

5.4 Overall Transformation Kinetics – TTT Diagram

❖ Johnson-Mehl-Avrami Kolmogorov Equation

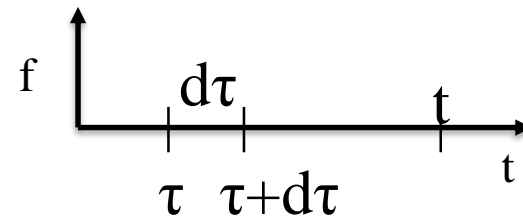
Assumption :

- Reaction produces by nucleation and growth
- Nucleation occurs randomly throughout specimen
- Reaction product grows radially until impingement

$\alpha \rightarrow \beta$

Define volume fraction transformed

$$f = \frac{\text{Vol. of new phase}}{\text{Vol. of specimen}}$$



$$df = \frac{\left(\begin{array}{l} \text{vol. of one particle nucleated} \\ \text{during } d\tau \text{ measured at time } t \end{array} \right) \times \left(\begin{array}{l} \text{number of nuclei} \\ \text{formed during } d\tau \end{array} \right)}{\text{volume of specimen}}$$

5.4 Johnson-Mehl-Avrami Kolmogorov Equation

$$\text{volume : } V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (vt)^3$$

v : growth rate (assumed const.)
 N : nucleation rate (const.)

$$V' = \frac{4}{3} \pi v^3 (t - \tau)^3$$

$$df = \frac{\frac{4}{3} \pi [v(t - \tau)]^3 \times (NV_0 d\tau)}{V_0}$$

$$f_e = \int_0^x d\hat{f} = \frac{4}{3} \pi N v^3 \int_0^t (t - \tau)^3 d\tau$$

$$f_e = \frac{\pi}{3} N v^3 t^4$$

→ do not consider impingement & repeated nucleation

→ only true for $f \ll 1$

f_e : extended volume fraction

❖ Johnson-Mehl-Avrami Kolmogorov Equation

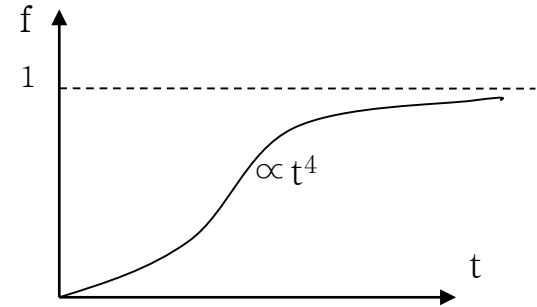
$$df = (1 - f) df_e \quad \rightarrow \quad 1 - f = \frac{df}{df_e}$$

$$f = 1 - \exp\left(-\frac{\pi}{3} N v^3 t^4\right)$$

$$f = 1 - \exp(-k t^n) \quad \dots \text{J-M-A Eq.}$$

k : sensitive to temp. (N, v)

n : $1 \sim 4$



For the case of 50% transform, $t_{0.5}$

$$\exp(-0.7) = 0.5$$

$$k t_{0.5}^n = 0.7$$

$$\text{i.e. } t_{0.5} = \frac{0.7}{k^{1/n}}$$

Example above.

$$t_{0.5} = \frac{0.9}{N^{1/4} v^{3/4}}$$

❖ Johnson-Mehl-Avrami Kolmogorov Equation

Other example:

1. Fixed nucleation site and active at the beginning.
2. Nucleation site depends on the time

$$I = I_o \exp(-pt) \quad (I_o : \text{number of active nucleation site / unit volume})$$

p : rate at which the individual sites are lost.

$$N(t) = -\frac{dI}{dt} = I_o p \exp(-pt) = pI$$

$$f = 1 - \exp \left\{ \left(-\frac{8\pi I_o v^3}{p^3} \right) \times [\exp(-pt) - 1 + pt - \dots] \right\}$$

