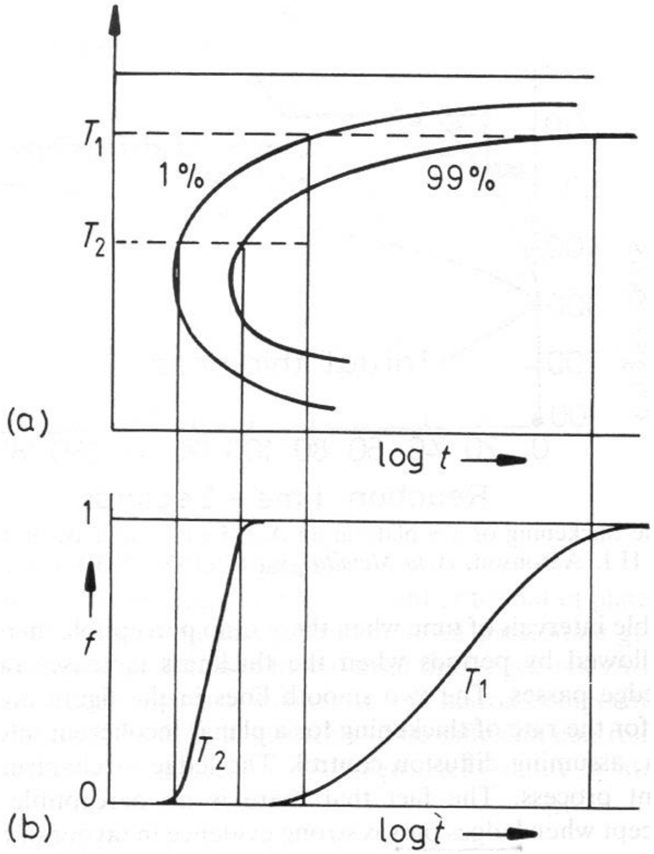
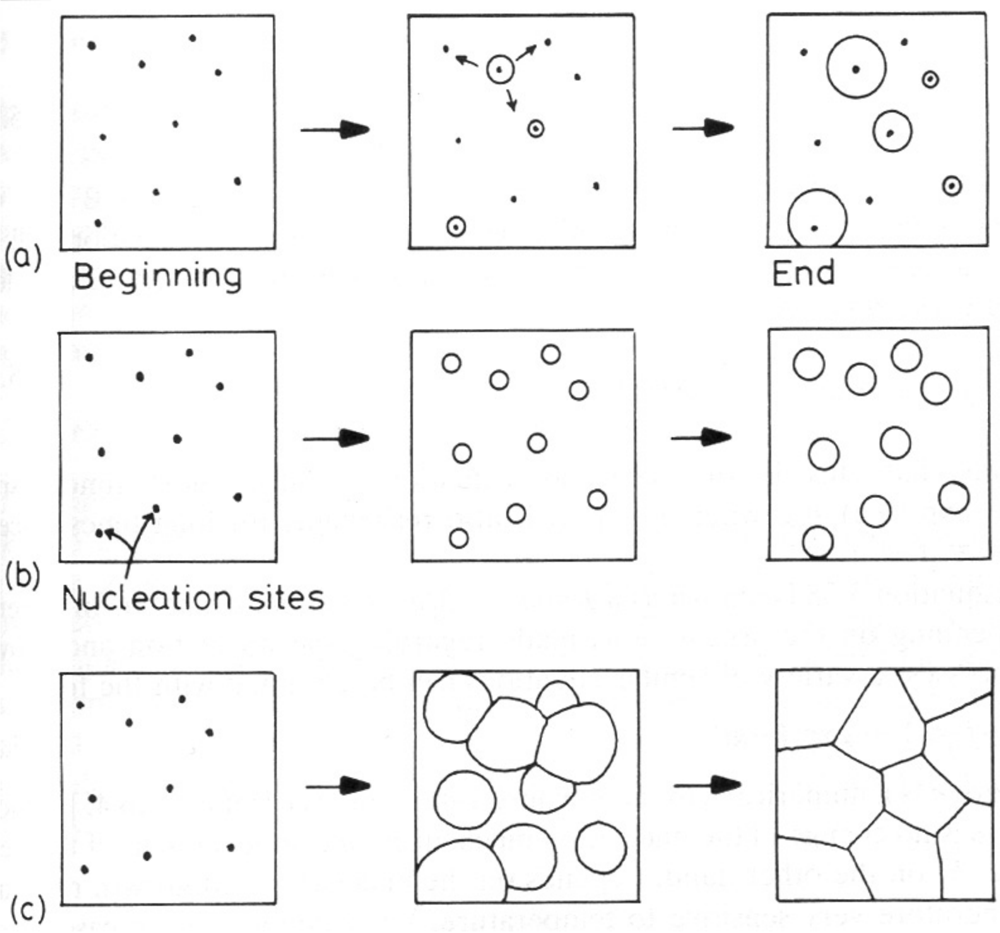
5.4 Overall Transformation Kinetics – TTT Diagram

❖ TTT Diagram :

the fraction of **T**ransformation (*f*) as a function of **T**ime and **T**emperature





