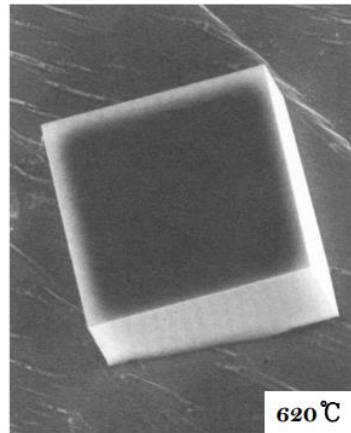


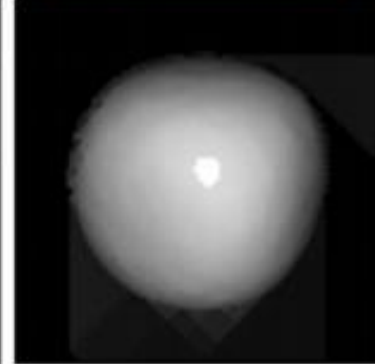
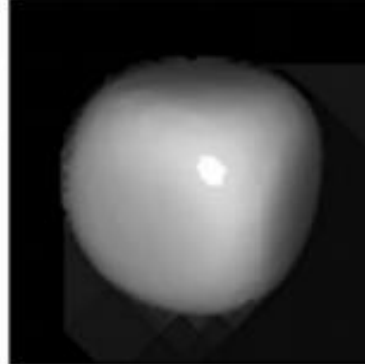
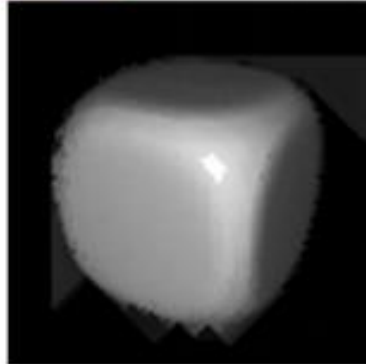
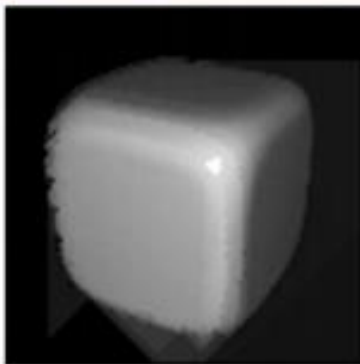
Smooth interface, Rough interface



J.C. Heyraud, J.J. Metois, J. Crystal Growth , 84, 503 (1987)

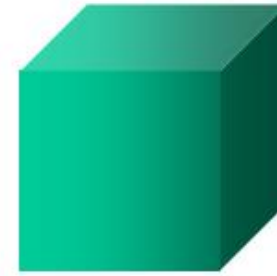
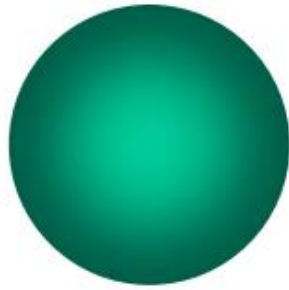
singular (smooth) interface

rough interface



Heating up to the roughening transition.

Two types of solid-liquid interfaces



How do you like to call them?

rough interface

singular (smooth) interface

What about the dependence of surface energy on crystal directions?

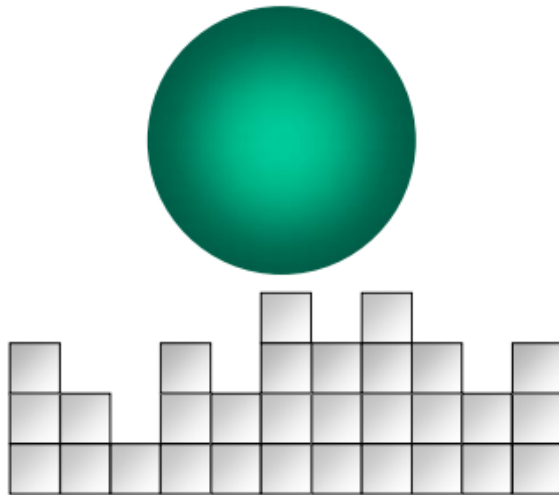
isotropic γ

anisotropic γ

Do not vary with crystallographic orientation,
i.e. γ -plots are spherical

Strong crystallographic effects,
: solidify with low-index close-packed facets

Rough vs. smooth interfaces



atomically-disordered

Ex) metallic systems



atomically-flat

nonmetals

Apply thermodynamics to this fact and derive more information.

