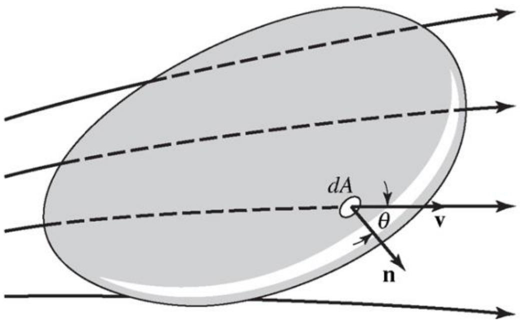
Control volume approach

Conservation of mass



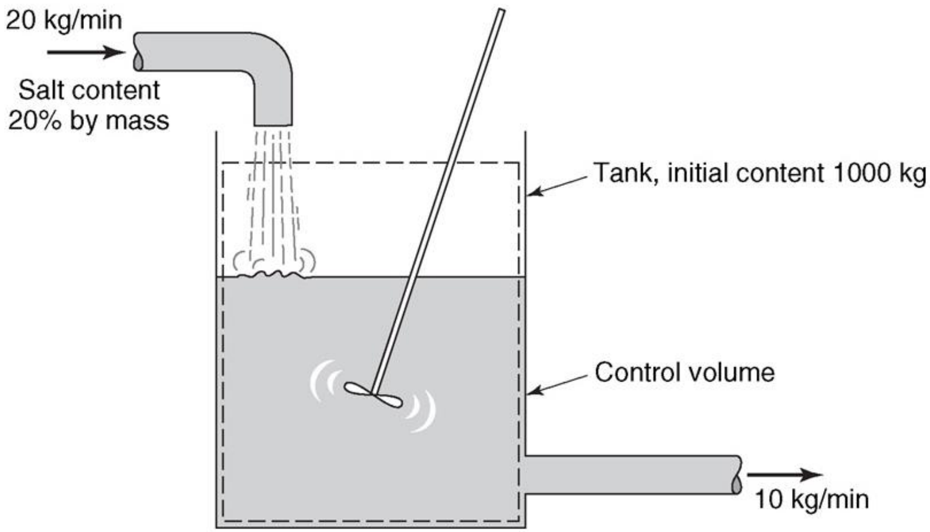
$$\begin{Bmatrix} \text{rate of mass} \\ \text{efflux from} \\ \text{control} \\ \text{volume} \end{Bmatrix} - \begin{Bmatrix} \text{rate of} \\ \text{flow into} \\ \text{control} \\ \text{volume} \end{Bmatrix} + \begin{Bmatrix} \text{rate of} \\ \text{accumulation} \\ \text{of mass within} \\ \text{control volume} \end{Bmatrix} = 0$$

$$\boxed{\iint_{\text{c.s.}} \rho (\mathrm{v} \cdot \mathrm{n})\, dA + \frac{\partial}{\partial t} \iiint_{\text{c.v.}} \rho \, dV = 0}$$

steady one-dimensional flow into and out of a control volume

$$\iint_{\mathrm{c.s.}} \rho(\mathbf{v}\cdot\mathbf{n})\,dA = -\iint_{\mathrm{A}_1} \rho v\,dA + \iint_{\mathrm{A}_2} \rho v\,dA = 0$$

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$



A tank initially contains 1000kg of brine containing 10% salt by mass. An inlet stream of the brine containing 20% salt flows into the tank at a rate of 20kg/min. The mixture in the tank is kept uniform by stirring. Brine is removed from the tank via an outlet pipe at a rate of 10kg/min. Find the amount of salt in the tank at any time t, and the elapsed time when the amount of salt in the tank is 200kg.

$$\boxed{\iint_{\text{c.s.}} \rho(\mathrm{v}\cdot\mathrm{n})\,dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho\; dV = 0}$$

Conservation of linear momentum

$$\Sigma \mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = \frac{d}{dt}\mathbf{P}$$

$$\left\{\begin{array}{c}\text{sum of}\\ \text{forces acting}\\ \text{on control}\\ \text{volume}\end{array}\right\} = \underbrace{\left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{out of control}\\ \text{volume}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{into control}\\ \text{volume}\end{array}\right\}}_{\text{net rate of momentum efflux from control volume}} + \left\{\begin{array}{c}\text{rate of}\\ \text{accumulation}\\ \text{of momentum}\\ \text{within control}\\ \text{volume}\end{array}\right\}$$

$$\boxed{\sum \mathbf{F} = \iint_{\text{c.s.}} \mathbf{v}\rho(\mathbf{v}\cdot\mathbf{n})\,dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho\mathbf{v}\,dV}$$

