Fundamentals of Microelectronics

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- CH2 Basic Physics of Semiconductors
- CH3 Diode Circuits
- CH4 Physics of Bipolar Transistors
- CH5 Bipolar Amplifiers
- CH6 Physics of MOS Transistors
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Chapter 4 Physics of Bipolar Transistors

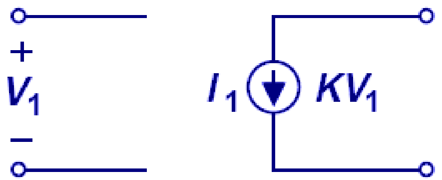
- 4.1 General Considerations
- 4.2 Structure of Bipolar Transistor
- 4.3 Operation of Bipolar Transistor in Active Mode
- 4.4 Bipolar Transistor Models
- 4.5 Operation of Bipolar Transistor in Saturation Mode
- 4.6 The PNP Transistor

Bipolar Transistor

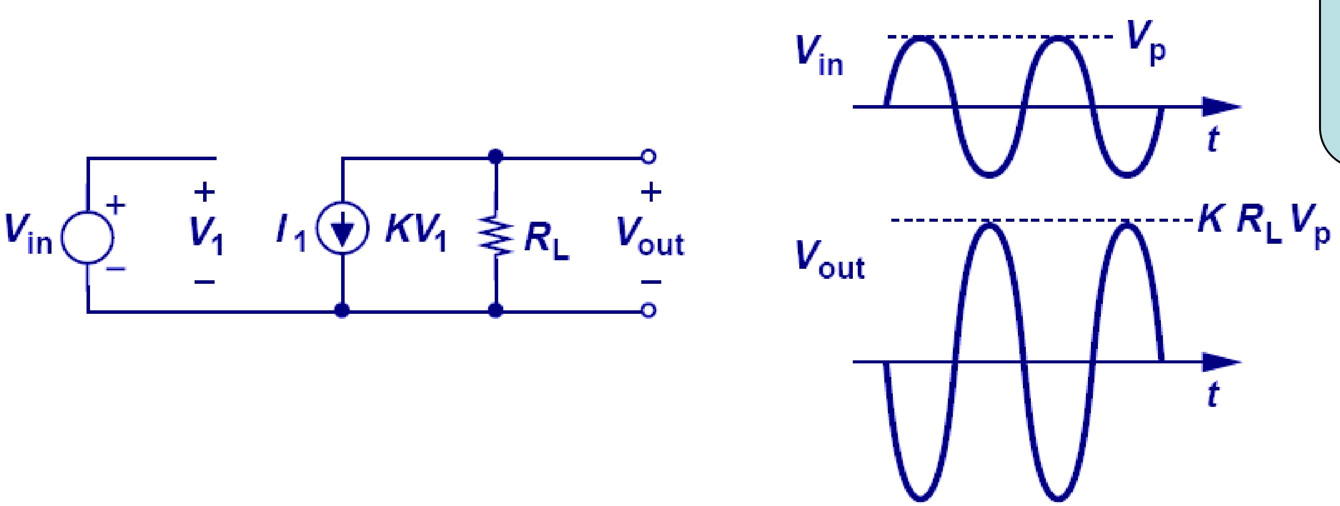
Voltage-Controlled Device as Amplifying Element ⇨ Structure of Bipolar Transistor ⇨ Operation of Bipolar Transistor ⇨ Large-Signal Model ⇨ Small-Signal Model

- **In the chapter, we will study the physics of bipolar transistor and derive large and small signal models.**

Voltage-Dependent Current Source



(a)

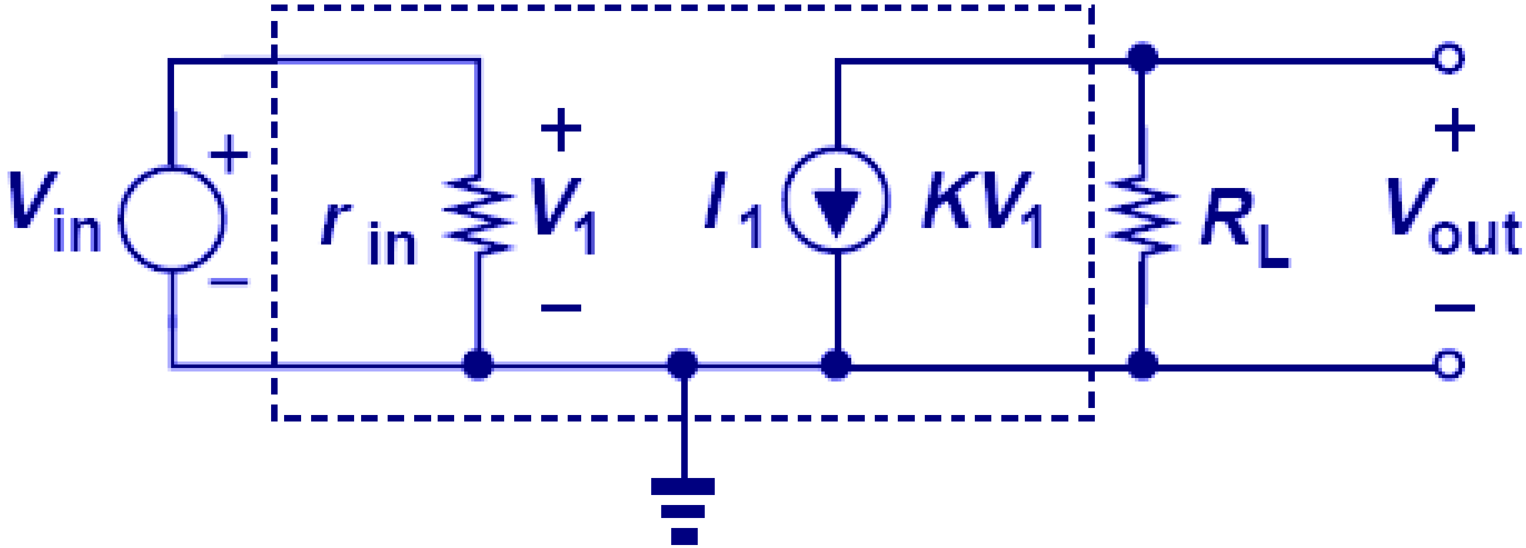


(b)

$$A_V = \frac{V_{out}}{V_{in}} = -KR_L$$

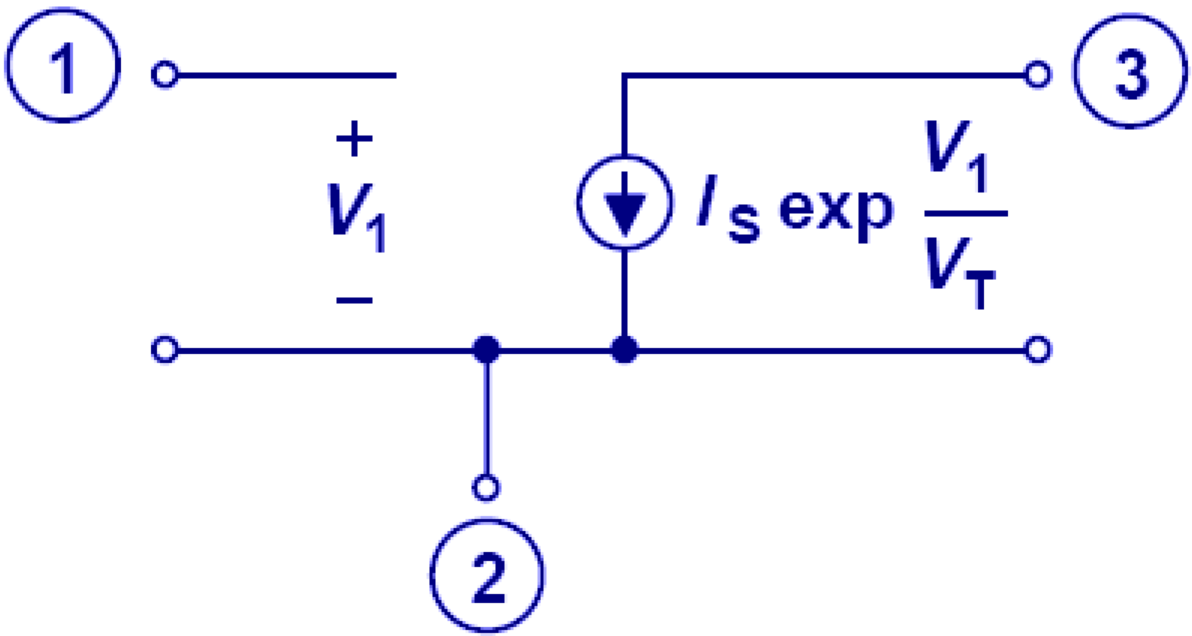
- A voltage-dependent current source can act as an amplifier.
- If KR_L is greater than 1, then the signal is amplified.

Voltage-Dependent Current Source with Input Resistance



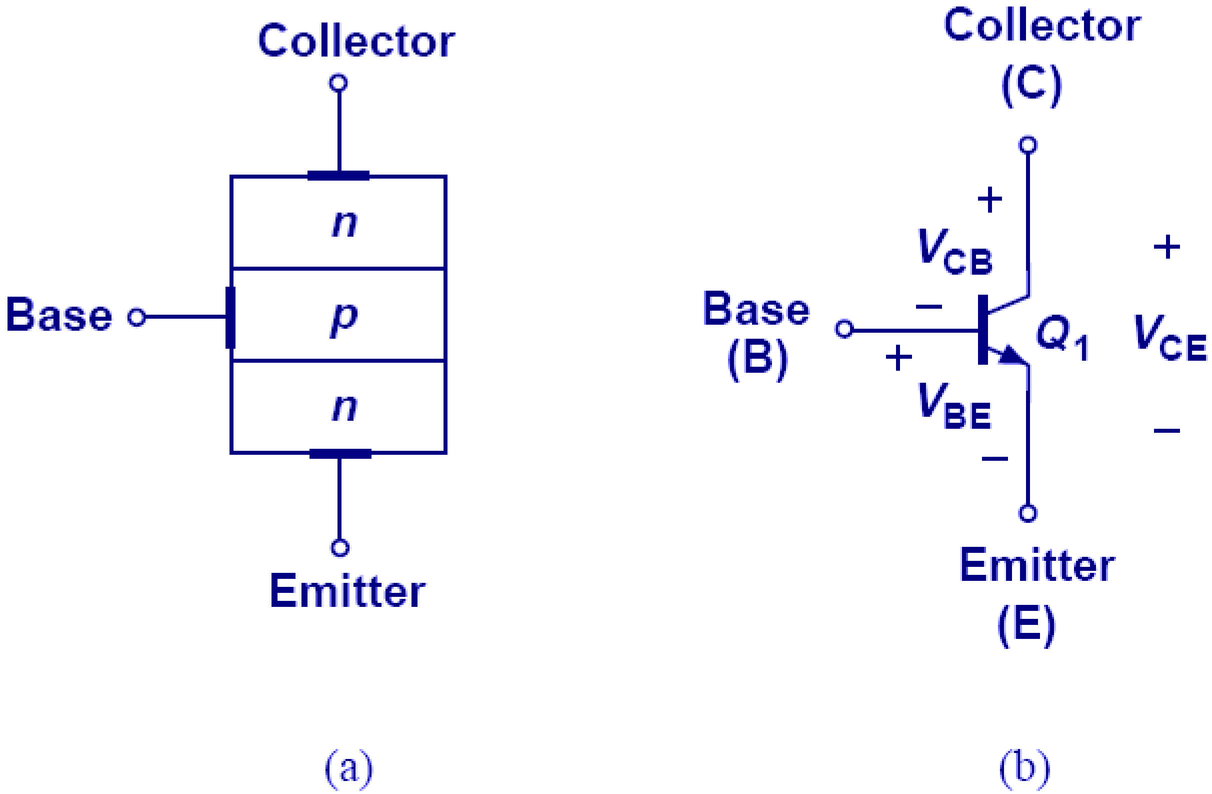
- **Regardless of the input resistance, the magnitude of amplification remains unchanged.**

Exponential Voltage-Dependent Current Source



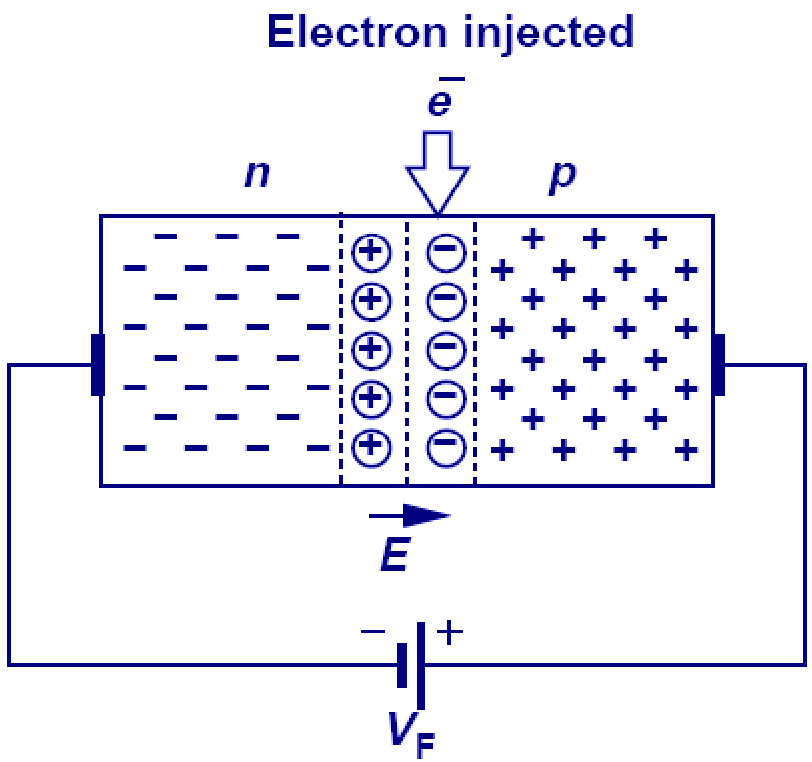
- A three-terminal exponential voltage-dependent current source is shown above.
- Ideally, bipolar transistor can be modeled as such.

Structure and Symbol of Bipolar Transistor



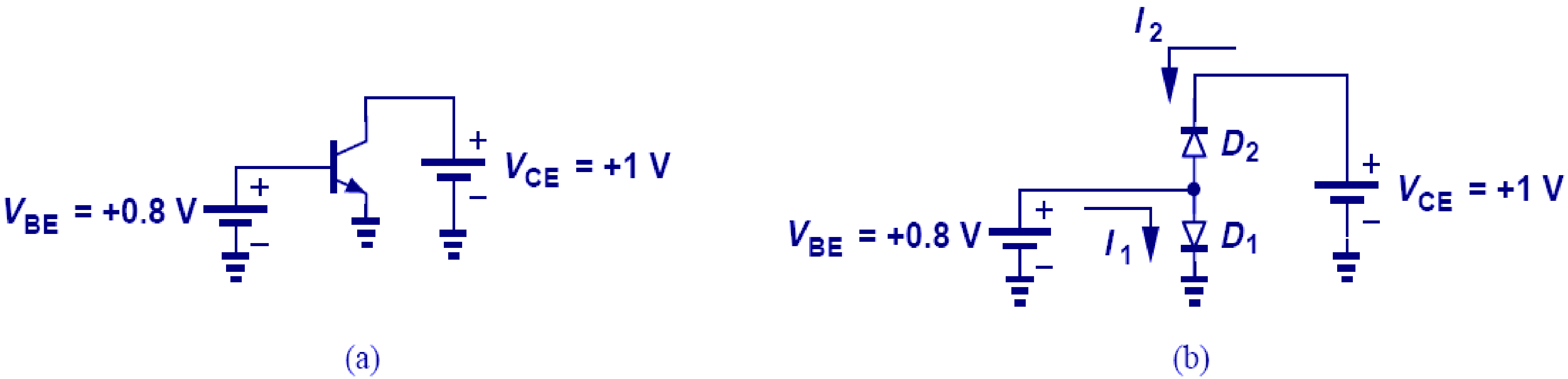
- Bipolar transistor can be thought of as a sandwich of three doped Si regions. The outer two regions are doped with the same polarity, while the middle region is doped with opposite polarity.