Entropy-dominant

weak bonding energy

stable at high T

Enthalpy-dominant

strong bonding energy

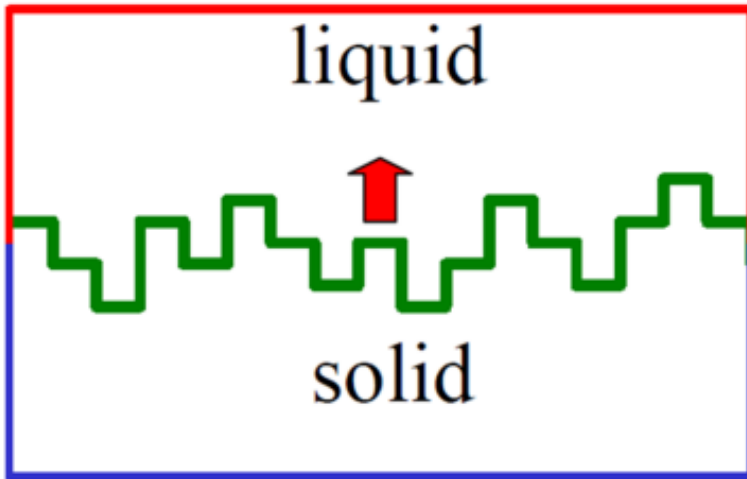
stable at low T



4.2 Growth of a Pure Solid

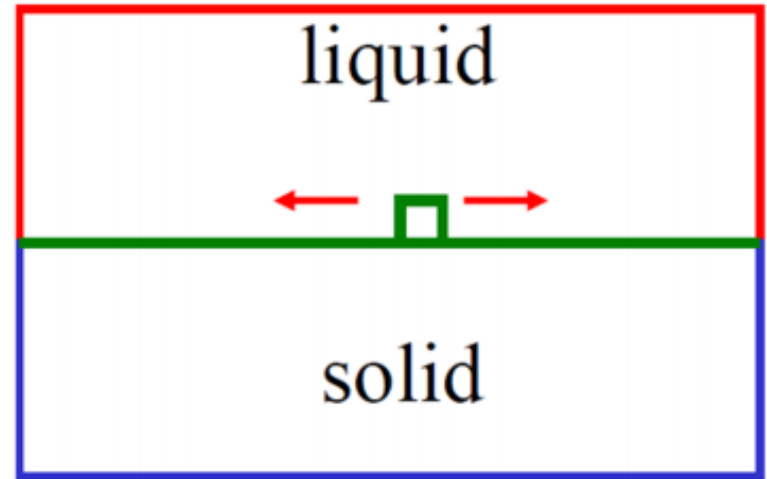
a) Continuous growth

: Atomically rough or diffuse interface



b) Lateral growth

: Atomically flat or sharply defined interface



4.2.1 Continuous Growth

- ❖ Solid/ liquid diffuse interface
≈ the migration of a random high angle grain boundary

- ❖ **Driving force for solidification** $\Delta G = \frac{L}{T_m} \Delta T_i$

(L : latent heat of melting, ΔT_i : undercooling of the interface)

- ❖ **Net rate of solidification**

$$v = k_1 \Delta T_i$$

Growth rate is usually a diffusion controlled process

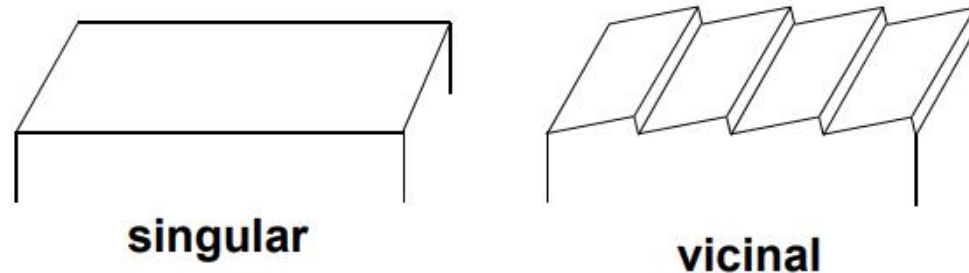
- ✓ Pure metal: grow at rate controlled by heat conduction
- ✓ Alloy grow at rate controlled by solute diffusion

4.2.1 Lateral Growth

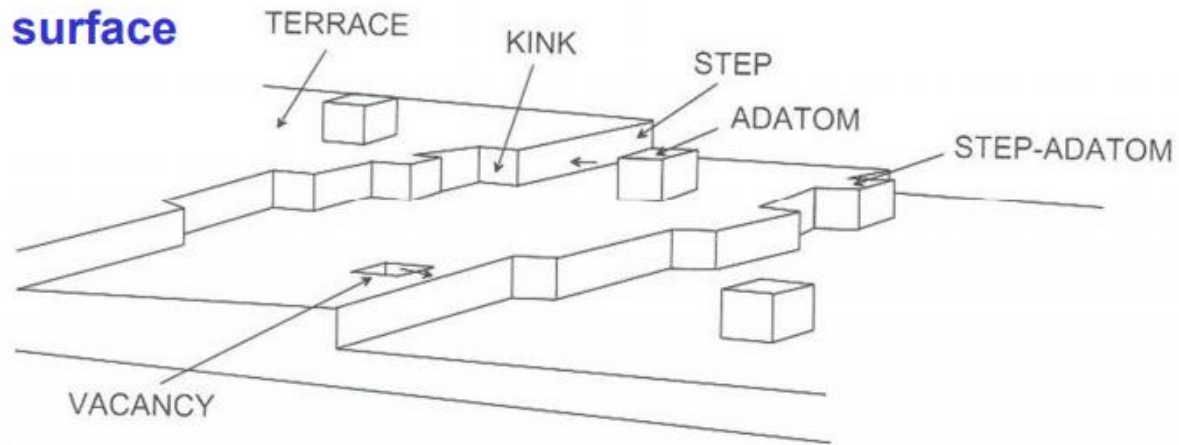
- ❖ Metals with a high entropy of melting prefer to form atomically smooth, close packed interface.