Middleman

$$\boxed{\frac{\partial}{\partial t}\int \rho\mathbf{u}\,d\mathrm{V} = \rho_1\langle U_1^2\rangle\mathbf{A}_1 - \rho_2\langle U_2^2\rangle\mathbf{A}_2 + P_1\mathbf{A}_1 - P_2\mathbf{A}_2 - \mathbf{F} + M\mathbf{g}}$$

Conservation of moment of momentum

$$\Sigma\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = \frac{d}{dt}\mathbf{P} \quad \Rightarrow \quad \sum \mathrm{M} = \frac{d}{dt}\mathrm{H}$$

$$\mathbf{r} \times \sum \mathbf{F} = \sum \mathbf{r} \times \mathbf{F} = \sum \mathbf{M}$$

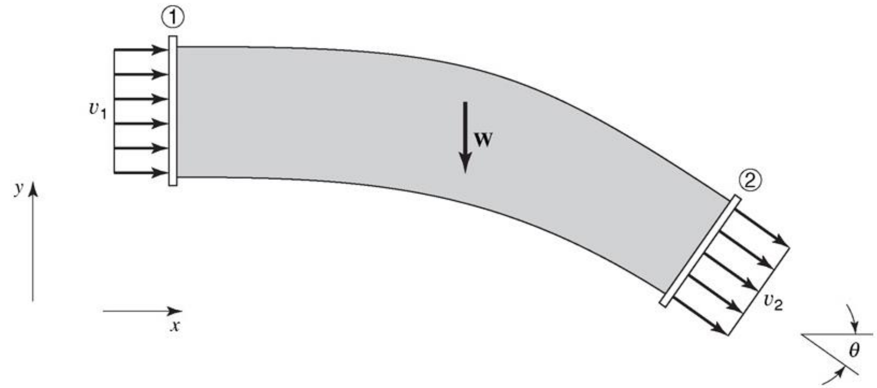
$$\mathbf{r} \times \frac{d}{dt} m\mathbf{v} = \frac{d}{dt}(\mathbf{r} \times m\mathbf{v}) = \frac{d}{dt}(\mathbf{r} \times \mathbf{P}) = \frac{d}{dt}\mathbf{H}$$

$$\boxed{\sum \mathbf{F} = \iint_{\text{c.s.}} \mathbf{v}\rho(\mathbf{v} \cdot \mathbf{n})\, dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho\mathbf{v}\, dV}$$

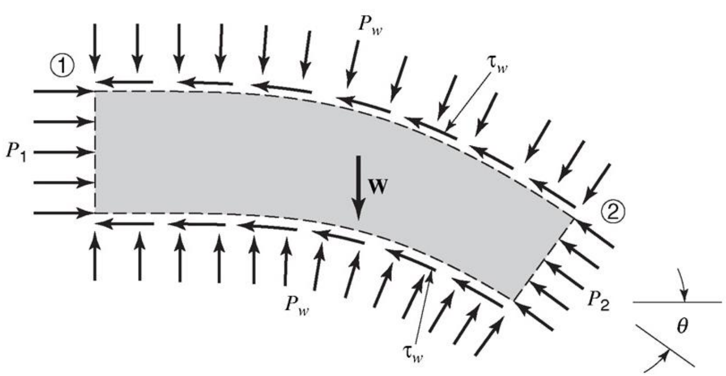
$$\Rightarrow \quad \sum \mathbf{M} = \iint_{\text{c.s.}} (\mathbf{r} \times \mathbf{v})\rho(\mathbf{v} \cdot \mathbf{n})\, dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} (\mathbf{r} \times \mathbf{v})\rho\, dV$$

applicable to pumps and turbines (having rotary motion) ex1; flow in a reducing pipe band

goal; find the force exerted on a reducing pipe bend from a flow of fluid in it



$$\boxed{\sum \mathbf{F} = \iint_{\text{c.s.}} \mathbf{v}\rho\left(\mathbf{v}\cdot\mathbf{n}\right)dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho\mathbf{v}dV}$$



the external forces imposed on the fluid;