5.4 Overall Transformation Kinetics – TTT Diagram

- **Johnson-Mehl-Avrami Kolmogorov Equation**

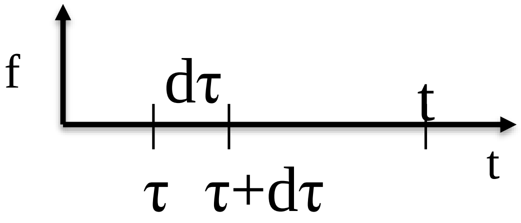
Assumption :

- Reaction produces by nucleation and growth
- Nucleation occurs randomly throughout specimen
- Reaction product grows radially until impingement

$$\alpha \rightarrow \beta$$

Define volume fraction transformed

$$f = \frac{Vol.\ of\ new\ phase}{Vol.\ of\ specimen}$$



$$\boxed{df = \frac{\begin{pmatrix} vol.\ of\ one\ particle\ nucleated \\ during\ d\tau\ measured\ at\ time\ t \end{pmatrix} \times \begin{pmatrix} number\ of\ nuclei \\ formed\ during\ d\tau \end{pmatrix}}{volume\ of\ specimen}}$$

5.4 Johnson-Mehl-Avrami Kolmogorov Equation

volume : $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (vt)^3$

v : growth rate (assumed const.)
N : nucleation rate (const.)

$$V' = \frac{4}{3}\pi v^3 (t-\tau)^3$$

$$df = \frac{\frac{4}{3}\pi[v(t-\tau)]^3 \times (NV_0 d\tau)}{V_0}$$

$$f_e = \int_0^x d\hat{f} = \frac{4}{3}\pi N v^3 \int_0^t (t-\tau)^3 d\tau$$

$$f_e = \frac{\pi}{3} N v^3 t^4$$

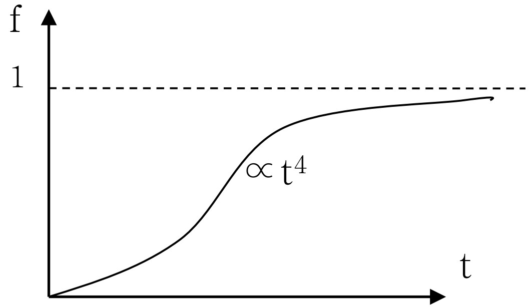
→ do not consider impingement & repeated nucleation
→ only true for f ≪ 1

f_e : extended volume fraction

❖ Johnson-Mehl-Avrami Kolmogorov Equation

$$df = (1 - f)\ df_e \quad \Rightarrow \quad 1 - f = \frac{df}{df_e}$$

$$f = 1 - exp\left(-\frac{\pi}{3} N v^3 t^4\right)$$



$$\boxed{f = 1 - exp(-k\,t^{\,n})}$$ ····· **J-M-A Eq.**

k : sensitive to temp. (N. v)
n : 1 ~ 4

For the case of 50% transform, $t_{0.5}$

$$exp\ (-0.7) = 0.5$$

$$k\ t_{0.5}^n = 0.7$$

i.e. $$t_{0.5} = \frac{0.7}{k^{1/n}}$$

$\Rightarrow$

Example above.

$$t_{0.5} = \frac{0.9}{N^{1/4}\, v^{3/4}}$$

❖ Johnson-Mehl-Avrami Kolmogorov Equation

Other example:

1. Fixed nucleation site and active at the beginning.
2. Nucleation site depends on the time

$I = I_o \, exp(-pt)$ (I_o : number of active nucleation site / unit volume)

p: rate at which the individual sites are lost.

$$N(t) = -\frac{dI}{dt} = I_o \, p \, exp(-pt) = pI$$

$$f = 1 - exp\left\{\left(-\frac{8\pi I_o v^3}{p^3}\right) \times \left[exp(-pt) - 1 + pt - \cdots\right]\right\}$$

limiting cases : pt small → same as J-M eq.

pt large → $\left\{\frac{p^3 t^3}{6}\right\}$ I quickly goes to zero.

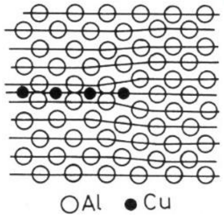
$$\boxed{f = 1 - exp\left(-\frac{\pi}{3} N_o v^3 t^3\right)}$$

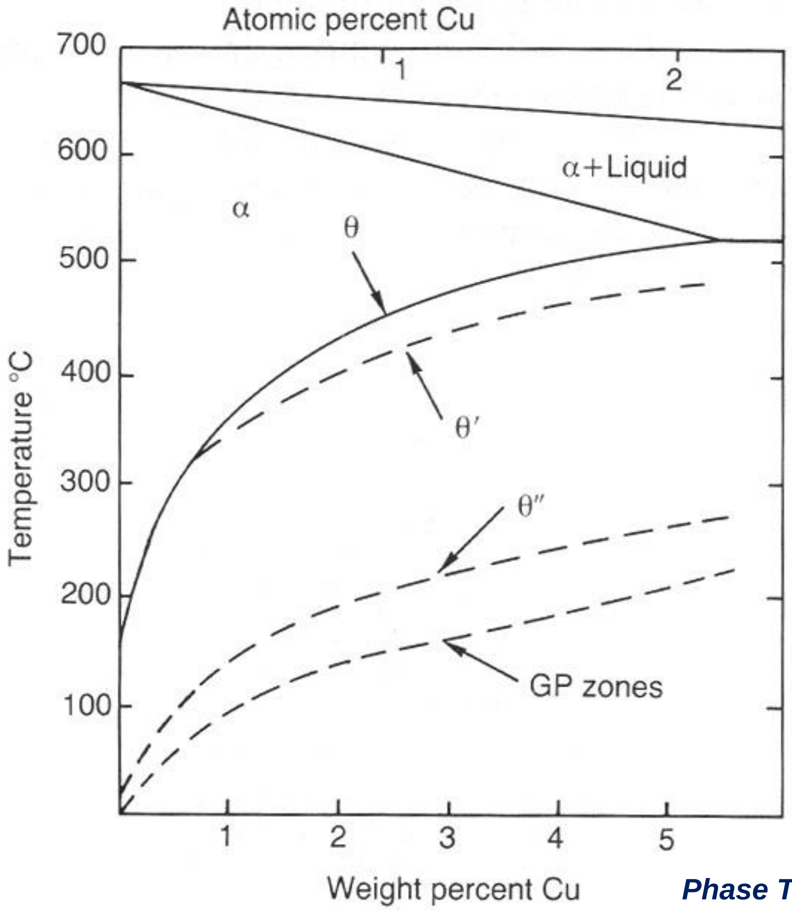
5.5 Precipitation in Age-Hardening Alloys