Atomic View

Ideal Surfaces

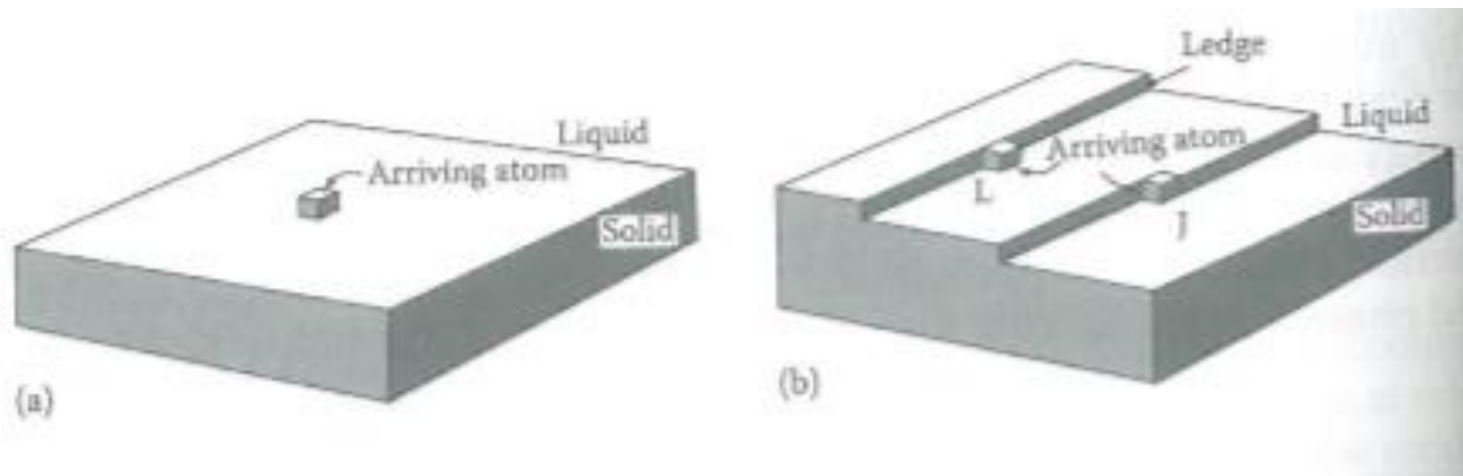


More realistic surface



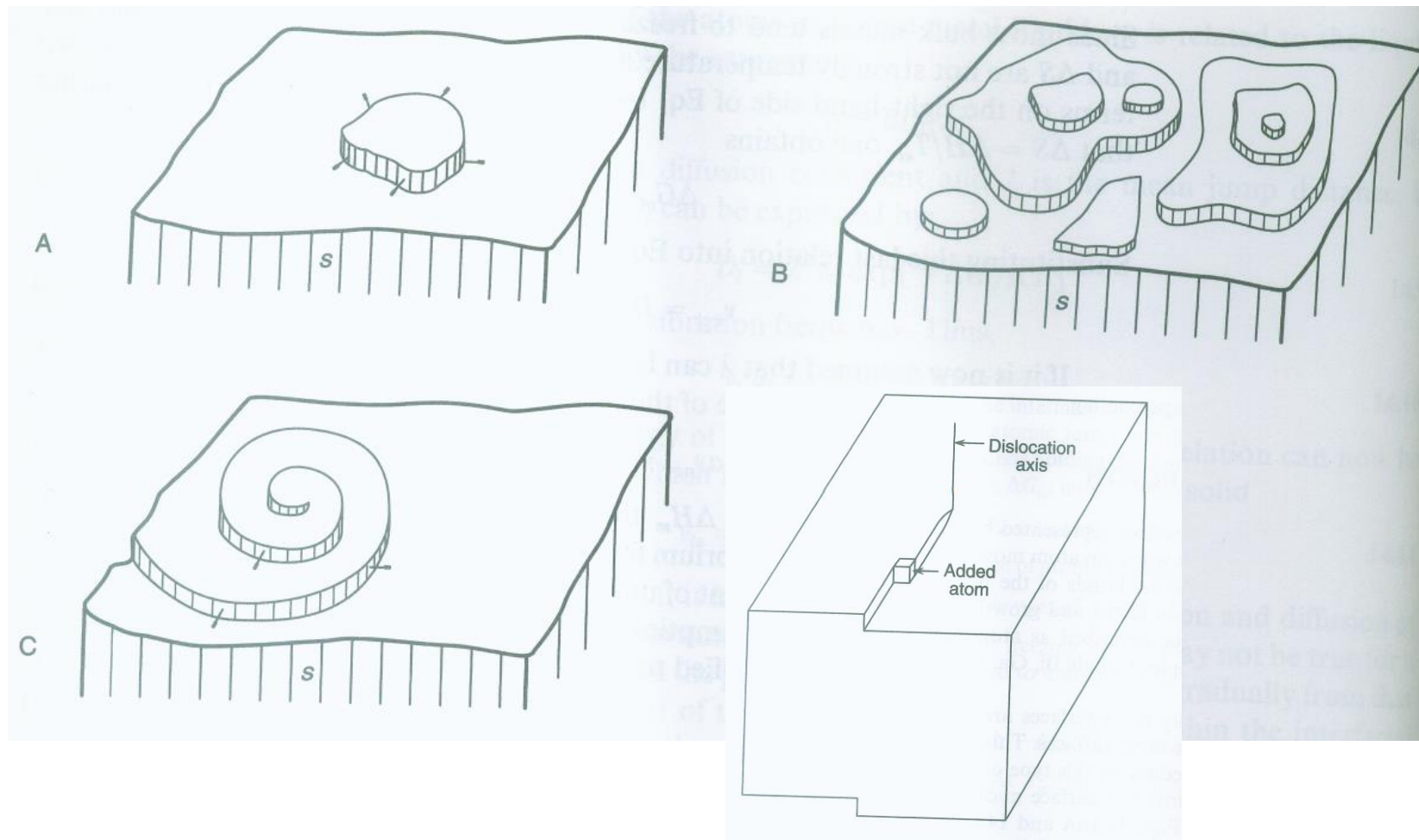
4.2.1 Lateral Growth

- ❖ G_{min} = Minimum interace energy, i.e. min. # of broken 'solid' bonds
- ❖ # of broken bonds for a single atom: flat surface – 4
edge -2
corner – no increase
- ❖ **Low accommodation rate**
 - ✓ Ledge + jog → liquid can join solid without increasing interfacial energy → lateral growth of ledge (coherent solid/ solid interface)



4.2 Growth of a Pure Solid

❖ Lateral Growth



4.2 Growth of a Pure Solid