1) pressure forces at section (1) and (2)
2) body force
3) forces due to pressure and shear stress exerted on the fluid by the pipe wall

$$\sum F_x = P_1A_1 - P_2A_2\cos\theta + B_x$$

$$\sum F_y = P_2A_2\sin\theta - W + B_y$$

$$\iint_{\text{c.s.}} v_x\rho\left(\mathbf{v}\cdot\mathbf{n}\right)dA = (v_2\cos\theta)(\rho_2v_2A_2) + (v_1)(-\rho_1v_1A_1)$$

$$\iint_{\text{c.s.}} v_y\rho\left(\mathbf{v}\cdot\mathbf{n}\right)dA = (-v_2\sin\theta)(\rho_2v_2A_2)$$

the force exerted on the pipe rather than on the fluid $\mathbf{R} = -\mathbf{B}$

$$R_x = -B_x = -v_2{}^2\rho_2 A_2 \cos\theta + v_1{}^2\rho_1 A_1 + P_1 A_1 - P_2 A_2 \cos\theta$$

$$R_y = -B_y = v_2{}^2\rho_2 A_2 \sin\theta + P_2 A_2 \sin\theta - W$$

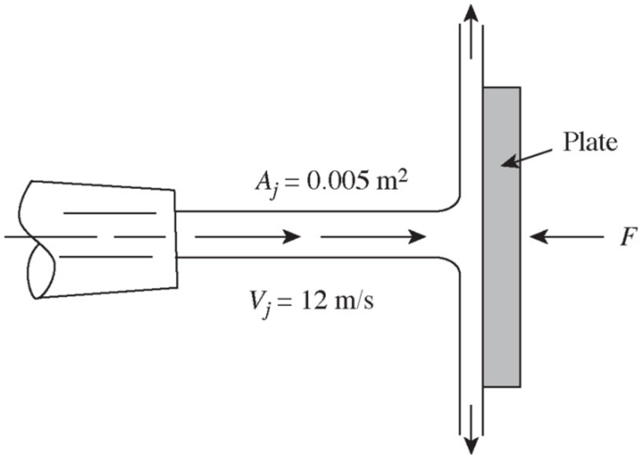
$$\dot{m} = \rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

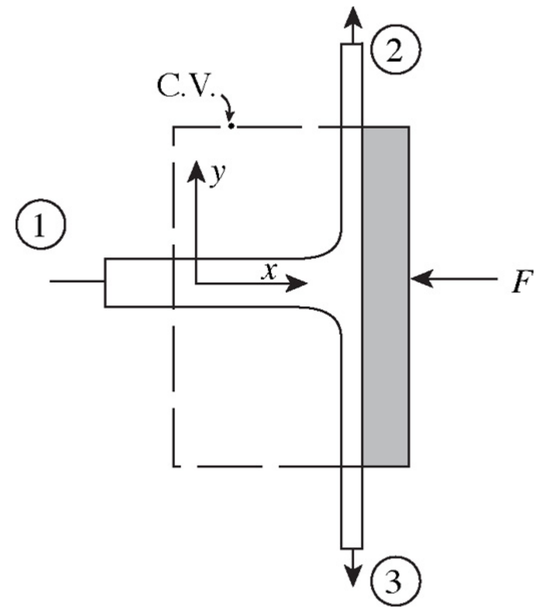
$$R_x = \dot{m}(v_1 - v_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta$$

$$R_y = \dot{m} v_2 \sin\theta + P_2 A_2 \sin\theta - W$$

ex3; a fluid jet striking a vertical plate

goal; force required to hold the plate stationary





$$\sum F_x = \iint_{\mathrm{c.s.}} v_x \rho(\mathbf{v}\cdot\mathbf{n})\,dA + \frac{\partial}{\partial t}\iiint_{\mathrm{c.v.}} \rho v_x\, dV$$

$$-F = \iint_{\mathrm{c.s.}} v_x \rho(\mathbf{v}\cdot\mathbf{n})\,dA = v_j \rho(-v_j A_j)$$