Injection of Carriers



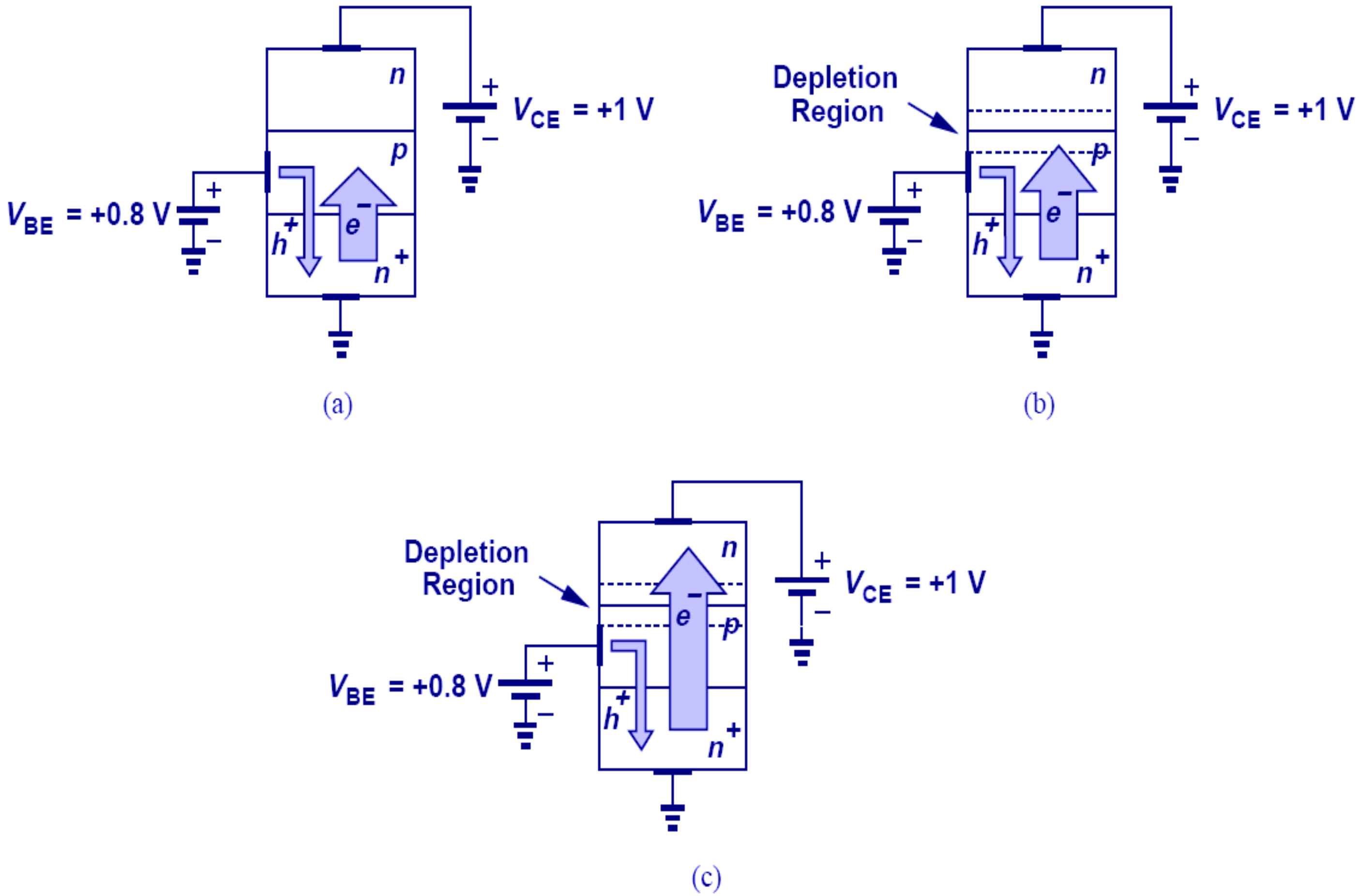
- Reverse biased PN junction creates a large electric field that sweeps any injected minority carriers to their majority region.
- This ability proves essential in the proper operation of a bipolar transistor.

Forward Active Region



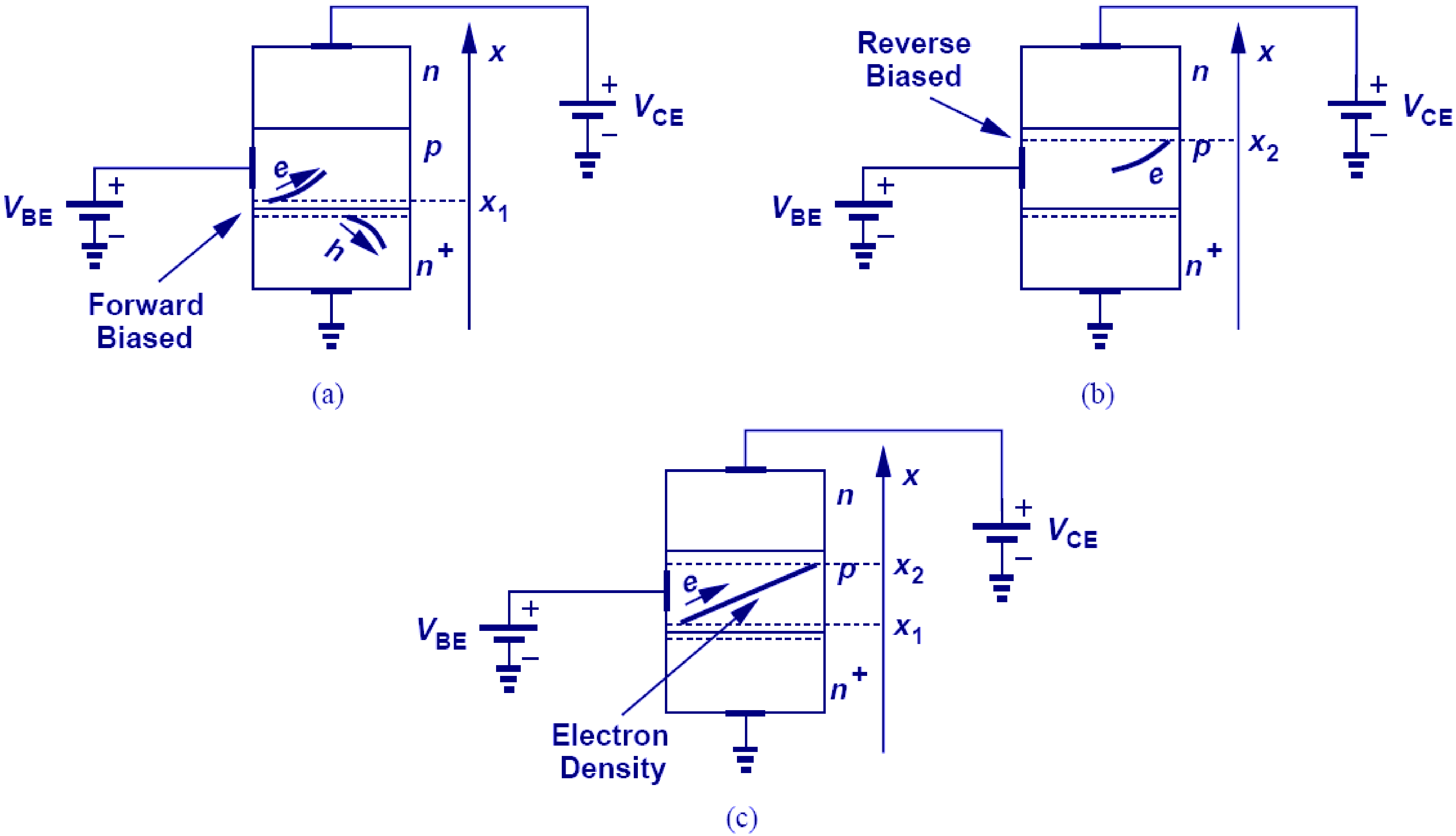
- **Forward active region: $V_{BE} > 0$, $V_{BC} < 0$.**
- **Figure b) presents a wrong way of modeling figure a).**

Accurate Bipolar Representation



- Collector also carries current due to carrier injection from base.

Carrier Transport in Base



Collector Current

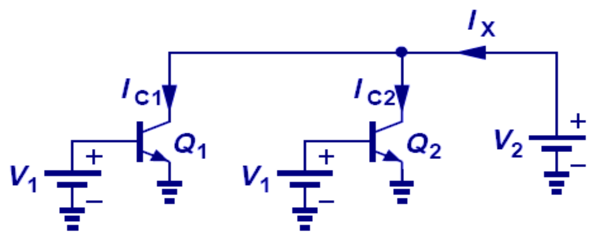
$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(\exp\frac{V_{BE}}{V_T} - 1\right)$$

$$I_C = I_S \exp\frac{V_{BE}}{V_T}$$

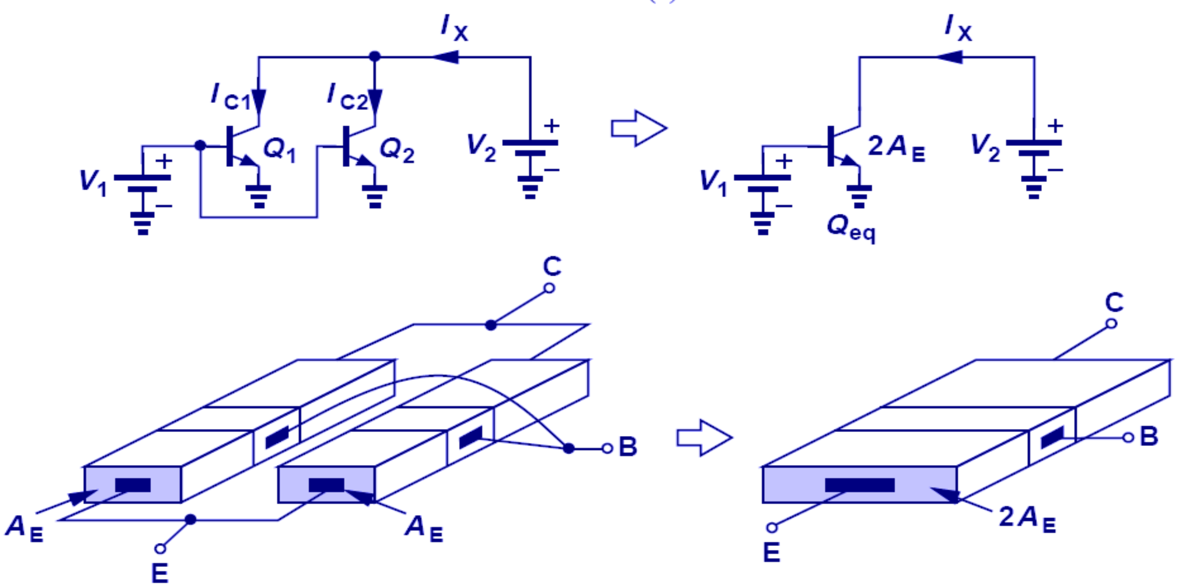
$$I_S = \frac{A_E q D_n n_i^2}{N_E W_B}$$

- **Applying the law of diffusion, we can determine the charge flow across the base region into the collector.**
- **The equation above shows that the transistor is indeed a voltage-controlled element, thus a good candidate as an amplifier.**

Parallel Combination of Transistors



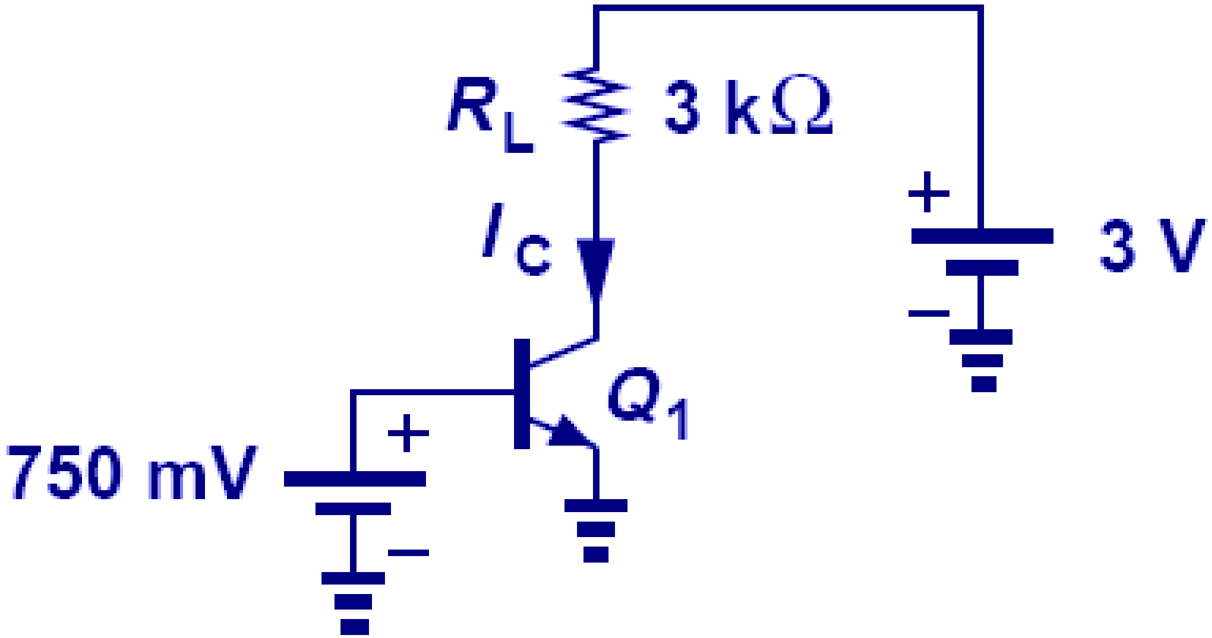
(a)



(b)

- **When two transistors are put in parallel and experience the same potential across all three terminals, they can be thought of as a single transistor with twice the emitter area.**

Simple Transistor Configuration



- **Although a transistor is a voltage to current converter, output voltage can be obtained by inserting a load resistor at the output and allowing the controlled current to pass thru it.**

Example 4.5

Example 4.5

Determine the output voltage in Fig. 4.10 if $I_S = 5 \times 10^{-16}$ A.

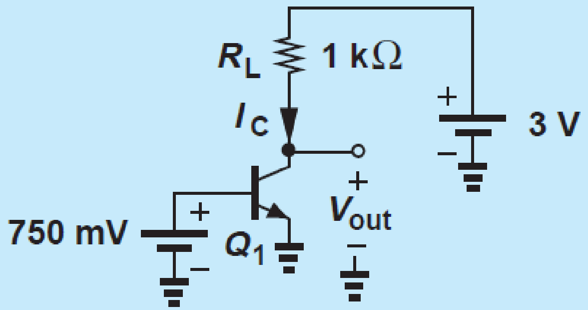
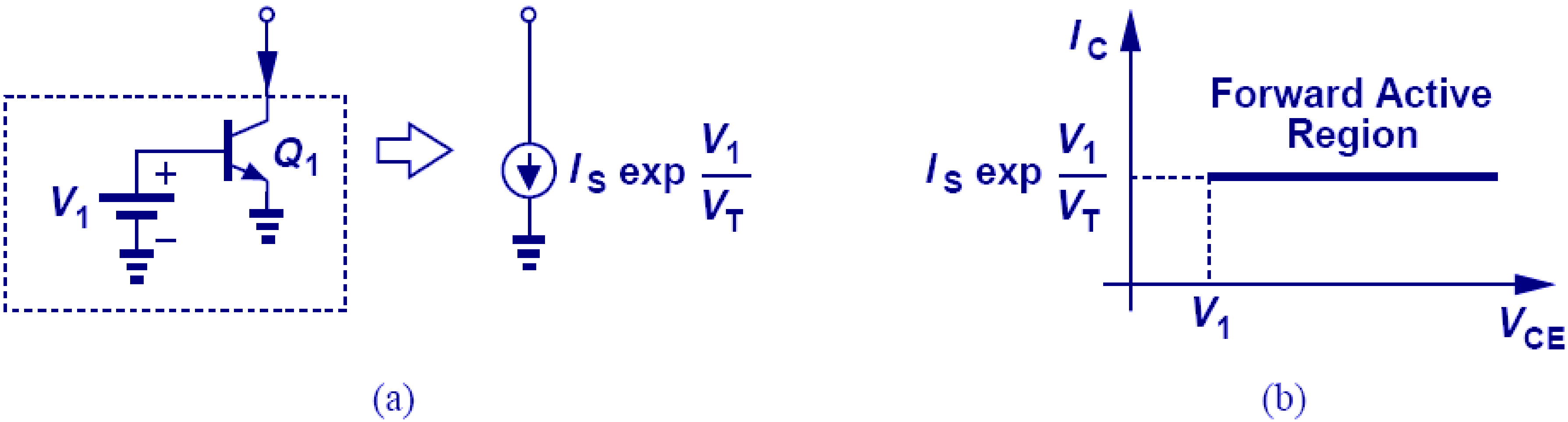


Figure 4.10 Simple stage with biasing.

Solution Using Eq. (4.9), we write $I_C = 1.69$ mA. This current flows through R_L, generating a voltage drop of 1 kΩ × 1.69 mA = 1.69 V. Since $V_{CE} = 3\text{ V} - I_C R_L$, we obtain

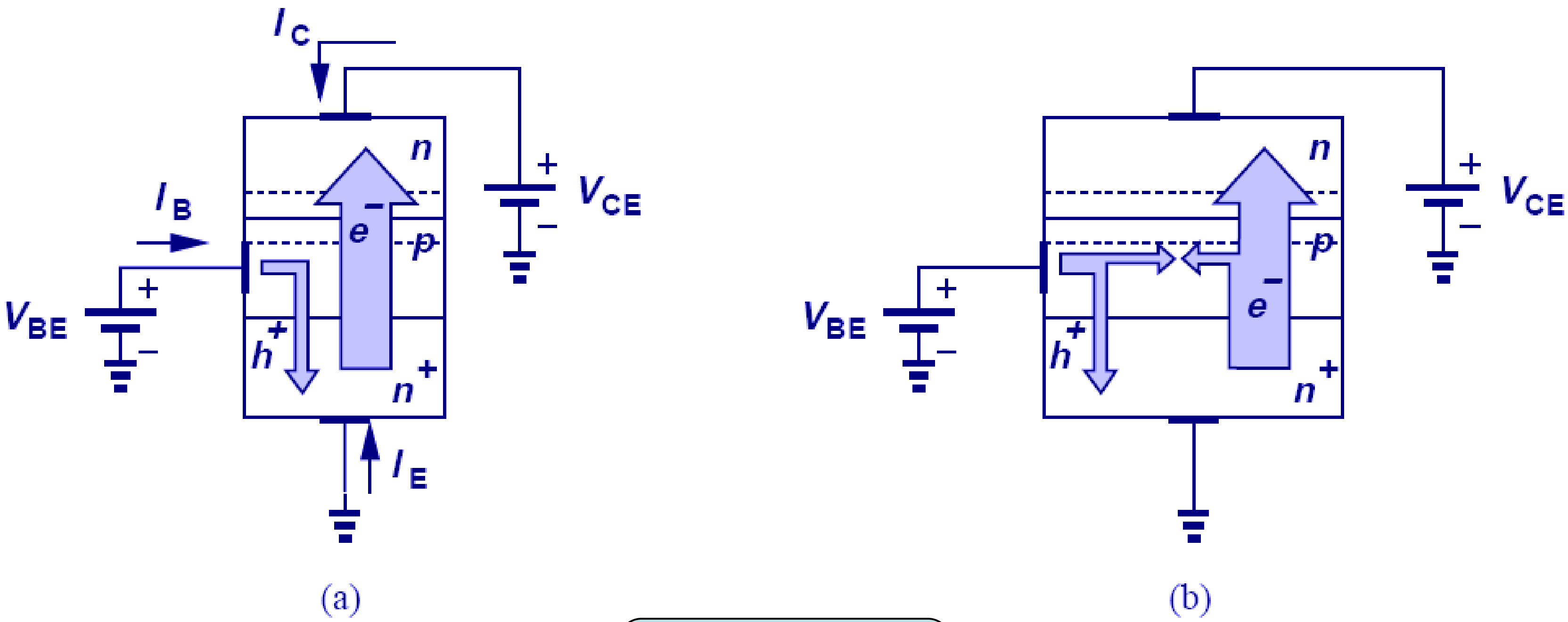
$$V_{out} = 1.31\text{ V}. \tag{4.19}$$

Constant Current Source



- Ideally, the collector current does not depend on the collector to emitter voltage. This property allows the transistor to behave as a constant current source when its base-emitter voltage is fixed.