1) Surface nucleation

- ✓ If large # of atoms → form a disc-shaped layer,
- ✓ Self-stabilized and continue to grow.
- ✓ The edges of disc + contributing to E → counterbalanced by ΔG_v .
- ✓ ΔT becomes large, $r^* \downarrow$.

$$v \propto \exp(-k^2 / \Delta T_i)$$

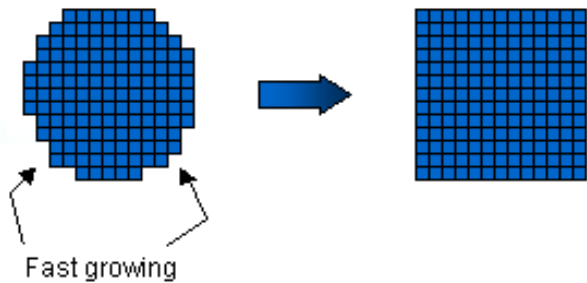
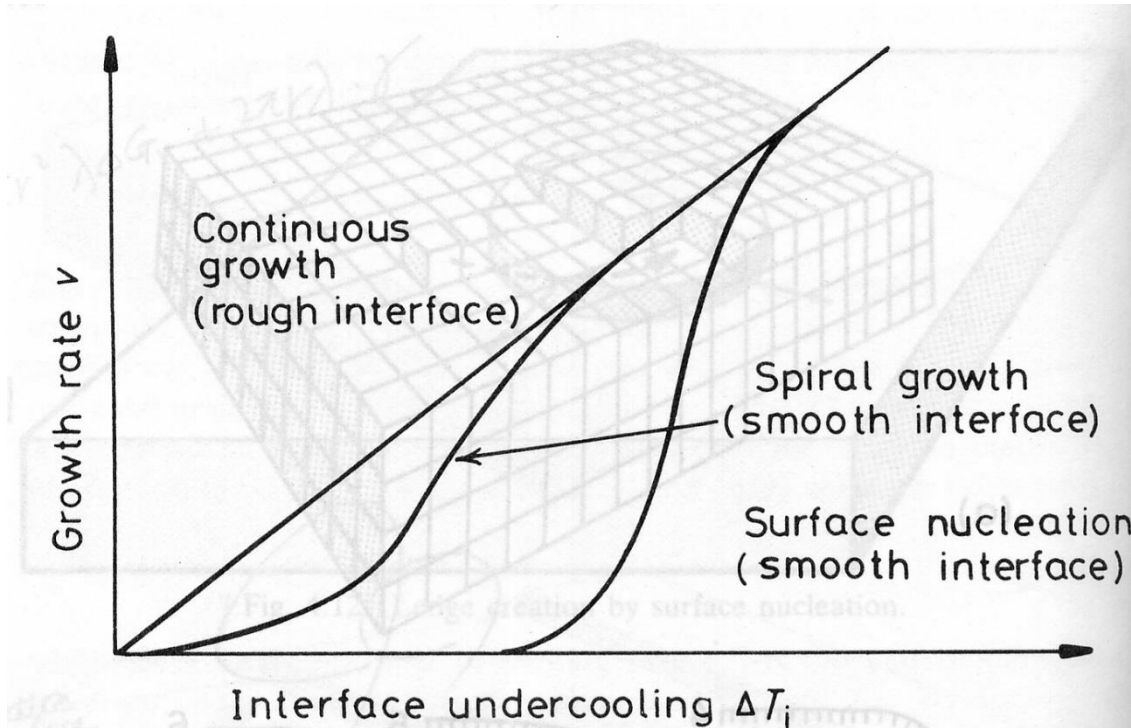
2) Spiral growth

- ✓ Addition of atoms to the ledge cause it to rotate around the axis of screw dislocation.
- ✓ The spiral tightens until it reaches a min radius of r^*
- ✓ v (growth rate) = $k_3(\Delta T_i)^2$