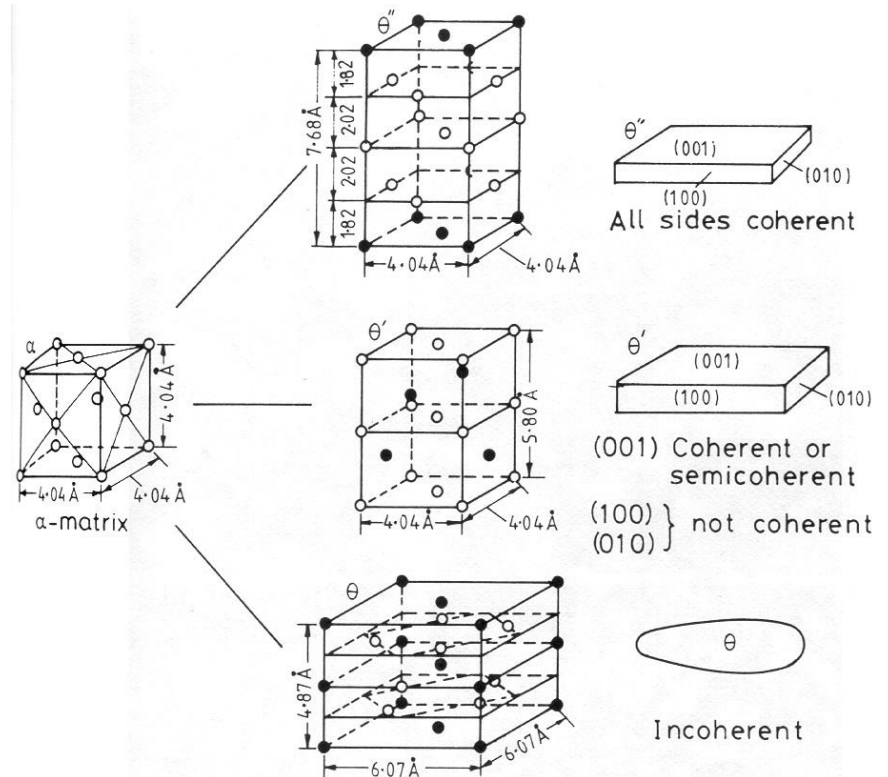
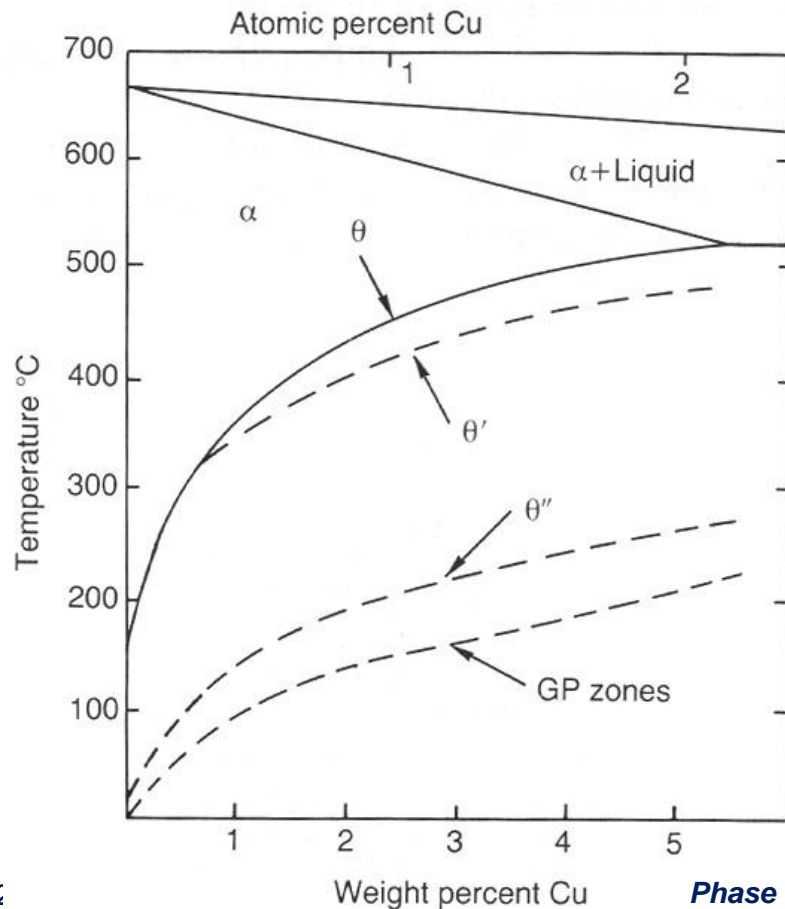
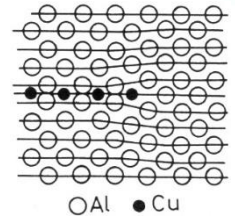
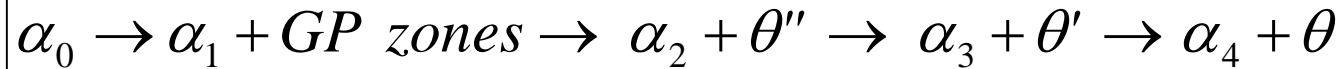
limiting cases : pt small \rightarrow same as J-M eq.
 pt large $\rightarrow \left\{ \frac{p^3 t^3}{6} \right\}$ I quickly goes to zero.