Base Current



$$I_C = \beta I_B$$

- **Base current consists of two components: 1) Reverse injection of holes into the emitter and 2) recombination of holes with electrons coming from the emitter.**

Emitter Current

$$I_E = I_C + I_B$$

$$I_E = I_C\left(1 + \frac{1}{\beta}\right)$$

$$\beta = \frac{I_C}{I_B}$$

- **Applying Kirchoff's current law to the transistor, we can easily find the emitter current.**

Summary of Currents

$$I_C = I_S \exp\frac{V_{BE}}{V_T}$$

$$I_B = \frac{1}{\beta} I_S \exp\frac{V_{BE}}{V_T}$$

$$I_E = \frac{\beta+1}{\beta} I_S \exp\frac{V_{BE}}{V_T}$$

$$\frac{\beta}{\beta+1} = \alpha$$

Example 4.6

Example 4.6 A bipolar transistor having $I_S = 5 \times 10^{-16}$ A is biased in the forward active region with $V_{BE} = 750$ mV. If the current gain varies from 50 to 200 due to manufacturing variations, calculate the minimum and maximum terminal currents of the device.

Solution For a given V_{BE}, the collector current remains independent of β:

$$I_C = I_S \exp \frac{V_{BE}}{V_T} \tag{4.26}$$

$$= 1.685 \text{ mA}. \tag{4.27}$$

The base current varies from $I_C/200$ to $I_C/50$:

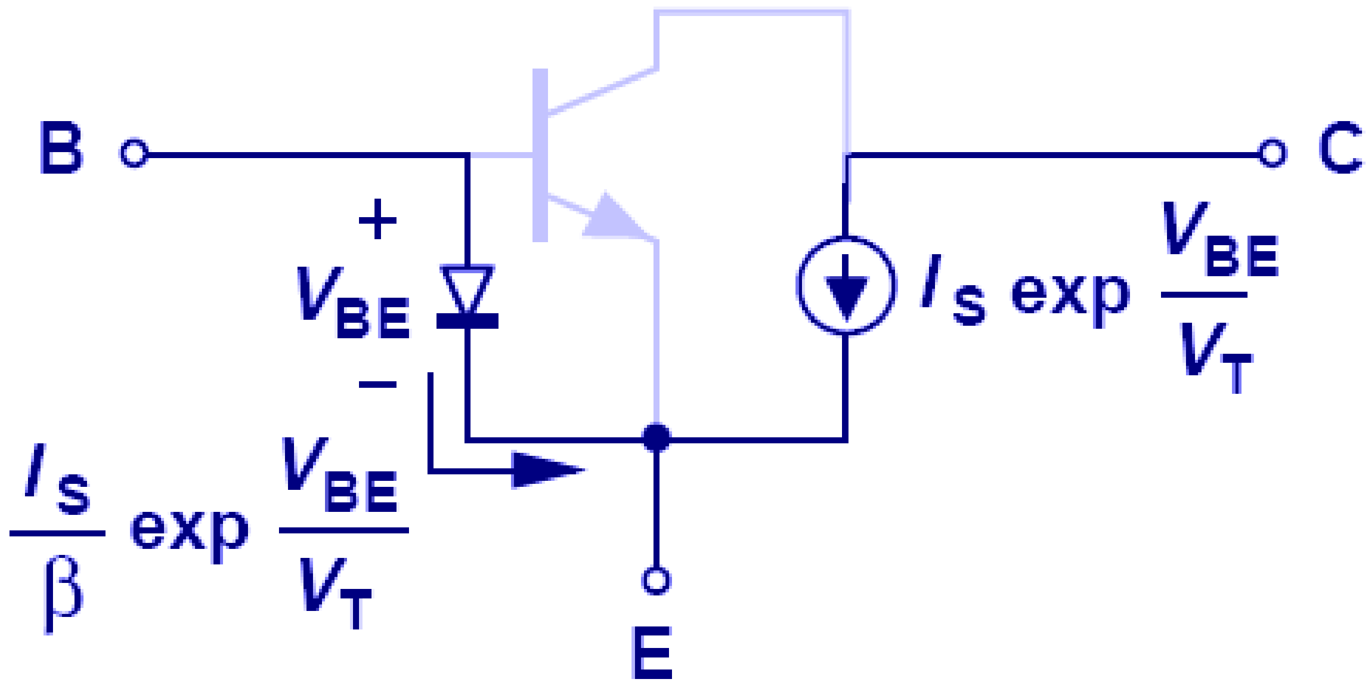
$$8.43\ \mu\text{A} < I_B < 33.7\ \mu\text{A}. \tag{4.28}$$

On the other hand, the emitter current experiences only a small variation because $(\beta + 1)/\beta$ is near unity for large β:

$$1.005 I_C < I_E < 1.02 I_C \tag{4.29}$$

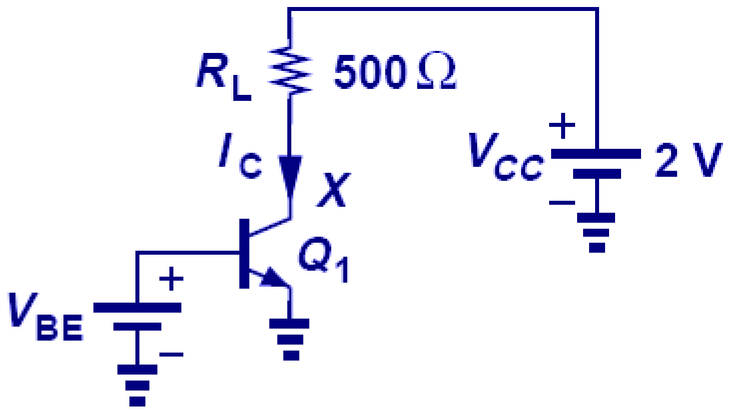
$$1.693 \text{ mA} < I_E < 1.719 \text{ mA}. \tag{4.30}$$

Bipolar Transistor Large Signal Model

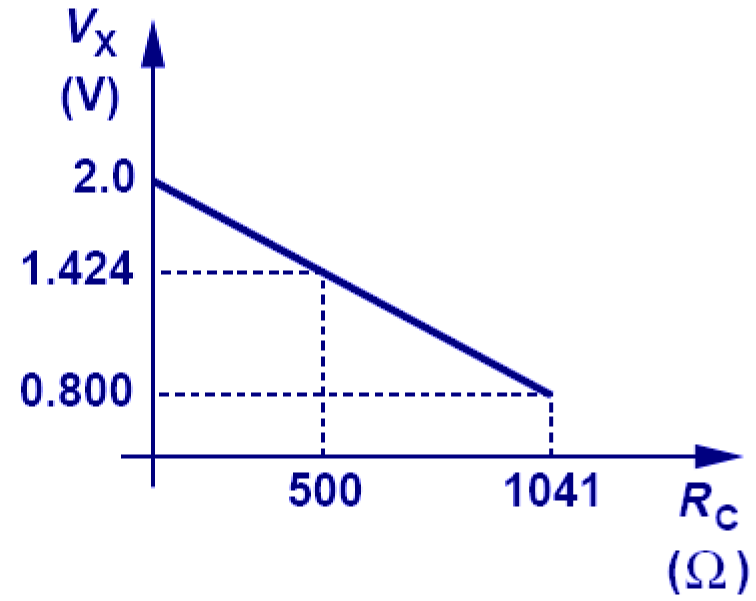


- **A diode is placed between base and emitter and a voltage controlled current source is placed between the collector and emitter.**

Example: Maximum R_L



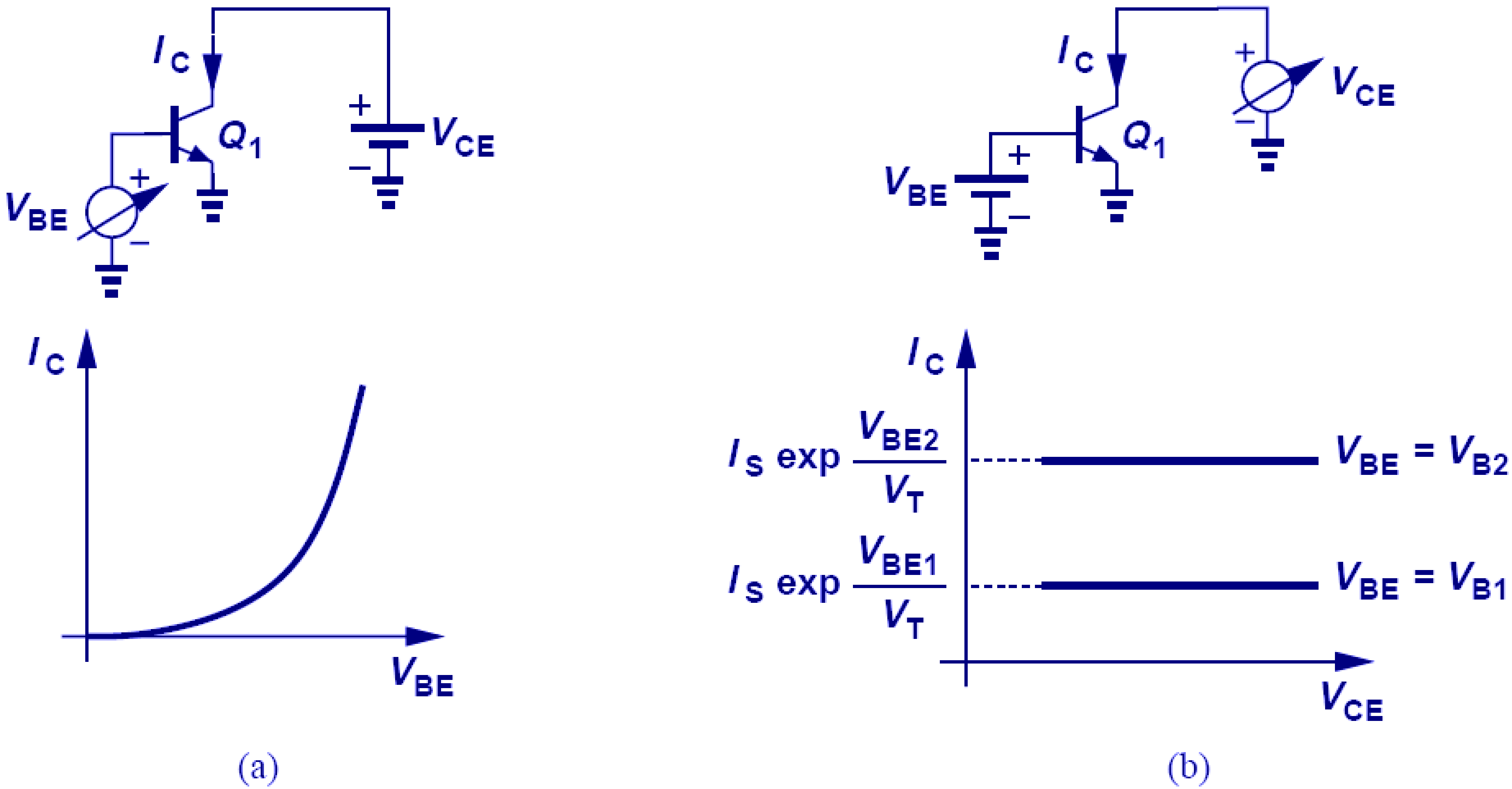
(a)



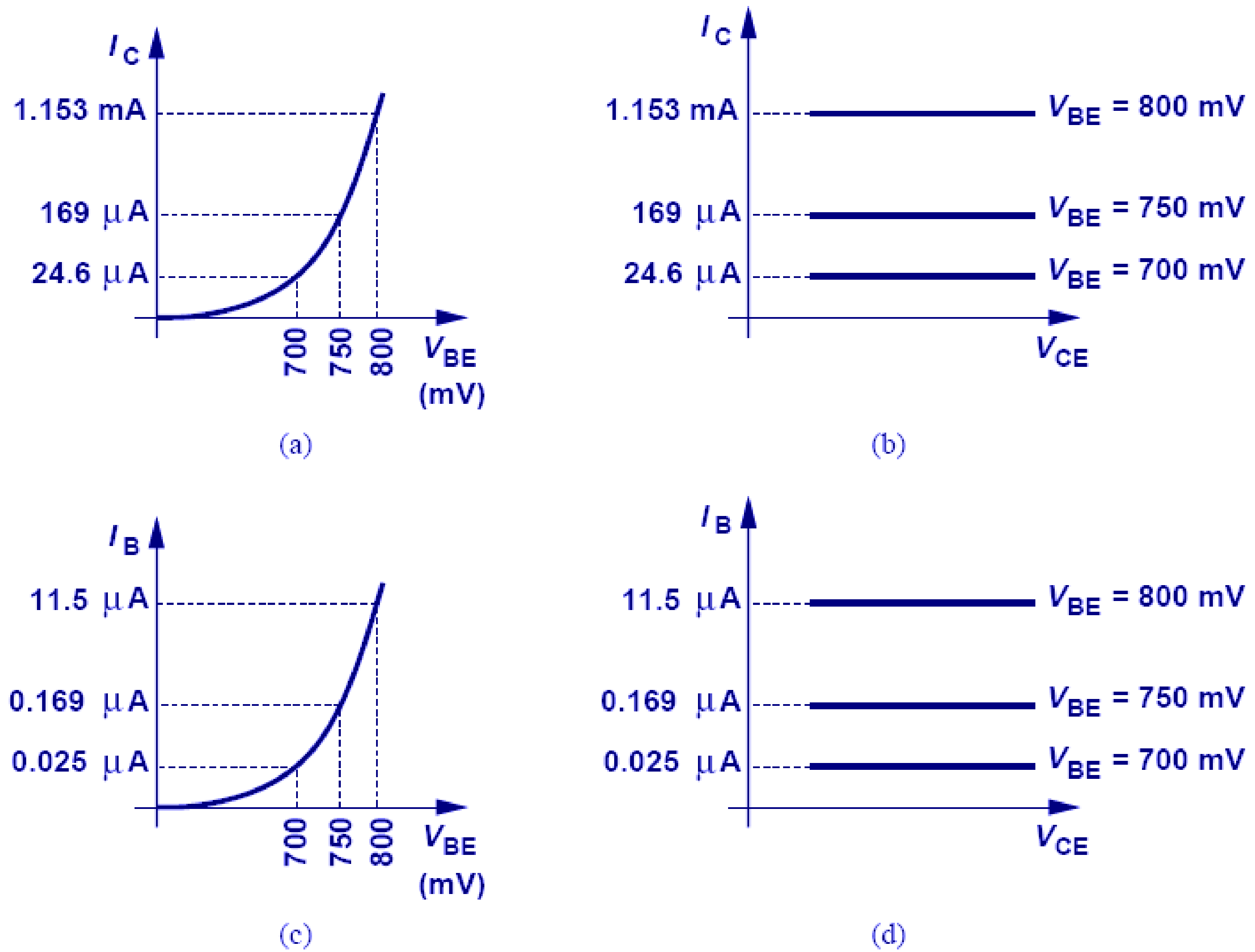
(b)

- As R_L increases, V_x drops and eventually forward biases the collector-base junction. This will force the transistor out of forward active region.
- Therefore, there exists a maximum tolerable collector resistance.