$$F = \rho A_j v_j^{\,2}$$

Conservation of energy

$$\left\{\begin{array}{c}\text{rate of addition}\\ \text{of heat to control}\\ \text{volume from}\\ \text{its surroundings}\end{array}\right\}-\left\{\begin{array}{c}\text{rate of work done}\\ \text{by control volume}\\ \text{on its surroundings}\end{array}\right\}=\left\{\begin{array}{c}\text{rate of energy}\\ \text{out of control}\\ \text{volume due to}\\ \text{fluid flow}\end{array}\right\}$$

$$-\left\{\begin{array}{c}\text{rate of energy into}\\ \text{control volume due}\\ \text{to fluid flow}\end{array}\right\}+\left\{\begin{array}{c}\text{rate of accumulation}\\ \text{of energy within}\\ \text{control volume}\end{array}\right\}$$

$$\frac{\delta Q}{dt}-\frac{\delta W}{dt}=\iint_{\text{c.s.}} e\rho(v\cdot n)\,dA+\frac{\partial}{\partial t}\iiint_{\text{c.v.}} e\rho\,dV$$

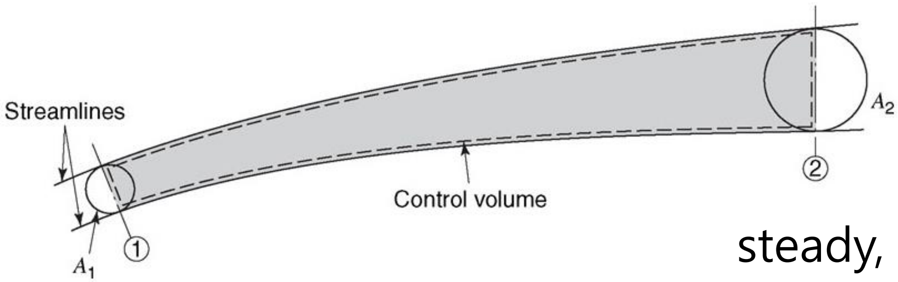
$$\frac{\delta W}{dt}=\frac{\delta W_s}{dt}+\frac{\delta W_{s\sigma}}{dt}+\frac{\delta W_\tau}{dt}=\frac{\delta W_s}{dt}-\iint_{\text{c.s.}} P(\mathbf{v}\cdot\mathbf{n})\,dA+\frac{\delta W_\mu}{dt}$$

$$\boxed{\frac{\delta Q}{dt}-\frac{\delta W_s}{dt}=\iint_{\text{c.s.}}\left(e+\frac{P}{\rho}\right)\rho(\mathrm{v}\cdot\mathrm{n})\,dA+\frac{\partial}{\partial t}\iiint_{\text{c.v.}} e\rho\,dV+\frac{\partial W_\mu}{dt}}$$

$$e=gy+\frac{v^2}{2}+u$$

energy per unit mass

Bernoulli equation



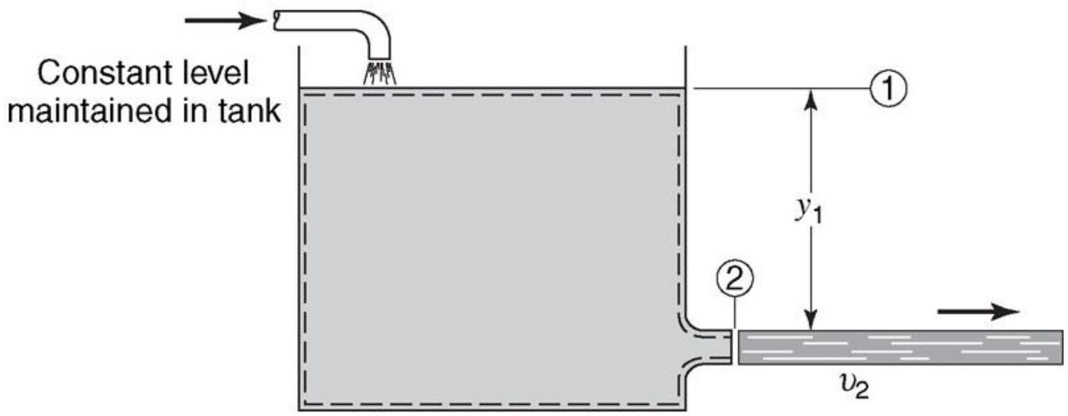
steady, incompressible, inviscid, isothermal, no heat transfer or work done