$$f = 1 - \exp \left(-\frac{\pi}{3} N_o v^3 t^3 \right)$$

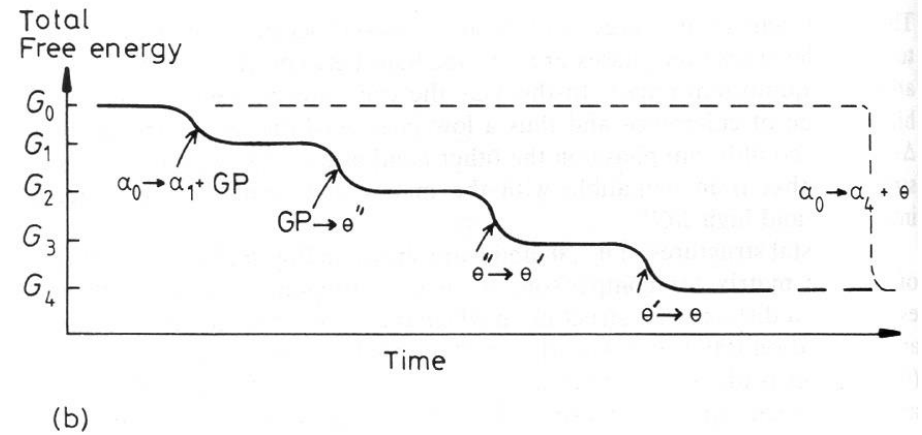
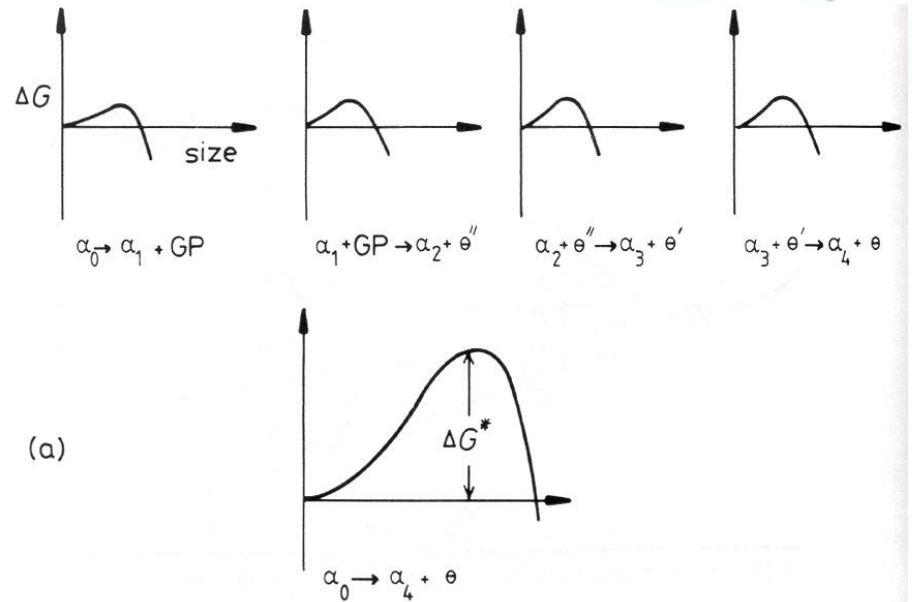
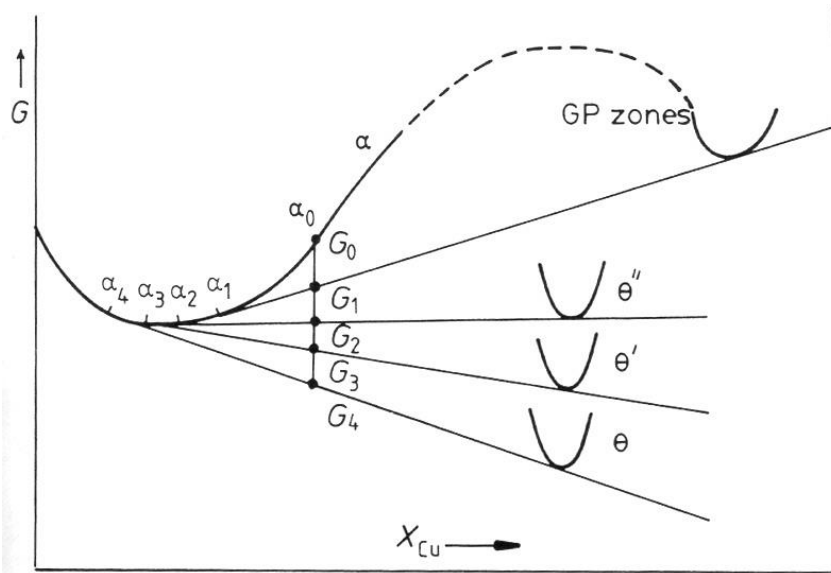
5.5 Precipitation in Age-Hardening Alloys