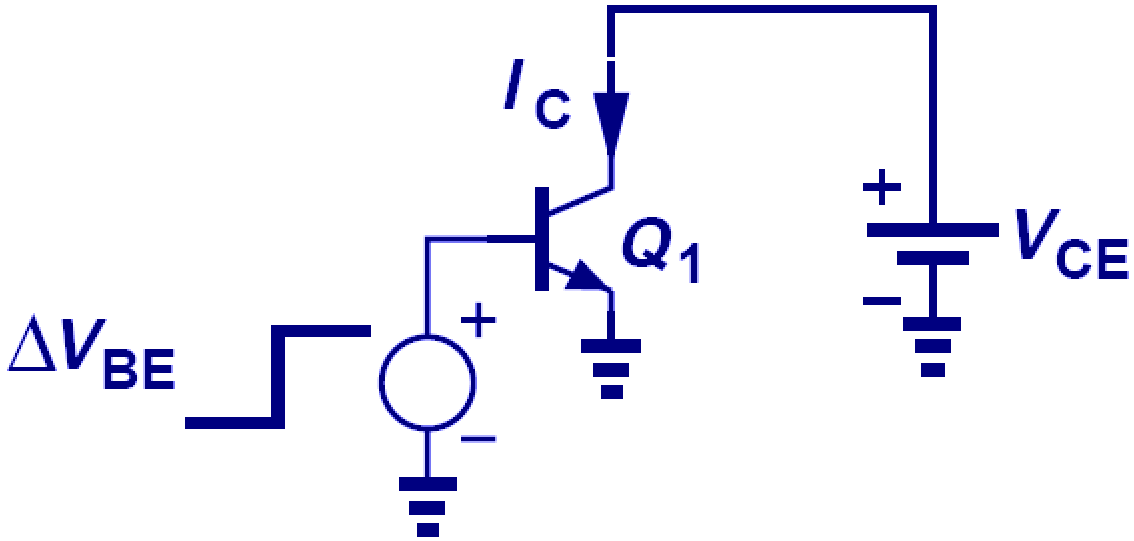
Characteristics of Bipolar Transistor



Example: IV Characteristics



Transconductance



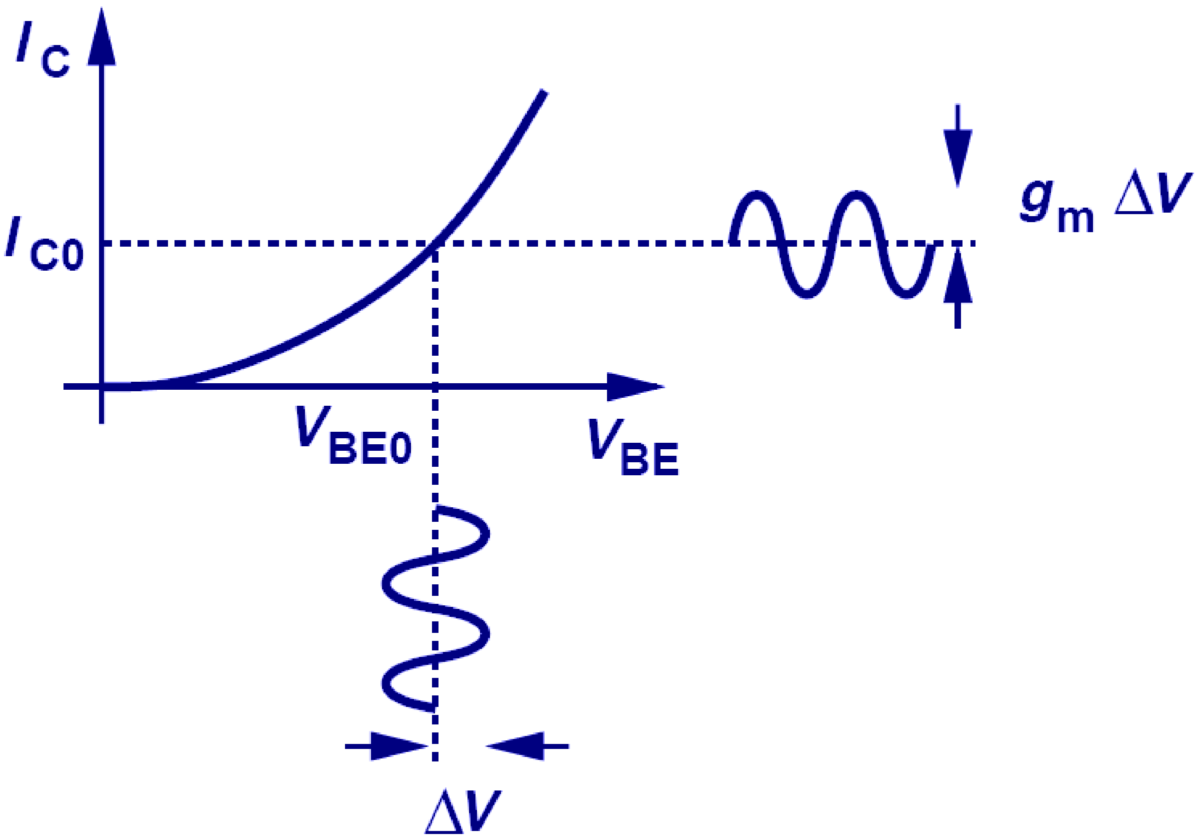
$$g_m = \frac{d}{dV_{BE}}\left(I_S \exp\frac{V_{BE}}{V_T}\right)$$

$$g_m = \frac{1}{V_T} I_S \exp\frac{V_{BE}}{V_T}$$

$$g_m = \frac{I_C}{V_T}$$

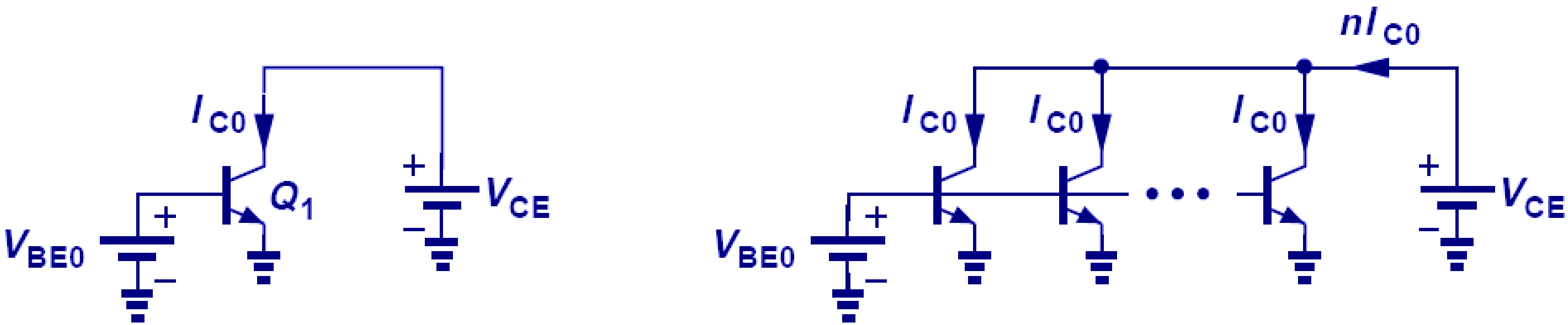
- **Transconductance, g_m shows a measure of how well the transistor converts voltage to current.**
- **It will later be shown that g_m is one of the most important parameters in circuit design.**

Visualization of Transconductance



- g_m can be visualized as the slope of I_C versus V_{BE}.
- A large I_C has a large slope and therefore a large g_m.

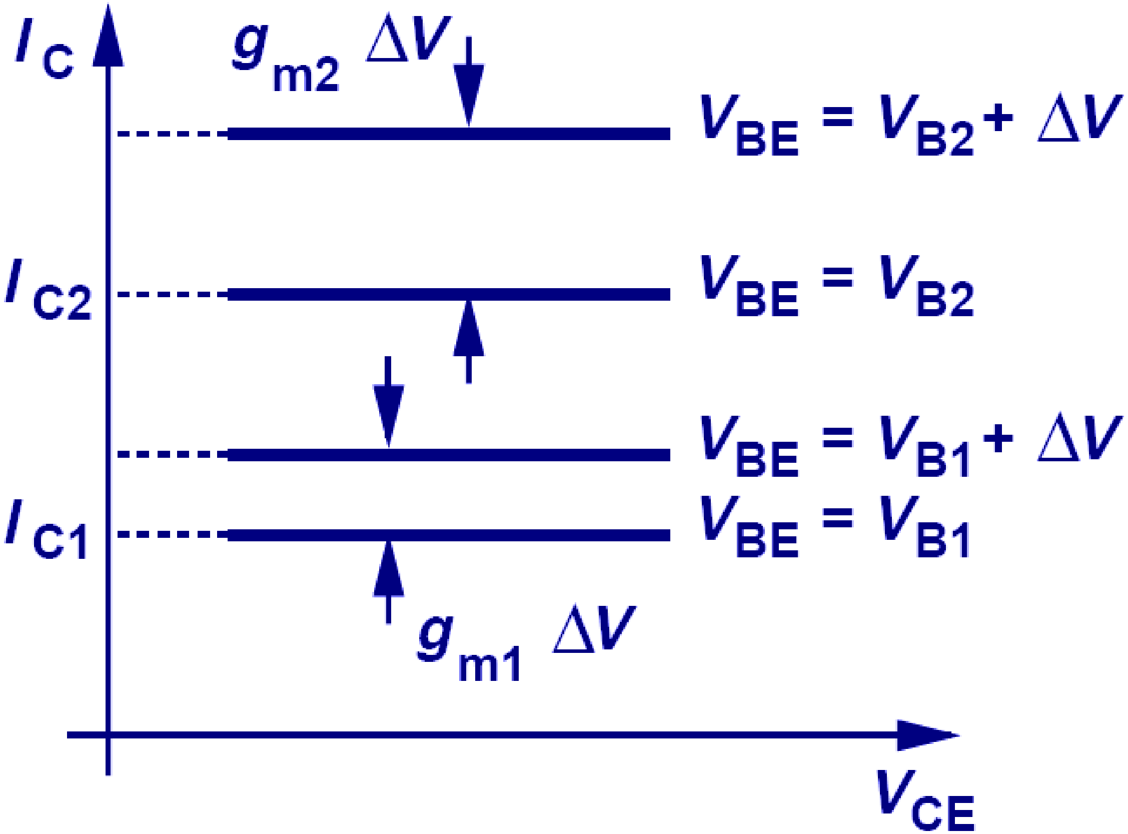
Transconductance and Area



(a) (b)

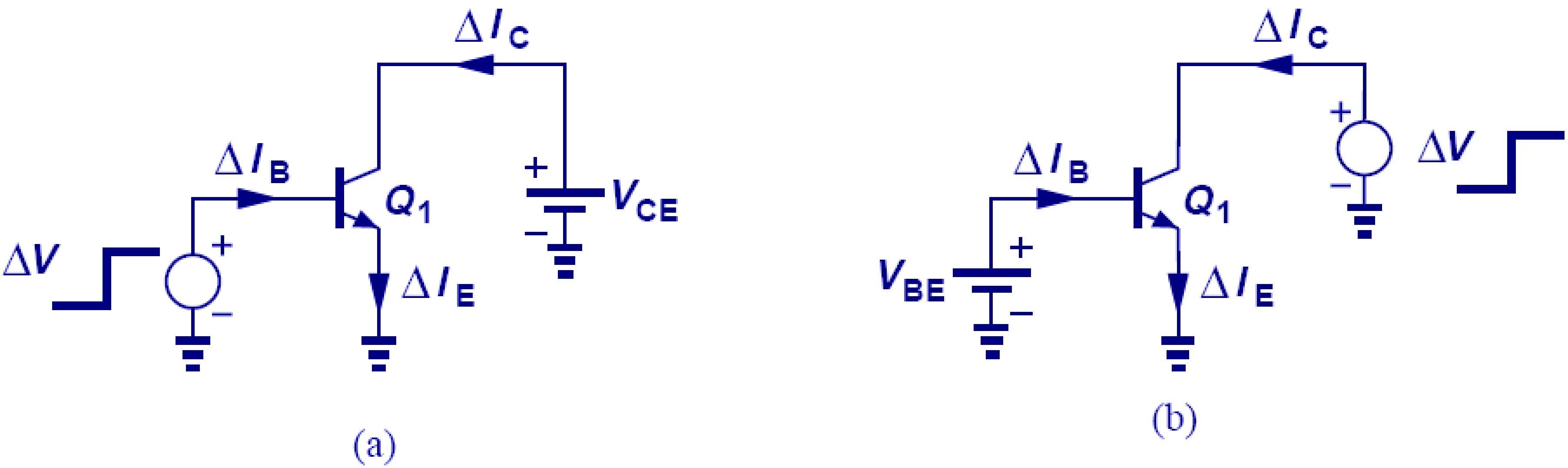
- When the area of a transistor is increased by n, I_S increases by n. For a <u>constant</u> $V_{BE,}$ I_C and hence g_m increases by a factor of n.

Transconductance and I_c



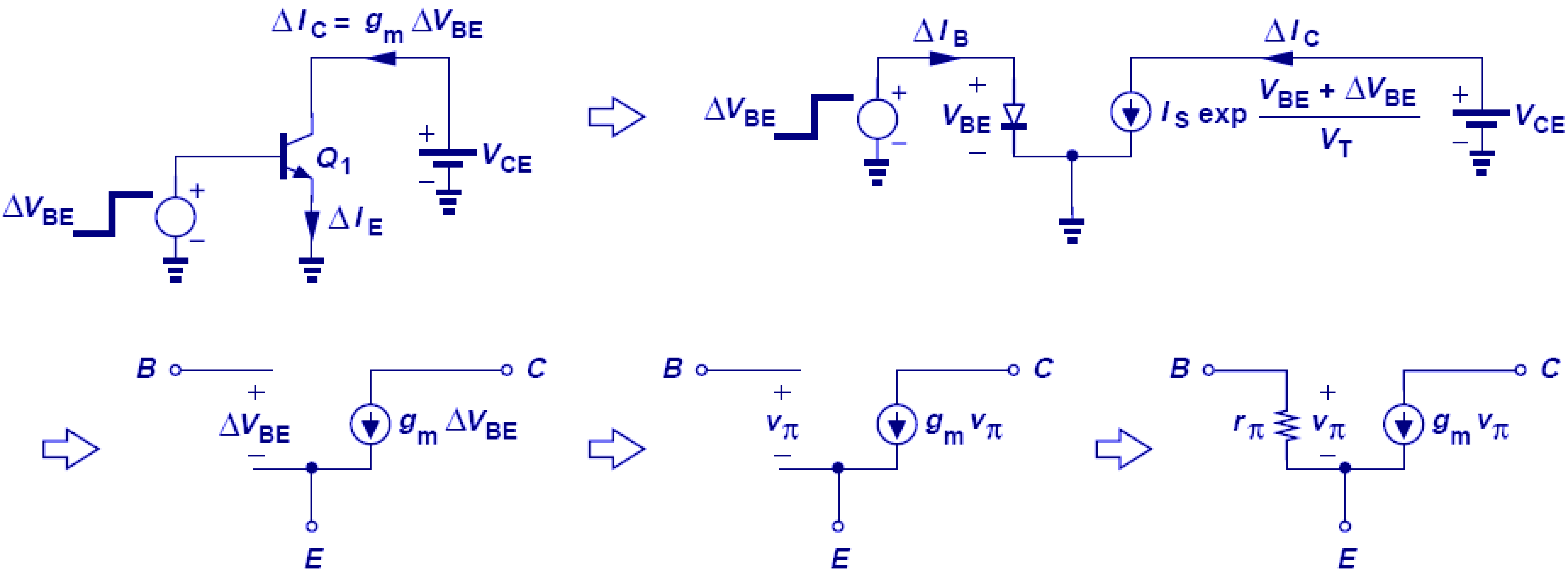
- **The figure above shows that for a given V_{BE} swing, the current excursion around I_{C2} is larger than it would be around I_{C1}. This is because g_m is larger with I_{C2}.**

Small-Signal Model: Derivation

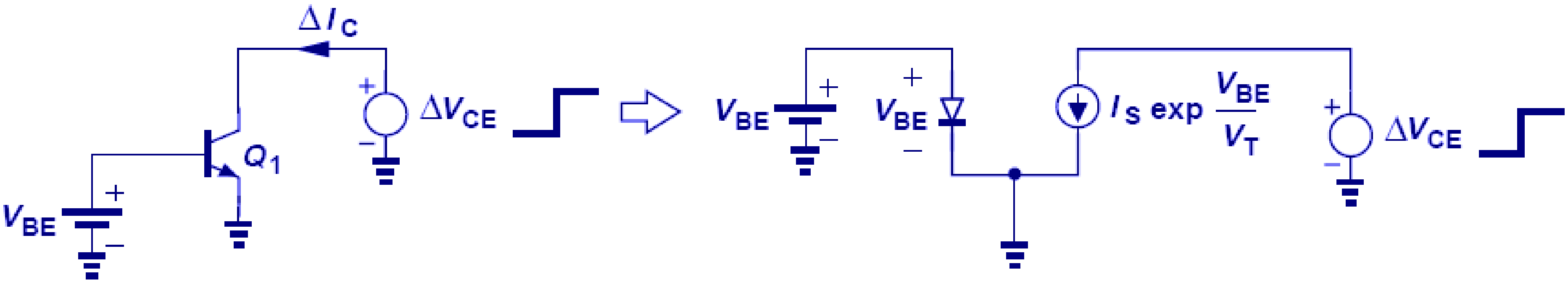


- Small signal model is derived by perturbing voltage difference every two terminals while fixing the third terminal and analyzing the change in current of all three terminals. We then represent these changes with controlled sources or resistors.