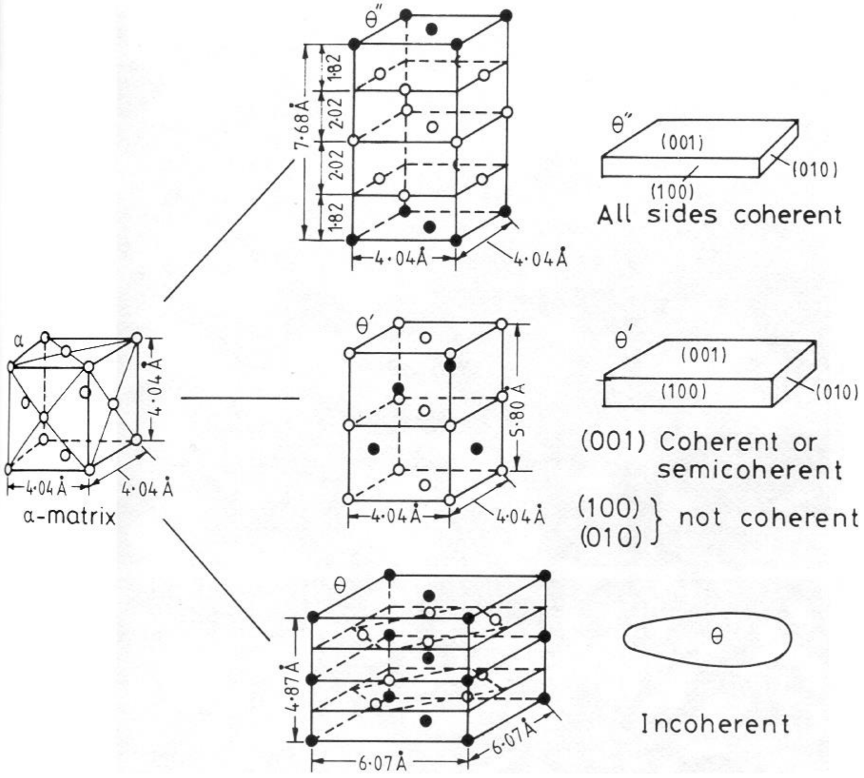
5.5.1 Precipitation in Aluminum-Copper Alloys

- **Transition phases**

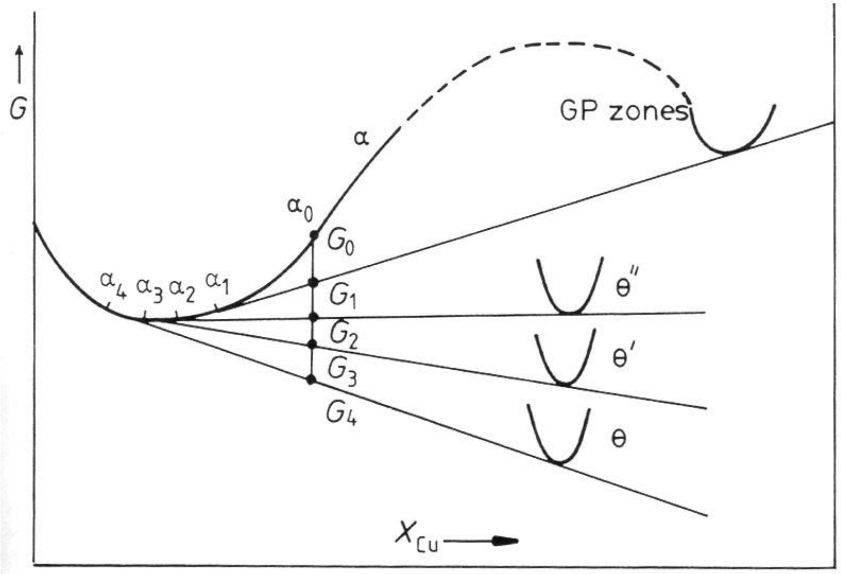
$$\boxed{\alpha_0 \rightarrow \alpha_1 + GP\ \ zones \rightarrow\ \alpha_2 + \theta'' \rightarrow\ \alpha_3 + \theta' \rightarrow \alpha_4 + \theta}$$