5.5.1 Precipitation in Aluminum-Copper Alloys

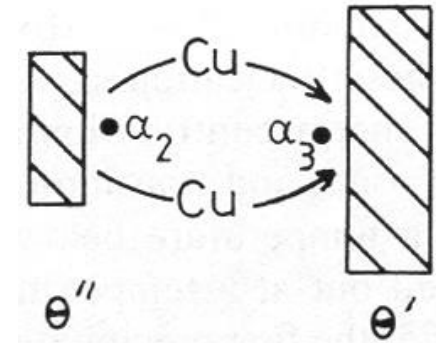
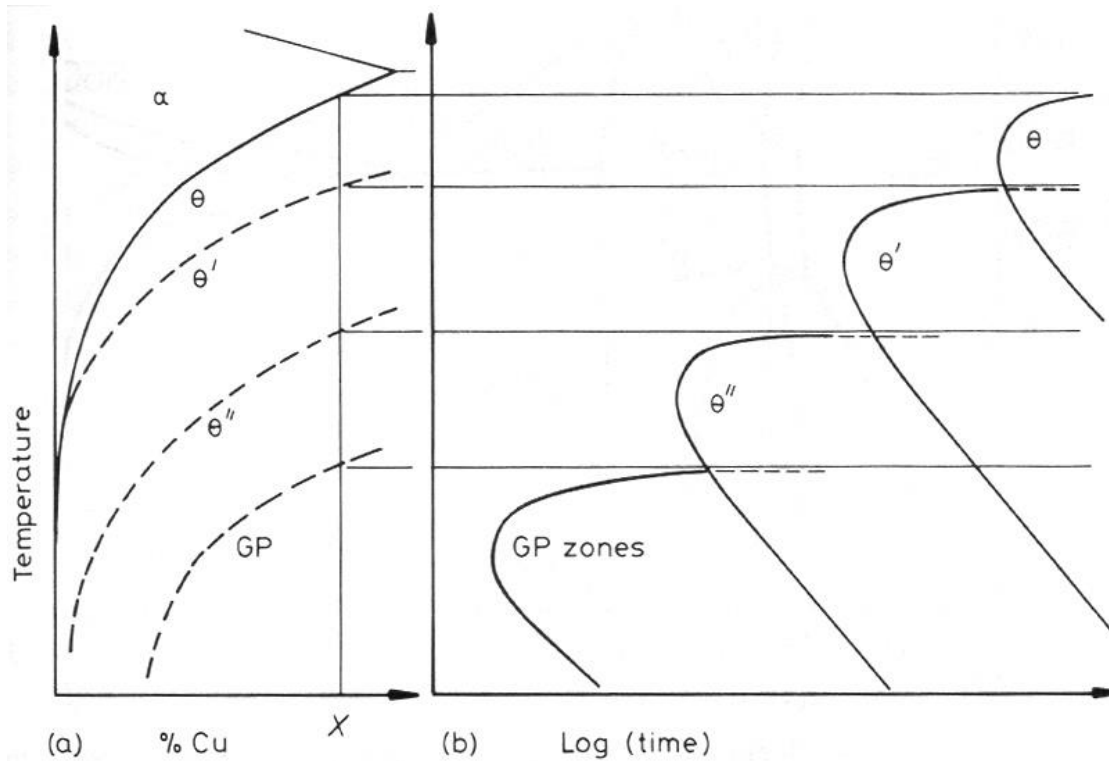
❖ Transition phases