3) Growth from twin boundary

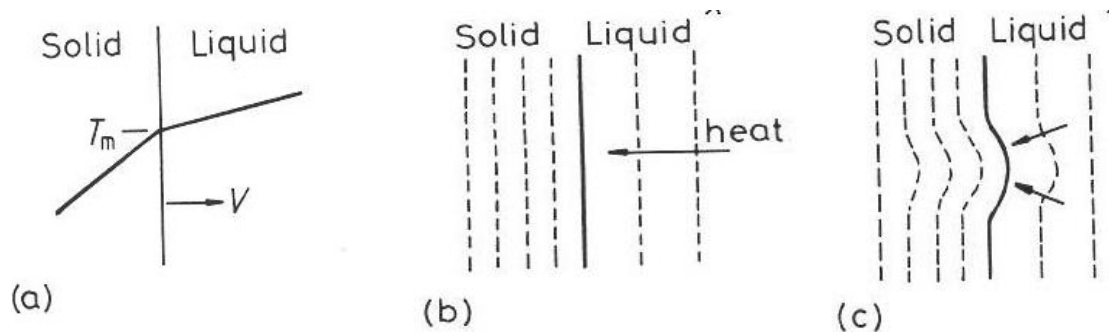
- ✓ Another permanent source of steps like spiral growth

4.2 Growth of a Pure Solid



4.2.3 Lateral Growth of a Pure Solid

1) Superheated liquid



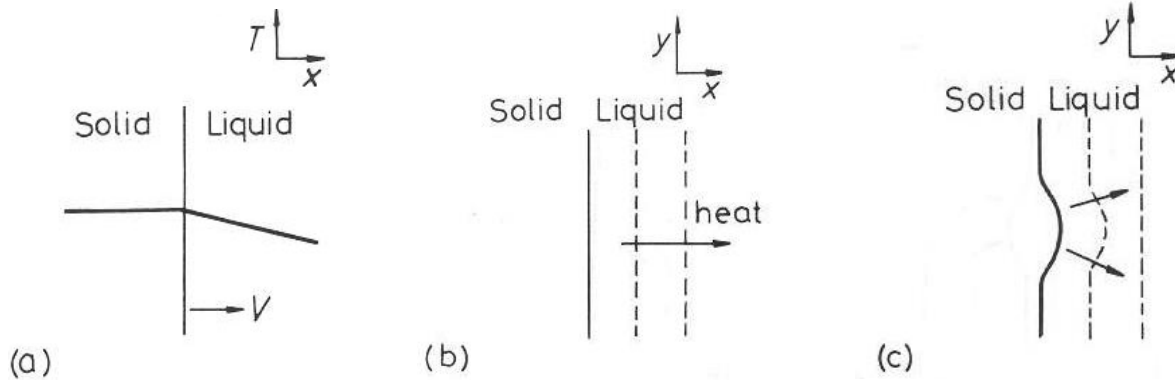
- ✓ Solid growing at v (planar)
- ✓ Heat from away from the interface through solid = heat inflow from liquid + L

$$K_S T'_S = K_L T'_L + vL_V$$

- ✓ If r is so large \rightarrow Gibbs-Thompson effect can be ignored the solid/liquid interface remain at T_m (r : radius of curvature of the protrusion)
- ✓ dT/dx in the liquid ahead of the protrusion will increase more positively.
- ✓ More heat to the protrusion \rightarrow melt away $V \downarrow$ to match other v in planar region

4.2 Growth of a Pure Solid

2) Solid growing into supercooled liquid



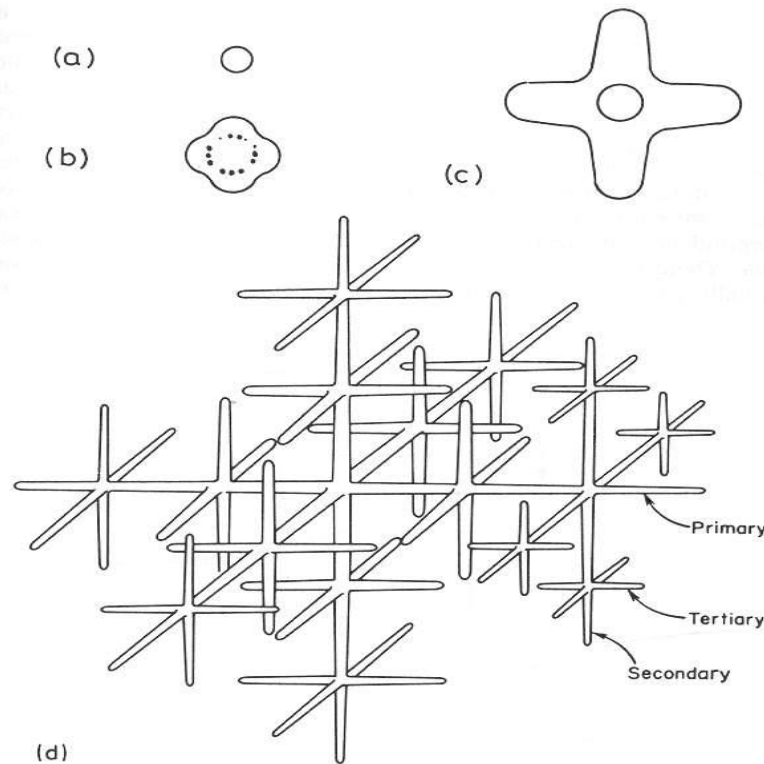
- ✓ Heat removed more effectively from the tip.
- ✓ Protrusion $\frac{dT_L'}{dX} < 0$ becomes more negative
- ✓ Heat flow from solid = the protrusion grows preferentially.
- ✓ Solidification begins in the bulk of the liquid.
- ✓ Supercooled liquid에서 solid 생성 \rightarrow 잠열이 liquid로 발산