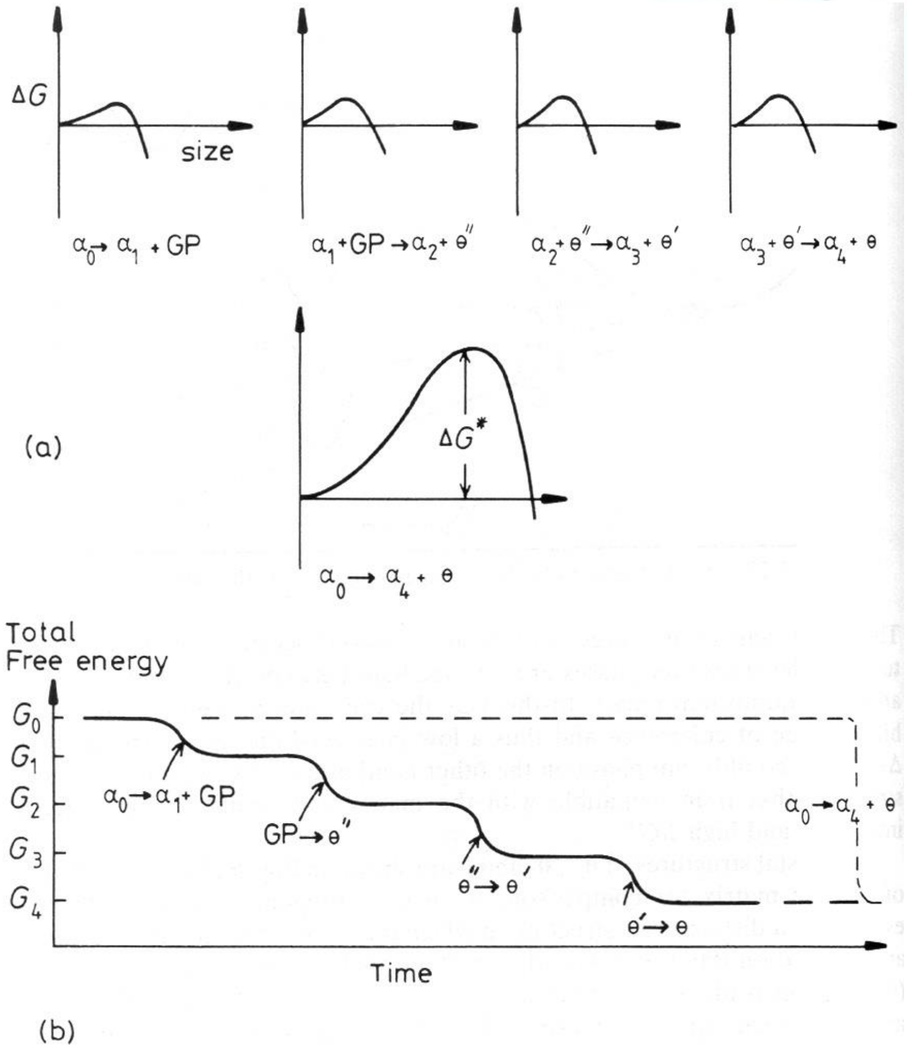




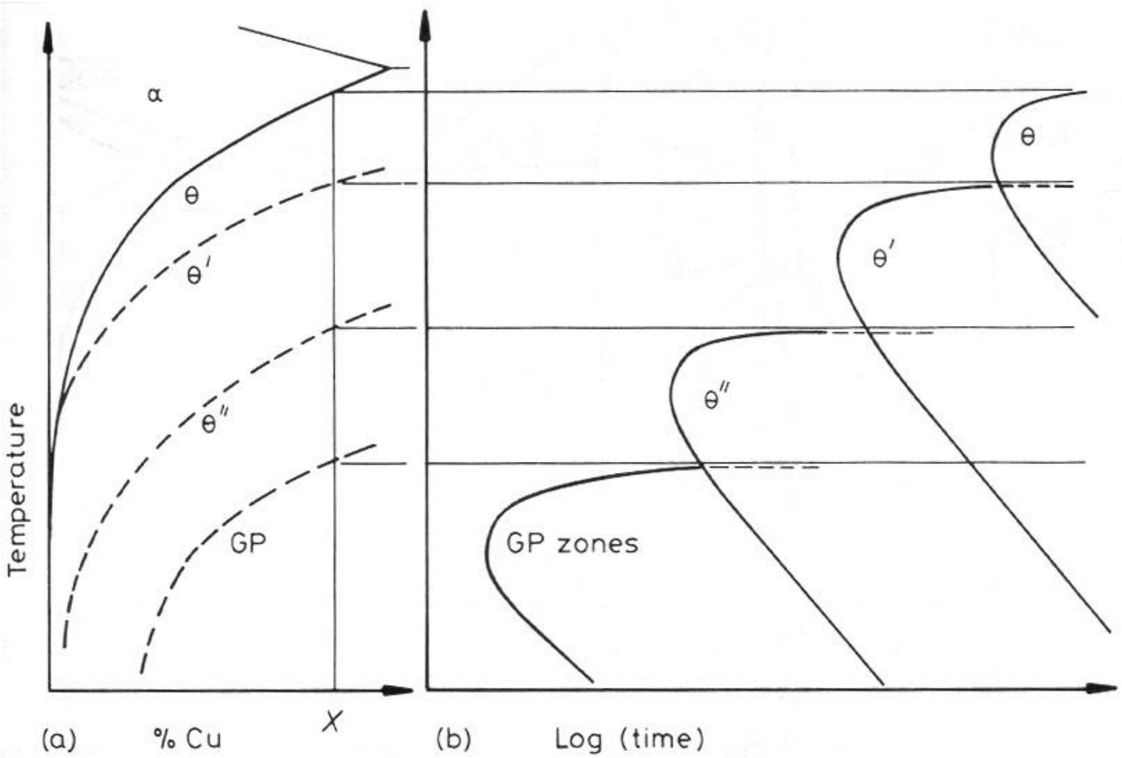


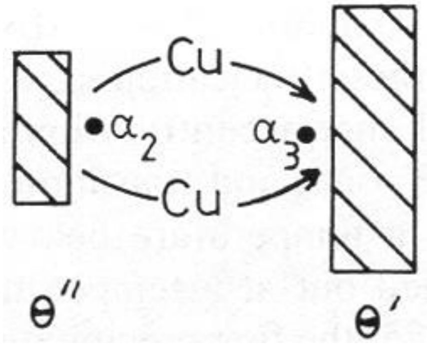
5.5 Precipitation in Age-Hardening Alloys