5.5 Precipitation in Age-Hardening Alloys

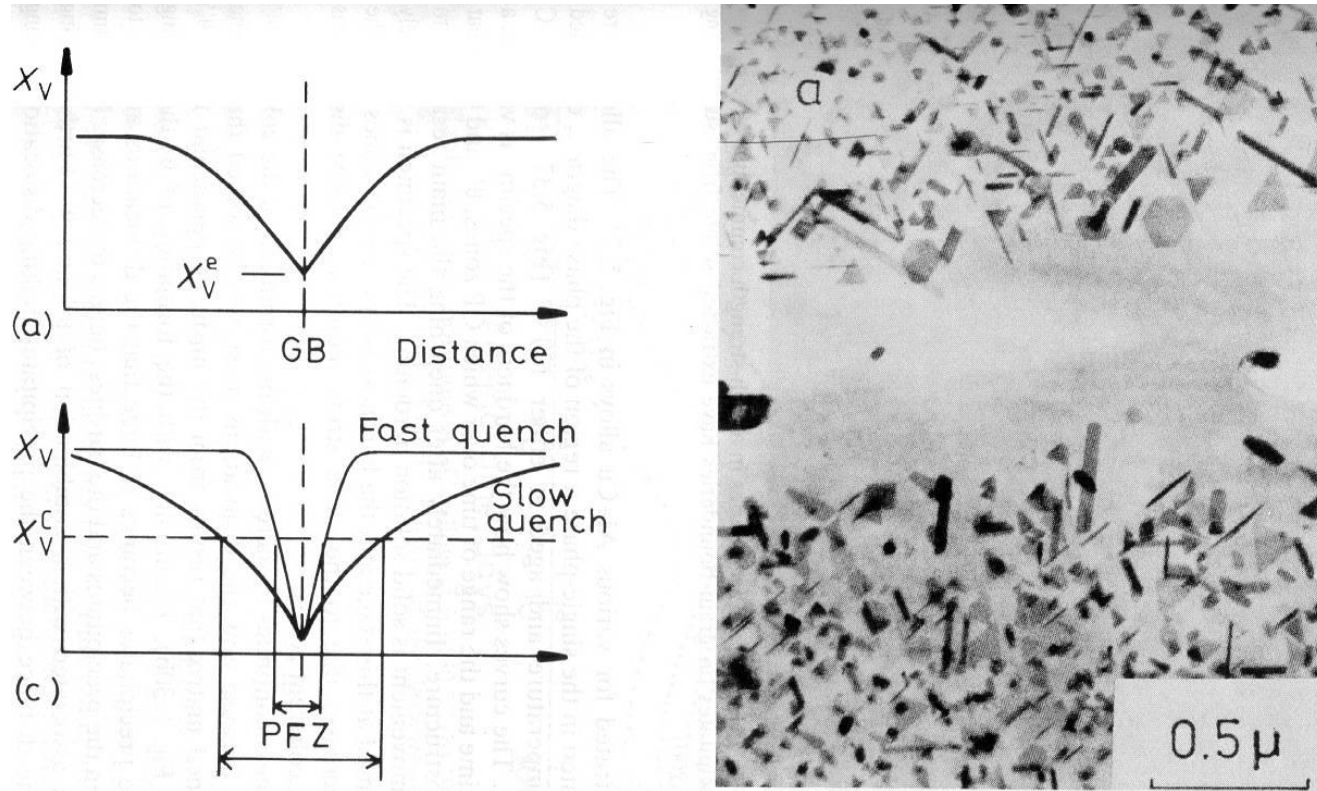


5.5 Precipitation in Age-Hardening Alloys



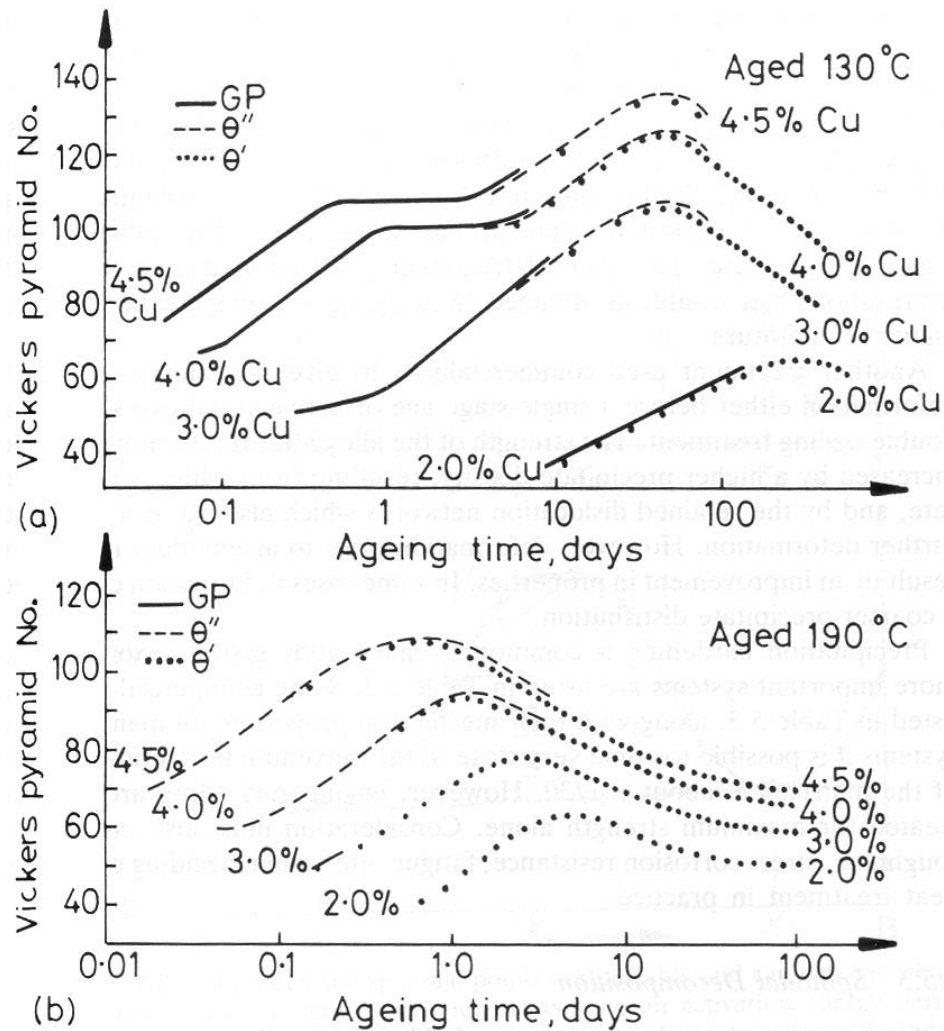
Growth of θ'
in the expense of θ''