4.2 Growth of a Pure Solid

❖ Development of Thermal Dendrite

cf. Constitutional supercooling

* shown at the beginning of solidification at the tip of dendrites



$$K_S T'_S = K_L T'_L + v L_V$$

Assume $T'_S \approx 0$

T'_L measured in the direction of v

$$\Delta T_c = T_i - T_\infty$$

4.2 Growth of a Pure Solid

$$v = \frac{-K_L T'_L}{L_V} \cong \frac{K_L}{L_V} \cdot \frac{\Delta T}{r}$$

✓ $v \uparrow$ as $r \downarrow$ due to enhanced heat conduction

$$\Delta T_r = \frac{2\gamma T_m}{L_V r} \quad \text{as a result of G-T eq.}$$

Min. radius r occur when $\Delta T_r = \Delta T_o = T_m - T_\infty$

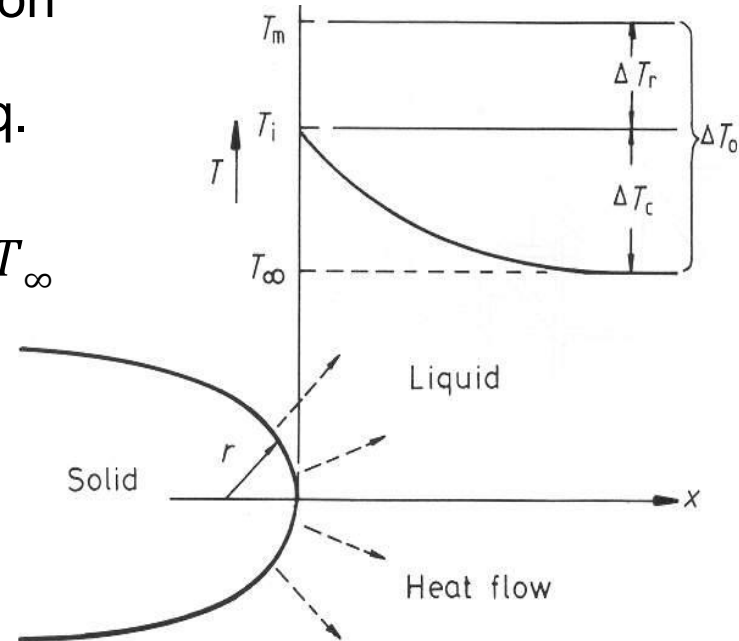
r_{min} = the critical nuclei radius

$$r^* = \frac{2\gamma T_m}{L_V \Delta T_o} \quad \Delta T_r = \frac{\Delta T_o r^*}{r}$$

$$v \cong \frac{K_L}{L_V} \cdot \frac{1}{r} \left(1 - \frac{r^*}{r} \right)$$

$V \rightarrow 0$ as $r \rightarrow r^*$ and as $r \rightarrow \infty$ heat conduction slow.

$v = v_{max}$ when $r = 2r^*$



4.3 Alloy Solidification

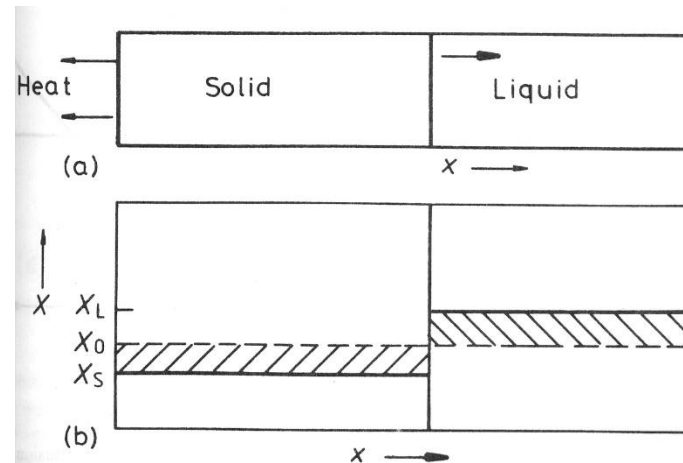
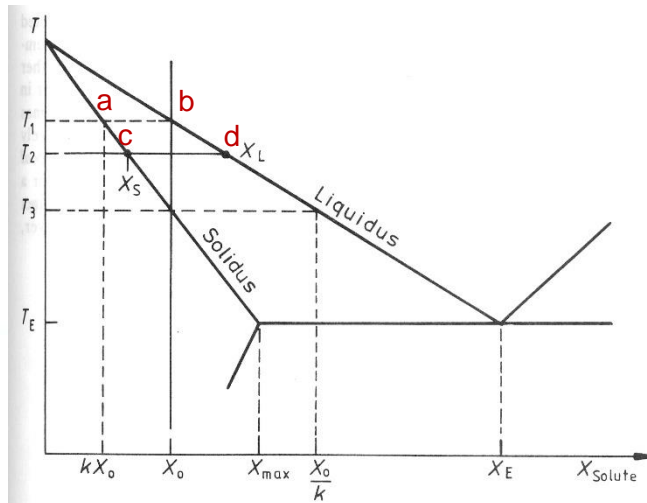
4.3.1 Solidification of single-phase alloys