Small-Signal Model: V_{BE} Change

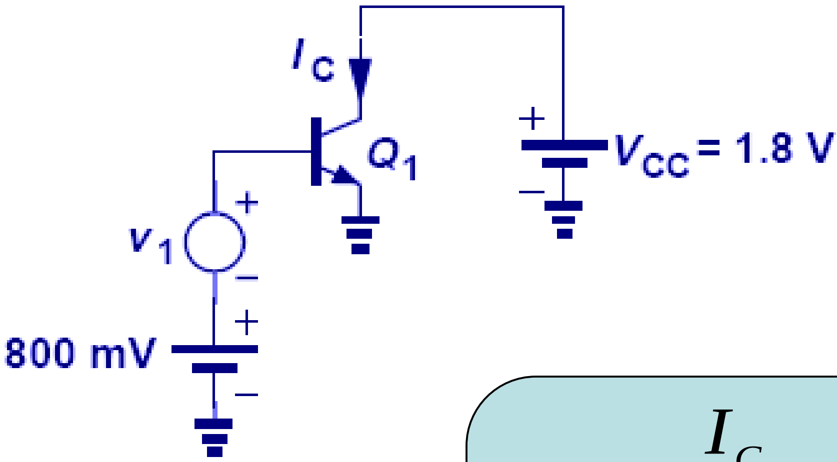


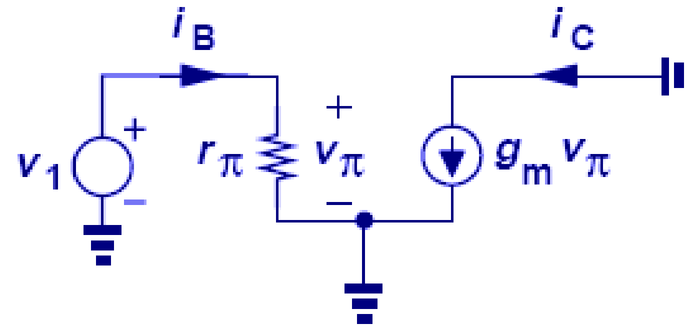
Small-Signal Model: V_{CE} Change



- Ideally, V_{CE} has no effect on the collector current. Thus, it will not contribute to the small signal model.
- It can be shown that V_{CB} has no effect on the small signal model, either.

Small Signal Example I





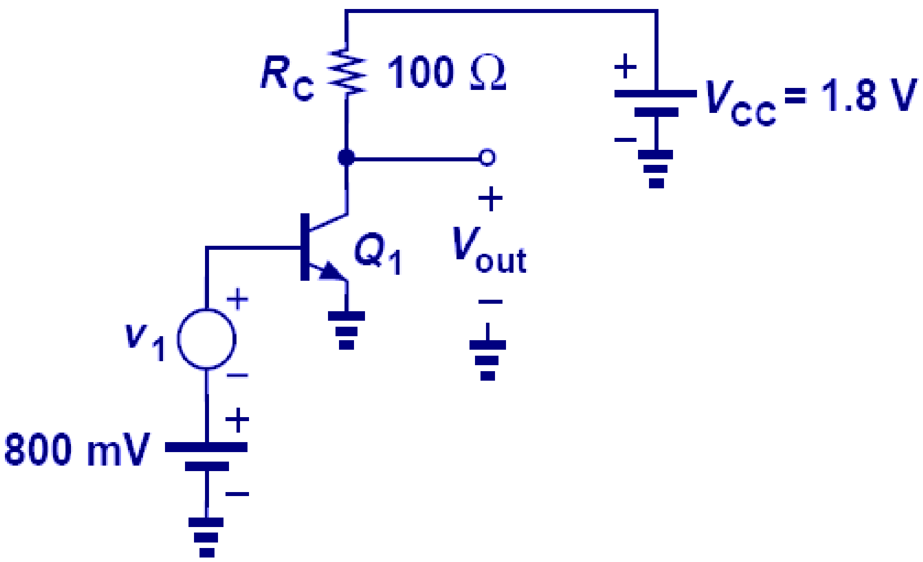
$$g_m = \frac{I_C}{V_T} = \frac{1}{3.75\Omega}$$

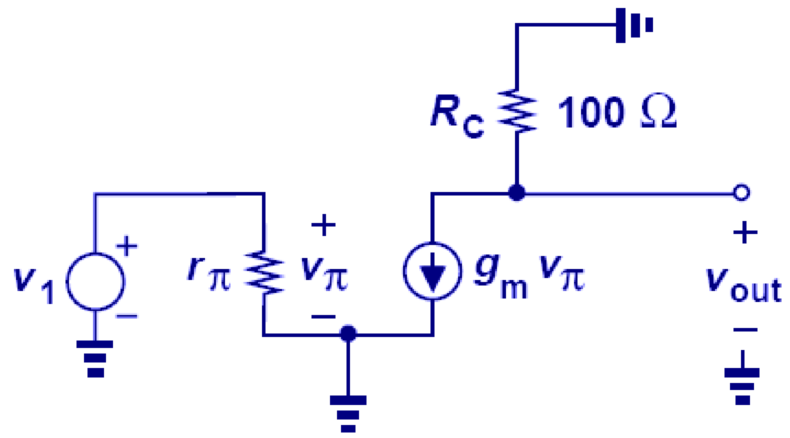
$$r_\pi = \frac{\beta}{g_m} = 375\Omega$$

I_C=7 mA
β=100

➢ **Here, small signal parameters are calculated from DC operating point and are used to calculate the change in collector current due to a change in V_{BE}.**

Small Signal Example II



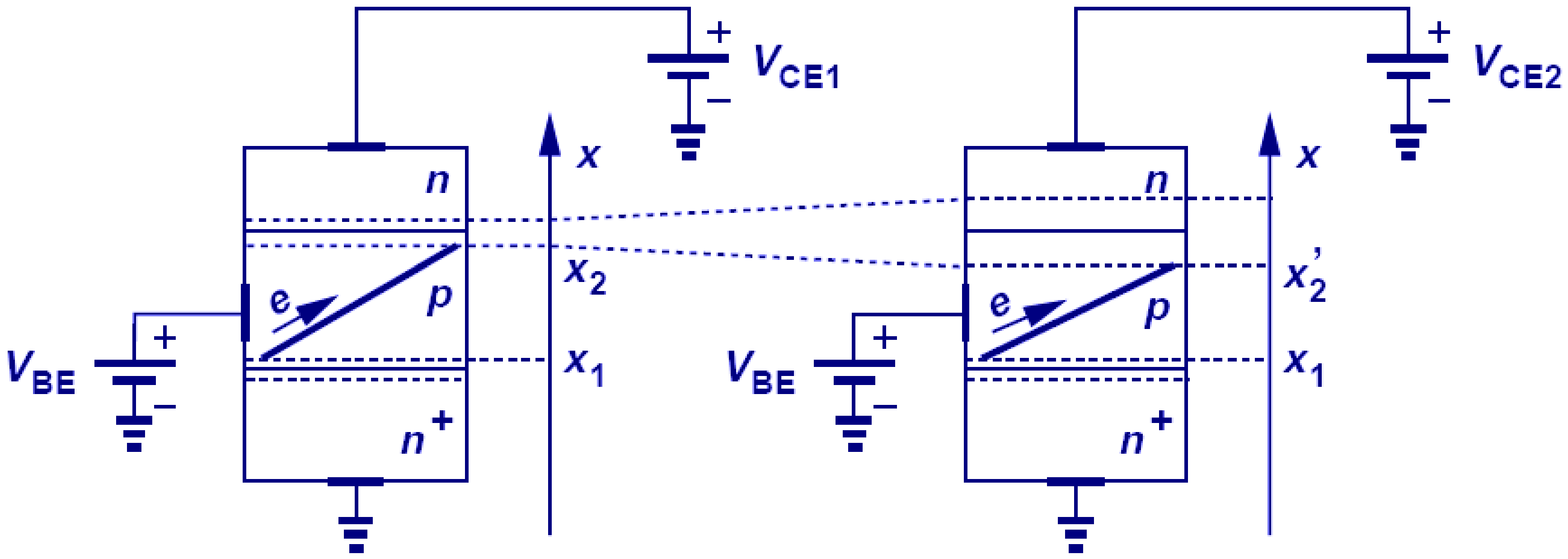


- In this example, a resistor is placed between the power supply and collector, therefore, providing an output voltage.

AC Ground

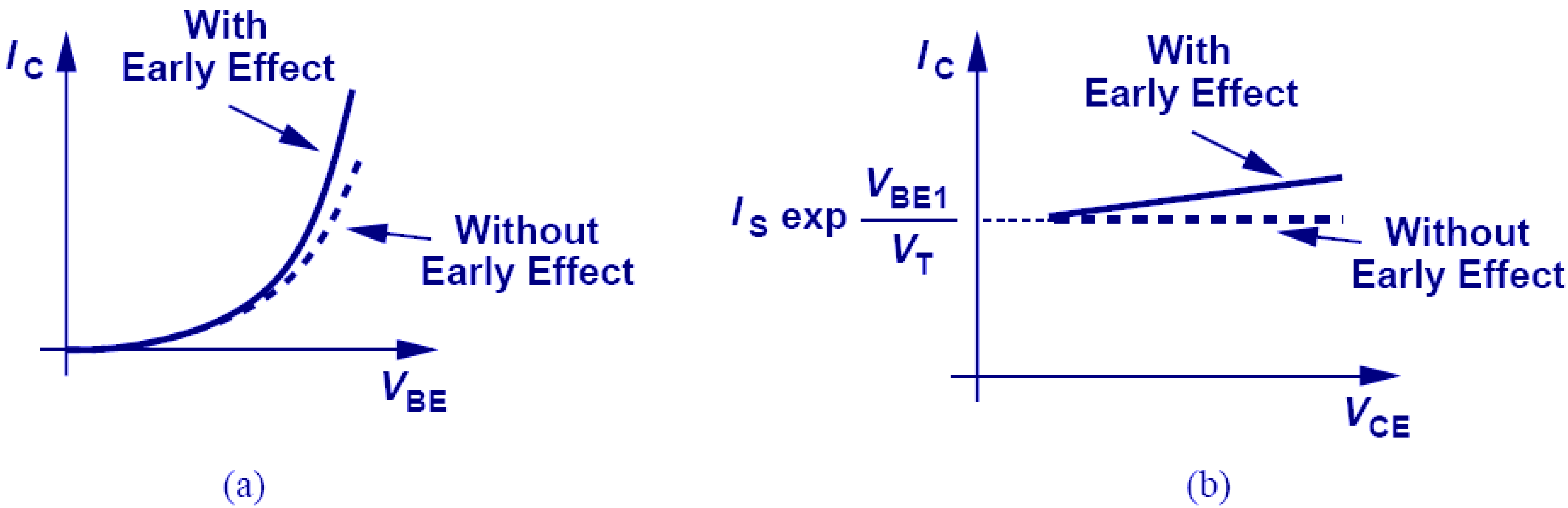
- **Since the power supply voltage does not vary with time, it is regarded as a ground in small-signal analysis.**

Early Effect



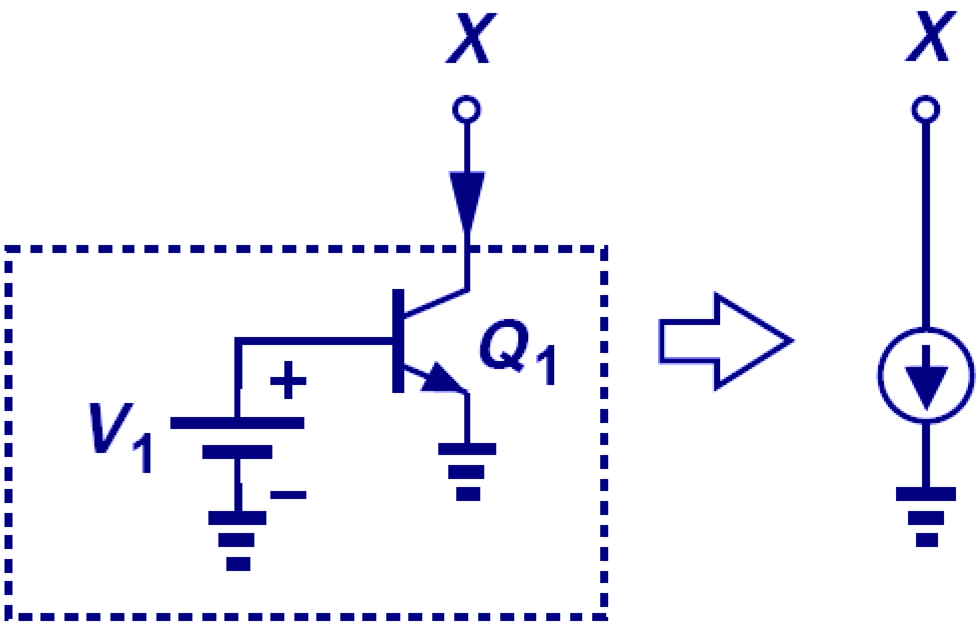
- **The claim that collector current does not depend on V_{CE} is not accurate.**
- **As V_{CE} increases, the depletion region between base and collector increases. Therefore, the effective base width decreases, which leads to an increase in the collector current.**

Early Effect Illustration



- With Early effect, collector current becomes larger than usual and a function of V_{CE}.

Early Effect Representation



$$\left(I_S \exp\frac{V_1}{V_T}\right)\left(1+\frac{V_X}{V_A}\right)$$

Example 4.13

Example 4.13 A bipolar transistor carries a collector current of 1 mA with $V_{CE} = 2$ V. Determine the required base-emitter voltage if $V_A = \infty$ or $V_A = 20$ V. Assume $I_S = 2 \times 10^{-16}$ A.

Solution With $V_A = \infty$, we have from Eq. (4.67)

$$V_{BE} = V_T \ln \frac{I_C}{I_S} \tag{4.70}$$

$$= 760.3\,\text{mV}. \tag{4.71}$$

If $V_A = 20$ V, we rewrite Eq. (4.67) as

$$V_{BE} = V_T \ln \left(\frac{I_C}{I_S} \frac{1}{1 + \frac{V_{CE}}{V_A}} \right) \tag{4.72}$$

$$= 757.8\,\text{mV}. \tag{4.73}$$

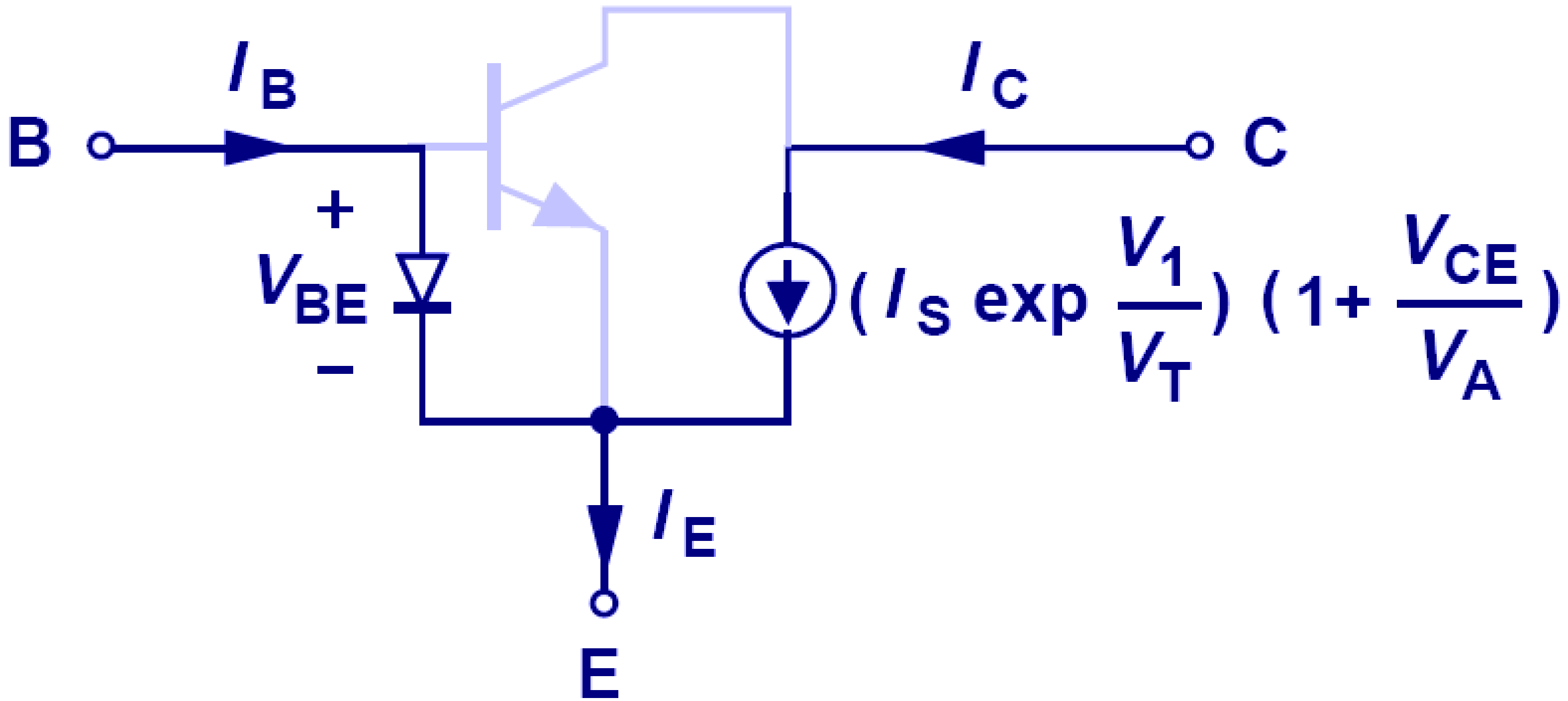
In fact, for $V_{CE} \ll V_A$, we have $(1 + V_{CE}/V_A)^{-1} \approx 1 - V_{CE}/V_A$

$$V_{BE} \approx V_T \ln \frac{I_C}{I_S} + V_T \ln \left(1 - \frac{V_{CE}}{V_A}\right) \tag{4.74}$$

$$\approx V_T \ln \frac{I_C}{I_S} - V_T \frac{V_{CE}}{V_A}, \tag{4.75}$$

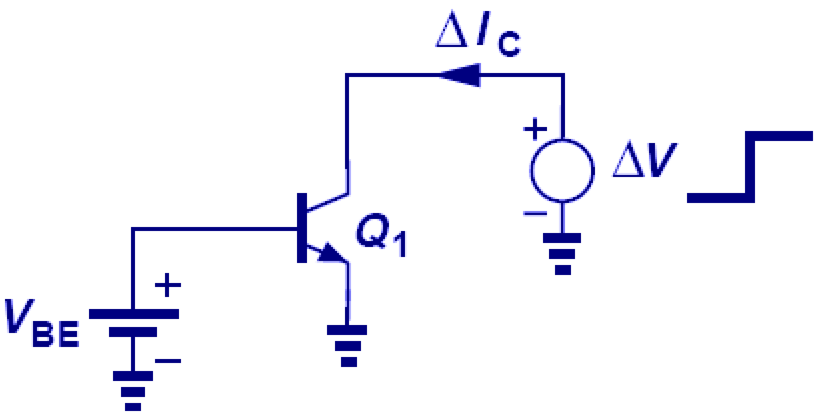
where it is assumed $\ln(1 - \epsilon) \approx -\epsilon$ for $\epsilon \ll 1$.