5.5.3 Quenched in Vacancies



Precipitation Free Zone: PFZ

5.5.4 Age Hardening

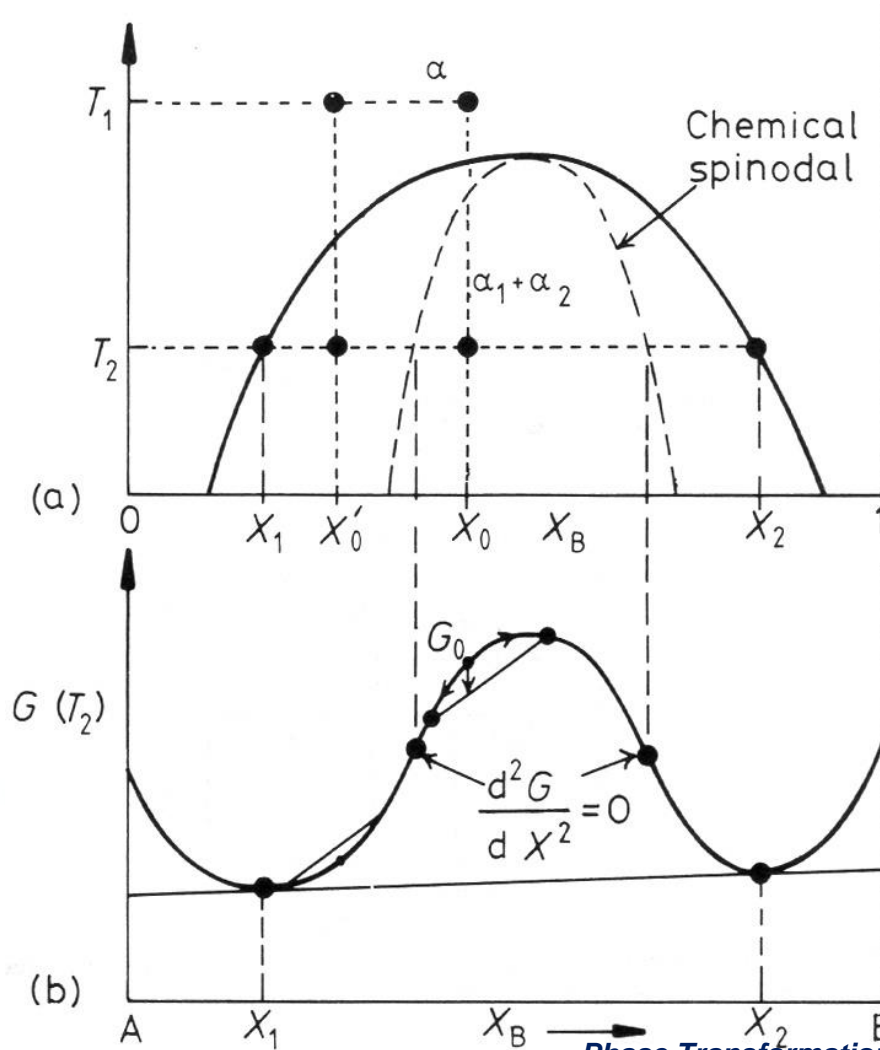


5.5.5 Spinodal Decomposition

No barrier for nucleation

Phase diagram with a miscibility gap

$$\Omega > 0$$

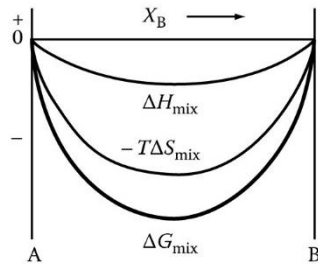


✓ $\frac{d^2G}{dX^2} < 0$, chemical spinodal

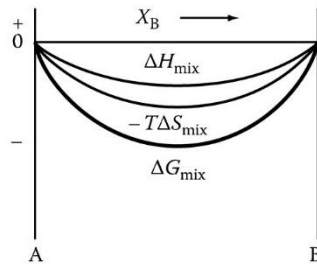
From Ch1

❖ The Effect of ΔH_{mix} and T on ΔG_{mix}