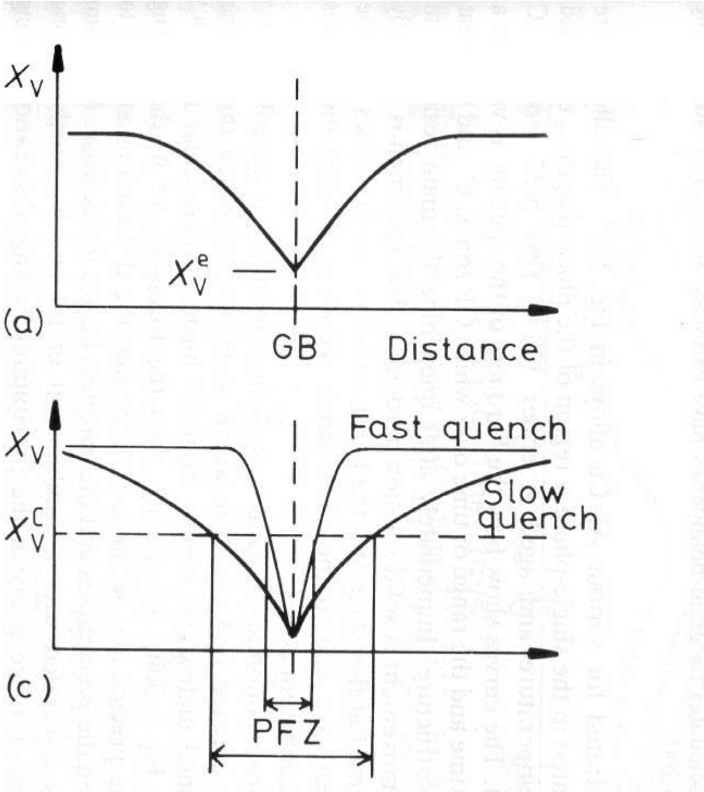
5.5 Precipitation in Age-Hardening Alloys





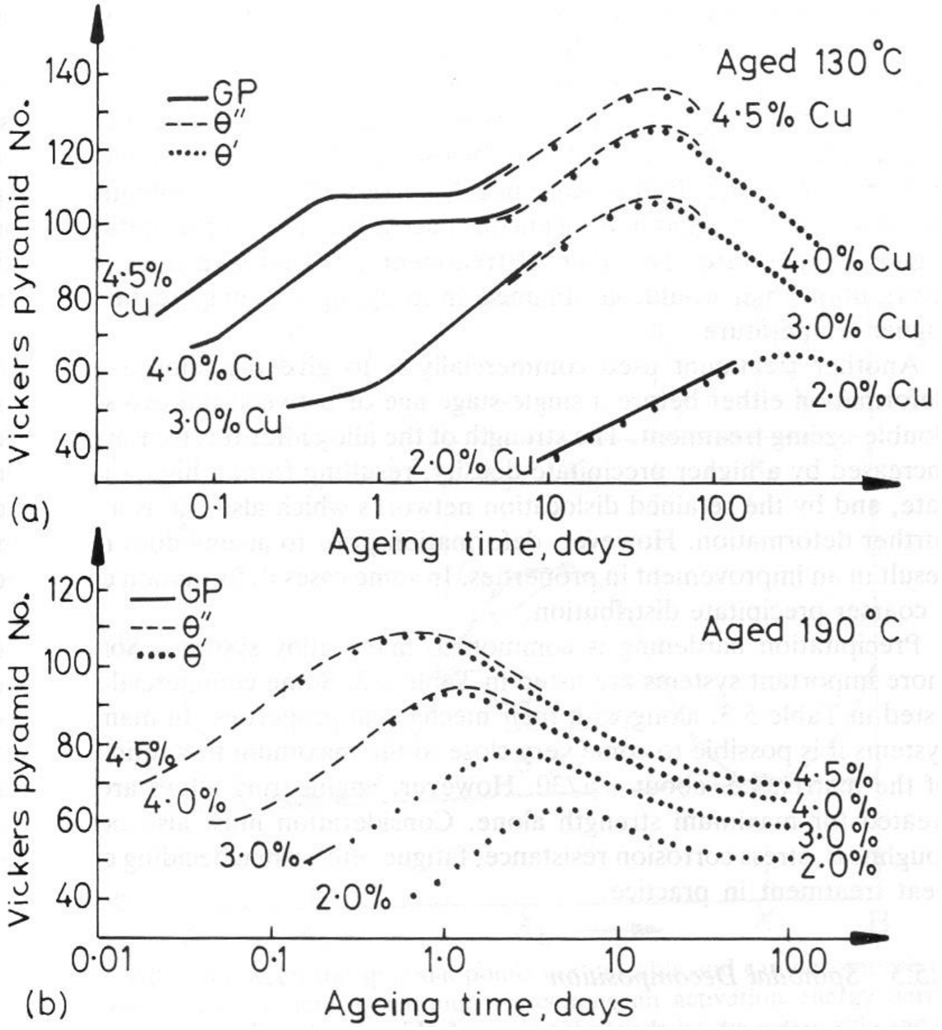
Growth of $\boldsymbol{\theta'}$
in the expense of $\boldsymbol{\theta}''$