Early Effect and Large-Signal Model

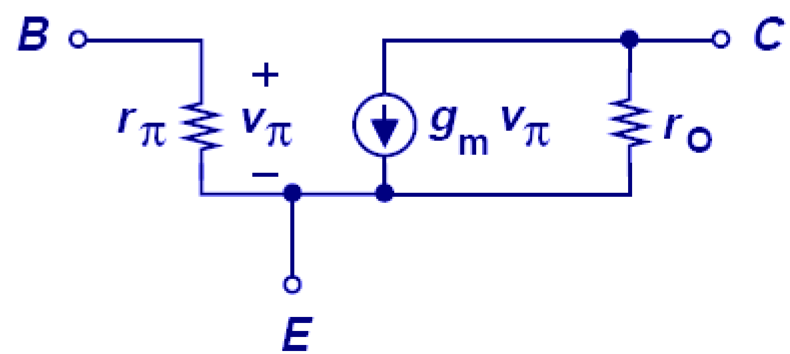


- Early effect can be accounted for in large-signal model by simply changing the collector current with a correction factor.
- In this mode, base current does not change.

Early Effect and Small-Signal Model



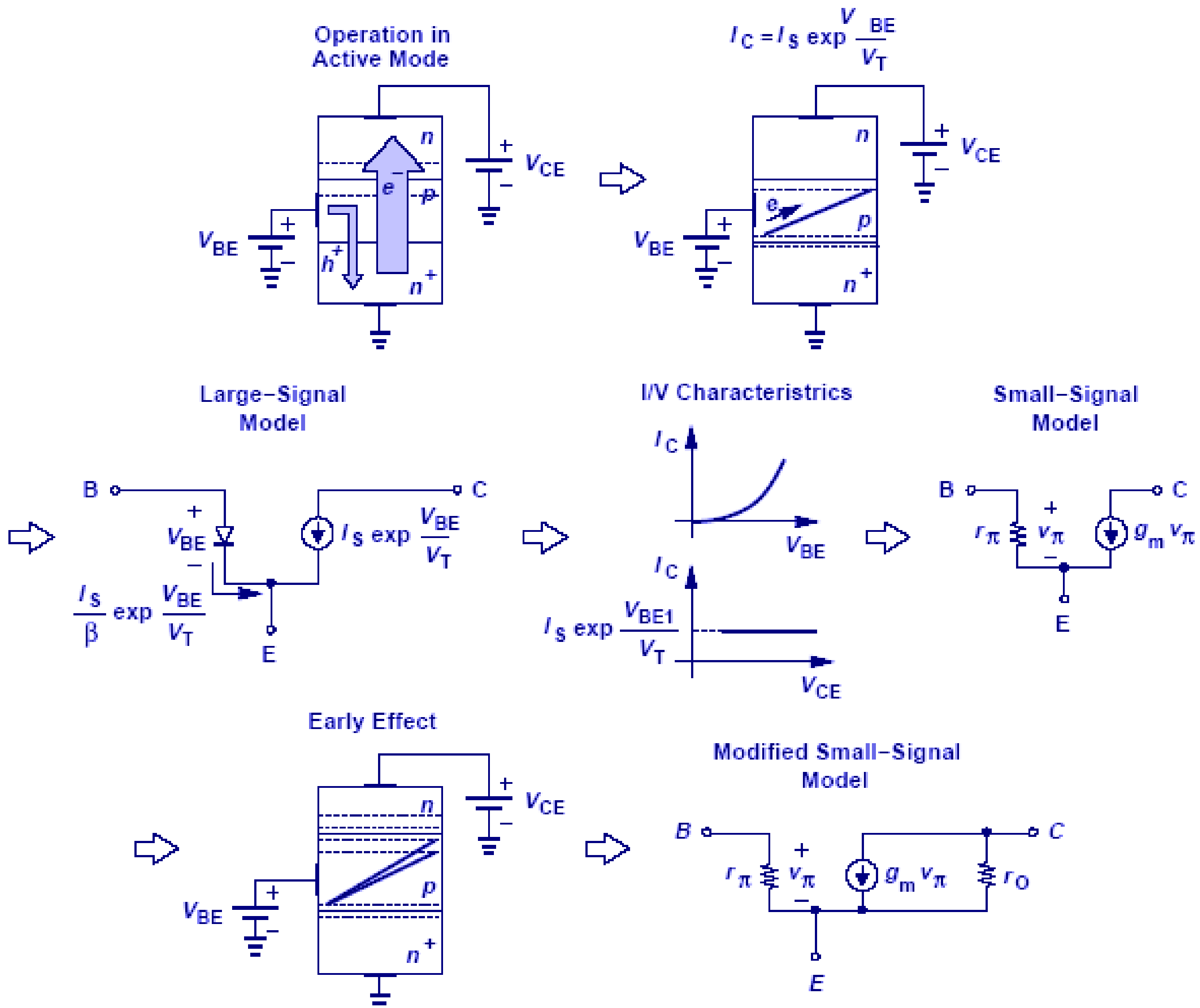
(a)



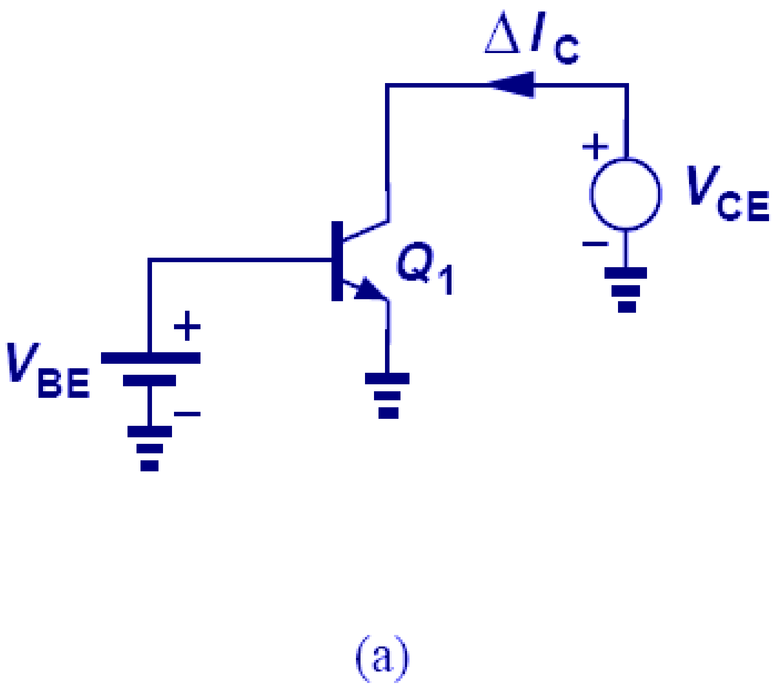
(b)

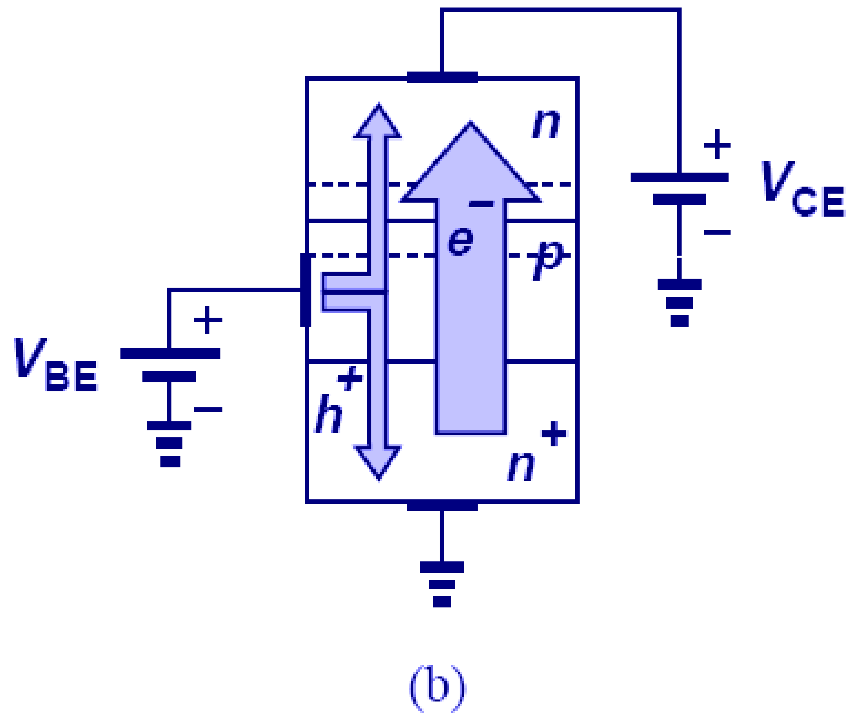
$$r_o = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{V_A}{I_S \exp \frac{V_{BE}}{V_T}} \approx \frac{V_A}{I_C}$$

Summary of Ideas



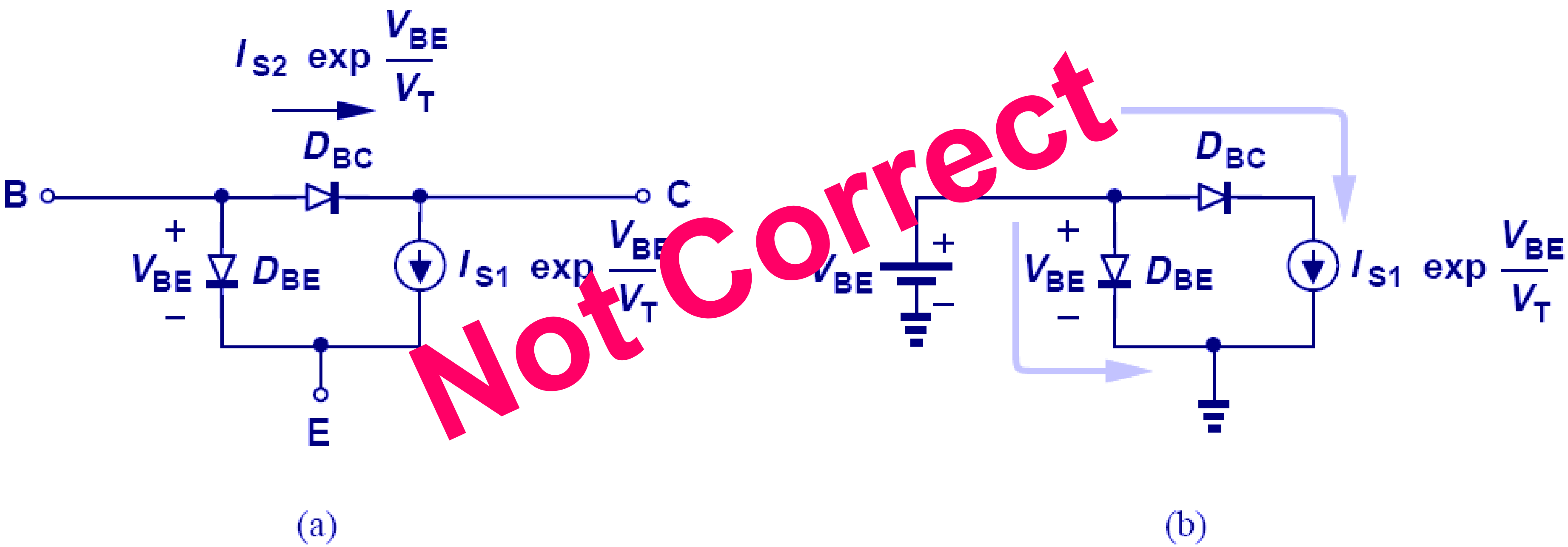
Bipolar Transistor in Saturation





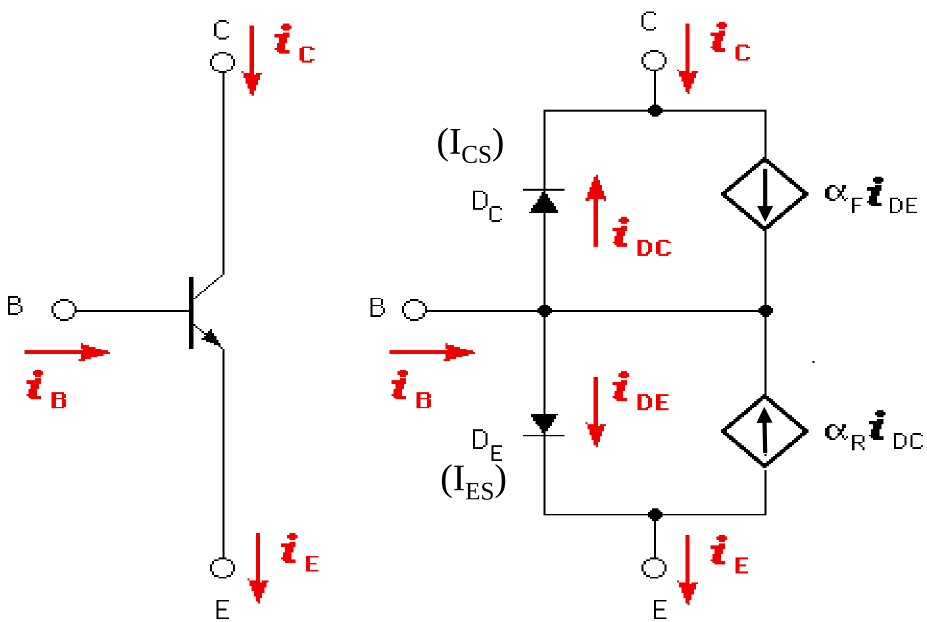
- When collector voltage drops below base voltage and forward biases the collector-base junction, base current increases and the current gain factor, β, decreases .

Large-Signal Model for Saturation Region



Ebers-Moll Model

- **Reciprocity holds: $\alpha_F I_{ES} = \alpha_R I_{CS} = I_S$**



$$i_E = \frac{I_S}{\alpha_F}\left[\exp\left(v_{BE}/V_T\right) - 1\right] - I_S\left[\exp\left(v_{BC}/V_T\right) - 1\right]$$

$$i_C = I_S\left[\exp\left(v_{BE}/V_T\right) - 1\right] - \frac{I_S}{\alpha_R}\left[\exp\left(v_{BC}/V_T\right) - 1\right]$$

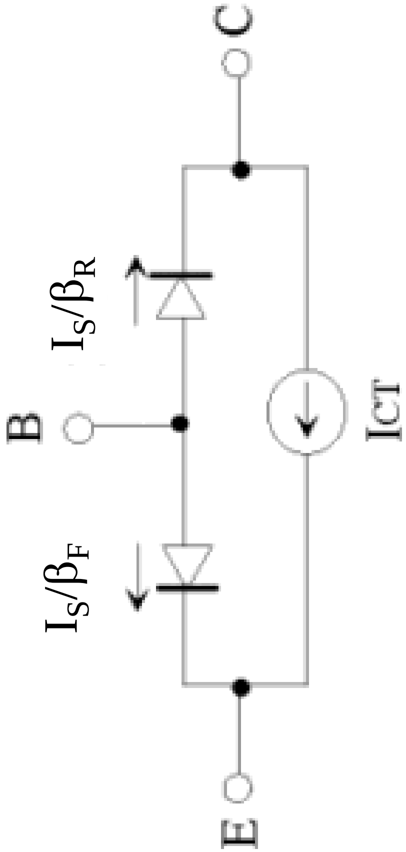
$$i_B = \frac{I_S}{\beta_F}\left[\exp\left(v_{BE}/V_T\right) - 1\right] + \frac{I_S}{\beta_R}\left[\exp\left(v_{BC}/V_T\right) - 1\right]$$

$$\beta_F = \frac{\alpha_F}{1 - \alpha_F} \qquad \beta_R = \frac{\alpha_R}{1 - \alpha_R}$$

$$\alpha_F = \frac{\beta_F}{\beta_F + 1} \qquad \alpha_R = \frac{\beta_F}{\beta_R + 1}$$