$\Omega < 0$

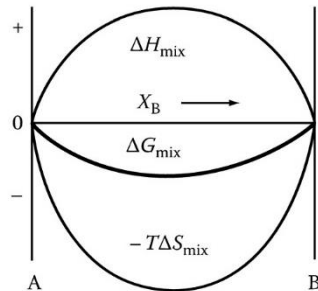


(a) $\Omega < 0$, high T

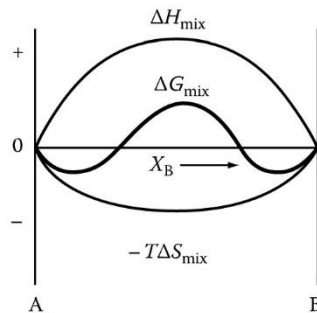


(b) $\Omega < 0$, low T

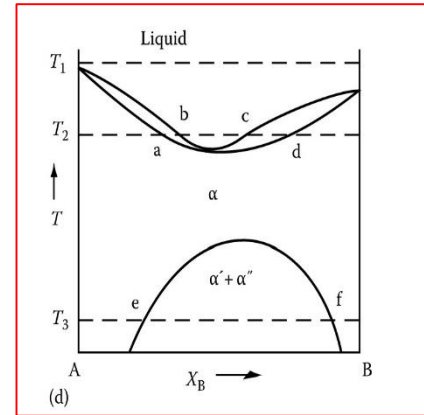
$\Omega > 0$



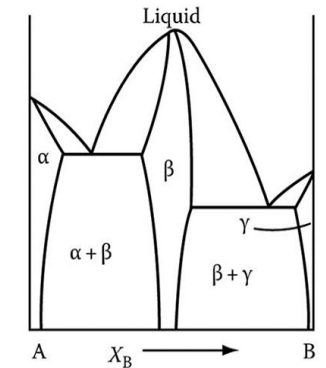
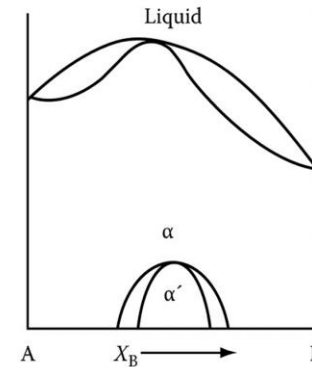
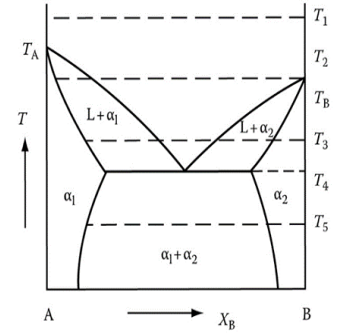
(c) $\Omega > 0$, high T



(d) $\Omega > 0$, low T

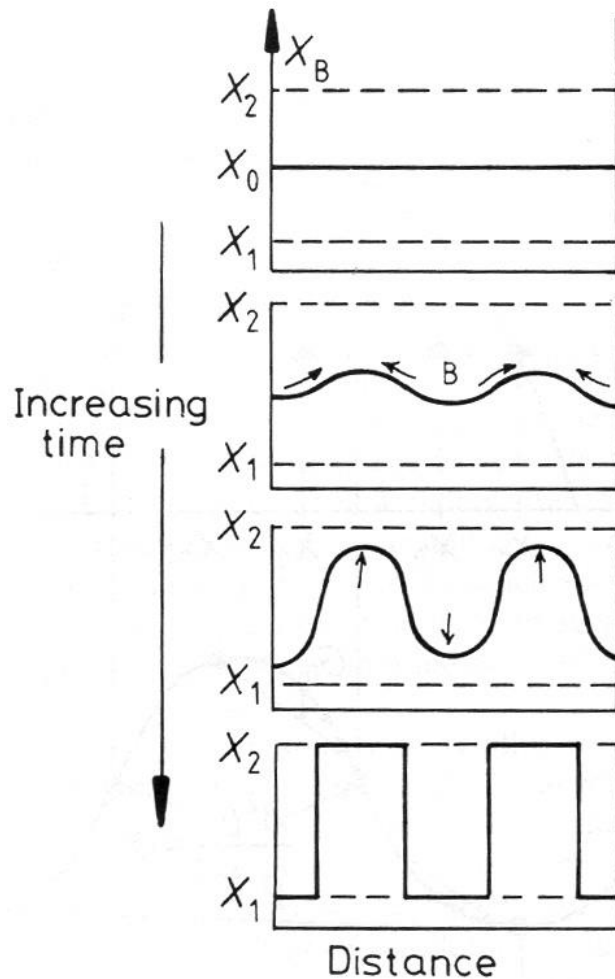


(d)