5.5.3 Quenched in Vacancies



Precipitation Free Zone: PFZ

5.5.4 Age Hardening

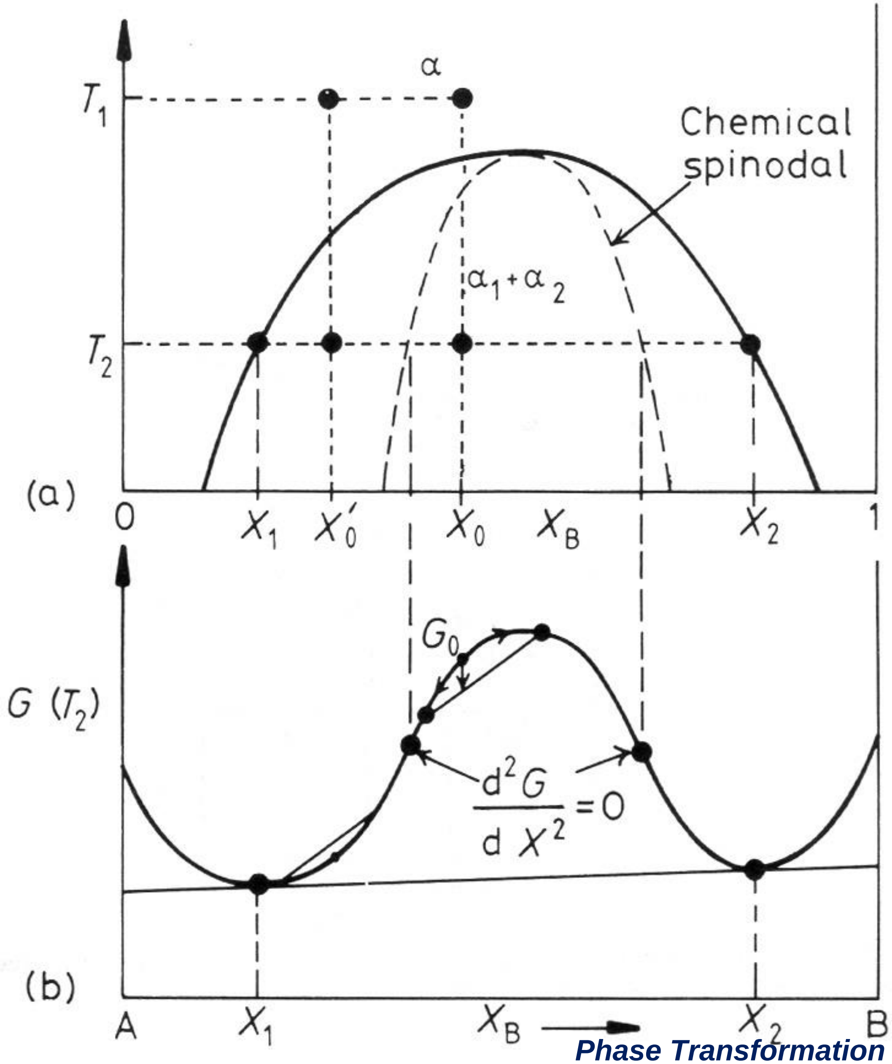


5.5.5 Spinodal Decomposition

No barrier for nucleation

Phase diagram with a miscibility gap

$\Omega > 0$

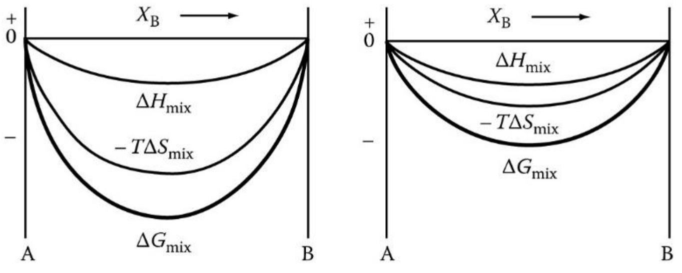


- ✓ $\frac{d^2G}{dX^2} < 0$, chemical spinodal

From Ch1

- **The Effect of ΔH_{mix} and T on ΔG_{mix}**

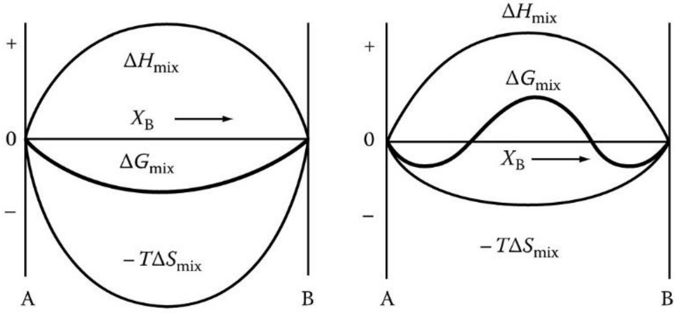
$\Omega < 0$



(a) Ω < O, high *T*

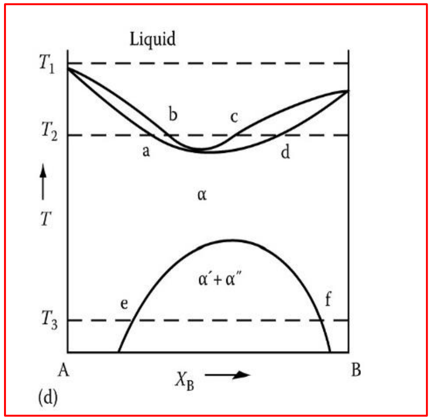
(b) Ω < O, low *T*