Transport Model

➢ Equivalent to Ebers-Moll Model



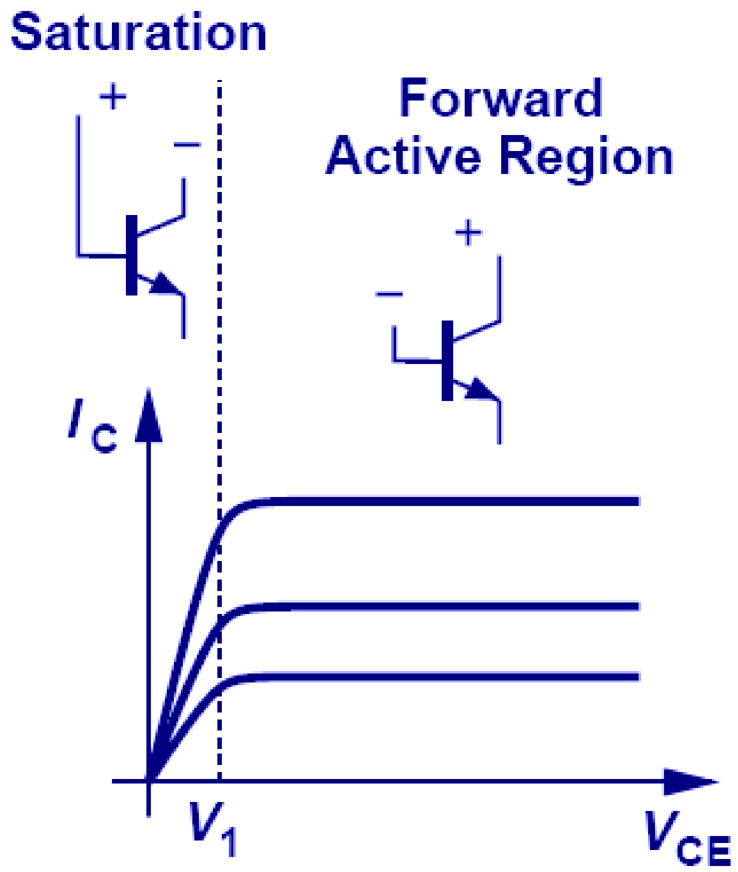
$$i_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - e^{\frac{V_{BC}}{V_T}}\right) - \frac{I_S}{\beta_R}\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$$

$$i_B = \frac{I_S}{\beta_F}\left(e^{\frac{V_{BE}}{V_T}} - 1\right) + \frac{I_S}{\beta_R}\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$$

$$i_E = I_S\left(e^{\frac{V_{BE}}{V_T}} - e^{\frac{V_{BC}}{V_T}}\right) + \frac{I_S}{\beta_F}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$

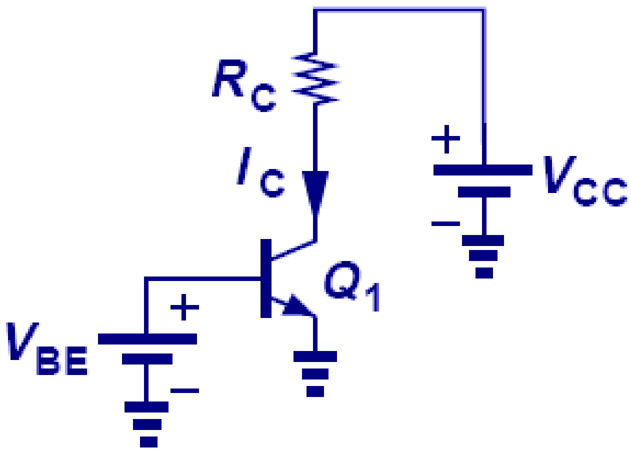
$$I_{CT} = I_S\left(e^{\frac{V_{BE}}{V_T}} - e^{\frac{V_{BC}}{V_T}}\right)$$

Overall I/V Characteristics

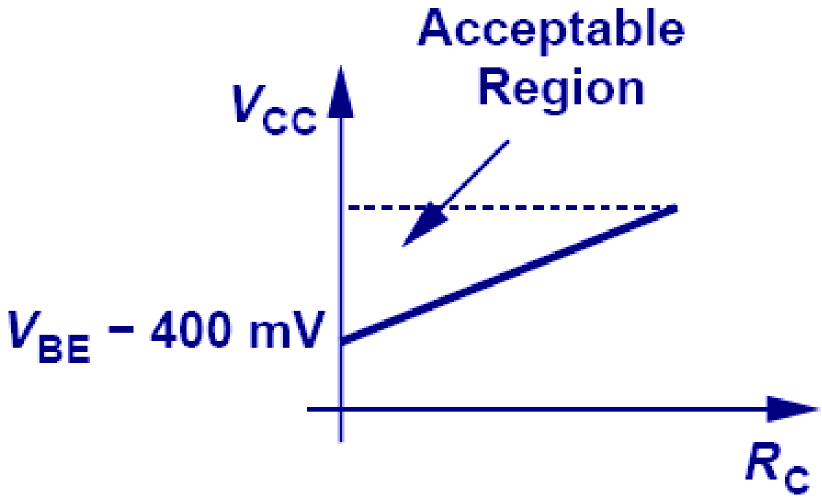


- The speed of the BJT also drops in saturation.

Example: Acceptable V_{CC} Region



(a)

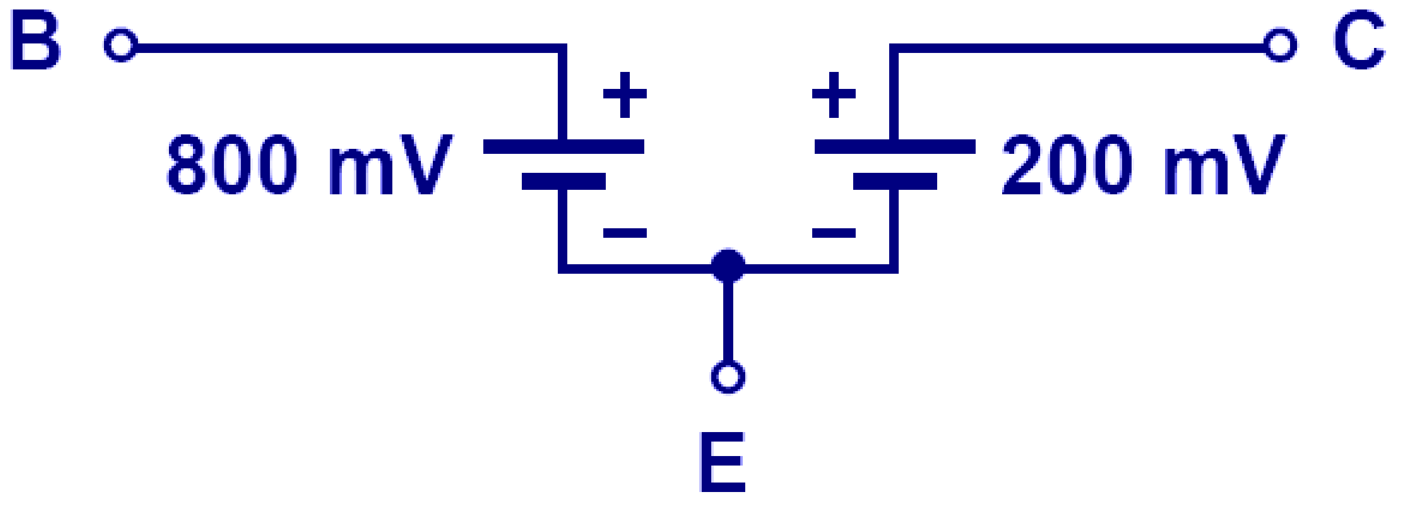


(b)

$$V_{CC} \geq I_C R_C + (V_{BE} - 400mV)$$

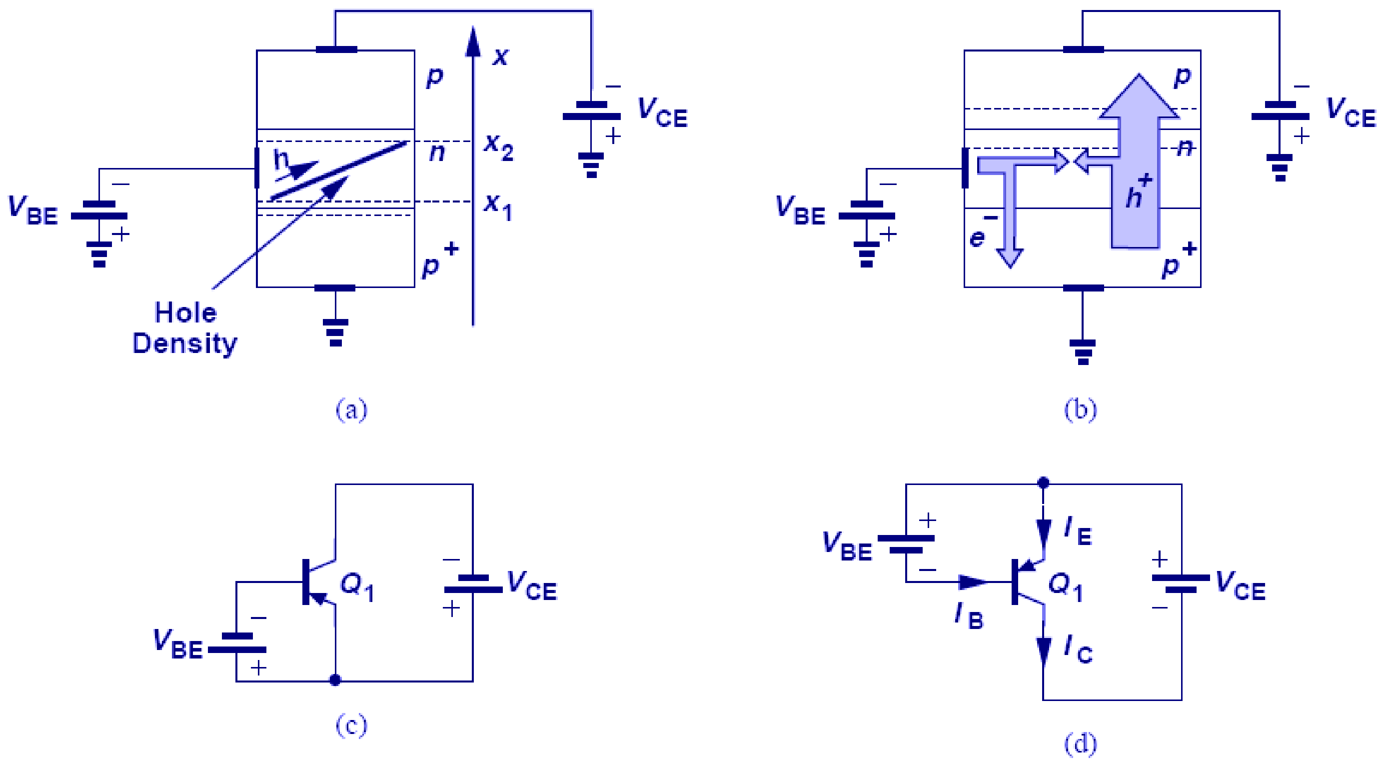
- **In order to keep BJT at least in soft saturation region, the collector voltage must not fall below the base voltage by more than 400mV.**
- **A linear relationship can be derived for V_{CC} and R_C and an acceptable region can be chosen.**

Deep Saturation



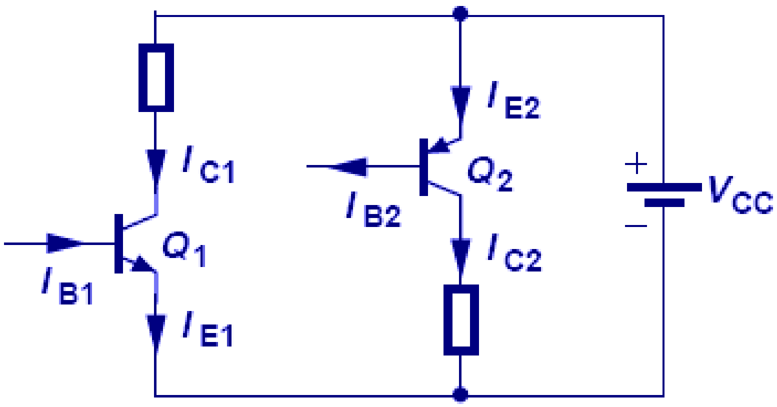
- In deep saturation region, the transistor loses its voltage-controlled current capability and V_{CE} becomes constant.

PNP Transistor

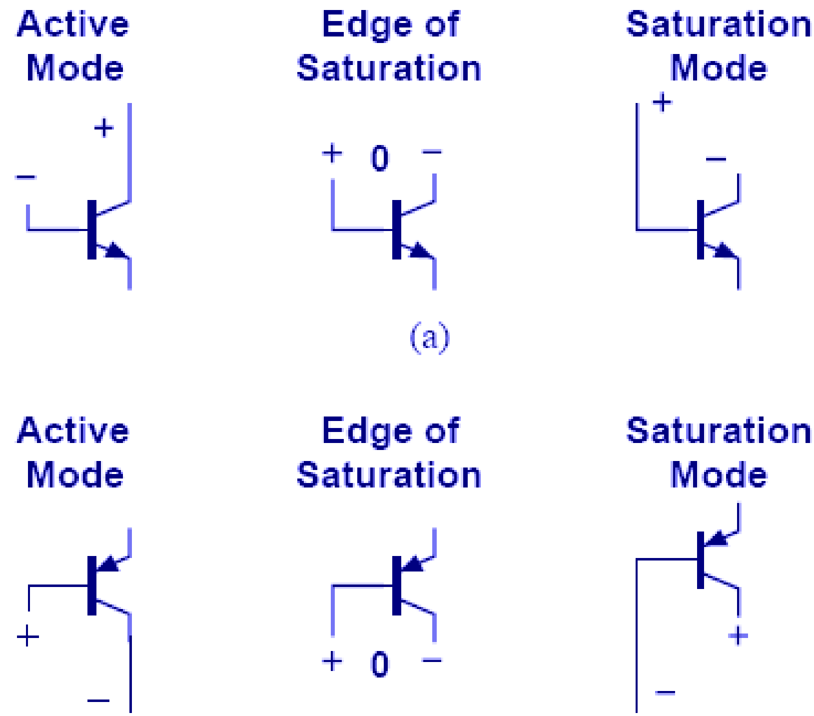


- **With the polarities of emitter, collector, and base reversed, a PNP transistor is formed.**
- **All the principles that applied to NPN's also apply to PNP's, with the exception that emitter is at a higher potential than base and base at a higher potential than collector.**

A Comparison between NPN and PNP Transistors



(a)



(b)

- **The figure above summarizes the direction of current flow and operation regions for both the NPN and PNP BJT's.**

PNP Equations

$$I_C = I_S \exp\frac{V_{EB}}{V_T}$$

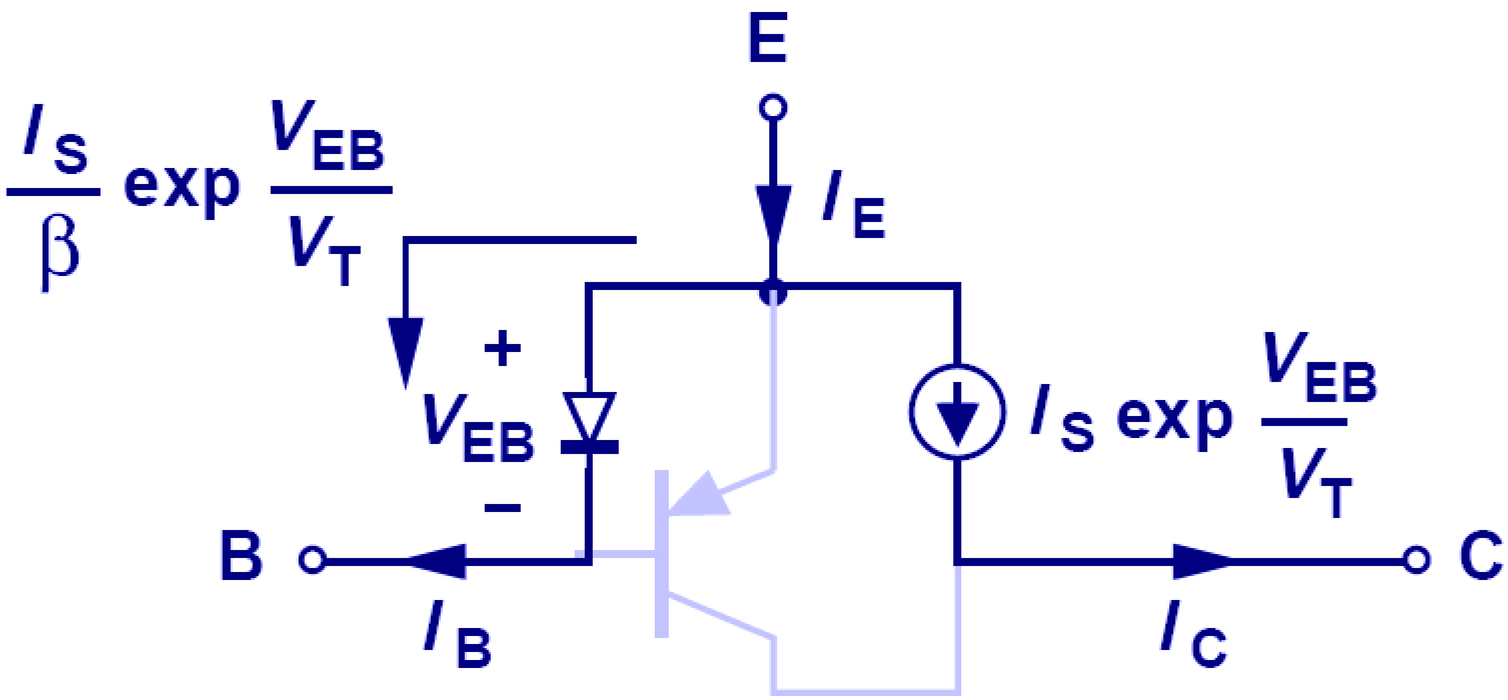
$$I_B = \frac{I_S}{\beta}\exp\frac{V_{EB}}{V_T}$$

$$I_E = \frac{\beta+1}{\beta} I_S \exp\frac{V_{EB}}{V_T}$$

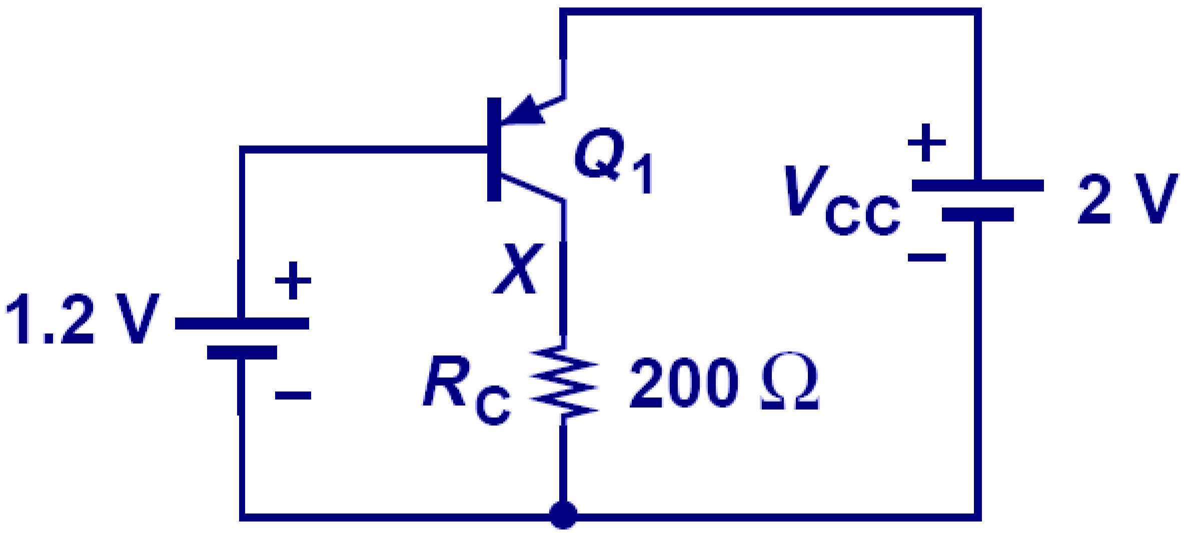
Early Effect $\Rightarrow$

$$I_C = \left(I_S \exp\frac{V_{EB}}{V_T}\right)\left(1+\frac{V_{EC}}{V_A}\right)$$

Large Signal Model for PNP



PNP Biasing



- **Note that the emitter is at a higher potential than both the base and collector.**

Example 4.17

Example 4.17

In the circuit shown in Fig. 4.41, determine the terminal currents of Q_1 and verify operation in the forward active region. Assume $I_S = 2 \times 10^{-16}$ A and $\beta = 50$, but $V_A = \infty$.