$$\boxed{\frac{\delta Q}{dt}-\frac{\delta W_s}{dt}=\iint_{\text{c.s.}}\left(e+\frac{P}{\rho}\right)\rho(\mathrm{v}\cdot\mathrm{n})\,dA+\frac{\partial}{\partial t}\iiint_{\text{c.v.}}e\rho\,dV+\frac{\partial W_\mu}{dt}}$$

$$\iint_{\text{c.s.}}\rho\left(e+\frac{P}{\rho}\right)(\mathrm{v}\cdot\mathrm{n})\,dA=\iint_{A_1}\rho\left(e+\frac{P}{\rho}\right)(\mathrm{v}\cdot\mathrm{n})\,dA+\iint_{A_2}\rho\left(e+\frac{P}{\rho}\right)(\mathrm{v}\cdot\mathrm{n})\,dA$$

$$=\left(gy_1+\frac{v_1^{\,2}}{2}+\frac{P_1}{\rho_1}\right)(-\rho_1\ v_1\ A_1)+\left(gy_2+\frac{v_2^{\,2}}{2}+\frac{P_2}{\rho_2}\right)(\rho_2\ v_2\ A_2)$$

$$\boxed{gy_1+\frac{v_1^{\,2}}{2}+\frac{P_1}{\rho}=gy_2+\frac{v_2^{\,2}}{2}+\frac{P_2}{\rho}}$$



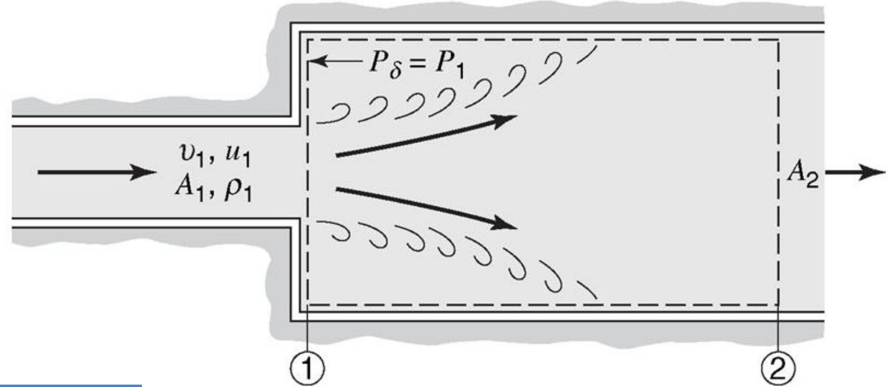
$$\boxed{gy_1 + \frac{v_1^{\,2}}{2} + \frac{P_1}{\rho} = gy_2 + \frac{v_2^{\,2}}{2} + \frac{P_2}{\rho}}$$

$$y_1 + \frac{P_{atm}}{\rho g} = \frac{v_2^{\,2}}{2g} + \frac{P_{atm}}{\rho g}$$

$$v_2 = \sqrt{2gy}$$

flow through a sudden expansion

goal; find the change in internal energy



conservation of mass

$$\boxed{\iint_{\text{c.s.}} \rho(\mathrm{v}\cdot\mathrm{n})\,dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho\, dV = 0}$$

$$\rho\, v_1 A_1 = \rho\, v_2 A_2$$

momentum

$$\boxed{\sum \mathbf{F} = \iint_{\text{c.s.}} \mathbf{v}\rho(\mathbf{v}\cdot\mathbf{n})\,dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} \rho\mathbf{v}\,dV}$$

$$P_1 A_1 - P_2 A_2 = \rho\, v_2{}^2 A_2 - \rho\, v_1{}^2 A_1$$

energy

$$\boxed{\frac{\delta Q}{dt} - \frac{\delta W_s}{dt} = \iint_{\text{c.s.}} \left(e + \frac{P}{\rho}\right)\rho(\mathrm{v}\cdot\mathrm{n})\,dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} e\rho\,dV + \frac{\partial W_\mu}{dt}}$$

$$(e_1 + \frac{P_1}{\rho})(\rho\, v_1\, A_1) = (e_2 + \frac{P_2}{\rho})(\rho\, v_2\, A_2)$$

$$e = gy + \frac{v^2}{2} + u$$