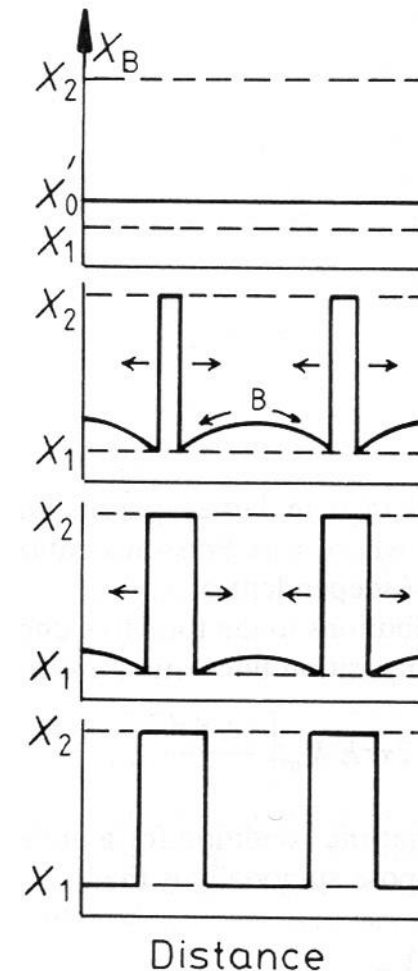


5.5.5 Spinodal Decomposition

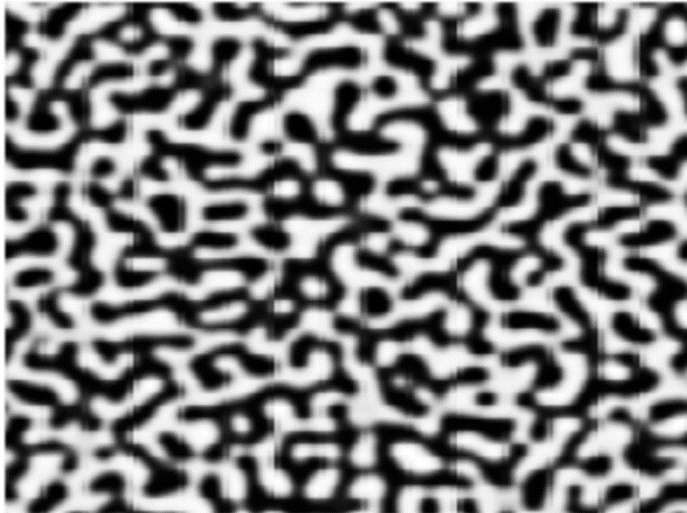
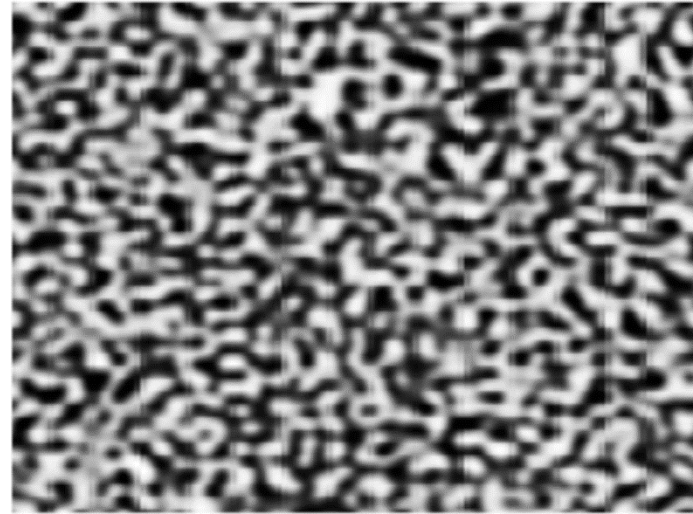
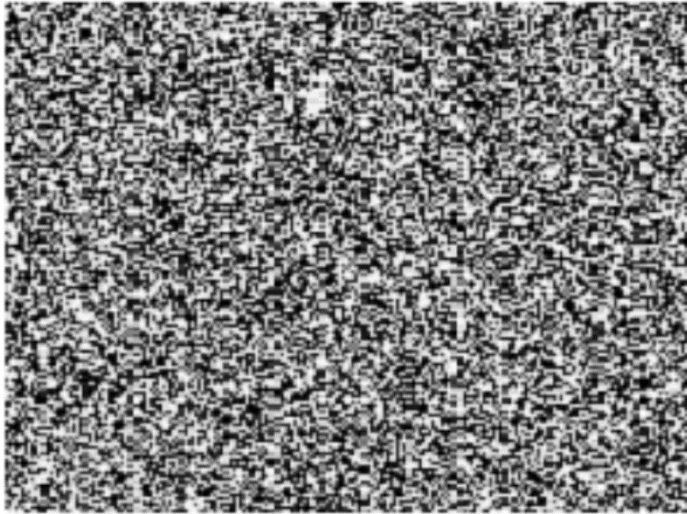
- ❖ Composition profiles in an alloy quenched **into the spinodal region**



- ❖ Composition profiles in an alloy **outside the spinodal points**



Spinodal decomposition



5.5.5 Spinodal Decomposition

❖ The Rate of Transformation

✓ Rate controlled by interdiffusion coefficient \tilde{D}

✓ Within the spinodal $\tilde{D} < 0$, composition fluctuation $\propto \exp(-\frac{t}{\tau})$

$$\tau = -\lambda^2 / 4\pi^2 D$$

- τ : characteristic time constant

- λ : wavelength of the composition modulation (1-D assumed)

transf. rate \uparrow as $\lambda \downarrow$

There is a minimum value of below which spinodal decomposition cannot occur

To calculate λ , it is need to take care of 1) interfacial energy 2) coherency strain energy