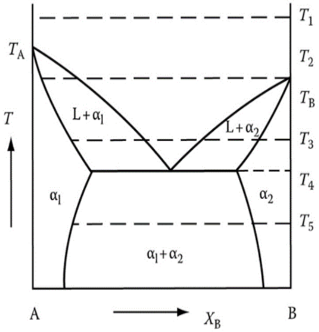
$\Omega > 0$

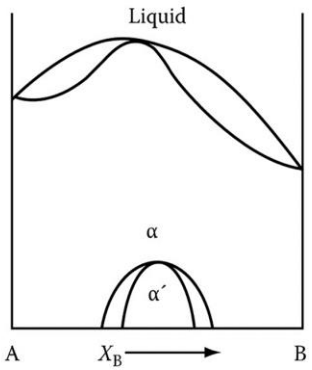


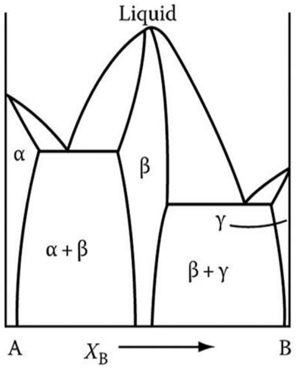
(c) Ω > O, high *T*

(d) Ω > O, low *T*



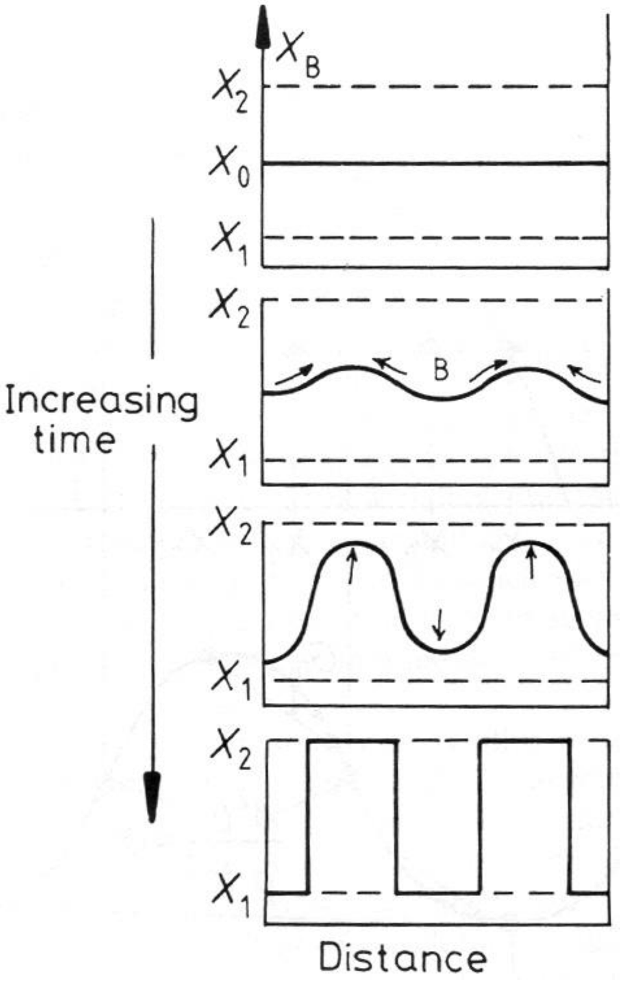




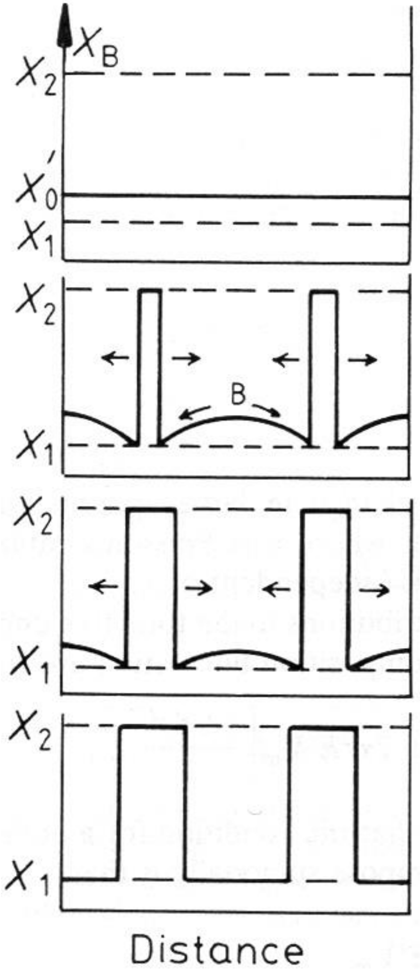


5.5.5 Spinodal Decomposition

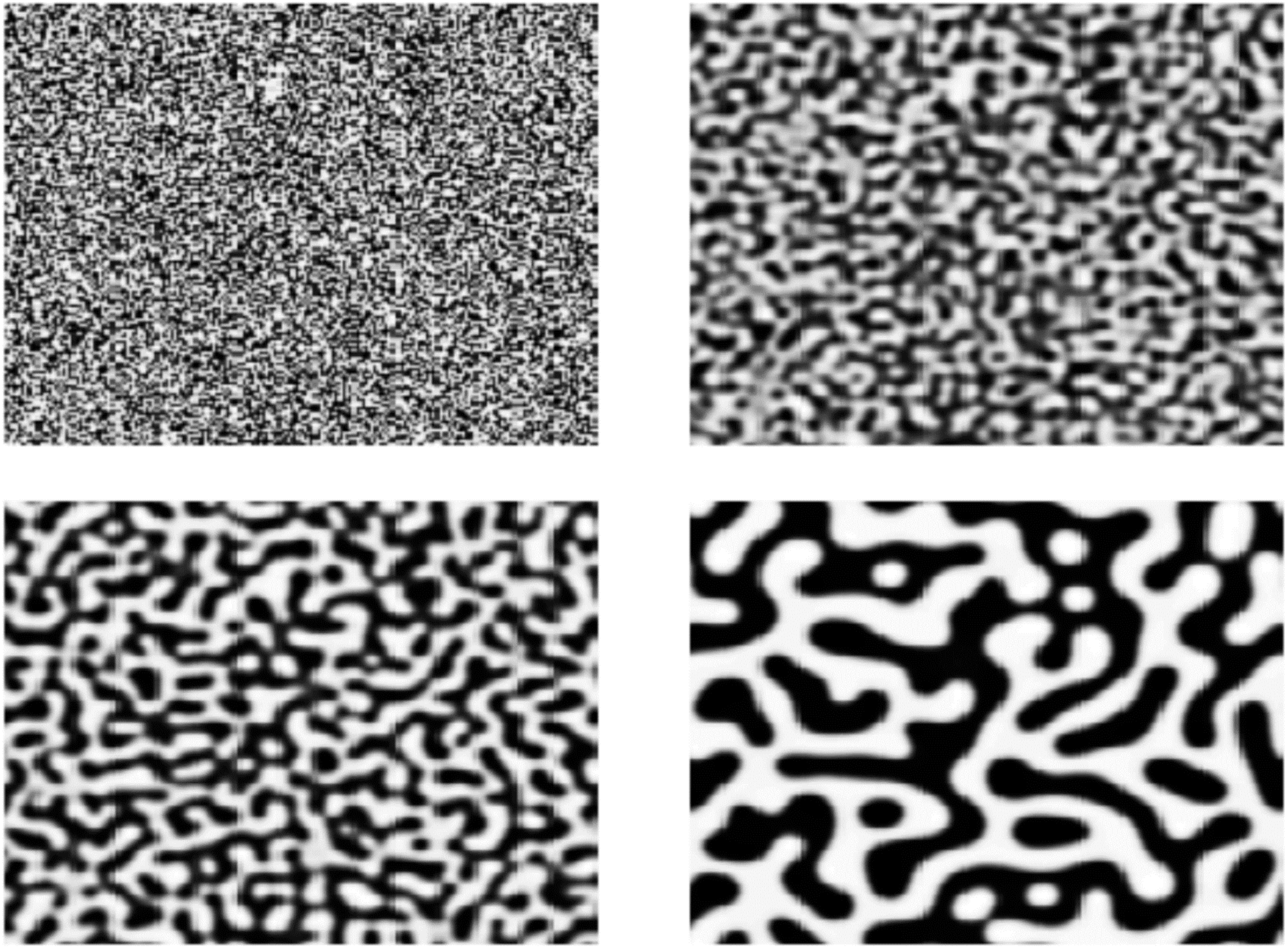
- Composition profiles in an alloy quenched into the spinodal region



- Composition profiles in an alloy outside the spinodal points



Spinodal decomposion



5.5.5 Spinodal Decomposition

❖ **The Rate of Transformation**

✓ Rate controlled by interdiffusion coefficient $\tilde{D}$

✓ Within the spinodal $\tilde{D} < 0$, composition fluctuation $\propto \exp(-\frac{t}{\tau})$

$$\tau = -\lambda^2 / 4\pi^2 D$$

- τ : characteristic time constant
- λ : wavelength of the composition modulation (1-D assumed)

transf. rate ↑ as λ ↓

There is a minimum value of below which spinodal decomposion cannot occur

To calculate λ, it is need to take care of 1) interfacial energy 2) coherency strain energy