$$k \text{ (partition coefficient)} = \frac{X_s}{X_L} < 1$$

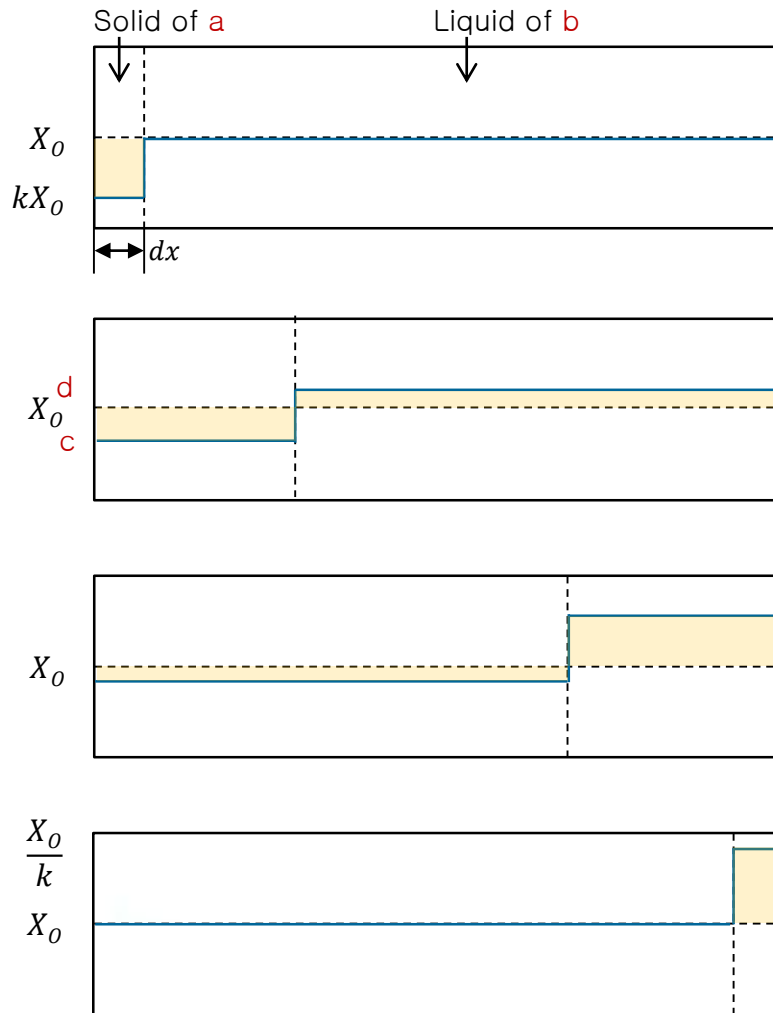
❖ Three limiting cases

1. Infinitely slow (equilibrium) solidification

- ✓ Sufficient time for diffusion in solid & liquid : kX_o, X_o
- ✓ Relative amount of solid and liquid : lever rule