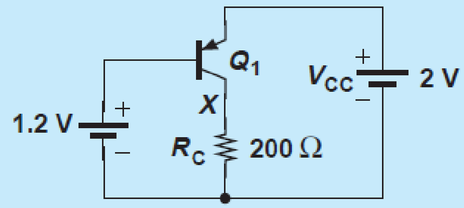


Figure 4.41 Simple stage using a *pnp* transistor.

Solution We have $V_{EB} = 2\text{ V} - 1.2\text{ V} = 0.8\text{ V}$ and hence

$$I_C = I_S \exp\frac{V_{EB}}{V_T} \tag{4.104}$$

$$= 4.61\text{ mA}. \tag{4.105}$$

It follows that

$$I_B = 92.2\ \mu\text{A} \tag{4.106}$$

$$I_E = 4.70\text{ mA}. \tag{4.107}$$

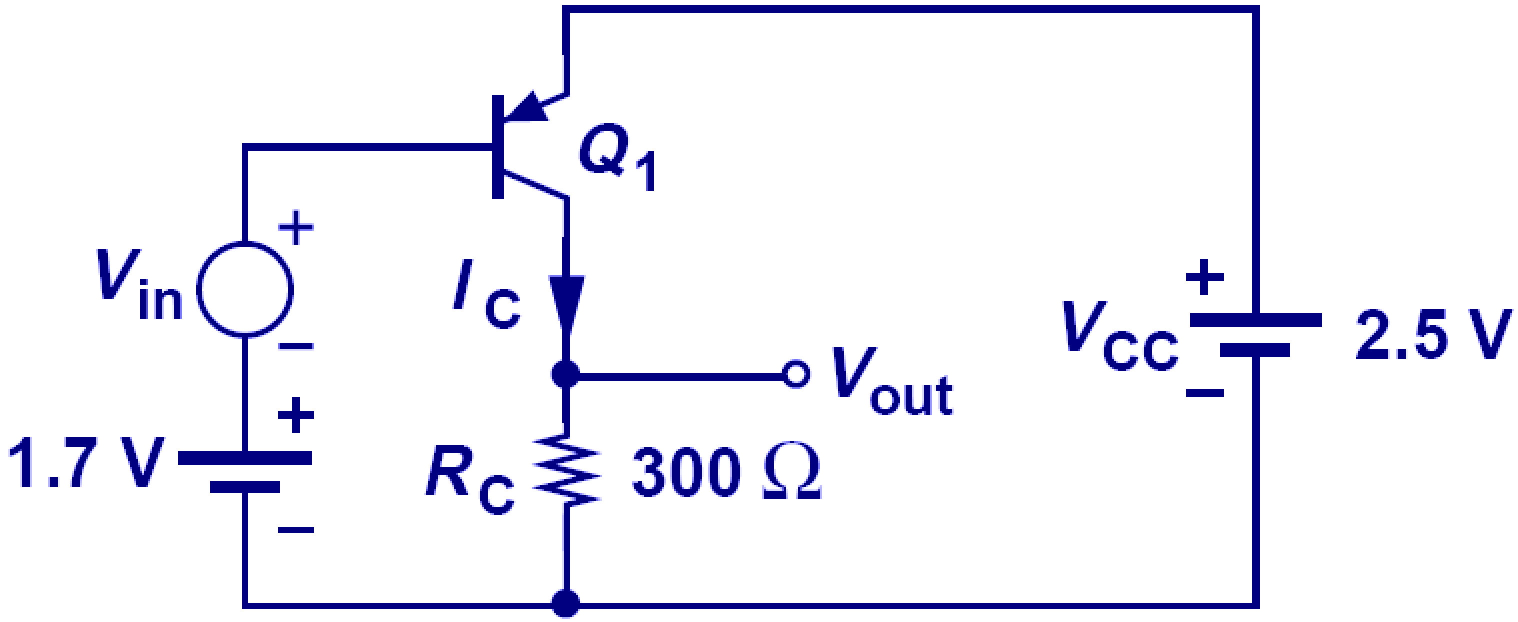
We must now compute the collector voltage and hence the bias across the B-C junction. Since R_C carries I_C,

$$V_X = R_C I_C \tag{4.108}$$

$$= 0.922\text{ V}, \tag{4.109}$$

which is *lower* than the base voltage. Invoking the illustration in Fig. 4.39(b), we conclude that Q_1 operates in the active mode and the use of equations (4.100)–(4.102) is justified.

Small Signal Analysis



Example 4.18

Example 4.18

In the circuit of Fig. 4.42, V_{in} represents a signal generated by a microphone. Determine V_{out} for $V_{in} = 0$ and $V_{in} = +5$ mV if $I_S = 1.5 \times 10^{-16}$ A.

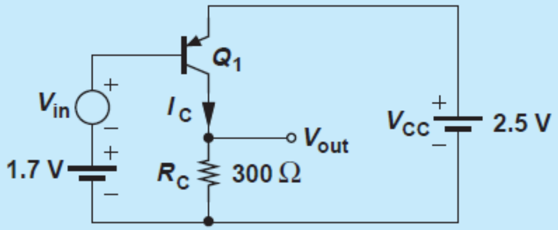


Figure 4.42 *PNP* stage with bias and small-signal voltages.

Solution For $V_{in} = 0$, $V_{EB} = +800$ mV and we have

$$I_C|_{V_{in}=0} = I_S \exp \frac{V_{EB}}{V_T} \tag{4.110}$$

$$= 3.46\ \text{mA}, \tag{4.111}$$

and hence

$$V_{out} = 1.038\ \text{V}. \tag{4.112}$$

If V_{in} increases to $+5$ mV, $V_{EB} = +795$ mV and

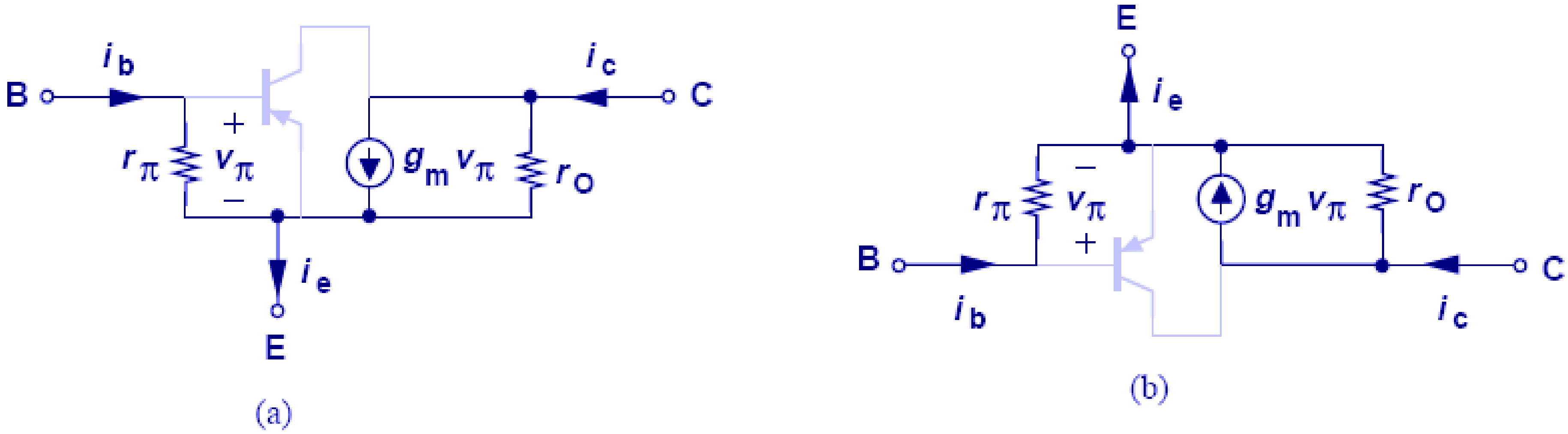
$$I_C|_{V_{in}=+5\ \text{mV}} = 2.85\ \text{mA}, \tag{4.113}$$

yielding

$$V_{out} = 0.856\ \text{V}. \tag{4.114}$$

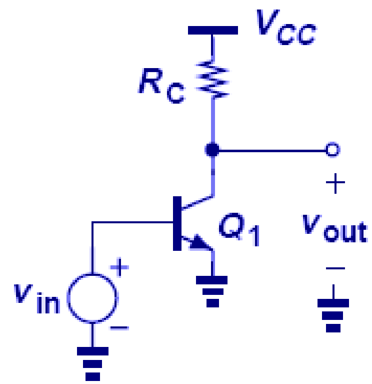
Note that as the base voltage *rises*, the collector voltage *falls*, a behavior similar to that of the *npn* counterparts in Fig. 4.25. Since a 5-mV change in V_{in} gives a 182-mV change in V_{out}, the voltage gain is equal to 36.4. These results are more readily obtained through the use of the small-signal model.

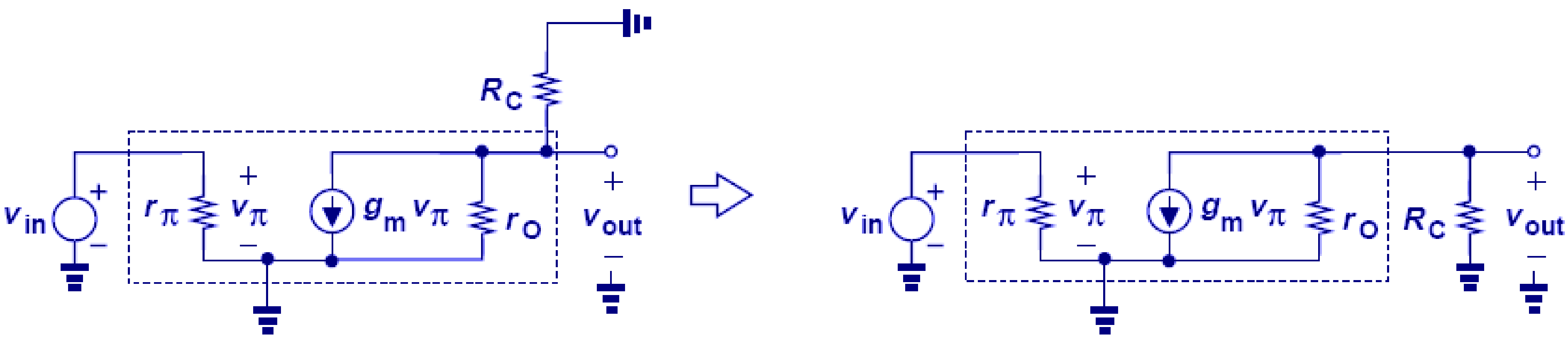
Small-Signal Model for PNP Transistor



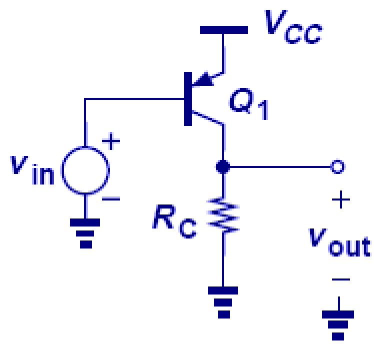
- The small signal model for PNP transistor is exactly IDENTICAL to that of NPN. This is not a mistake because the current direction is taken care of by the polarity of V_{BE}.

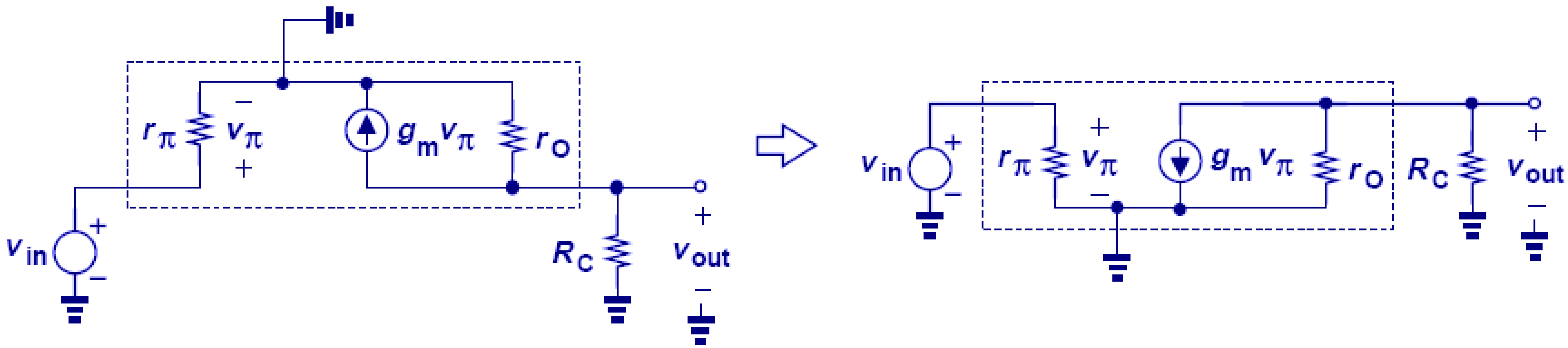
Small Signal Model Example I





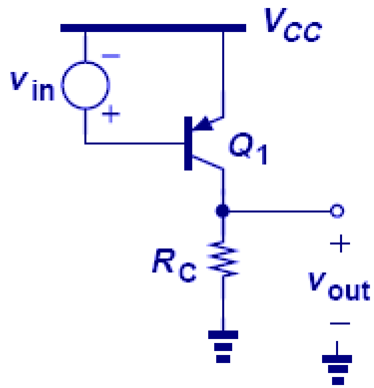
Small Signal Model Example II



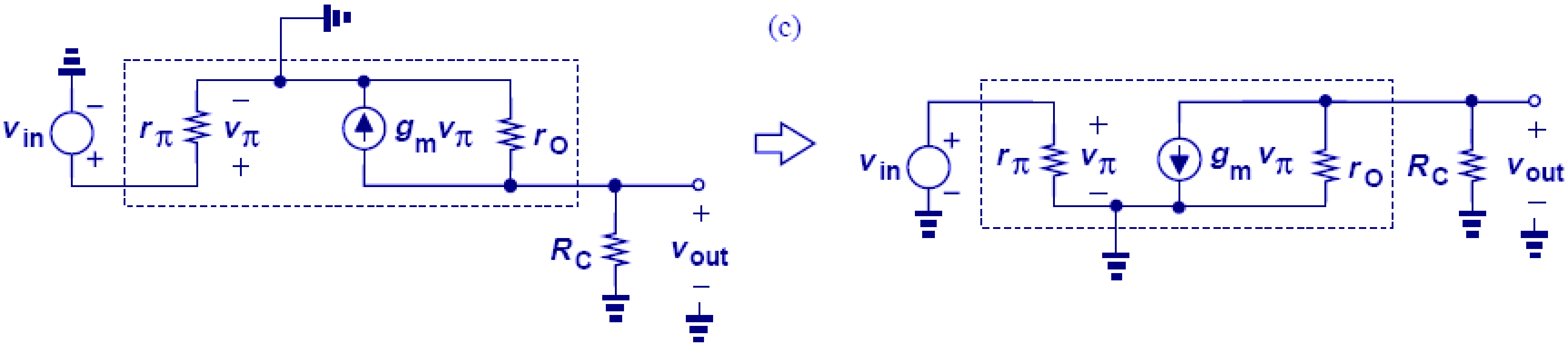


- Small-signal model is identical to the previous ones.

Small Signal Model Example III



(c)



- Since during small-signal analysis, a constant voltage supply is considered to be AC ground, the final small-signal model is identical to the previous two.

Small Signal Model Example IV