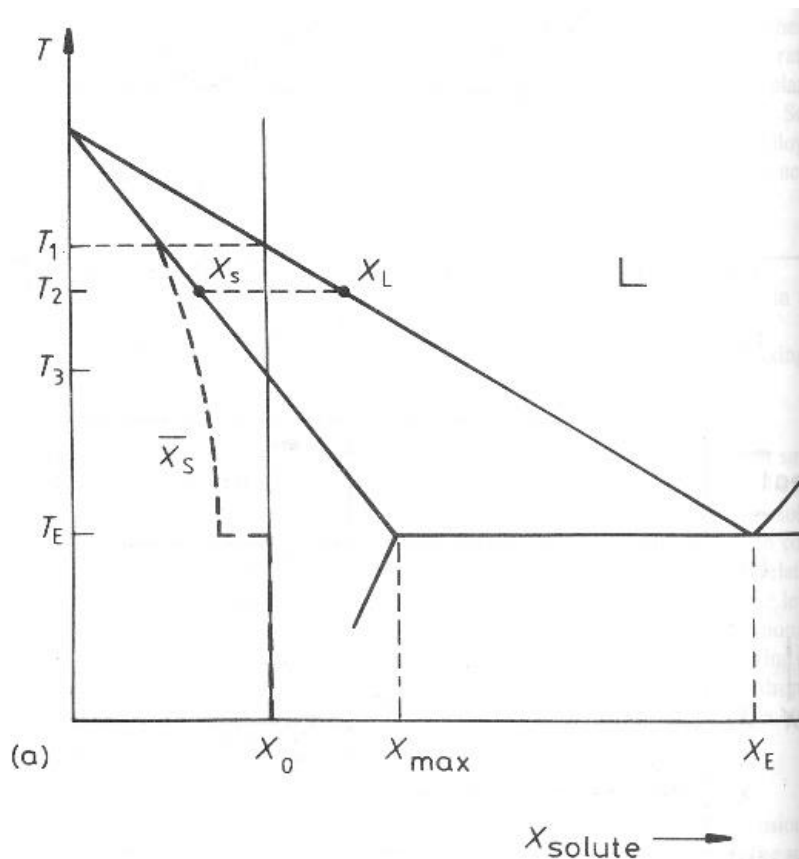
4.3 Alloy Solidification



- ✓ Solidification starts at T_1 (solid kX_0)
- ✓ Liquid 상의 조성 증가 → Solidification이 더 낮은 온도 ($T < T_1$)에서 생김
- ✓ Liquid와 solid의 양: lever rule
- ✓ 마지막 T_3 에서 응고 종료
- ✓ 전체 bar가 X_0 조성으로 응고

4.3 Alloy Solidification

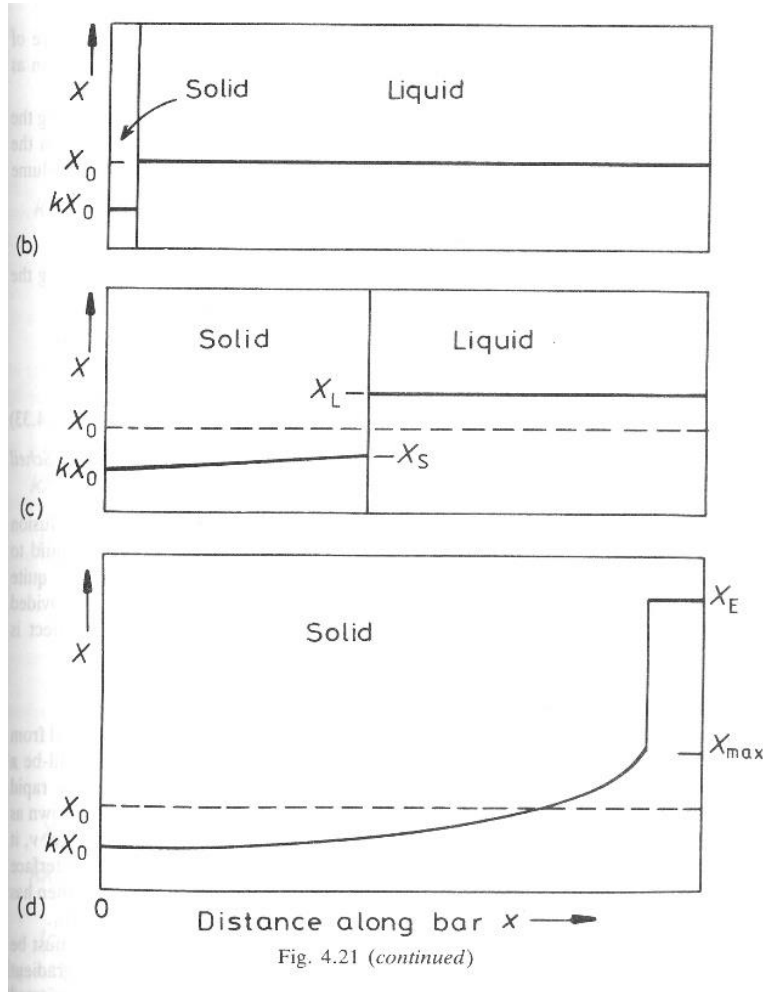
2. No diffusion in solid, perfect mixing in liquid



- ✓ Separate layers of solid retain their original compositions mean comp. of the solid $(\bar{X}_s) < X_s$
- ✓ Liquid become richer than $\frac{X_0}{k} \rightarrow X_E$
- ✓ Variation of X_s : solute rejected to the liquid

4.3 Alloy Solidification

2. No diffusion in solid, perfect mixing in liquid



- ✓ T_1 에 도달하면 응고시작
- ✓ 계면온도는 T_1 이하로 감소
- ✓ Liquid/solid의 계면은 항상 “local equilibrium”
- ✓ Mean composition of solid (\bar{X}_S)
- ✓ 계면에서 liquid와 solid의 양 (\bar{X}_S 와 X_L 의 lever rule)
- ✓ Liquid는 $\frac{X_0}{k}$ 를 넘어서 X_E 까지 갈 수 있다.

4.3 Alloy Solidification

$$\text{Mass balance : } (X_L - X_S)df_S = (1 - f_S)dX_L$$

$$\text{B.C. : } X_S = kX_O \text{ when } f_S=0$$

$$\int_0^{f_S} \frac{df_S}{1 - f_S} = \int_{X_O}^{X_L} \frac{dX_L}{X_L - X_S} = \int \frac{dX_L}{X_L(1 - k_O)}$$

$$\int_0^{f_S} (1 - k)(-1)d \ln(1 - f_S) = \int_{X_O}^{X_L} d \ln X_L$$

$$\ln \frac{X_L}{X_O} = (k - 1) \ln(1 - f_S)$$

$$\therefore X_L = X_O f_S^{(k-1)} \quad \leftarrow \text{non-eq. lever rule. (Scheil eq.)}$$

If $k < 1$: predicts that if no diff. in solid, some eutectic always exist to solidify